Frontiers of Theoretical High Energy Physics

Andre Lukas
(updated by Andrei Starinets)

At the Second International Congress of Mathematics in Paris on August 8, 1900, David Hilbert offered a list of 10 major problems of mathematics (later extended to 23 or 24)

David Hilbert
(1862-1943)

1. Cantor's problem of the cardinal number of the continuum.

2. The compatibility of the axioms of arithmetic.

3. Give two tetrahedra that cannot be decomposed into congruent tetrahedra directly or by adjoining congruent tetrahedra.

4. Find geometries whose axioms are closest to those of Euclidean geometry if the ordering and incidence axioms are retained, the congruence axioms weakened, and the equivalent of the parallel postulate omitted.

5. Can the assumption of differentiability for functions defining a continuous transformation group be avoided?

6. Can physics be axiomatised?

7. Let $\alpha \neq 1 \neq 0$ $be\ algebraic\ and\ \beta\ irrational. Is\ \alpha^{\beta}\ then\ transcendental?$

8. Prove the Riemann hypothesis.

9. ….

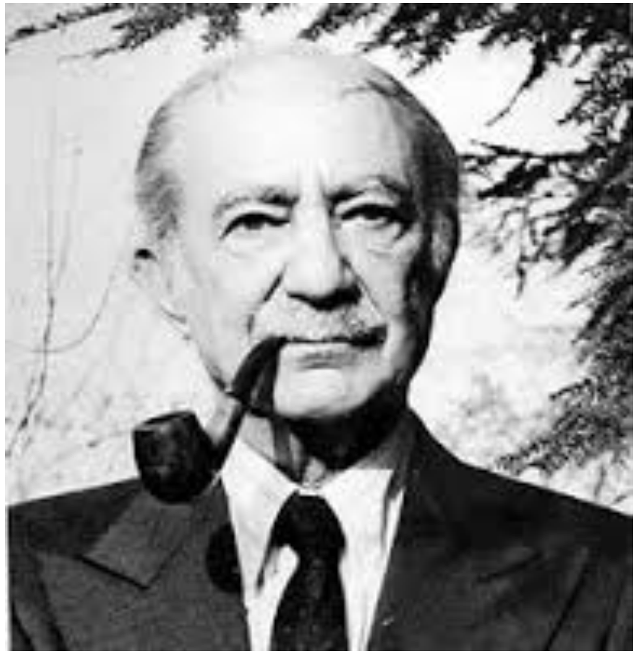
Frontiers of knowledge: a pessimistic paradigm

Incompleteness theorem(s) (Kurt Gödel, 1931)

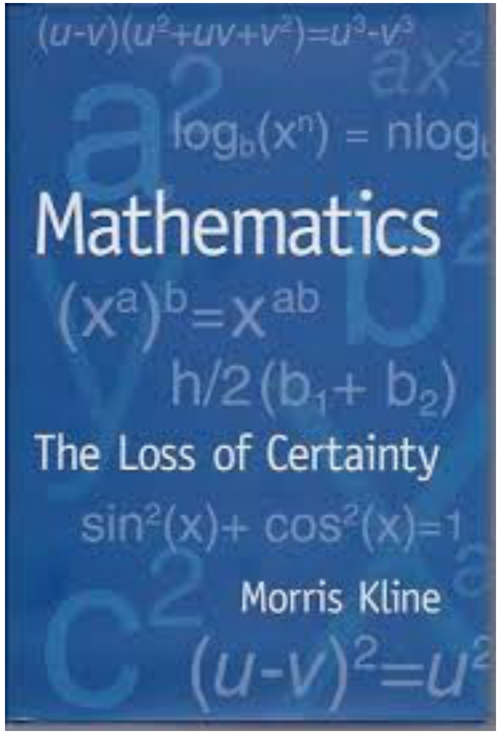
For any computable axiomatic system that is powerful enough to describe the arithmetic of the natural numbers:

1) If the system is consistent, it cannot be complete.

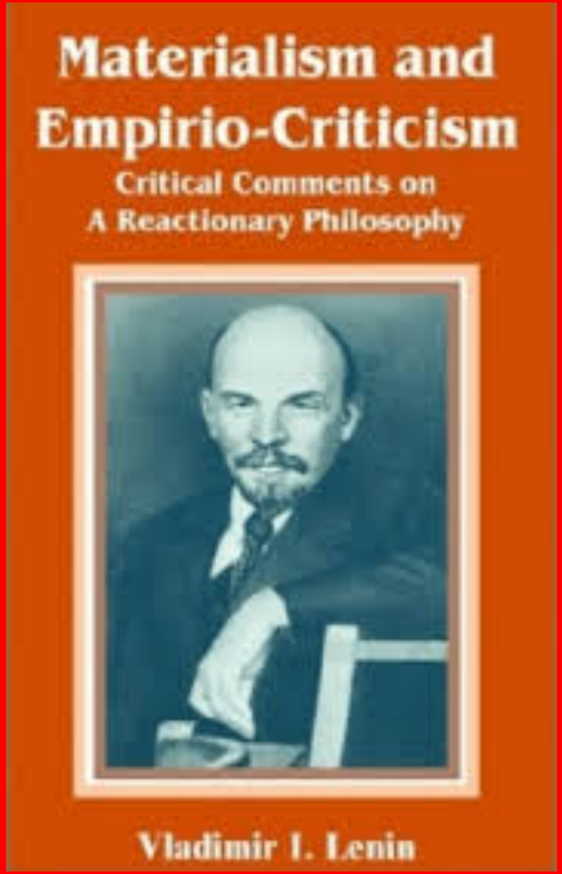
2) The consistency of the axioms cannot be proved within the system.



Morris Kline

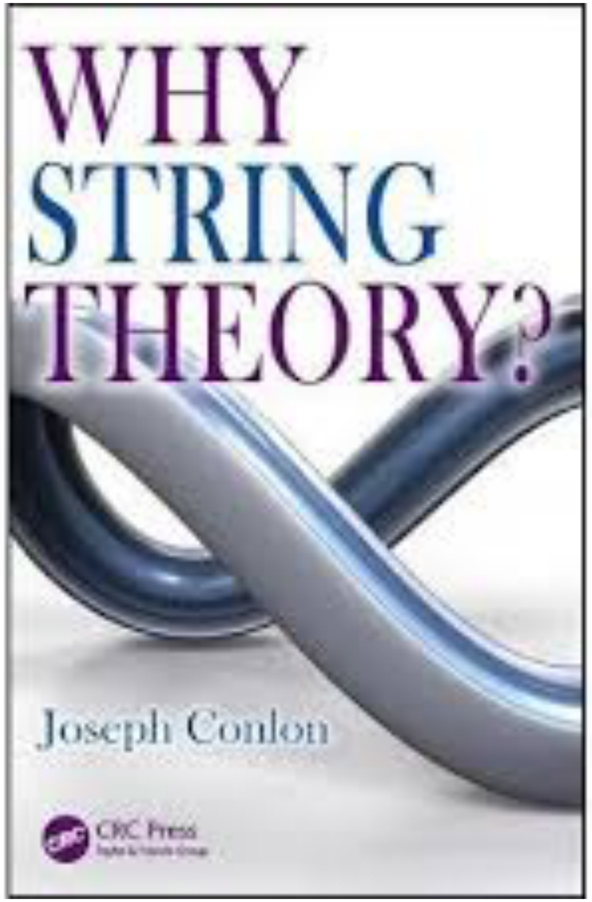


Frontiers of knowledge: an optimistic paradigm



"The electron is as *inexhaustible* as the atom, nature is infinite, but it *exists* infinitely…"
V. I. Lenin, "Materialism and Empirio-Criticism" (1908)

Modern (2015), popular and honest account of the current state of affairs in string theory



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Higgs: $$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}_{(1)}$$

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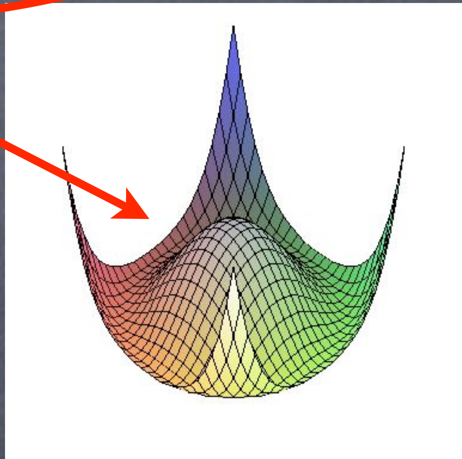

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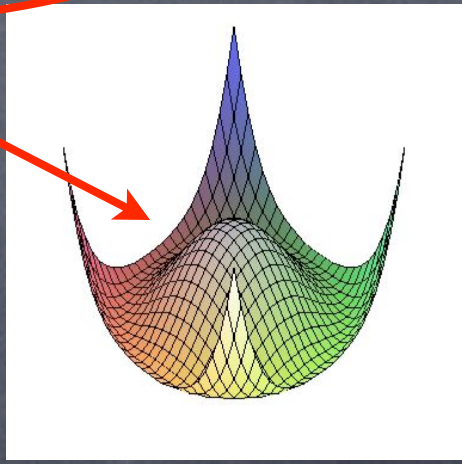

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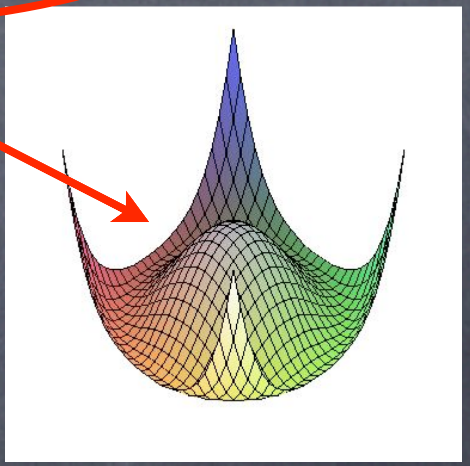

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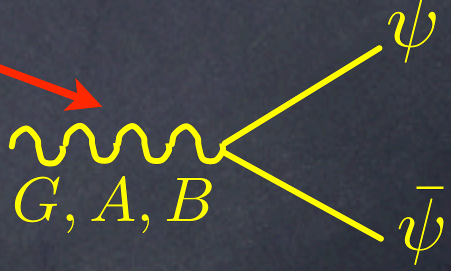

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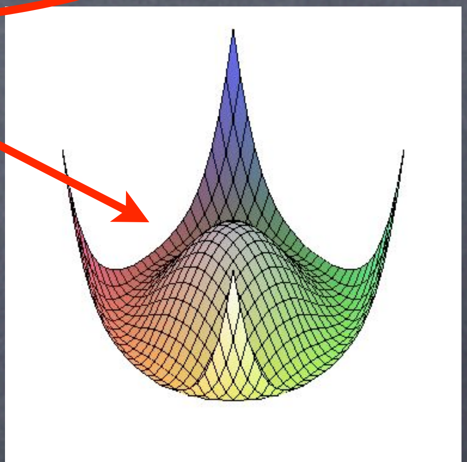

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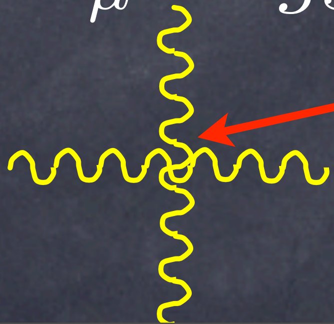

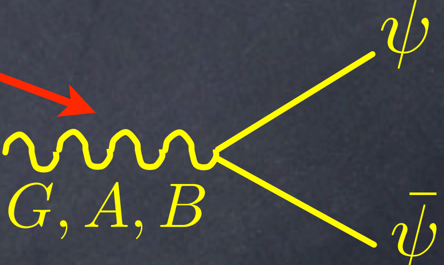

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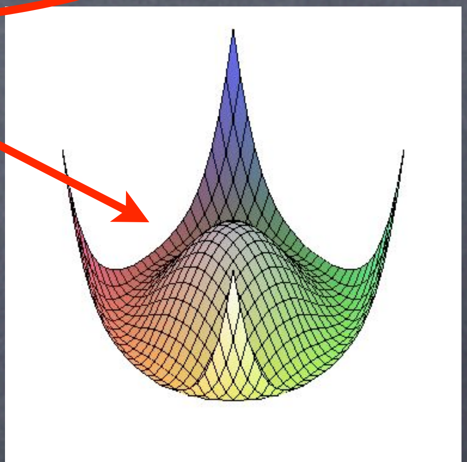

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Astrophysics

Cosmology

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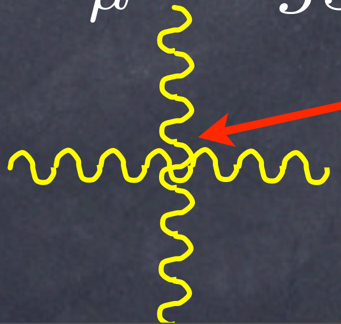

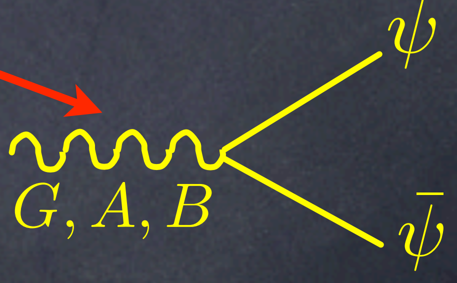

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- No candidate inflaton in the standard model -> additional scalar fields,...

Theoretical problems

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- Standard model is a quantum theory. What about quantum gravity?

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Consider Einstein equation :

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Attempts to quantise gravity by standard quantum field theory methods run into serious problems.

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Currently, string/M-theory is the only known theory with a chance of satisfying this "definition" of a fundamental theory.

String theory

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Warm-up: world-line of a relativistic particle

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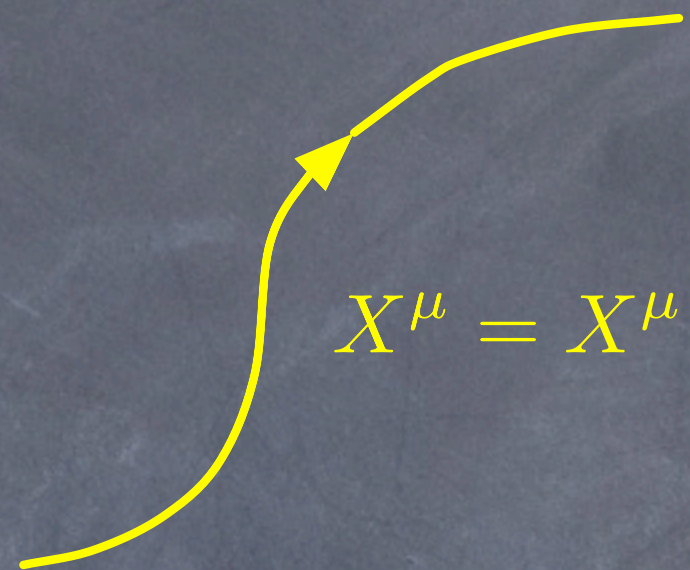

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String theory

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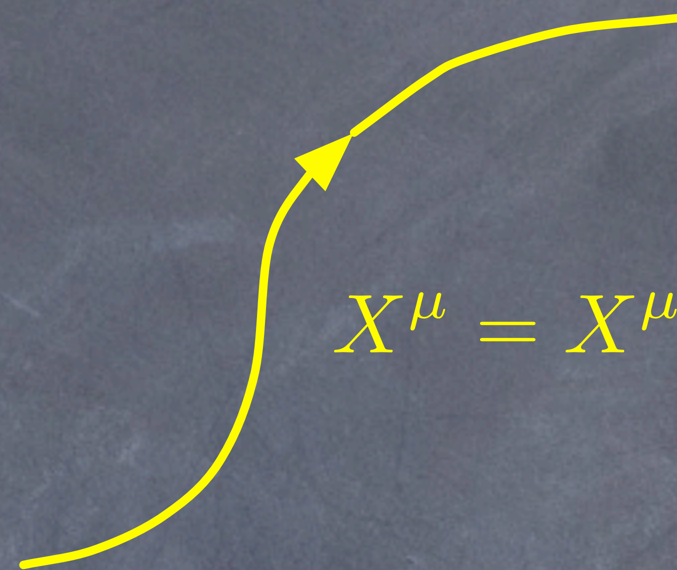

$$X^\mu = X^\mu(\tau)$$

World-line action :

$$S = -m \int d\tau \sqrt{-\frac{dX^\mu}{d\tau}\frac{dX^\nu}{d\tau}\eta_{\mu\nu}} \qquad \rightarrow \qquad \frac{d^2 X^\mu}{d\tau^2} = 0$$

