# Transport Properties of Holographic P-Wave Superfluids

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# Why is this model interesting?

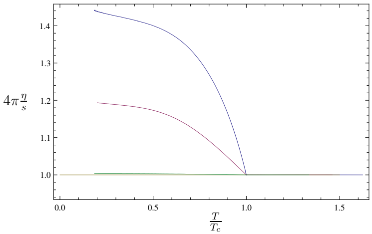
- breaking of spacetime symmetry
  (interesting for condensed matter applications)

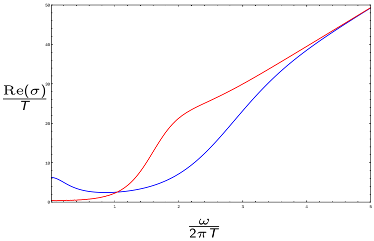
- transport effects not present in isotropic systems

- transport still determined by ODEs rather than PDEs (also at finite spatial momentum)

# Take-home message!

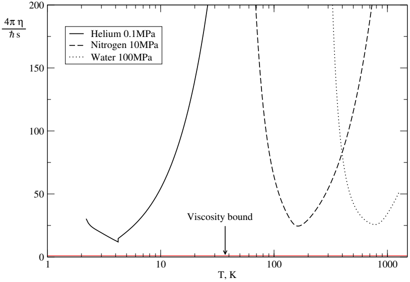
- 2 shear viscosities
  one "universal", one "non-universal", i.e. temperature dependent
- 2 optical conductivities with different low frequency behaviour



# Why are these important?

- temperature dependent shear viscosity
- Drude-like behaviour of conductivity at low frequencies




Kovtun, Son, Starinets 2004

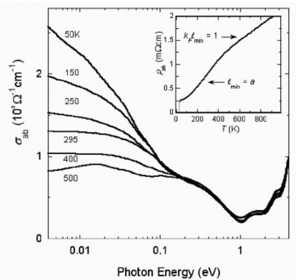



FIG. 25. $La_{1.9}Sr_{0.1}CuO_4$ at specified temperatures. The inset shows the in-plane resistivity data for $La_{1.9}Sr_{0.1}CuO_4$ up to 1000 K. From Hussey *et al.*, 2004.

Basov, Timusk 2005

# Outline

Very Brief Review of GGD and Hydrodynamics

P-Wave Superfluids

Perturbations in this Model $\Rightarrow$ Transport

# Gauge/Gravity Duality – Overview

- finite temperature $\Leftrightarrow$ black hole solutions

Witten 1998

- finite charge density
  $\Leftrightarrow$ gravity solutions with gauge field $A_t \neq 0$ (AdS RN)

Chamblin, Emparan, Johnson, Myers 1998

- holographic superfluid:

several papers by Gubser, Hartnoll, Herzog, Horowitz, Pufu 2008

s-wave: field theory: SSB global $U(1)$
$\Leftrightarrow$ gravity: SSB gauged $U(1)$

p-wave: field theory: SSB global $U(1)$ and spatial $SO(3)$
$\Leftrightarrow$ gravity: SSB gauged $U(1)$ and spatial $SO(3)$

# Hydrodynamics – Overview

- long wavelength, small frequency fluctuations about thermal equilibrium
- effective theory: macroscopic behaviour of the system
- response of system: transport coefficients
  e.g. shear viscosity, bulk viscosity, diffusion constants, conductivity
- constitutive equations

$$T^{\mu\nu} = T^{\mu\nu}_{\text{eq.}} + \Pi^{\mu\nu} \text{ and } J^{\mu} = J^{\mu}_{\text{eq.}} + \Upsilon^{\mu}$$

with $\Pi_{ij} \sim \eta \left(\partial_i u_j + \partial_j u_i\right)$

# Different holographic p-wave models

- $SU(2)$ gauge field in the bulk with vector condensate (discussed in this talk)
- helical superfluid phases (using two-forms)

Donos, Gauntlett, Pantelidou 2011-13

# Gravitational Setup for holographic p-wave Superfluid

- $SU(2)$ Einstein-Yang-Mills theory in (4+1)-dimensional asymptotically AdS Space

$$S = \frac{1}{2{\kappa_5}^2} \int \mathrm{d}^5x \sqrt{-g} \left[ R - \Lambda - \frac{\alpha^2}{2} F^a_{MN} F^{aMN} \right] + S_{\mathrm{bdy}}$$

- with

$$\alpha \equiv \frac{\kappa_5}{g_{\mathrm{YM}}}$$

- $\alpha$ measures the backreaction of gauge fields on geometry

# Black Hole Ansatz with Vector Hair

- metric ansatz

$$\mathrm{d}s^2 = -N(r)\sigma(r)^2\mathrm{d}t^2 + \frac{1}{N(r)}\mathrm{d}r^2 + \frac{r^2}{f(r)^4}\mathrm{d}x^2 + r^2 f(r)^2 \left(\mathrm{d}y^2 + \mathrm{d}z^2\right)$$

