Magnetic monopoles in spin ice

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Workshop on Highly Frustrated Magnets and Strongly Correlated Systems

ICTP, Trieste, August 9, 2007.
Outline

- **Spin ice** in a nutshell

- A useful perspective: decomposing dipoles into “dumbells”

- Dipole *fractionalisation* and the emergence of *magnetic monopoles*

- **Monopole physics**: From induced liquid-gas phase transitions to the Stanford monopole experiment
The Structure of Spin Ice

- **pyrochlore lattice** of rare earth atoms (Dy$_2$Ti$_2$O$_7$ with spin $S = 15/2$, and Ho$_2$Ti$_2$O$_7$ with spin $S = 8$)

- Crystal field anisotropy along the local [111] axis $\Delta_{CEF} \sim 200$ K $\rightarrow$ **classical Ising spins** below $T \sim 10$ K
The Phase Diagram [Melko, Gingras, J. Phys.: C. M. 16, R1277 (2004)]

Exchange and dipolar interactions

\[ H = \frac{J}{3} \sum_{\langle ij \rangle} S_i S_j + D a^3 \sum_{\langle ij \rangle} \left[ \hat{e}_i \cdot \hat{e}_j \frac{1}{|\mathbf{r}_{ij}|^3} - 3 \frac{(\hat{e}_i \cdot \mathbf{r}_{ij})(\hat{e}_j \cdot \mathbf{r}_{ij})}{|\mathbf{r}_{ij}|^5} \right] S_i S_j \]

where \( S_i = \pm 1 \) are the normalized Ising spins, \( \hat{e}_i \) is the unit vector in the local [111] direction, and \( J \sim 1 - 2 \) K and \( D = 1.41 \) K.
• replace each dipole $\vec{d}$ by two equal and opposite magnetic charges $\pm q$ separated by the bond length $a$ ($q = d/a$)

• adjust the onsite Coloumb interaction so as to give the correct nearest-neighbour interaction between dipoles:

$$v(r_{ij}) = \begin{cases} \frac{\mu_0 q_i q_j}{4\pi r_{ij}} & r_{ij} \neq 0 \\ v_0 q_i q_j & r_{ij} = 0 \end{cases}$$

$$v_0 q_i q_j = \left\{ \frac{J}{3} + \frac{4D}{3} \left( 1 + \sqrt{\frac{2}{3}} \right) \right\} \frac{q_i q_j}{(\mu/a)^2}$$

• The dumbbell model reproduces the dipolar interaction energy quantitatively (up to quadrupolar corrections).
with a simple resummation \( q_i \rightarrow Q_\alpha = \sum_{i \in \text{tretrahedron}} \alpha q_i \), the energy of a generic dumbell configuration can be rewritten as

\[
v(r_{ij}) \rightarrow V(r_{\alpha\beta}) = \begin{cases} 
\frac{\mu_0}{4\pi} \frac{Q_\alpha Q_\beta}{r_{\alpha\beta}} & \alpha \neq \beta \\
\frac{1}{2}v_o Q_\alpha^2 & \alpha = \beta 
\end{cases}
\]

\( \rightarrow \) the 2-in, 2-out ice rules appear naturally: the lowest energy configurations are the ones with \( Q_\alpha = 0 \) everywhere (\( \alpha = \beta \) term is big enough)!

\( \rightarrow \) degeneracy of ice states follows trivially (terms with \( \alpha \neq \beta \) vanish)
Low-temperature defects (i.e., violations of the ice rule)

- single spin flip in a spin-ice configuration: naively a dipolar excitation

- two neighbouring opposite charges $Q_\alpha = \pm 2d/a$ in the dumbbell picture.

- the two charges can be separated at the expense of a purely Coulomb interaction (not confining in 3D) $\Rightarrow$ the dipolar excitation fractionalise into two magnetic monopoles!
Monopole basics

\[ E = \frac{\mu_0 Q^2}{4\pi r} \]

can be explicitly checked by numerical evaluation in specific spin-ice configurations

⇒ tension of ‘Dirac string’ connecting monopoles vanishes

Intuitive picture: separating the two poles of a magnet, while the string in between is energetically immaterial
The Physics of Magnetic Monopoles in Spin Ice

- they are real monopoles (sinks and sources of the magnetic field $\vec{H}$), i.e., they would be felt by independent test charges

- they are classical in nature, in that the Dirac string is energetically immaterial but observable (thus no quantisation condition)

- How can we observe them?
  
  - indirectly, by looking for signatures in spin-ice physics (e.g., ionic liquid behaviour)
  
  - directly, e.g., via scattering or Stanford search type experiments
A Liquid-Gas Phase Transition in Spin Ice

- As the field $\vec{B}$ is increased, the system attains first a partial ordering known as kagome ice, within the ice rules (Left Panel).

- When the field is strong enough, every spin acquires a positive projection in the direction of the field and we obtain densely packed magnetic monopoles (Right Panel).
• ionic liquid of monopoles as a function of temperature and chemical potential $(B) \rightarrow$ liquid-gas behavior (M. E. Fisher et al.)

• confirmed numerically – observed in actual experiments! (Hiroi, Maeno groups)
Detection Using a Superconducting Coil

- the passage of a monopole through a superconducting coil induces a **long-lived current**, whose strength is proportional to the magnetic charge $q_m = 2d/a$ [Cabrera, PRL 48, 1378 (1982)]

- this current sets up a **magnetic flux** equal and opposite to the one carried by the **dipole string** connecting the two monopoles, which is strictly confined within the spin-ice slab

Spin-ice monopoles can (in principle) be detected in much the same way as ‘ordinary’ monopoles would!
Conclusions

• the low-energy excitations in spin ice are classical magnetic monopoles

• these monopoles interact via a magnetic Coulomb interaction $V(r) = \mu_0 Q^2 / (4\pi r)$

• they are sources and sinks of the magnetic field $\vec{H}$

• the monopoles live at the end points of observable, yet tensionless Dirac strings, hence they are not quantised

• they can be experimentally observed, either indirectly (e.g., in the recently discovered liquid-gas phase transition) or directly (e.g., monopole-search type experiments)

• other experiments: scattering, transport and noise measurements, flux detection – further ideas welcome