World-sheet of free relativistic string

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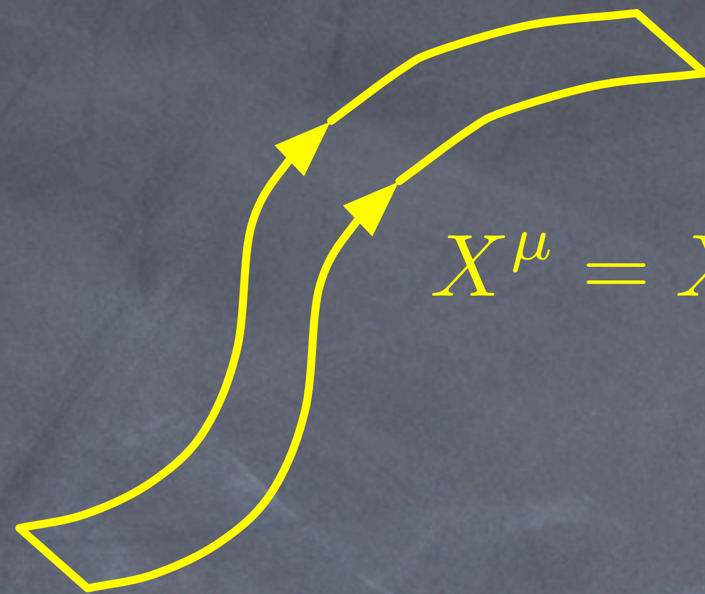
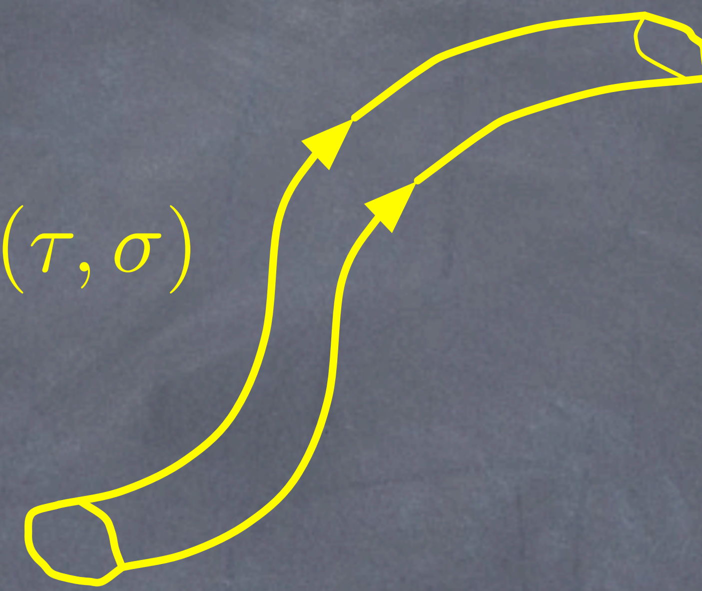


$$X^\mu = X^\mu(\tau, \sigma)$$



open string

closed string

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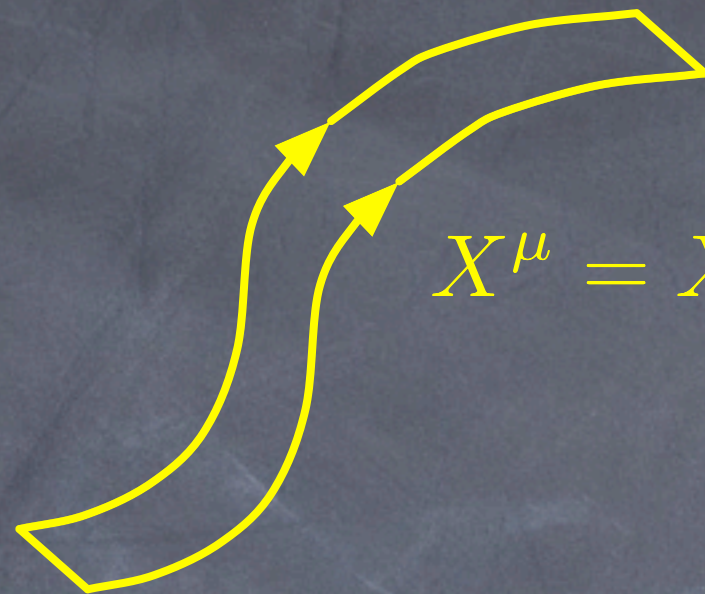

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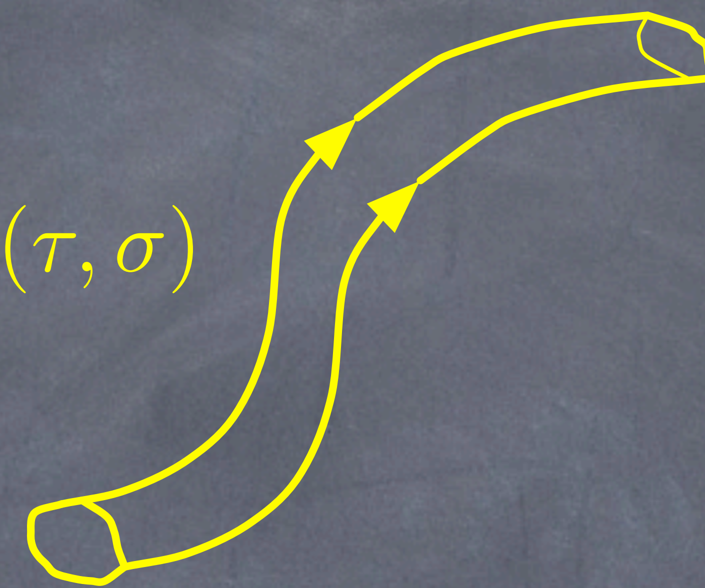


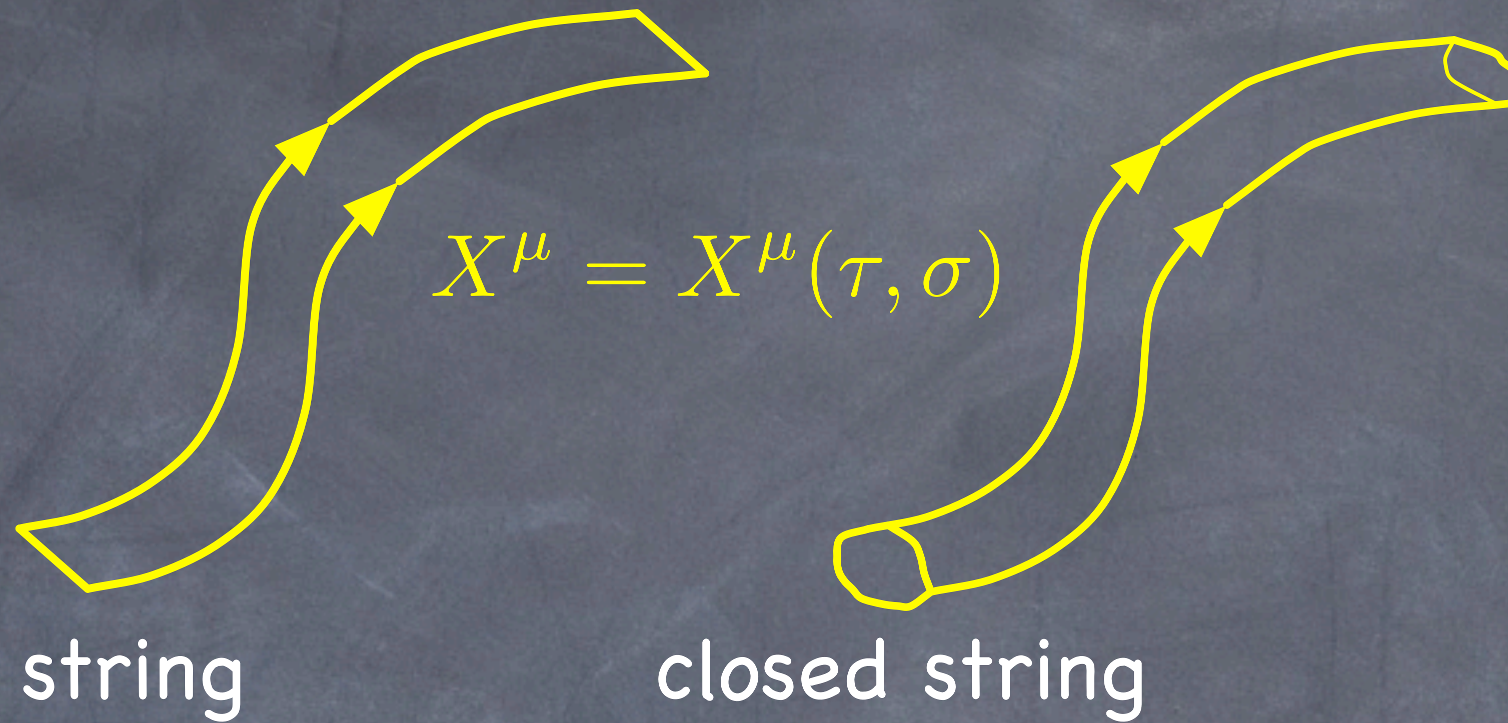
open string

closed string

World-sheet action:

$$S = -\frac{1}{2\pi\alpha'}\int d^2\sigma \sqrt{-\det\left(\frac{\partial X^\mu}{\partial\sigma^\alpha}\frac{\partial X^\nu}{\partial\sigma^\beta}\eta_{\mu\nu}\right)}$$

World-sheet of free relativistic string



World-sheet action:

string tension

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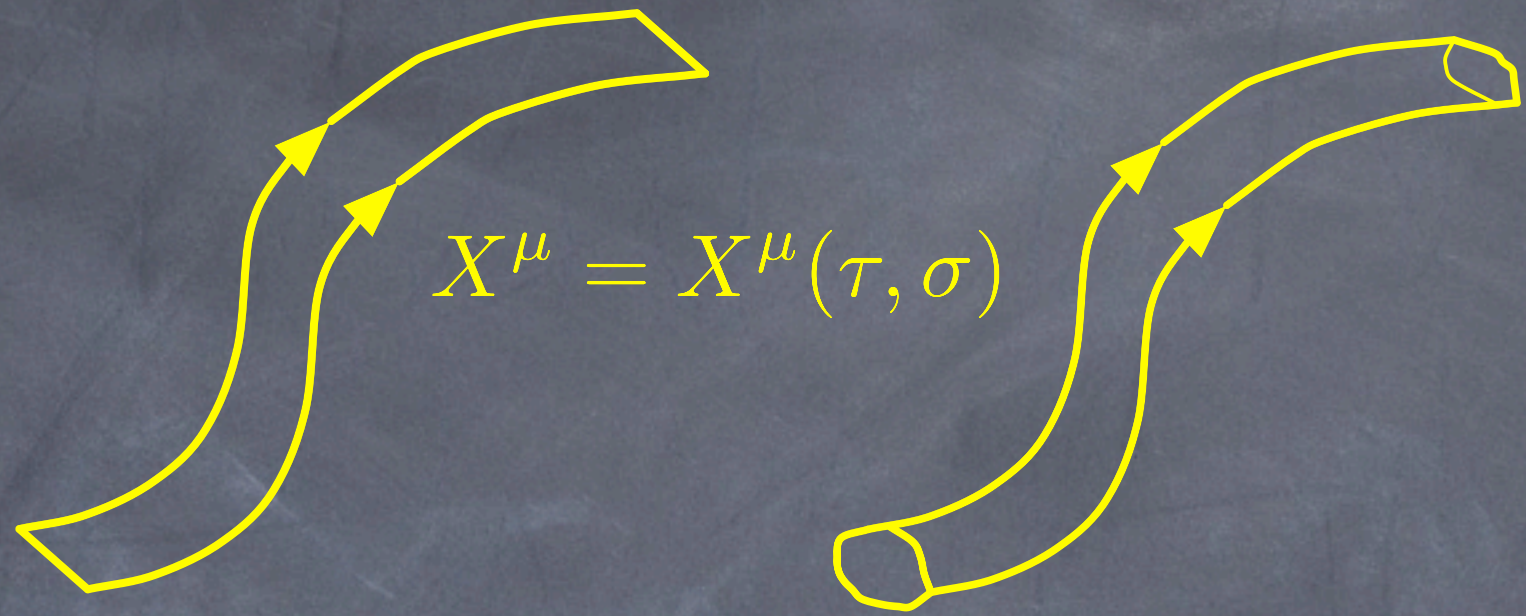

open string   closed string

World-sheet action: string tension

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Spectrum: $\alpha' m^2 = n \in \mathbf{Z} \begin{cases} n = 0 & \rightarrow \quad \text{observed particles} \\ n \neq 0 & \rightarrow \quad \text{supermassive} \end{cases}$

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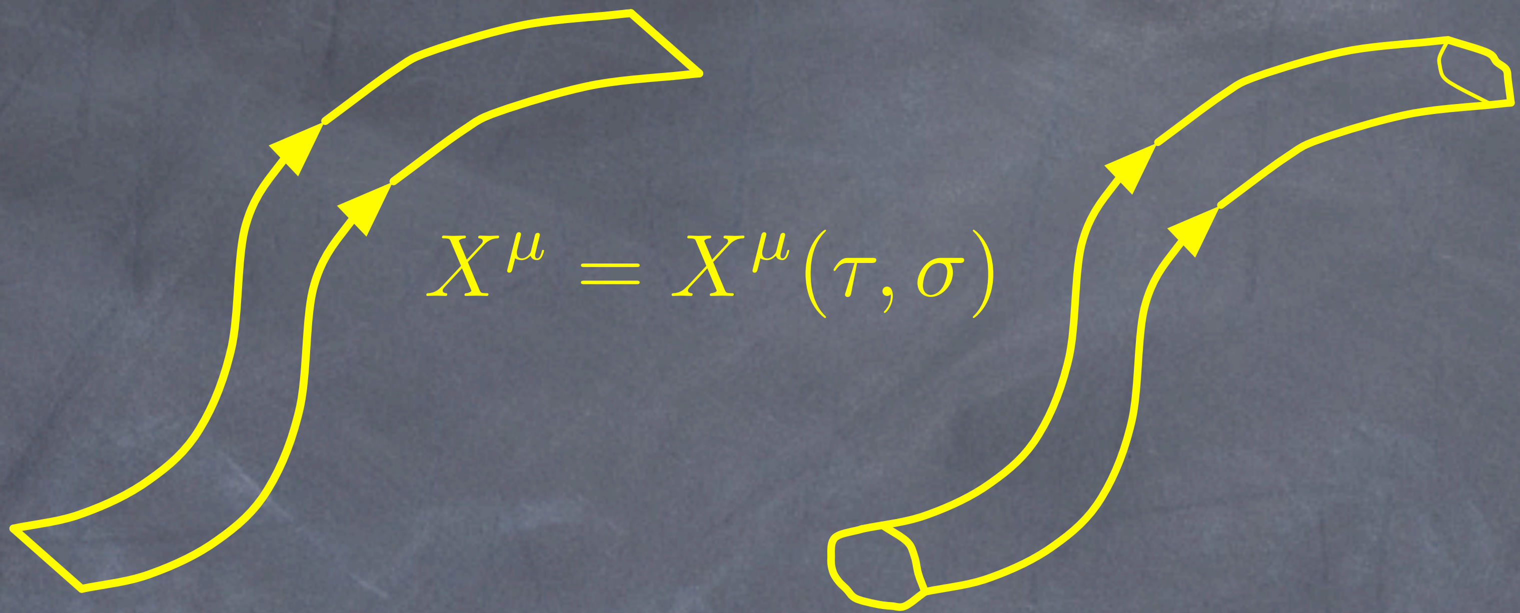