with

$$N(r) = -\frac{2m(r)}{r^2} + r^2$$

AdS boundary $r = r_{\text{bdy}} \to \infty$ & black hole horizon $r = r_h$

- gauge field ansatz

$$A = \phi(r)\tau^3\mathrm{d}t + w(r)\tau^1\mathrm{d}x$$

Manvelyan, Radu, Tchrakian 2008

# Looking for solutions with...

| Field Theory | $\Leftrightarrow$ | Gravity |
|---|---|---|
| finite temperature $T$ | | black hole solutions |
| isospin chemical potential $\mu$ | | $A_t^3 = \phi(r) \neq 0$ |
| $\Rightarrow$ breaks $SU(2) \to U(1)_3$ | | $\Rightarrow$ breaks $SU(2) \to U(1)_3$ |
| $\langle \mathcal{J}_1^x \rangle \neq 0$ | | $A_x^1 = w(r) \neq 0$ |
| $U(1)_3 \to \mathbb{Z}_2$, $SO(3) \to SO(2)$ | | $U(1)_3 \to \mathbb{Z}_2$, $SO(3) \to SO(2)$ |

- $w(r_{\text{bdy}}) = 0 \Rightarrow$ SSB $U(1)_3 \to \mathbb{Z}_2$ & $SO(3) \to SO(2)$

$\Rightarrow$ holographic p-wave superfluid with backreaction

Ammon, Erdmenger, Grass, Kerner, O'Bannon 2009

# Hairy Black Hole Solution

Depending on the temperature $T$, we find the following thermodynamically preferred solutions:

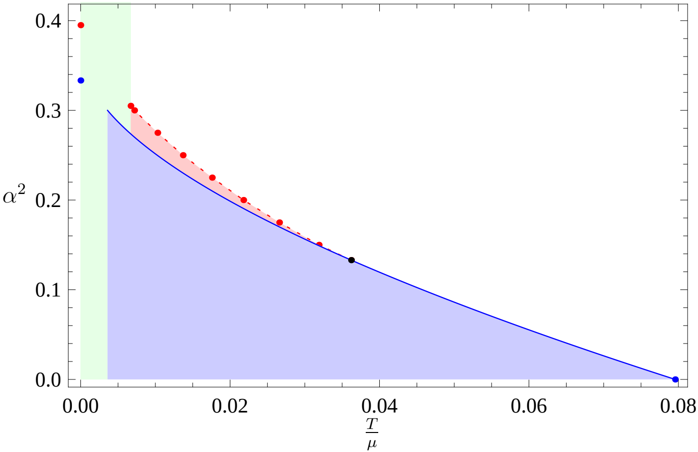
- $w(r) = 0$, AdS Reissner-Nordström black hole solution ($T > T_c$), or
- $w(r) \neq 0$, numerical solution ($T < T_c$).

Depending on the backreaction parameter $\alpha$ we find a

- 2nd order phase transition ($\alpha < \alpha_{\text{crit}}$), or
- 1st order phase transition ($\alpha > \alpha_{\text{crit}}$)

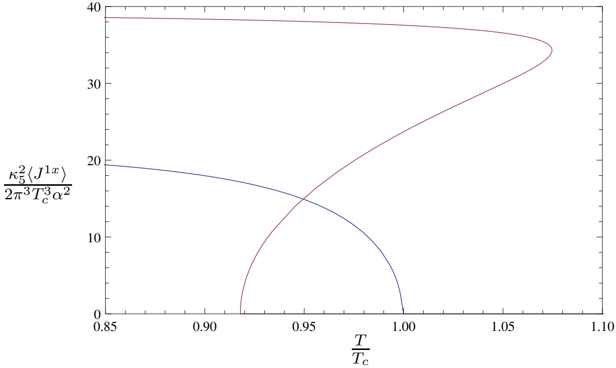
between the solutions.

# Hairy Black Hole Solution




preferred ground state:
blue: broken phase, $w(r) \neq 0$; white: Reissner-Nordström black hole