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open string — closed string

World-sheet action: (string tension: $\frac{1}{2\pi\alpha'}$)

$$S = -\frac{1}{2\pi\alpha'}\int d^2\sigma \sqrt{-\det\left(\frac{\partial X^\mu}{\partial \sigma^\alpha}\frac{\partial X^\nu}{\partial \sigma^\beta}\eta_{\mu\nu}\right)}$$

Spectrum: $\alpha' m^2 = n \in \mathbf{Z} \begin{cases} n = 0 & \rightarrow \quad \text{observed particles} \\ n \neq 0 & \rightarrow \quad \text{supermassive} \end{cases}$

Further reading: Barton Zwiebach, "A first course in string theory"

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AdS/CFT correspondence

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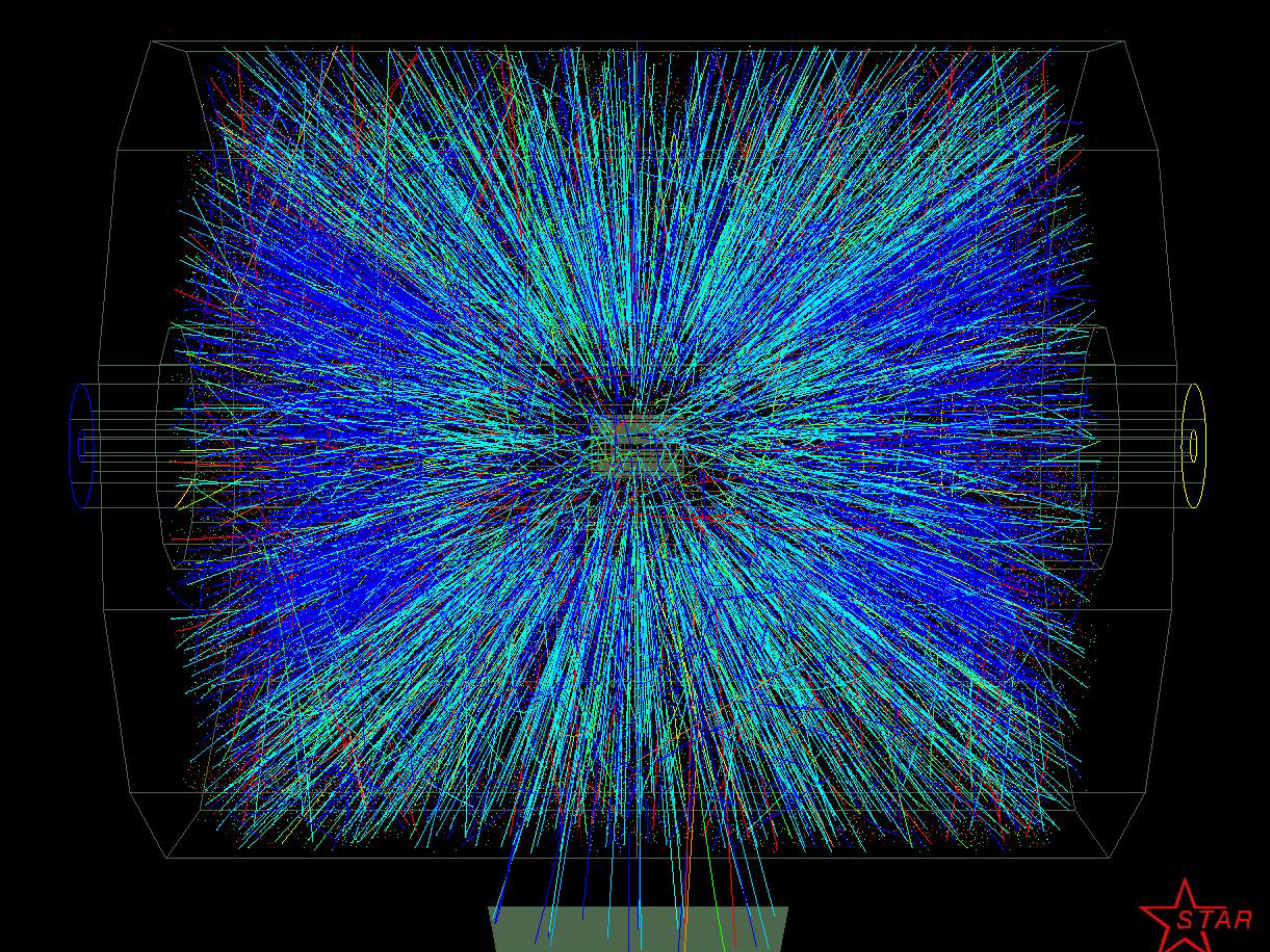
A 10-dim. theory of strings is equivalent to a 4-dim. gauge theory not unlike QCD!

Non-perturbative physics: a simple example

physics of heavy ion collisions


STAR

Heavy ion collision experiments at RHIC (2000-current) and LHC (2010-??) create hot and dense nuclear matter known as the "quark-gluon plasma"

(note: qualitative difference between p-p and Au-Au collisions)

Evolution of the plasma "fireball" is described by relativistic fluid dynamics (relativistic Navier-Stokes equations)

Need to know

thermodynamics (equation of state)
kinetics (first- and second-order transport coefficients)
in the regime of intermediate coupling strength:

$$\alpha_s(T_{\mathrm{RHIC}}) \sim O(1)$$

initial conditions (initial energy density profile)

thermalization time (start of hydro evolution)

freeze-out conditions (end of hydro evolution)

Energy density vs temperature for various gauge theories

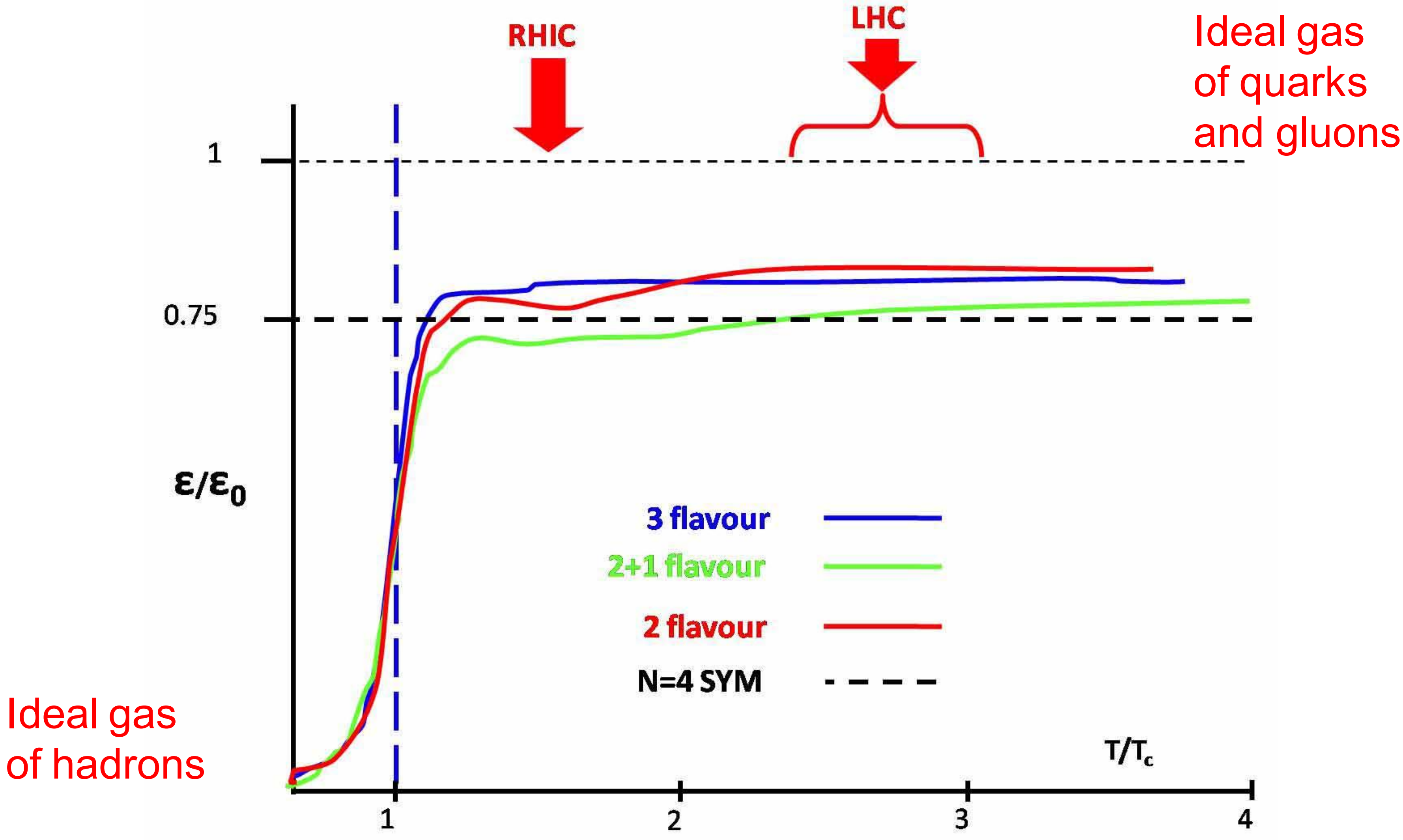
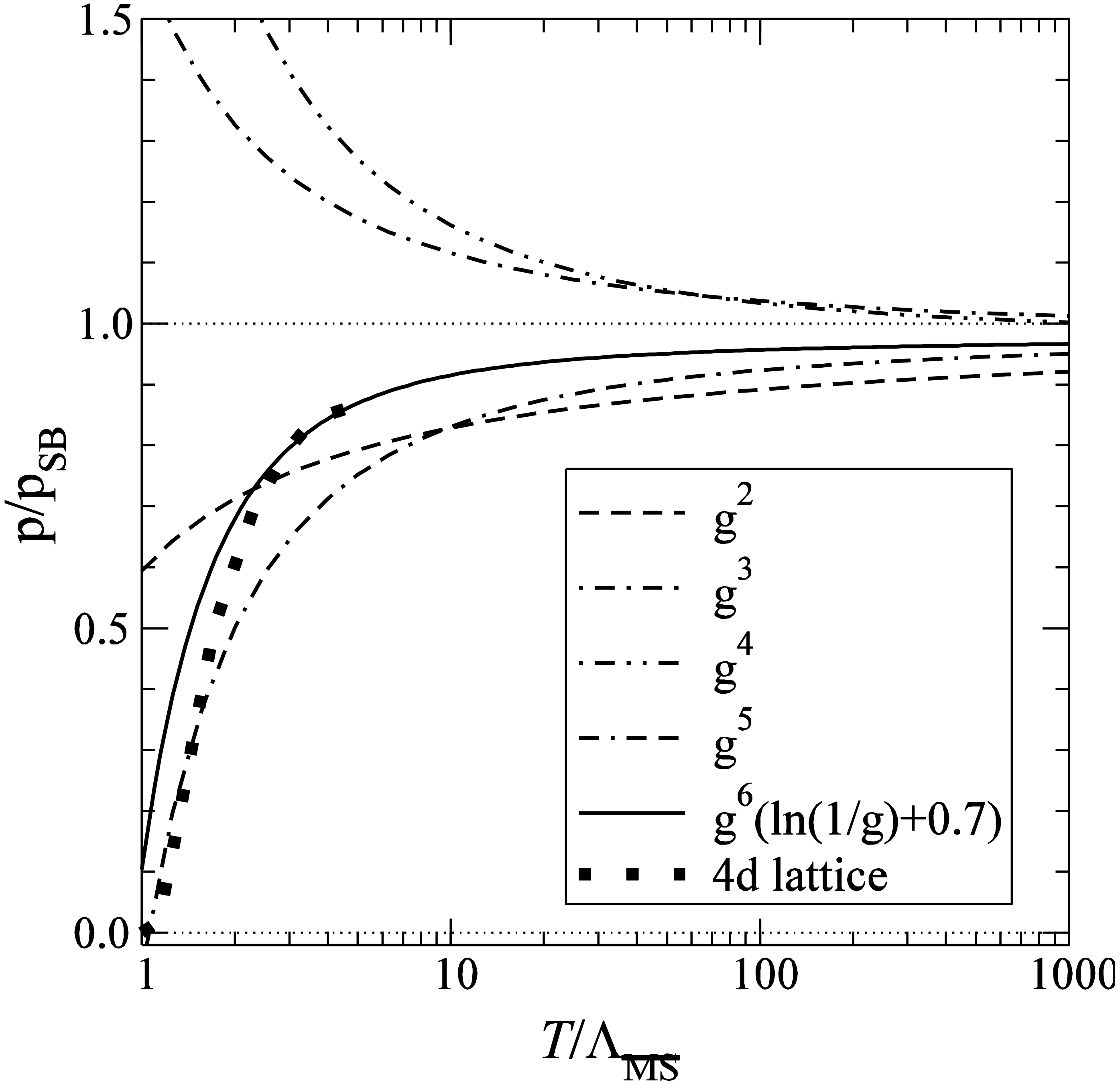
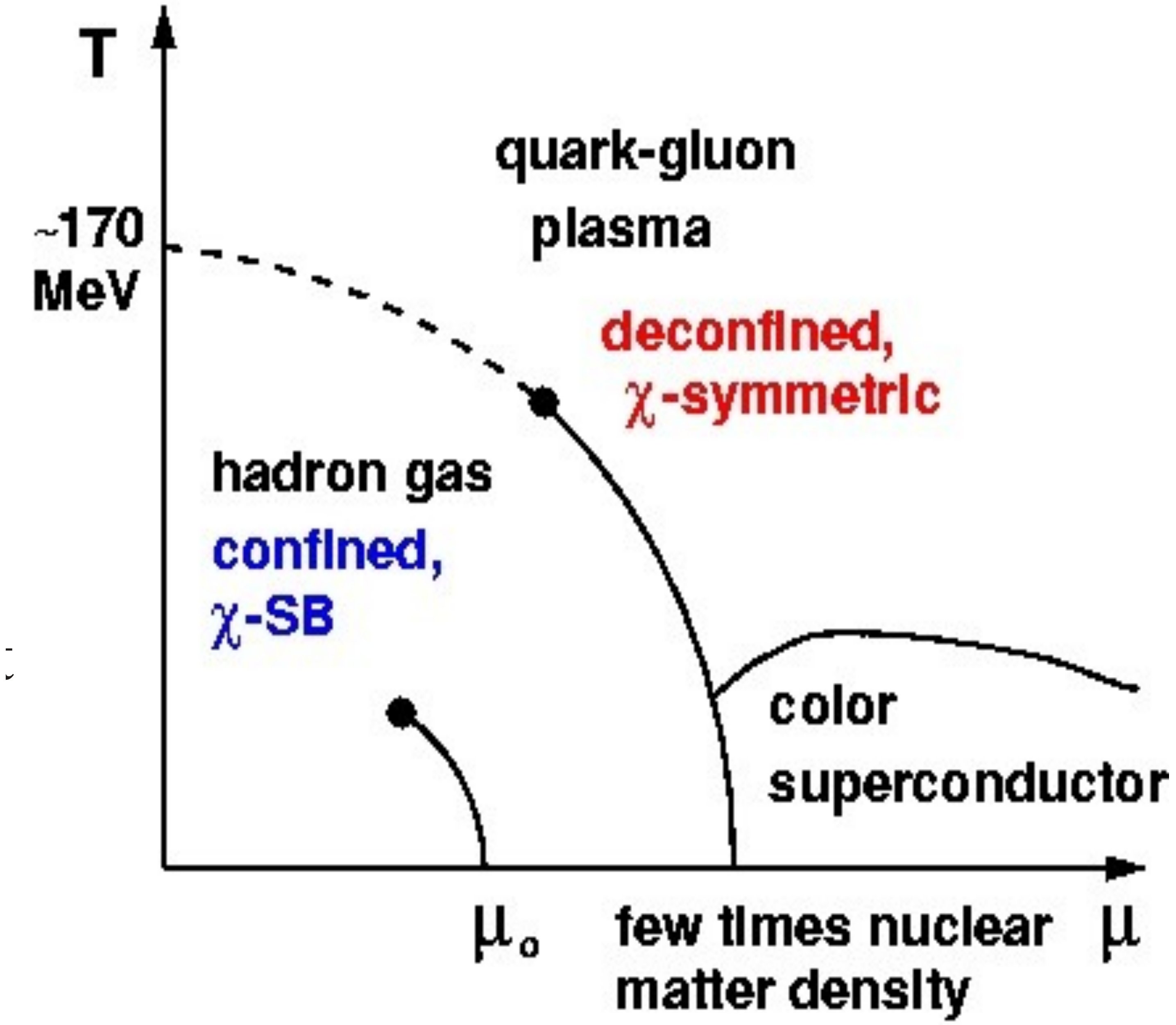


Figure: an artistic impression from Myers and Vazquez, 0804.2423 [hep-th]

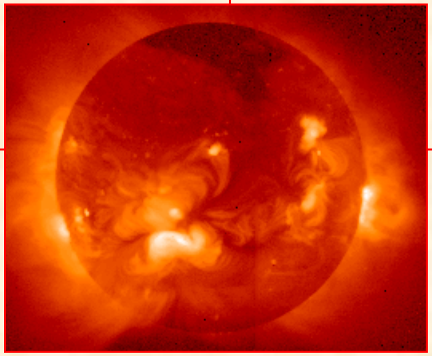
Pressure in perturbative QCD



QCD phase diagram



Quantum field theories at finite temperature/density



Equilibrium

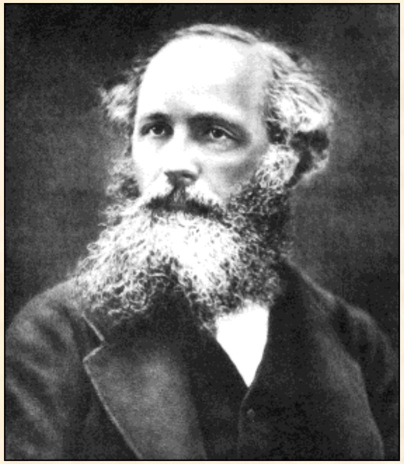
entropy
equation of state
....

perturbative

pQCD

non-perturbative

Lattice



Near-equilibrium

transport coefficients
emission rates
....

perturbative

kinetic theory

non-perturbative

????

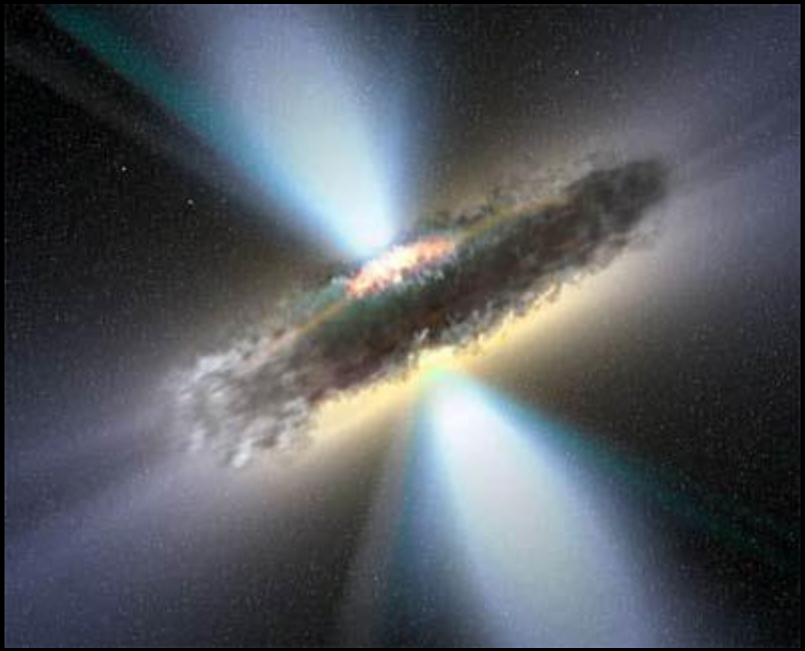
First-order transport (kinetic) coefficients

Shear viscosity	η
Bulk viscosity	ζ
Charge diffusion constant	D_Q
Supercharge diffusion constant	D_s
Thermal conductivity	κ_T
Electrical conductivity	σ

* Expect Einstein relations such as $\frac{\sigma}{e^2 \Xi} = D_{U(1)}$ to hold

Hydrodynamic properties of strongly interacting hot plasmas in 4 dimensions

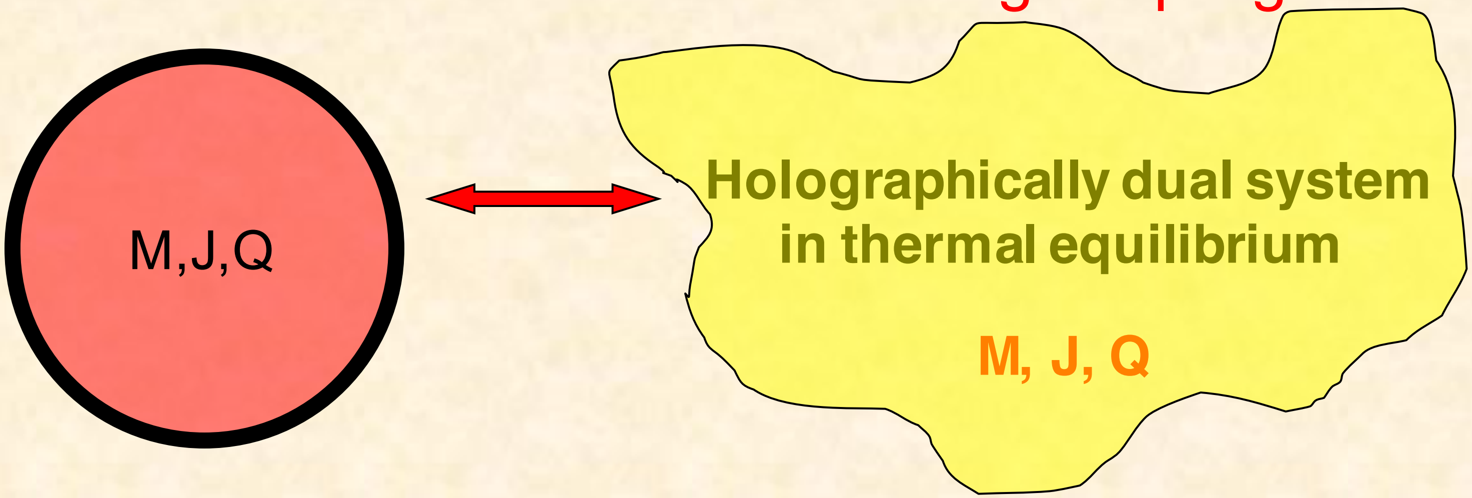
can be related (for certain models!)



to fluctuations and dynamics of 5-dimensional black holes

10-dim gravity

4-dim gauge theory – large N, strong coupling



T_{Hawking} $S_{\text{Bekenstein-Hawking}}$ $\Longleftrightarrow$ T S

Gravitational+electromag fluctuations $\Longleftrightarrow$ Deviations from equilibrium

$g^{(0)}_{\mu\nu} + h_{\mu\nu}$ $A^0_\mu + a_\mu$ $\Longleftrightarrow$????

$$j_i = -D\partial_i j^0 + \cdots$$

$"\Box" \, h_{\mu\nu} = 0$ and B.C. $\Longleftrightarrow$

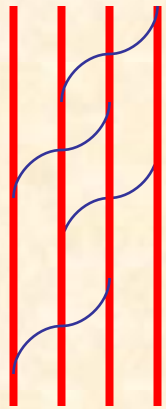
$$\partial_t j^0 + \partial_i j^i = 0$$

$$\partial_t j^0 = D\nabla^2 j^0$$

Quasinormal spectrum $\Longleftrightarrow$

$$\omega = -iDq^2 + \cdots$$

From brane dynamics to AdS/CFT correspondence



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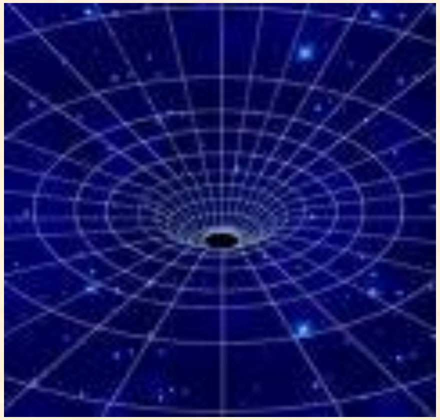


Open strings picture:
dynamics of N_c coincident D3 branes
at low energy is described by

$\mathcal{N} = 4$ supersymmetric
$SU(N_c)$ YM theory in 4 dim

⟺

conjectured
exact equivalence

Closed strings picture:
dynamics of N_c coincident D3 branes
at low energy is described by

type IIB superstring theory
on $AdS_5 \times S^5$ backgrond

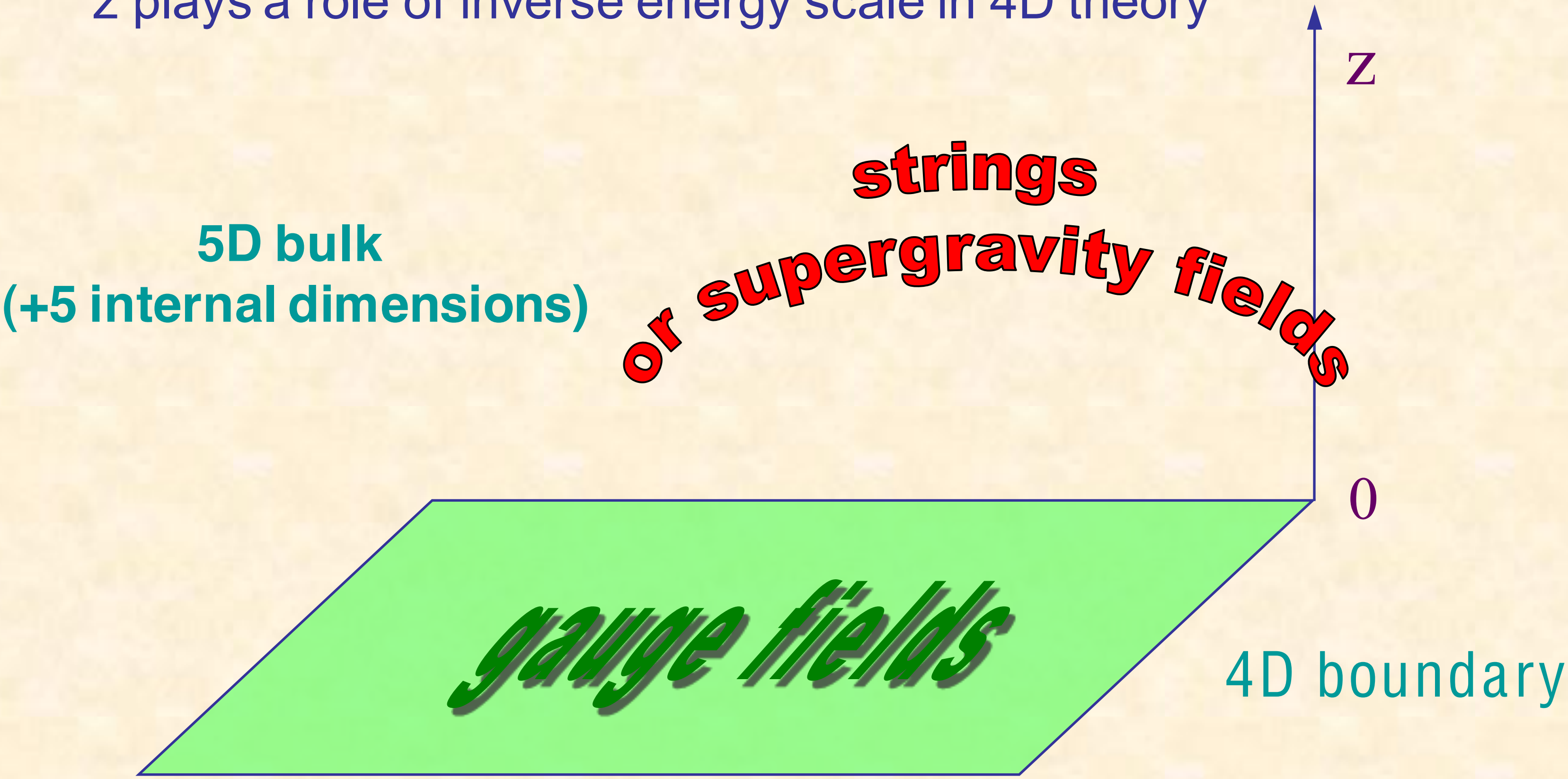
Maldacena (1997); Gubser, Klebanov, Polyakov (1998); Witten (1998)

The bulk and the boundary in AdS/CFT correspondence

$$ds^2 = \frac{\eta_{\mu\nu}\, dx^\mu\, dx^\nu + dz^2}{z^2}$$

UV/IR: the AdS metric is invariant under $z \to \Lambda z \quad x \to \Lambda x$

z plays a role of inverse energy scale in 4D theory



AdS/CFT correspondence

$\mathcal{N} = 4$ supersymmetric $SU(N_c)$ YM theory in 4 dim ⟺ type IIB superstring theory on $AdS_5 \times S^5$backgrond

conjectured
exact equivalence

$$Z_{\text{SYM}}[J] = \langle e^{-\int J\, \mathcal{O}\, d^4x} \rangle_{\text{SYM}} = Z_{\text{string}}[J]$$

Generating functional for correlation functions of gauge-invariant operators ⟺ String partition function

$$\langle \mathcal{O}\ \mathcal{O}\ \cdots \mathcal{O} \rangle$$

In particular

$$Z_{\text{SYM}}[J] = Z_{\text{string}}[J] \simeq e^{-S_{\text{grav}}[J]}$$

$$\lambda \equiv g_{YM}^2\, N_c \gg 1$$

$$N_c \gg 1$$

Classical gravity action serves as a generating functional for the gauge theory correlators

Sound and supersymmetric sound in $4d\ \mathcal{N}=4$ SYM

In 4d CFT

$\epsilon = 3P$

$\zeta = 0$

$\Longrightarrow$

$$v_s = \sqrt{\frac{\partial P}{\partial \epsilon}} = \frac{1}{\sqrt{3}}$$

$$v_{SS} = \frac{P}{\epsilon} = \frac{1}{3}$$

Sound mode: $\omega = \pm \frac{q}{\sqrt{3}} - i \frac{2\eta}{3sT} q^2 + \cdots$

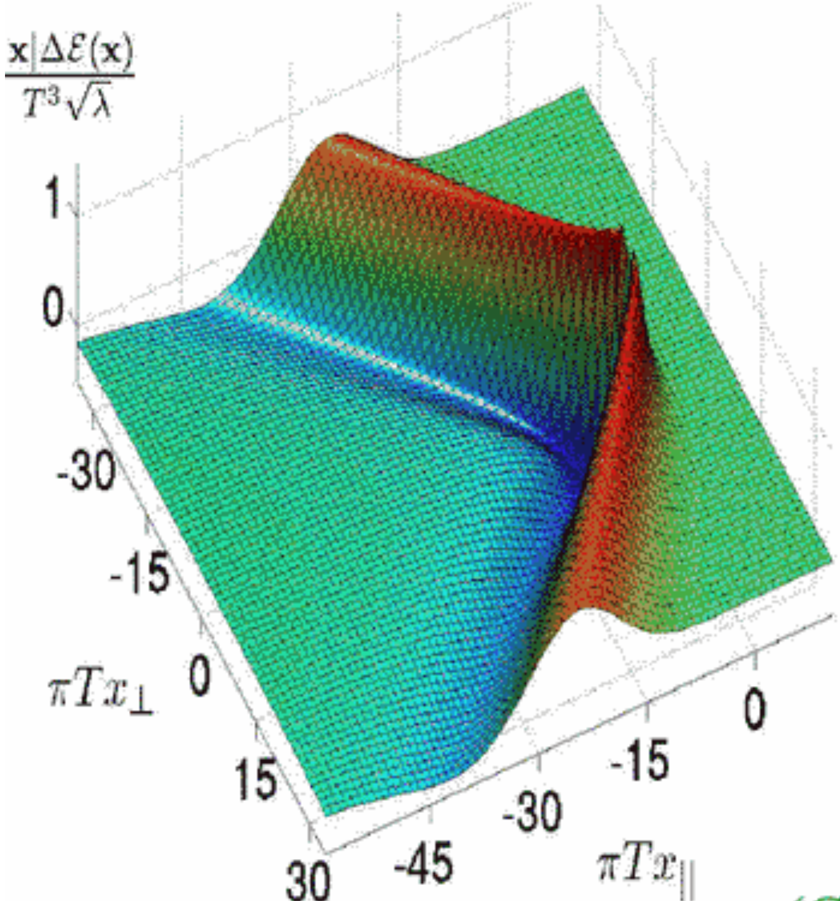
Supersound mode: $\omega = \pm \frac{q}{3} - i D_s q^2 + \cdots$