# Condensate $\langle \mathcal{J}_1^x \rangle$

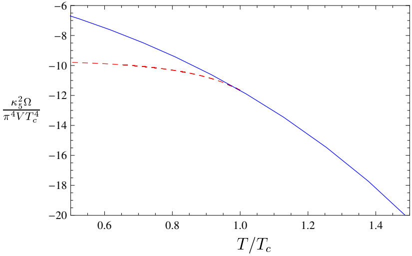


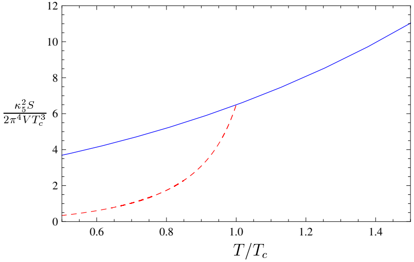

colour coding: $\alpha = 0.316 < \alpha_c$ and $\alpha = 0.447 > \alpha_c$

$\langle \mathcal{J}_1^x \rangle \propto (1 - T/T_c)^{1/2}$ for $\alpha = 0.316$

Ammon, Erdmenger, Grass, Kerner, O'Bannon 2009

# Potential $\Omega$ and Entropy density $s$ for $\alpha = 0.316 < \alpha_c$

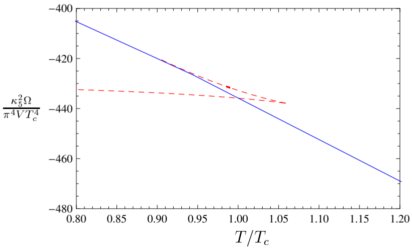


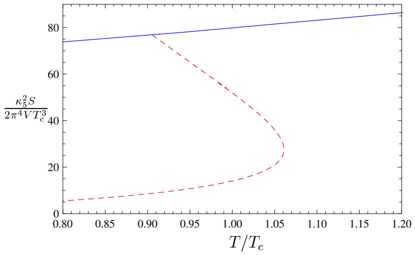




colour coding: Reissner-Nordström solution and $w(r) \neq 0$ solution

Ammon, Erdmenger, Grass, Kerner, O'Bannon 2009

# Potential $\Omega$ and Entropy density $s$ for $\alpha = 0.447 > \alpha_c$




colour coding: Reissner-Nordström solution and $w(r) \neq 0$ solution

Ammon, Erdmenger, Grass, Kerner, O'Bannon 2009

# Interpretation of $\alpha$ on Field Theory Side (rule of thumb)

$$\frac{1}{\kappa_5^2} \propto \text{\# of total degrees of freedom}$$

$$\frac{1}{g_{\mathsf{YM}}^2} \propto \text{\# of charged degrees of freedom}$$

$$\Rightarrow \alpha^2 = \frac{\kappa_5^2}{g_{\mathsf{YM}}^2} \propto \frac{\text{\# of charged degrees of freedom}}{\text{\# of total degrees of freedom}}$$

# Perturbations about the Thermodynamic Equilibrium

small perturbations:

- metric $\hat{g}_{MN} = g_{MN}(r) + h_{MN}(x^\mu, r)$
- gauge field $\hat{A}^a_M = A^a_M(r) + a^a_M(x^\mu, r)$
- $SO(2)$ symmetry in broken phase
  $\Rightarrow$ two distinct momenta needed: $k_\parallel$ and $k_\perp$

# Classification of Perturbations

- simplification: $k_\perp = 0$ (no further symmetry breaking)

$\Rightarrow$ classification under $SO(2)$ rotational symmetry around $x$-axis possible:

| | dynamical fields | constraints | # physical modes |
|---|---|---|---|
| helicity 2 | $h_{yz}, h_{yy} - h_{zz}$ | none | 2 |
| helicity 1 | $h_{ty}, h_{xy}; a^a_y$ | $h_{yr}$ | 4 |
| | $h_{tz}, h_{xz}; a^a_z$ | $h_{zr}$ | 4 |
| helicity 0 | $h_{tt}, h_{xx}, h_{yy} + h_{zz}, h_{xt}; a^a_t, a^a_x$ | $h_{tr}, h_{xr}, h_{rr}; a^a_r$ | 4 |

- gauge choice $h_{Mr} = 0$ and $a^a_r = 0 \Rightarrow$ 14 "physical" modes

- further simplification: $k_{\|} \to 0$ limit

# Today we concentrate on...

| | dynamical fields | constraints | # physical modes |
|---|---|---|---|
| helicity 2 | $h_{yz}, h_{yy} - h_{zz}$ | none | 2 |
| helicity 1 | $h_{ty}, h_{xy}; a^a_y$ | $h_{yr}$ | 4 |
| | $h_{tz}, h_{xz}; a^a_z$ | $h_{zr}$ | 4 |
| helicity 0 | $h_{tt}, h_{xx}, h_{yy} + h_{zz}, h_{xt}$; $a^a_t, a^a_x$ | $h_{tr}, h_{xr}, h_{rr}; a^a_r$ | 4 |

# Spatial Metric Fluctuations $h_{ij}$

- related to field theory correlators

$$G^{ij,kl}(\omega, \vec{k}) = \int \mathrm{d}^4 x \; \mathrm{e}^{-ik_\mu x^\mu} \theta(t) \; \langle [T^{ij}(t, \vec{x}), T^{kl}(0)] \rangle$$

$\Rightarrow$ 3 transport coefficients in agreement with expectations from symmetry

$$h_{yy} - h_{zz},\; h_{yz} \Leftrightarrow \eta_{yz}$$
$$h_{x\perp} \Leftrightarrow \eta_{x\perp}$$
$$h_{xx} - 1/2(h_{yy} + h_{zz}) \Leftrightarrow \lambda$$

# The Viscosity Tensor $\eta_{ijkl}$

- anisotropic systems:
  21 independent components
- isotropic systems:
  2 independent components (1 shear + 1 bulk viscosity)
- transversely isotropic systems ($SO(2)$ rotational symmetry around preferred axis):
  5 independent components (2 shear viscosities)

# Viscosity Tensor and Dissipation

- dissipative part of energy-momentum tensor:

$$\Pi^{ij} = -\eta^{ijkl} u_{kl}$$

Landau, Lifshitz

with $u_{ij} = \frac{1}{2}\left(\nabla_i u_j + \nabla_j u_i\right)$

- transversely isotropic case:

$$\begin{aligned} \eta^{xxxx} &= \zeta_x + \frac{4}{3}\lambda\,, & \eta^{yyyy} &= \eta^{zzzz} = \zeta_y + \frac{\lambda}{3} + \eta_{yz}\,, \\ \eta^{xxyy} &= \eta^{xxzz} = -\frac{2}{3}\lambda\,, & \eta^{yyzz} &= \zeta_y + \frac{\lambda}{3} - \eta_{yz}\,, \\ \eta^{yzyz} &= \eta_{yz}\,, & \eta^{xyxy} &= \eta^{xzxz} = \eta_{x\perp}\,. \end{aligned}$$