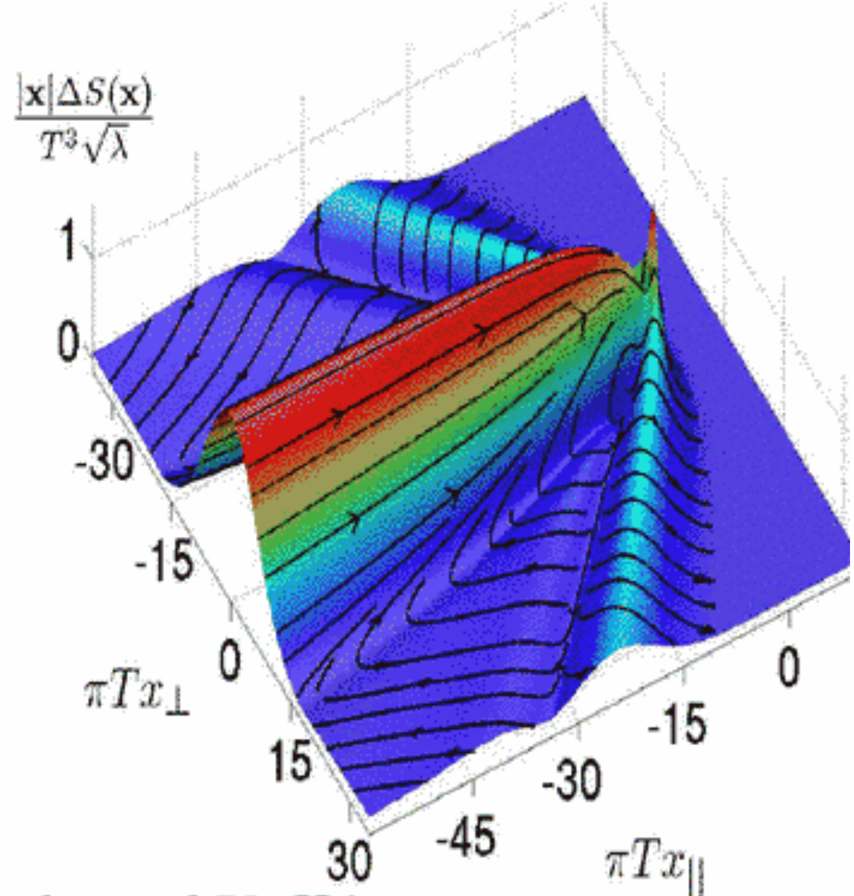
Quasinormal modes in dual gravity

Graviton: $\omega = \pm \frac{q}{\sqrt{3}} - i \frac{1}{6\pi T} q^2 + \cdots \Longrightarrow \frac{\eta}{s} = \frac{1}{4\pi}$

Gravitino: $\omega = \pm \frac{q}{3} - i \frac{2\sqrt{2}}{9\pi T} q^2 + \cdots \Longrightarrow D_s = \frac{2\sqrt{2}}{9\pi T}$

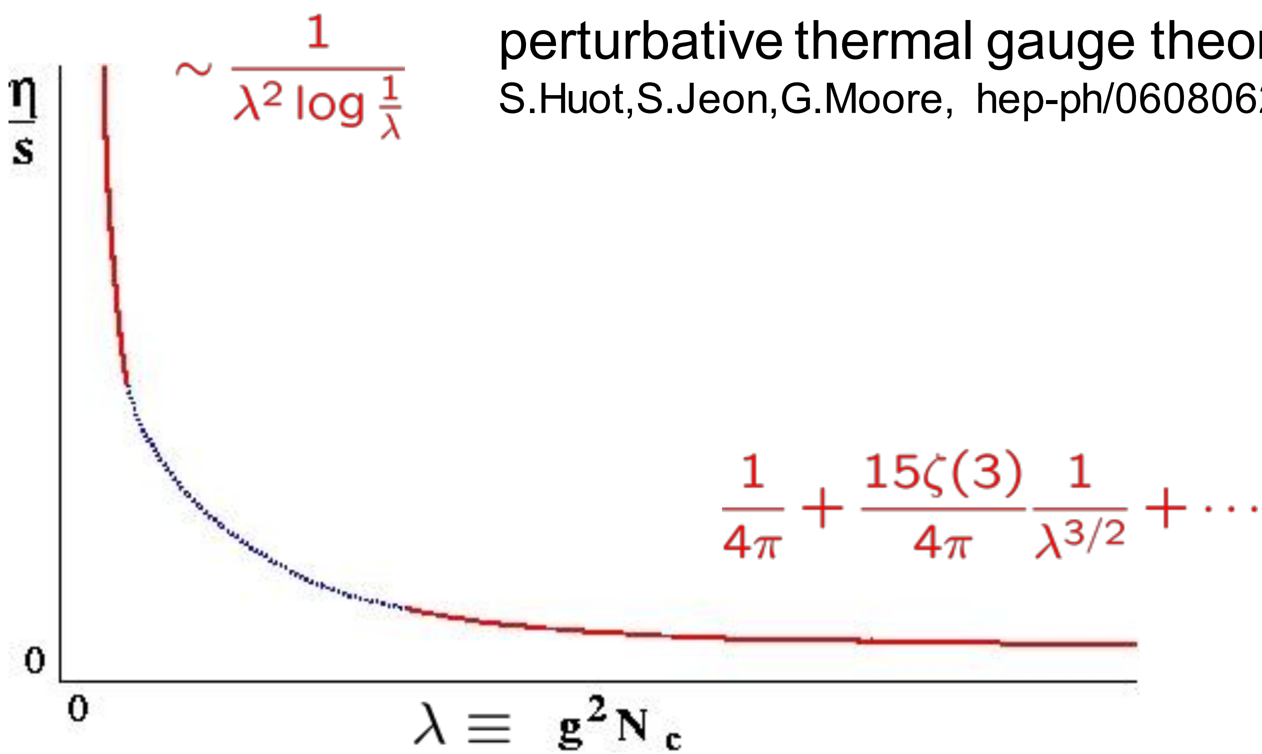
Energy and Momentum Density





(Chesler and Yaffe)

Shear viscosity in $\mathcal{N} = 4$ SYM



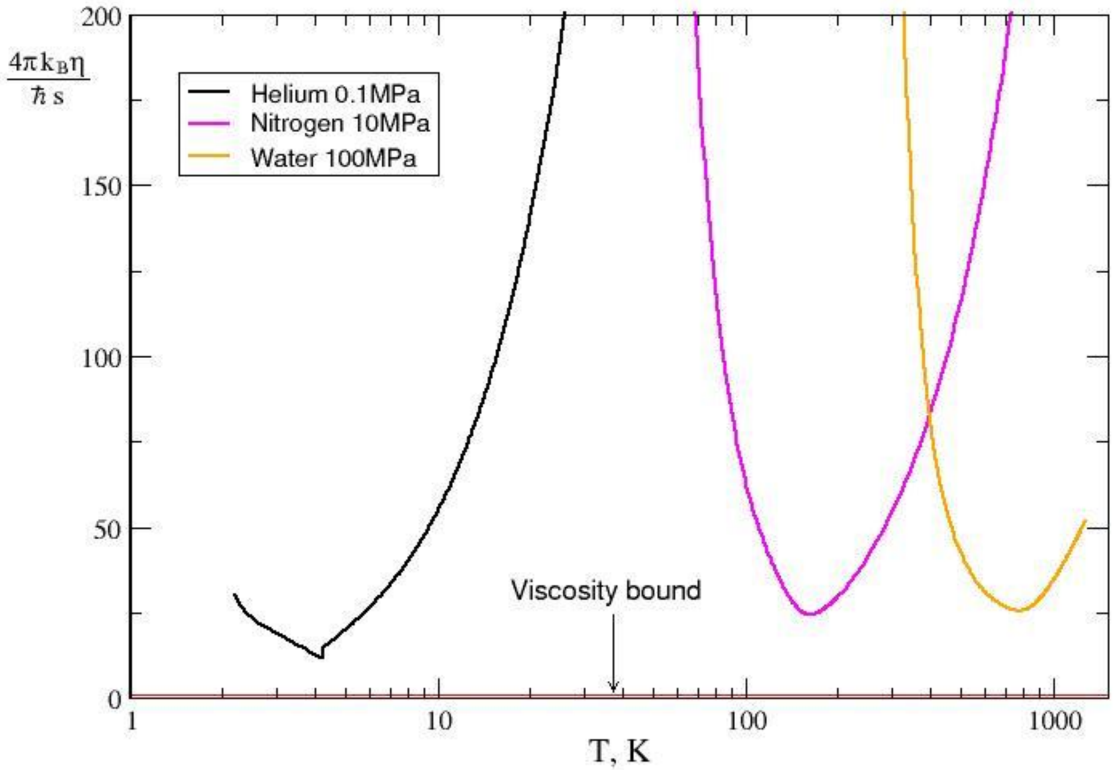
perturbative thermal gauge theory
S.Huot, S.Jeon, G.Moore, hep-ph/0608062

Correction to $1/4\pi$: Buchel, Liu, A.S., hep-th/0406264

Buchel, 0805.2683 [hep-th]; Myers, Paulos, Sinha, 0806.2156 [hep-th]

A viscosity bound conjecture

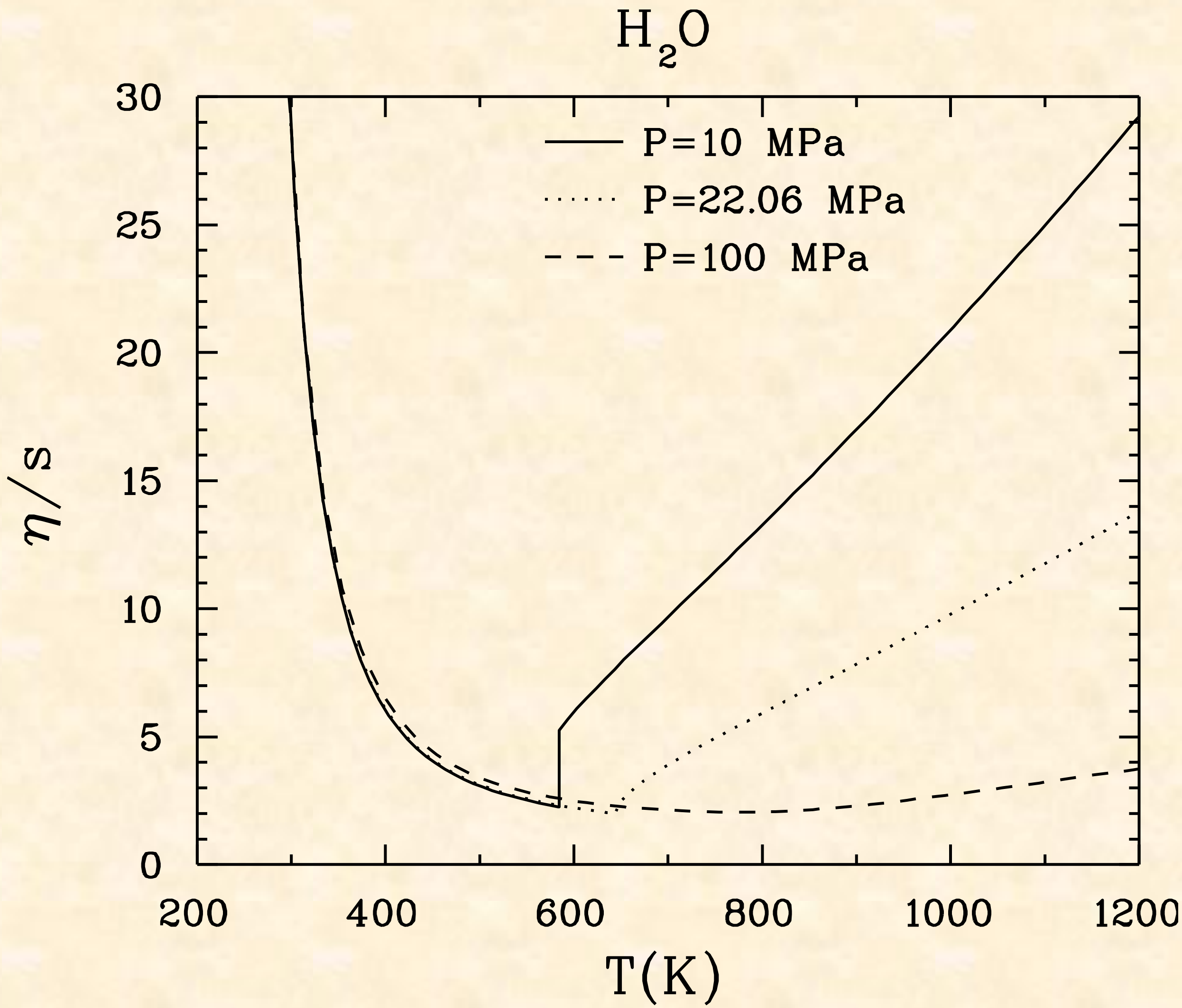
$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13}\, K \cdot s$$



Minimum of $\frac{\eta}{s}$ in units of $\frac{\hbar}{4\pi k_B}$

Xe	84
Kr	57
CO_2	32
H_2O	25
C_2H_5OH	22
Ne	17
He	8.8

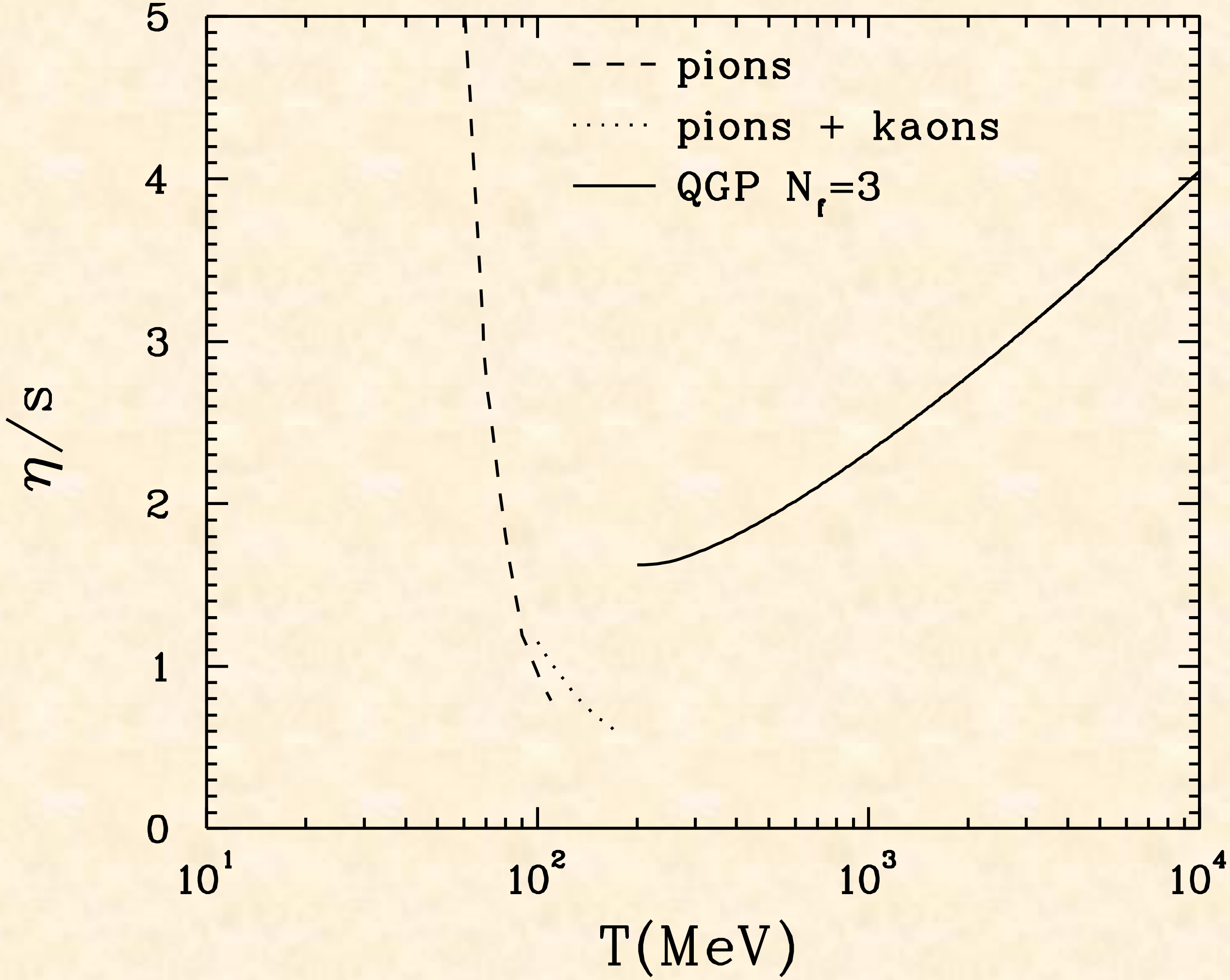
P.Kovtun, D.Son, A.S., hep-th/0309213, hep-th/0405231



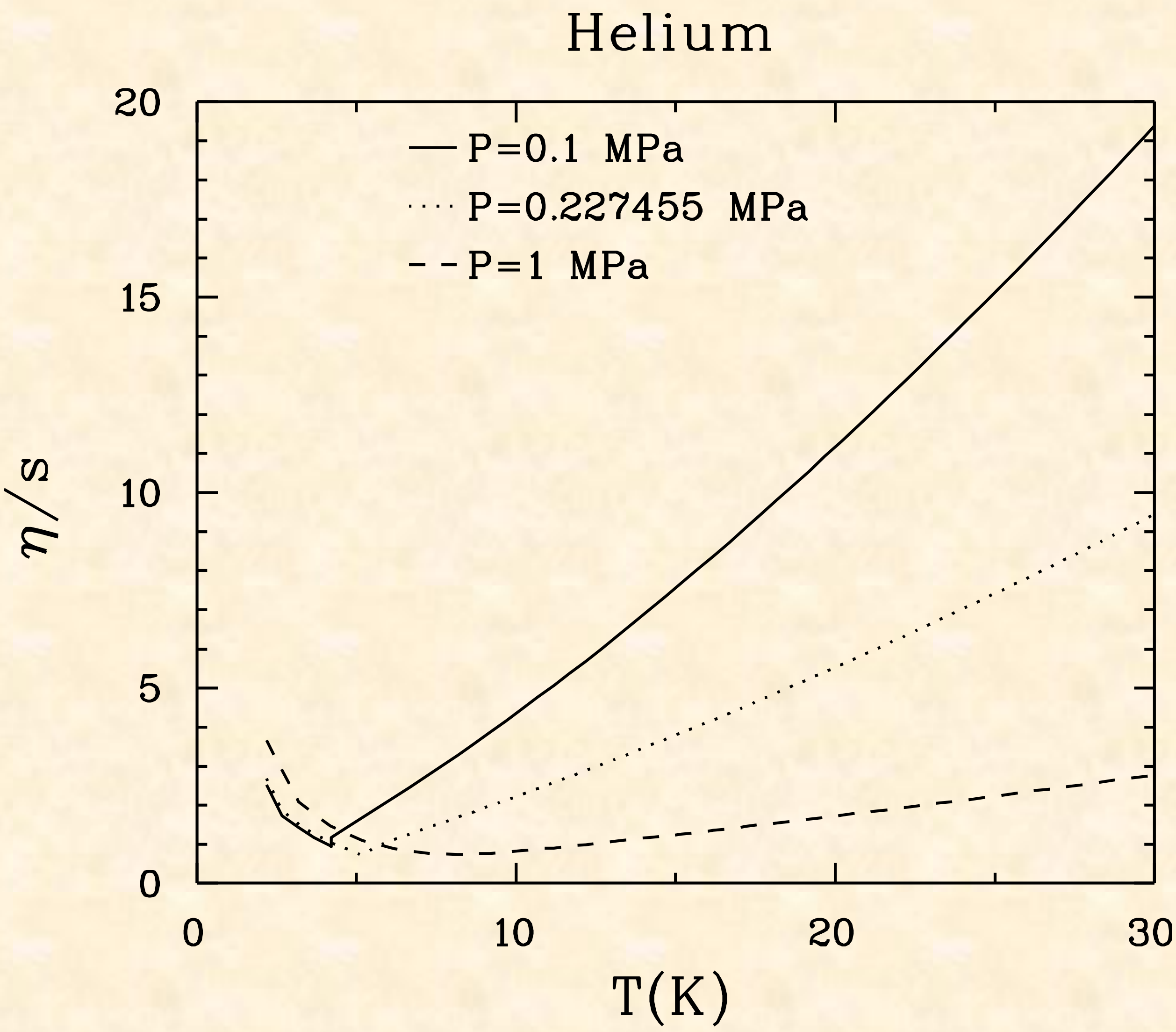
$(\eta/s)_{\text{min}} \sim 25$ in units of $\frac{\hbar}{4\pi k_B}$

Chernai, Kapusta, McLerran, nucl-th/0604032

QCD



Chernai, Kapusta, McLerran, nucl-th/0604032



$(\eta/s)_{\text{min}} \sim 8.8$ in units of $\dfrac{\hbar}{4\pi k_B}$

Chernai, Kapusta, McLerran, nucl-th/0604032

First-order transport coefficients in N = 4 SYM

in the limit $N_c \to \infty\,, \quad g_{YM}^2 N_c \to \infty$

Shear viscosity $\eta = \frac{\pi}{8} N_c^2 T^3 \left[1 + O\left(\frac{1}{(g^2 N_c)^{3/2}}, \frac{1}{N_c^2} \right) \right]$

Bulk viscosity $\zeta = 0$

for non-conformal theories see Buchel et al; G.D.Moore et al Gubser et al.

Charge diffusion constant $D_R = \frac{1}{2\pi T} + \cdots$

Supercharge diffusion constant $D_s = \frac{2\sqrt{2}}{9\pi T}$

Thermal conductivity $\frac{\kappa_T\ \mu^2}{\eta\ T} = 8\pi^2 + \cdots$

Electrical conductivity $\sigma = e^2 \frac{N_c^2 T}{16\,\pi} + \cdots$

M-theory and branes

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All five string theories are related and part of a single theory : M-theory

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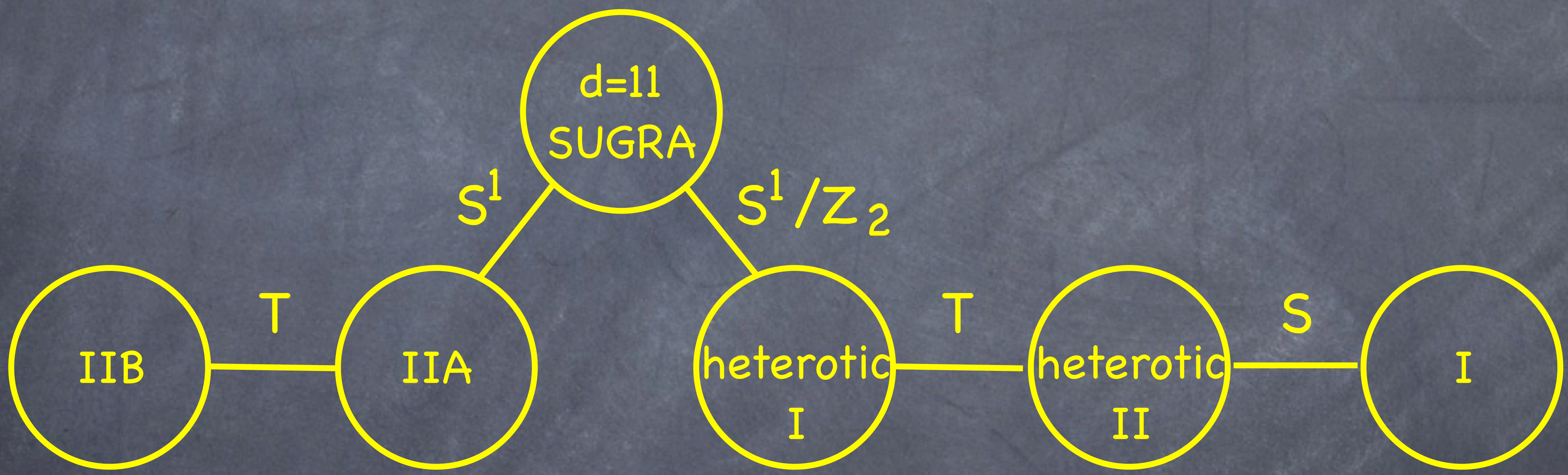

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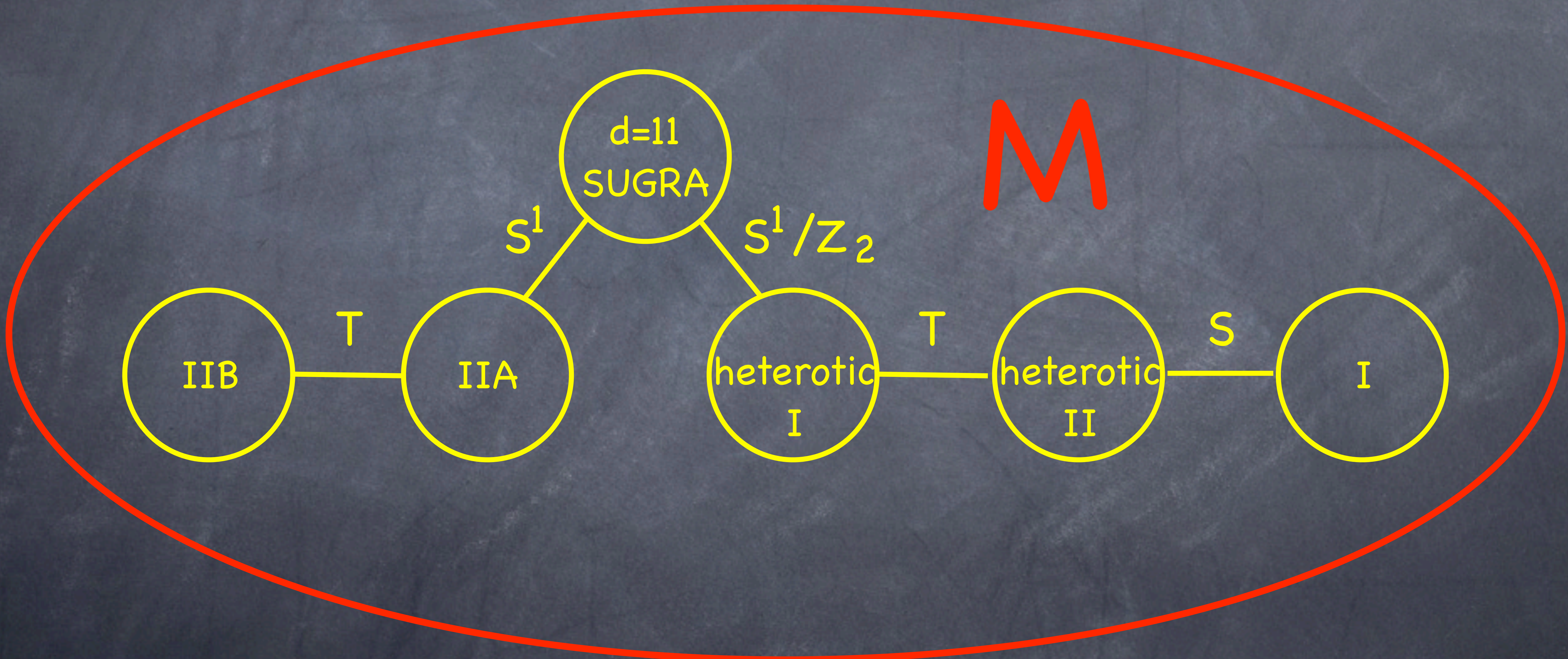

Branes

M-theory contains not just strings but extended objects (p-branes) of all dimensions !

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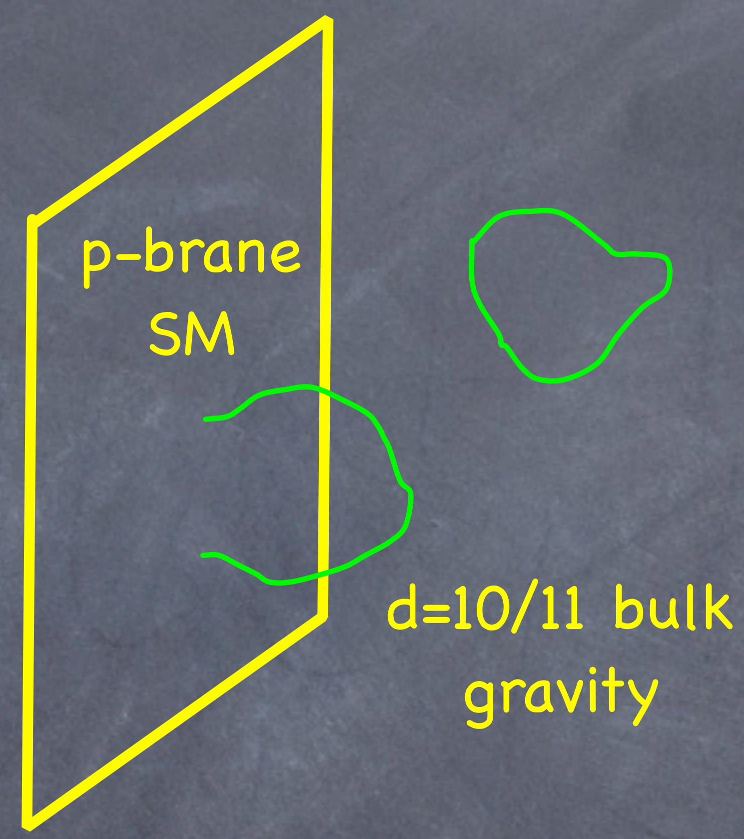

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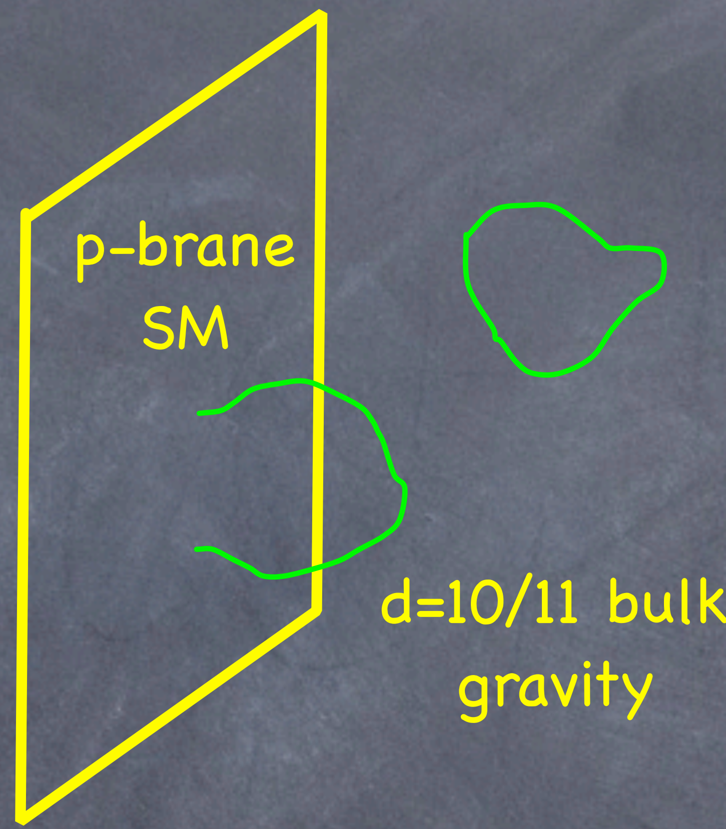

bulk -> gravity (closed strings)

brane -> gauge theories (open strings)