- conformal fluid ($\Pi^i{}_i = 0$) $\Rightarrow \zeta_x = 0 = \zeta_y$

$\Rightarrow$ 3 coefficients left: $\eta_{yz}$, $\eta_{x\perp}$ and $\lambda$

# Off-Diagonal Metric Perturbations

- perturbation of action:

$$\delta\mathcal{L} \sim T^{xy} h_{xy} + T^{xz} h_{xz} + T^{yz} h_{yz} + \dots,$$

  $h_{ij}$ small perturbations
  $T^{ij} = T^{ij}_{\text{contact}} + \Pi^{ij}$

- transversely isotropic case:

$$\Pi_{xy} = \Pi_{xz} \sim \eta_{x\perp} h_{x\perp}$$
$$\Pi_{yz} \sim \eta_{yz} h_{yz}$$

- isotropic case $\eta_{x\perp} = \eta_{\text{iso}} = \eta_{yz}$

# Isotropic Holographic Systems

- isotropic holographic duals to Einstein gravity (also at finite chemical potential):

$$\frac{\eta}{s} = \frac{1}{4\pi},$$

  in units $\hbar = 1 = k_B$
- very good agreement with measurements of quark-gluon plasma
- conjectured to be a lower bound for substances found in nature

Kovtun, Son, Starinets 2004

- What happens if we break the rotational symmetry?

# Hydrodynamics from $h_{yz}$ (Helicity 2 mode)

- $T_{yz} = -(T_{yy}^{\text{eq}} + \mathrm{i}\omega\eta_{yz})h_{yz}$

- Kubo formula:

$$\eta_{yz} = -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}(G_{yz,yz})$$

- from numerics:

$$\frac{\eta_{yz}}{s} = \frac{1}{4\pi} \pm 0.5\%$$

- $h_{yz}$ minimally coupled scalar $\Rightarrow$ result expected

Kovtun, Son, Starinets 2004; Iqbal, Liu 2008;...

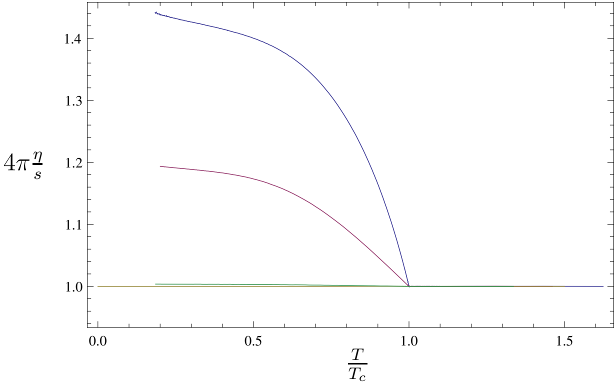
# Hydrodynamics from $h_{x\perp}$ (Helicity 1 Mode)

- $T_{x\perp} = -(T_{xx}^{\text{eq}} + \mathrm{i}\omega\eta_{x\perp})h_{x\perp}$
- Kubo formula:

$$\eta_{x\perp} = -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\left(G_{x\perp,x\perp}\right)$$

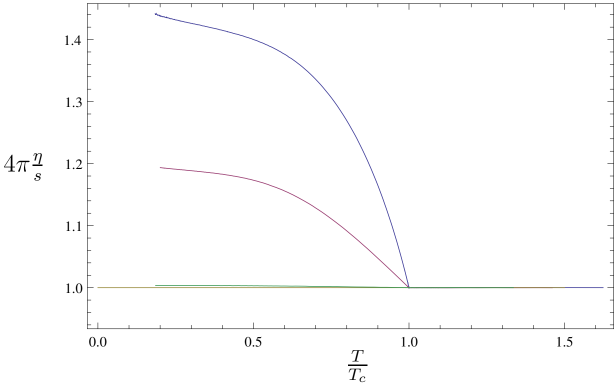
- $h_{x\perp}$ not a minimally coupled scalar
- coupling to gauge field perturbations, therefore $\eta_{x\perp}$ not universal

# Non-universal $\frac{\eta_{x\perp}}{s}$ for $\alpha < \alpha_c$




colour coding: $\frac{\eta_{yz}}{s} = \frac{1}{4\pi}$ and $\frac{\eta_{x\perp}}{s}$: $\alpha_c > \alpha_1 > \alpha_2 > \alpha_3$
at 0th order in large $N_c$ and large 't Hooft coupling corrections

# Non-universal $\frac{\eta_{x\perp}}{s}$ for $\alpha < \alpha_c$




$$1 - 4\pi\frac{\eta_{x\perp}}{s} \propto \left(1 - \frac{T}{T_c}\right)^{\beta} \quad \text{with} \quad \beta = 1.00 \pm 3\%$$

confirmed analytically by Basu, Oh 2011

# Other examples of deviations of $\eta/s$ from "universality":

- corrections in 1/'t Hooft coupling

  Buchel, Liu, Starinets 2004; Benincasa, Buchel 2005; Buchel 2008

- corrections in $1/N_c$

  Kats, Petrov 2007; Buchel, Myers, Sinha 2008

- Gauss-Bonnet Gravity and causality arguments
  $\Rightarrow$ new bound below $1/4\pi$