Compactification

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Need to compactify six or seven dimensions to obtain d=4 theory :

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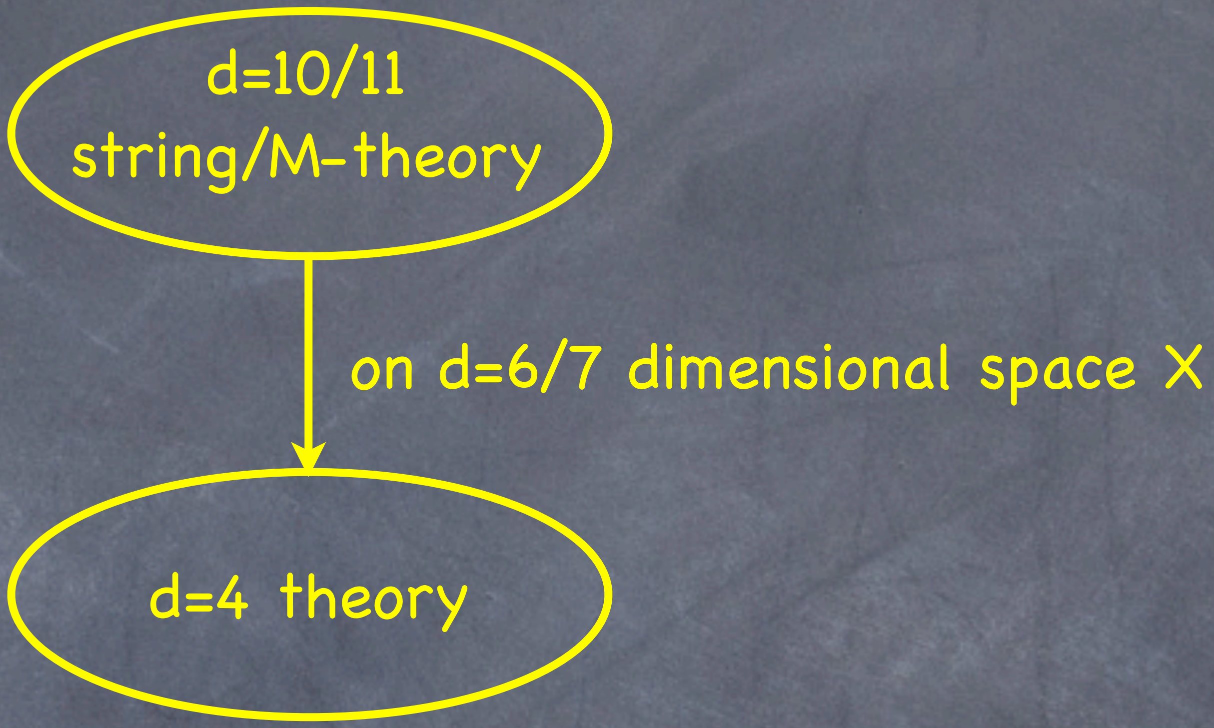

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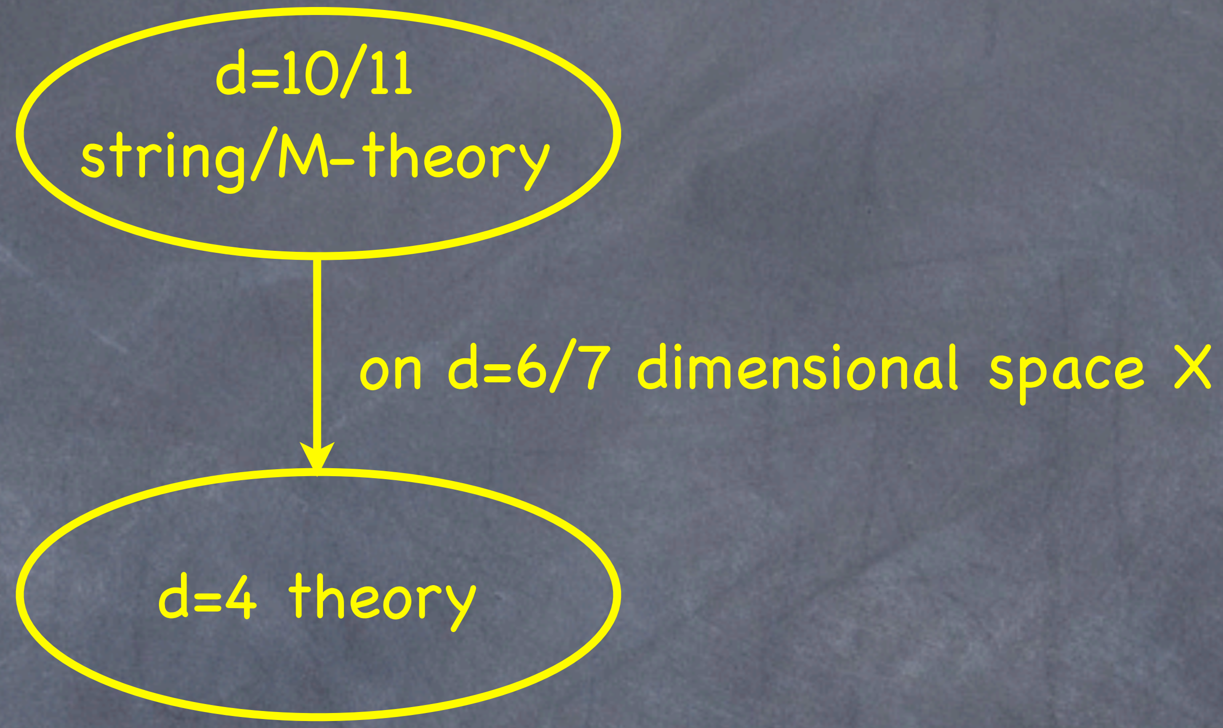

Two-fold degeneracy in space X: continuous one in size and shape (moduli), discrete one topology

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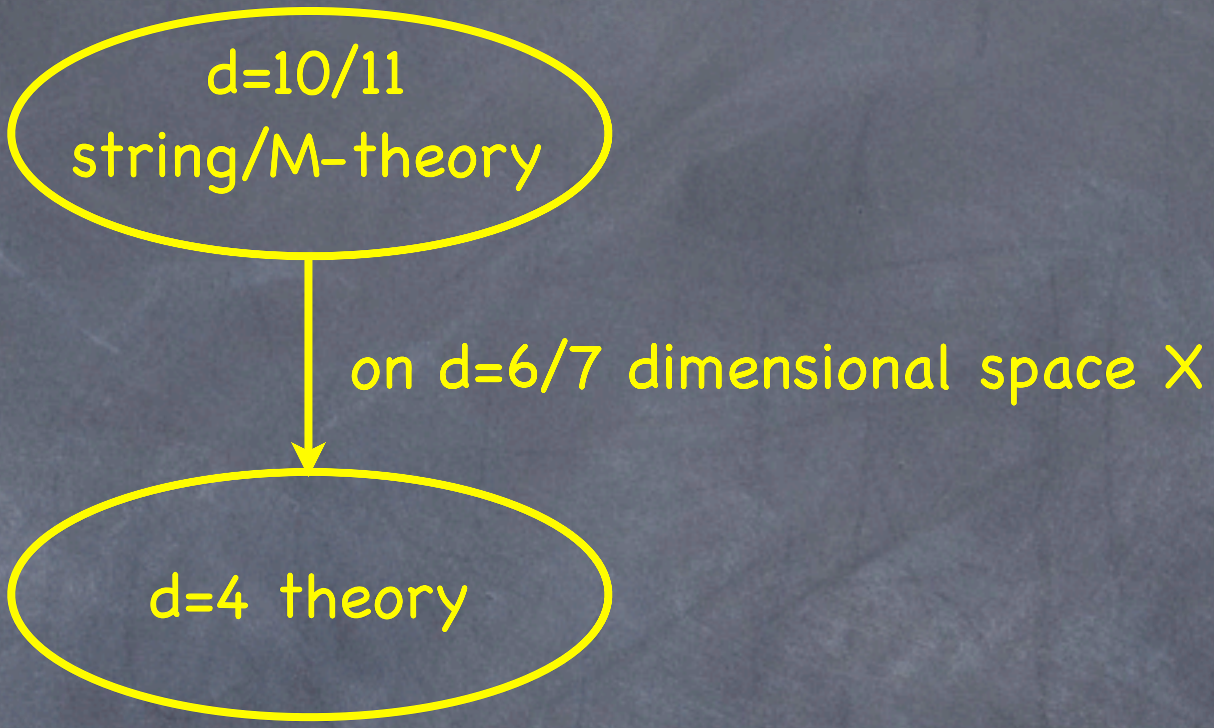

Two-fold degeneracy in space X: continuous one in size and shape (moduli), discrete one topology

But d=4 theory depends on space X ...

M-theory and the real world (?)

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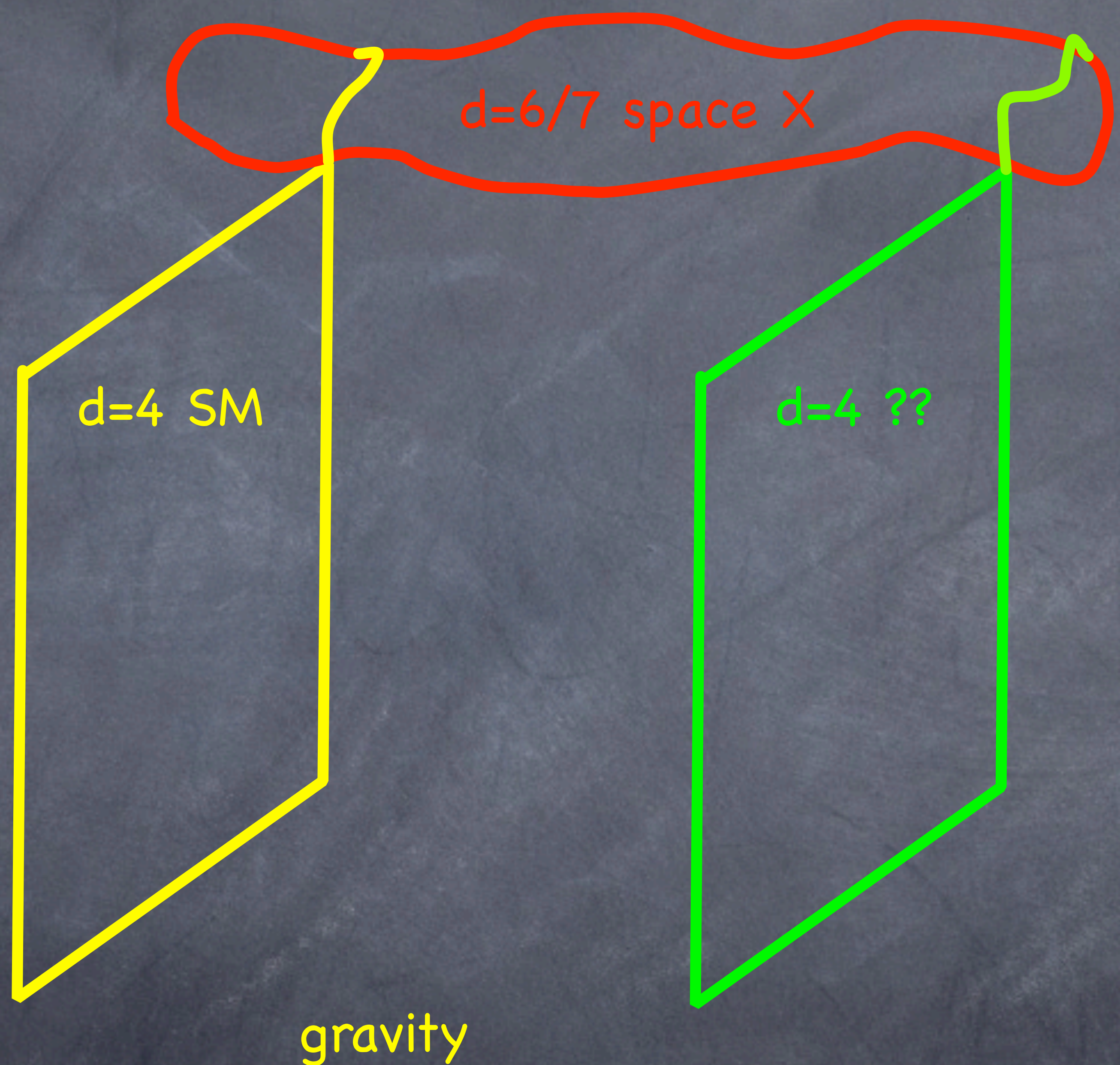

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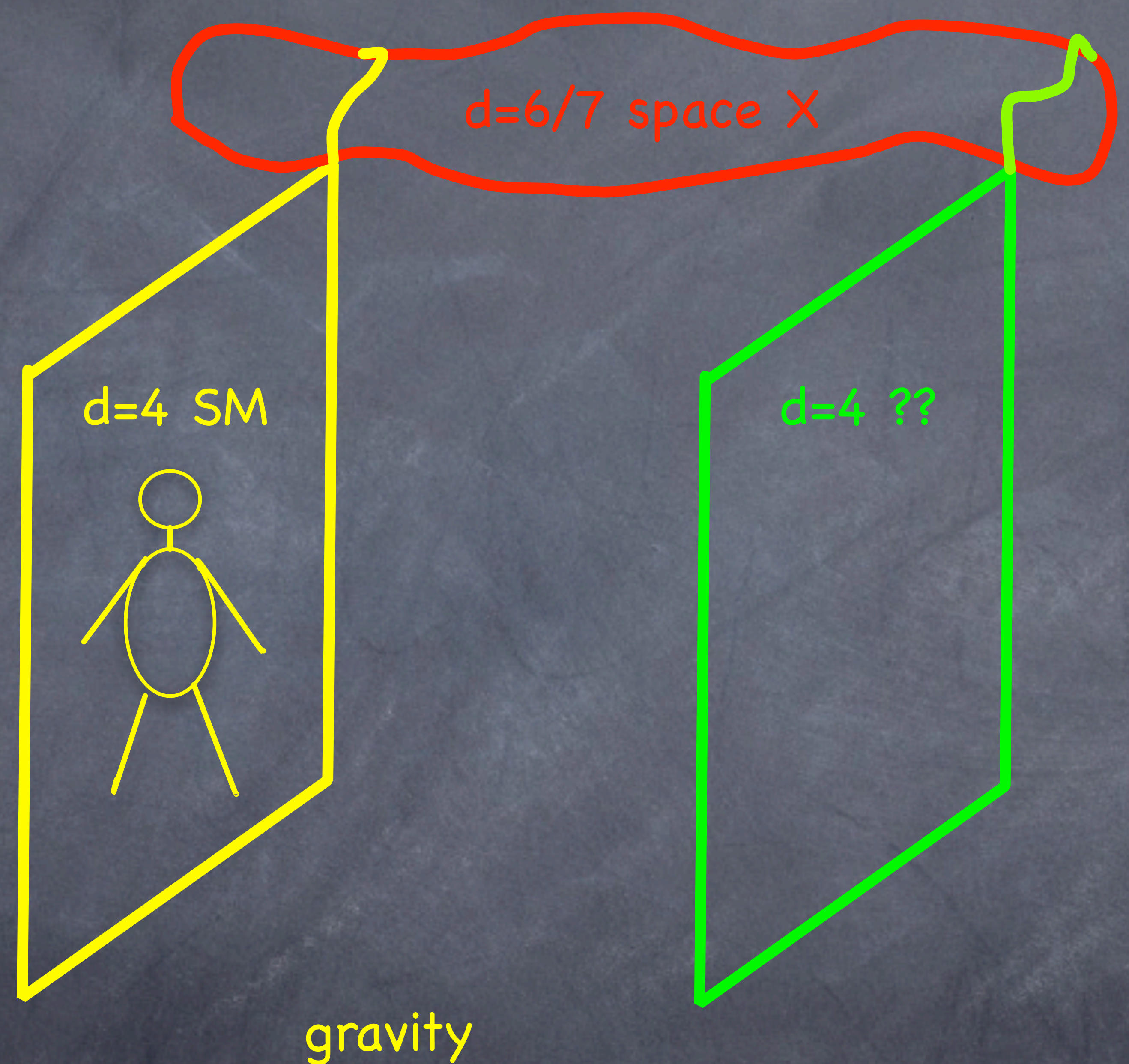