  Brigante, Liu, Myers, Shenker, Yaida 2008

- explicit symmetry breaking in Axion-Dilaton Gravity
  $\Rightarrow$ values below $1/4\pi$ possible

  Rheban, Steineder 2011

# Diagonal Metric Perturbations

- perturbation of action:

$$\delta\mathcal{L} \sim T^{xx}h_{xx} + T^{yy}h_{yy} + T^{zz}h_{zz} + \ldots,$$

with $h_{ij}$ small perturbations

- rewriting of these perturbations leads to (only dissipative part)

$$\begin{aligned} &= \frac{1}{2}\left(\Pi^{xx} - (\Pi^{yy} + \Pi^{zz})\right)\left(h_{xx} - \frac{1}{2}(h_{yy} + h_{zz})\right) \\ &+ \frac{1}{2}\left(\Pi^{xx} + \Pi^{yy} + \Pi^{zz}\right)\left(h_{xx} + \frac{1}{2}(h_{yy} + h_{zz})\right) \\ &+ \frac{1}{2}\left(\Pi^{yy} - \Pi^{zz}\right)(h_{yy} - h_{zz}) + \ldots \end{aligned}$$

# Transversely Isotropic vs. Isotropic Case

- (conformal) transversely isotropic case:

$$
\begin{aligned}
&(\Pi_{xx} - (\Pi_{yy} + \Pi_{zz})) \sim \lambda \, (h_{xx} - 1/2 \, (h_{yy} + h_{zz})) \\
&(\Pi_{xx} + \Pi_{yy} + \Pi_{zz}) = 0 \\
&(\Pi_{yy} - \Pi_{zz}) \sim \eta_{yz} \, (h_{yy} - h_{zz})
\end{aligned}
$$

- (conformal) isotropic case:

$$
\begin{aligned}
&(\Pi_{xx} - (\Pi_{yy} + \Pi_{zz})) \sim \eta_{\text{iso}} \, (h_{xx} - 1/2 \, (h_{yy} + h_{zz})) \\
&(\Pi_{xx} + \Pi_{yy} + \Pi_{zz}) = 0 \\
&(\Pi_{yy} - \Pi_{zz}) \sim \eta_{\text{iso}} \, (h_{yy} - h_{zz})
\end{aligned}
$$

- phase transition from transversely isotropic to isotropic system:

$$\lambda \to \eta_{\text{iso}}$$

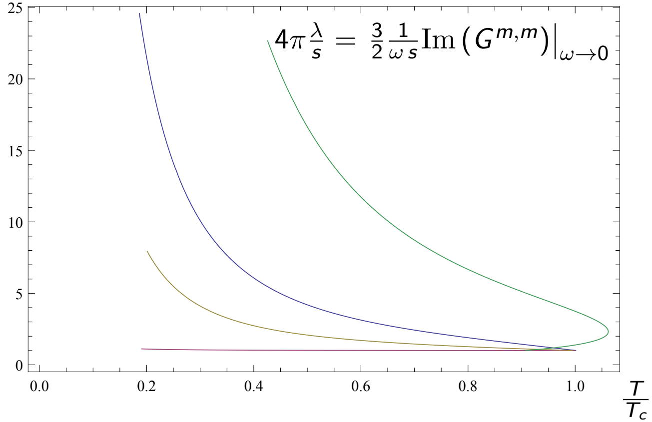
# Hydrodynamics from $h_{xx} - 1/2\,(h_{yy} + h_{zz})$ (Helicity 0 Mode)

- $T_{xx} - (T_{yy} + T_{zz}) = \left(\text{contact terms} + \mathrm{i}\frac{2\omega}{3}\lambda\right)\left(h_{xx} - \frac{1}{2}\,(h_{yy} + h_{zz})\right)$
- $h_{xx} - 1/2\,(h_{yy} + h_{zz})$ corresponds to gauge inv. field at the boundary $r = r_{\text{bdy}}$
- Kubo formula:

$$\lambda = \lim_{\omega \to 0} \frac{3}{2\omega} \mathrm{Im}\,(G^{m,m})$$

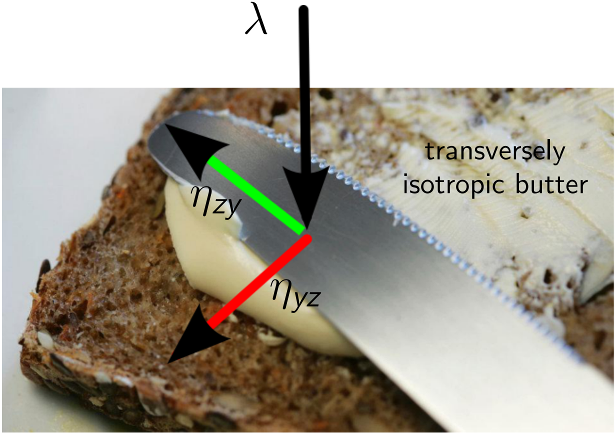
- $\lambda$ is normalised to match $\eta_{\text{iso}}$ at $T_c$

# Viscosity Tensor Component $\frac{\lambda}{s}$




colour coding: $\alpha_1 > \alpha_c > \alpha_2 > \alpha_3 > \alpha_4$

# Crude picture of viscosity tensor components




For $\eta_{xy}$ we would have to turn the knife!

# Optical Conductivities

- gauge field fluctuations $a_i^3$
- related to optical conductivity
- coupled to $h_{ti}$, related to heat conductivity
- $i = \perp$ helicity 1 mode
  $i = x$ helicity 0 mode

# Optical Conductivity and Thermoelectric Effect

- field theory (Thermoelectric effect):

$$\begin{pmatrix} \langle J^i \rangle \\ \langle Q^i \rangle \end{pmatrix} = \begin{pmatrix} \sigma^{ii} & T\alpha^{ii} \\ T\alpha^{ii} & T\bar{\kappa}^{ii} \end{pmatrix} \begin{pmatrix} E_i \\ -(\nabla_i T)/T \end{pmatrix}$$

with $\langle Q^i \rangle = \langle T^{ti} \rangle - \mu \langle J^i \rangle$

- identification of external fields with fluctuations

$$E_i \sim \text{linear combination of } h_{ti} \text{ and } a_i^3$$

$$\frac{\nabla_i T}{T} \sim h_{ti}$$

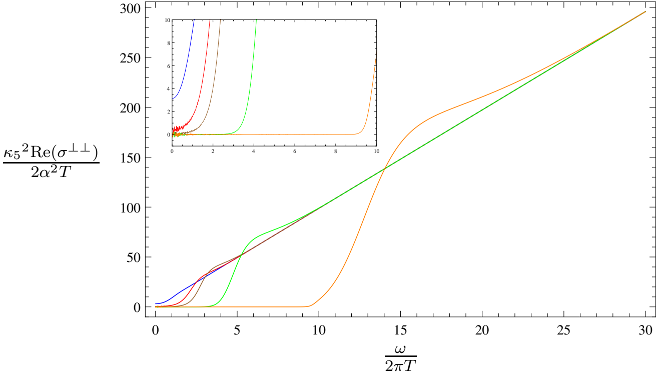
e.g. Hartnoll 2009

- relation between conductivity and Green's function

$$\sigma^{ii} = -\frac{\mathrm{i} G_{3,3}^{i,i}}{\omega} \, ,$$

$T\alpha^{ii}$ and $T\bar{\kappa}^{ii}$ determined by $\sigma^{ii}$

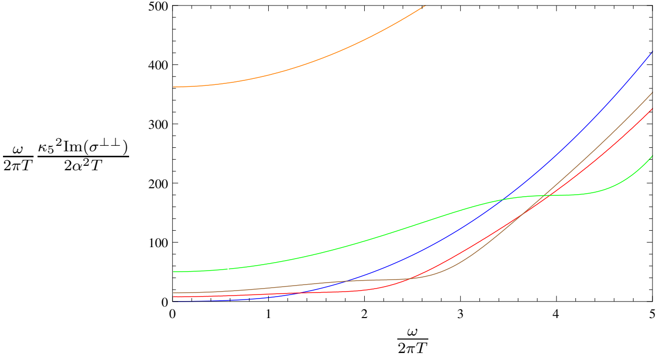
# Optical Conductivity $\mathrm{Re}(\sigma^{\perp\perp})$ at $\alpha < \alpha_c$




colour coding:

$T = \infty > T = 1.00\,T_c >$
$T = 0.88\,T_c > T = 0.50\,T_c > T = 0.19\,T_c$

# $\omega \mathrm{Im}(\sigma^{\perp\perp})$ at $\alpha < \alpha_c$

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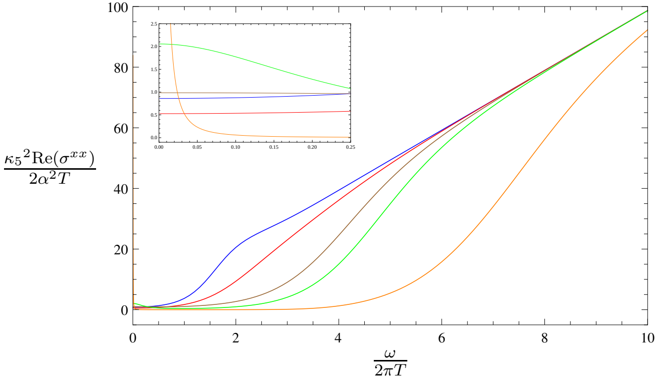
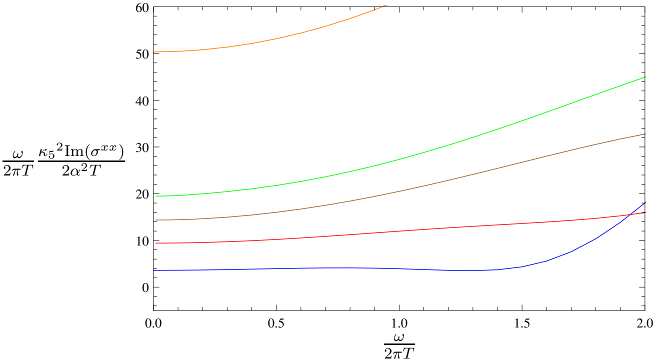
# Optical Conductivity $\mathrm{Re}(\sigma^{xx})$ at $\alpha < \alpha_c$




colour coding:
$T = 1.63\,T_c > T = 0.98\,T_c >$
$T = 0.88\,T_c > T = 0.78\,T_c > T = 0.50\,T_c$