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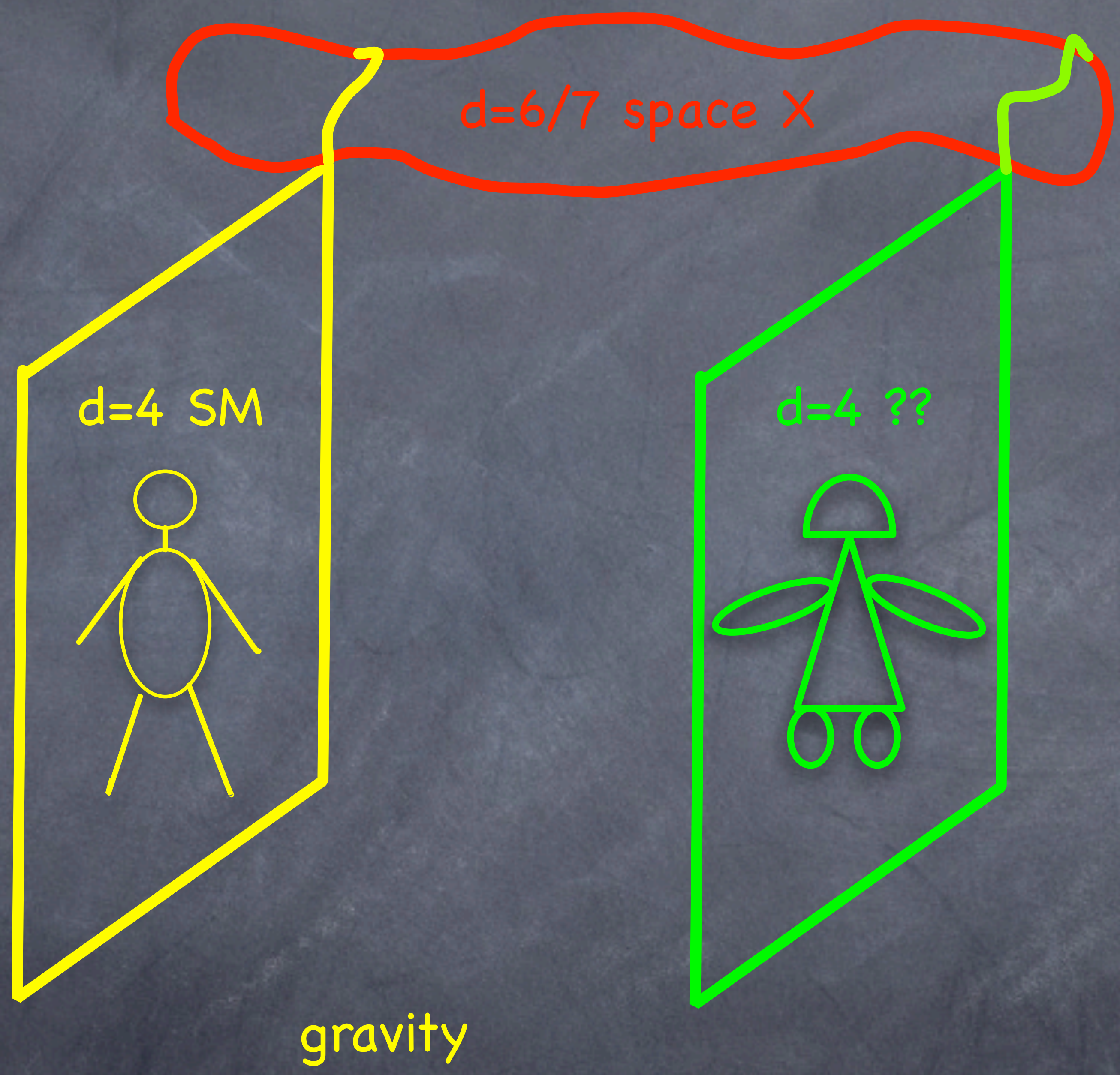

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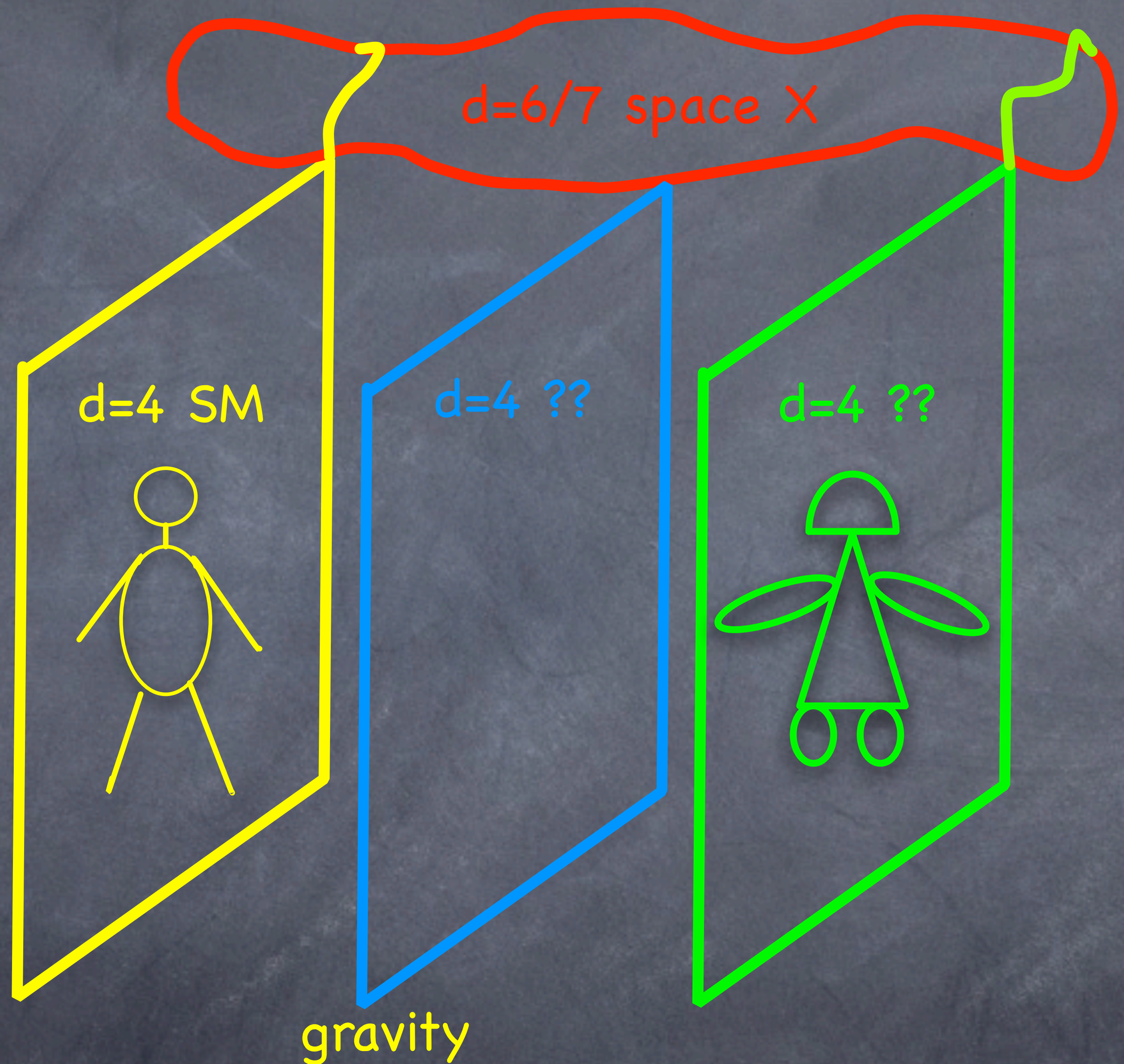

The enemy

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different moduli :

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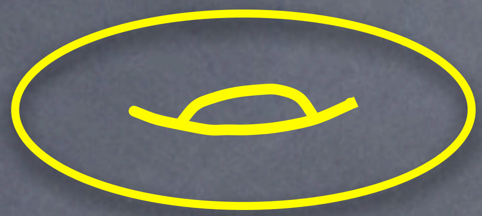

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-> many different four-dimensional theories

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We now have a better understanding of how to fix moduli.

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Problem of many different topologies remains !

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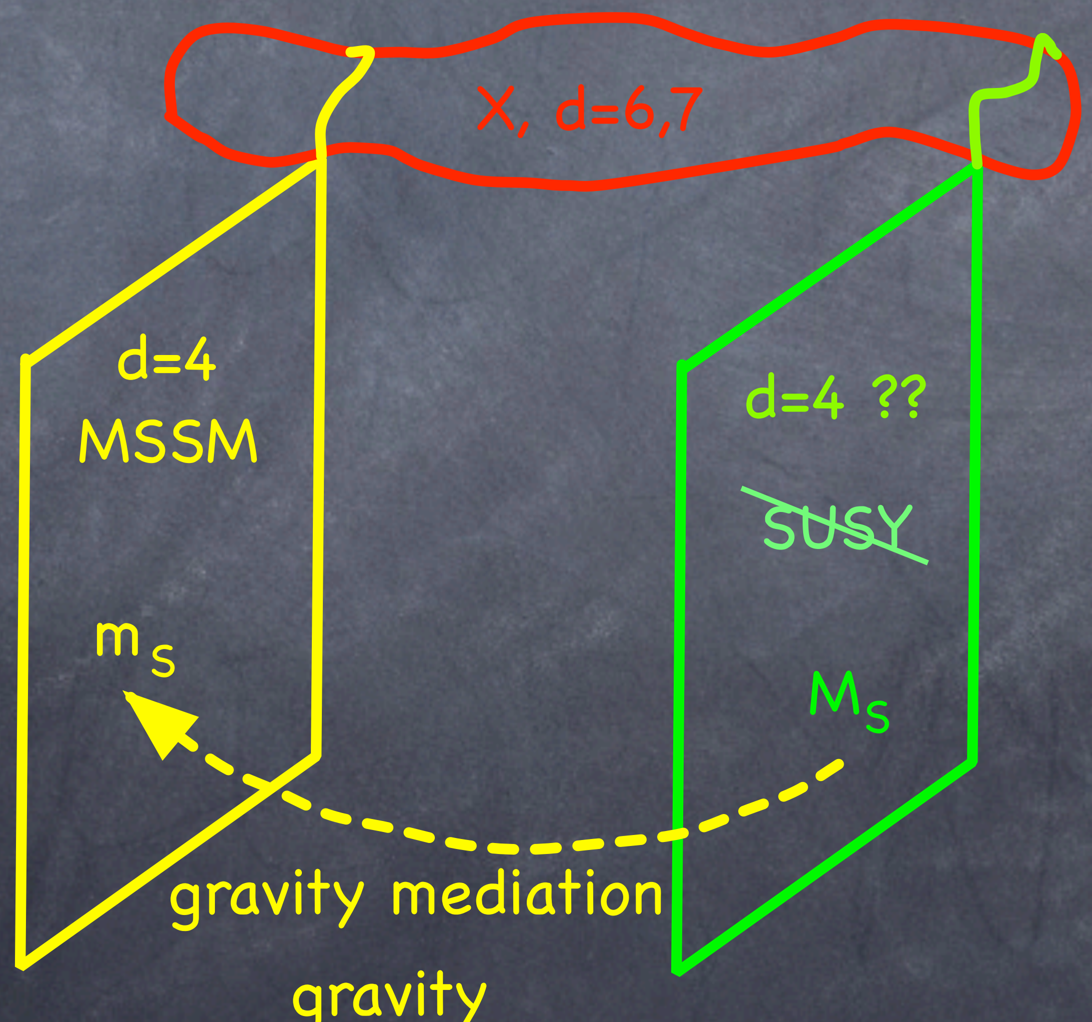

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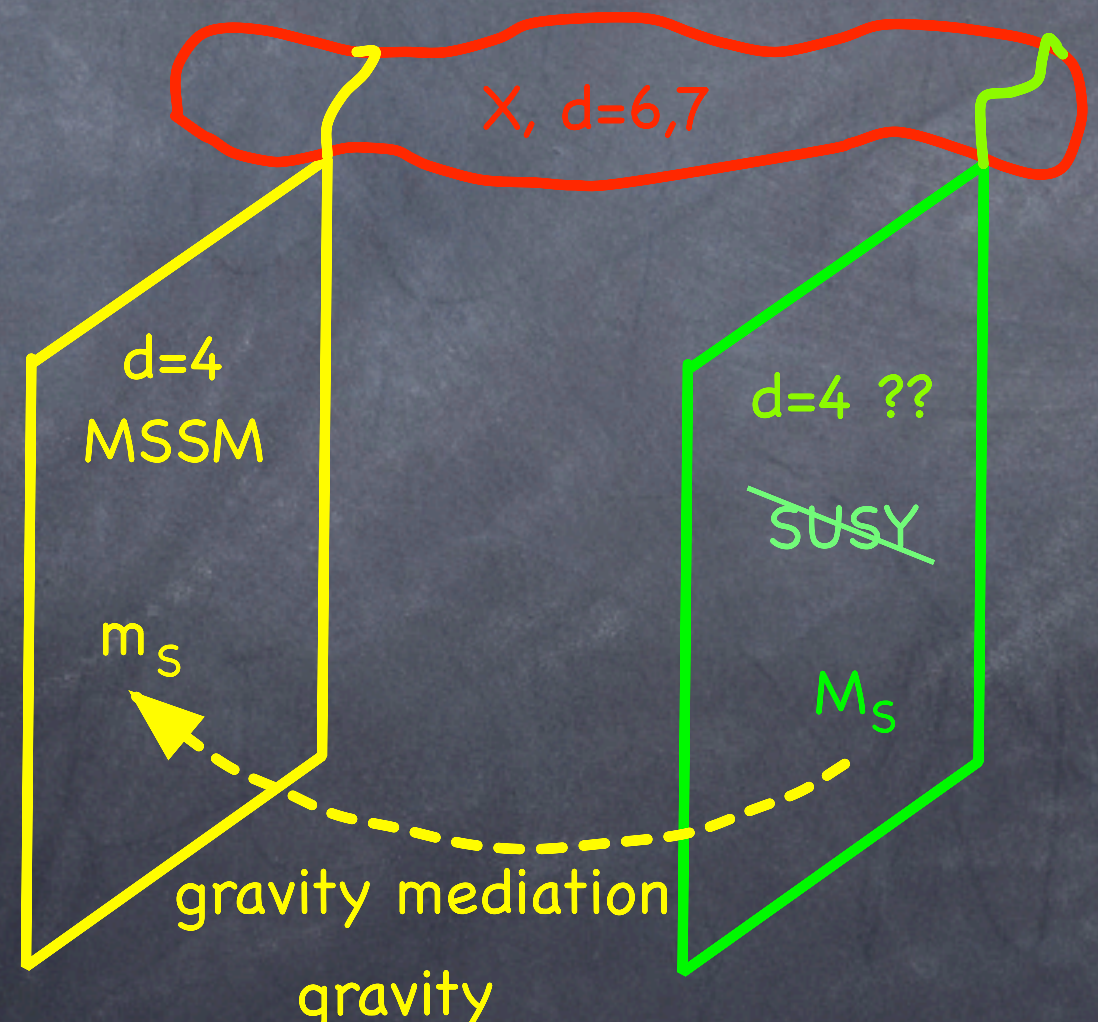

$m_s = M_S^2/M_P$ so for $m_s \sim 1\,\mathrm{TeV}$ we need $M_s \sim 10^{11}\,\mathrm{GeV}$

Summary

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Summary

- String/M-theory is currently the only candidate for a fundamental theory.
- The basic structure of the theory is not yet fully understood.
- It contains (quantum) gravity and standard model-like theories.
- Most theoretical ideas beyond the standard model have a place within M-theory.
- M-theory has an enormously rich structure.

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If you want to do it, you have to be (seriously) good at Maths!