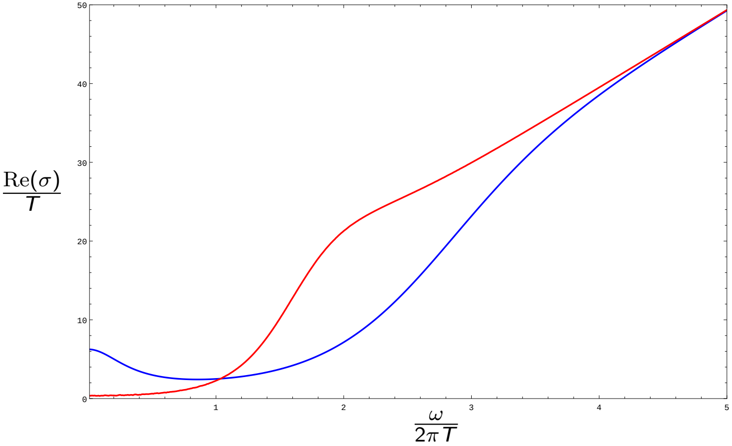
# $\omega \mathrm{Im}(\sigma^{xx})$ at $\alpha < \alpha_c$




colour coding:

$T = 1.63\,T_c > T = 0.98\,T_c >$
$T = 0.88\,T_c > T = 0.78\,T_c > T = 0.50\,T_c$

# Comparison between $\mathrm{Re}(\sigma^{xx})$ and $\mathrm{Re}(\sigma^{\perp\perp})$




$\mathrm{Re}(\sigma^{xx})$ parallel and $\mathrm{Re}(\sigma^{\perp\perp})$ perpendicular to condensate

# Comparison to real system

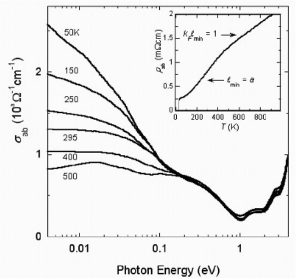



FIG. 25. $La_{1.9}Sr_{0.1}CuO_4$ at specified temperatures. The inset shows the in-plane resistivity data for $La_{1.9}Sr_{0.1}CuO_4$ up to 1000 K. From Hussey *et al.*, 2004.

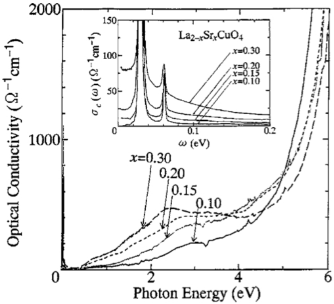
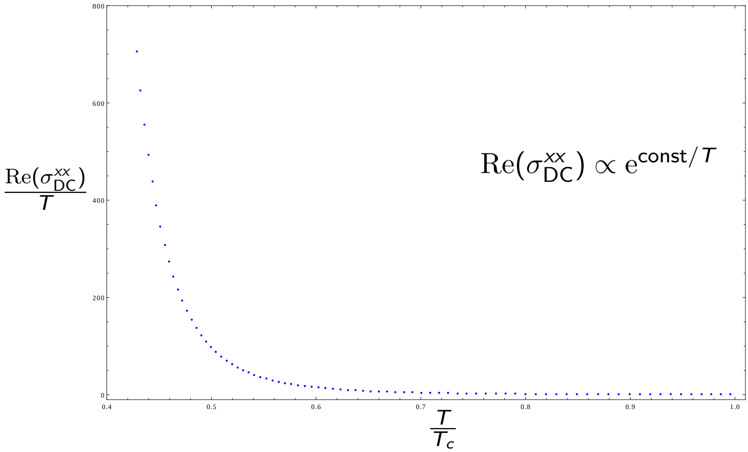



FIG. 11. Evolution of the *c*-axis optical conductivity spectra below 6 eV for $La_{2-x}Sr_xCuO_4$. The inset displays infrared region in more detail. From Uchida *et al.*, 1996.

Basov, Timusk 2005

# $\mathrm{Re}(\sigma^{xx}_{\mathrm{DC}})$




- there is a $\delta(\omega)$-term not considered here
- $\mathrm{Re}(\sigma^{\perp\perp}_{\mathrm{DC}}) \propto \mathrm{e}^{-\mathrm{const}/T}$ (again not considering $\delta(\omega)$)

# Comments on DC Conductivities

- both have $\delta(\omega)$-terms also in AdS RN phase due to momentum conservation

  e.g. Hartnoll, Herzog, Horowitz 2008

- both get (different!) contributions to $\delta(\omega)$-terms from superfluid density in broken phase

- finite part of $\mathrm{Re}(\sigma_{\mathrm{DC}})$ and low freq. behaviour:

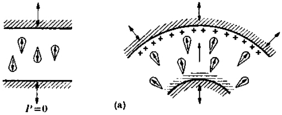
  perpendicular to condensate: exponential suppression with decreasing temperature (similar to s-wave case, see HHH 08)

  parallel to condensate: broad Drude peak and
  DC cond. exponentially "enhanced"
  (similar to "metallic" phase of Donos, Hartnoll 2012)

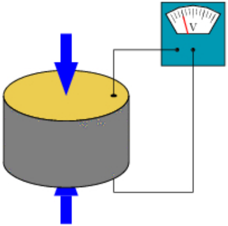
# Other Effects due to Broken Rot. Sym.

- helicity 1: coupling between $a_\perp^+$, $a_\perp^-$ and $h_{x\perp}$
  helicity 0: coupling between $\Phi_+ \, , \Phi_-$ and $\Phi_m$

- helicity 1 case resembles flexoelectric effect
  helicity 0 case resembles piezoelectric effect



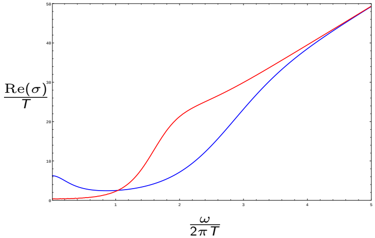

de Gennes

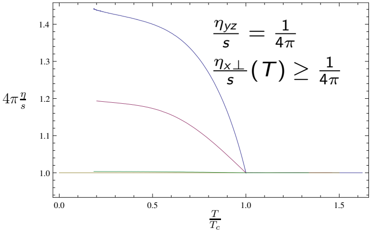



Wikipedia

# Conclusion

- hol. backreacted p-wave superfluid is a nice toy model modelling many different effects
- non universal behaviour of shear viscosity
  two different optical conductivities




- other effects:
  thermoelectric, flexoelectric and piezoelectric effect

# Future Directions

- What happens if we turn on finite momentum? (instabilities $\rightarrow$ spatially modulated phase)
- How does the far from equilibrium response (quantum quenching of order parameter) change due to the rot. sym. breaking?

  Bhaseen, Gauntlett, Simons, Sonner, Wiseman 2012
- Construct constitutive relations using the fluid/gravity duality.

Thank you!

# The "Physical" Modes – I

- 2 helicity 2 modes: $h_{yz}$ and $h_{yy} - h_{zz}$
- equations of motion of minimally coupled scalar in $AdS_5$
- correlators are related to the viscosity tensor component $\eta_{yz}$

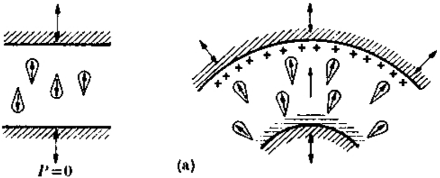
# The "Physical" Modes – II

- 4 physical helicity 1 modes decoupling into 2 blocks: $h_{x\perp}$, $a_\perp^+$, $a_\perp^-$ and $a_\perp^3$
- $h_{x\perp}$ correlator related to viscosity tensor component $\eta_{x\perp}$
- $a_\perp^3$ related to optical conductivity $\sigma^{\perp\perp}$ perpendicular to condensate

# The "Physical" Modes – III

- 4 physical helicity 0 modes decoupling into 2 blocks: $\Phi_{+,-,m}$ and $\Phi_4$
- $\Phi_+ = \Phi_+(a_x^+, h_{yy})$, $\Phi_- = \Phi_-(a_x^-, a_t^3, h_{tt}, h_{yy})$ and $\Phi_m = \Phi_m(h_{xx}, h_{yy})$
- $\Phi_m$ is related to a third component of shear viscosity tensor $\lambda$
- $\Phi_4 = \Phi_4(a_x^3, a_t^1, a_t^2, h_{tx})$ related to optical conductivity $\sigma^{xx}$ in direction of condensate

# Flexoelectric Effect



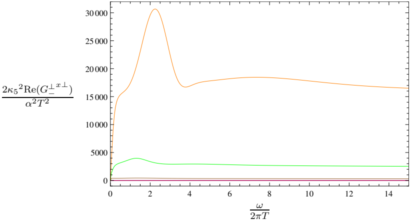

de Gennes

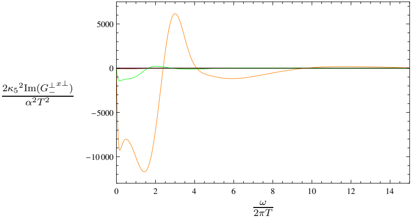
- strain leads to effective polarisation
- electric field leads to stress
- our system: coupling between flavour fields and strain

# $G_{-}^{\perp x\perp}$ for $\alpha = 0.316$

$$G_{\text{hel1}} = \begin{pmatrix} G_{+,+}^{\perp,\perp} & G_{+,-}^{\perp,\perp} & G_{+}^{\perp x\perp} \\ G_{-,+}^{\perp,\perp} & G_{-,-}^{\perp,\perp} & G_{-}^{\perp x\perp} \\ G^{x\perp}{}_{+}^{\perp} & G^{x\perp}{}_{-}^{\perp} & -T_{xx}^{\text{eq}} - \mathrm{i}\omega\eta_{x\perp} \end{pmatrix}$$

# $G_{-}^{\perp x \perp}$ for $\alpha = 0.316$

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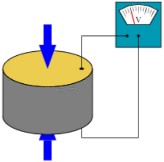
$T = 1.00\,T_c > T = 0.88\,T_c > T = 0.50\,T_c$

# Piezoelectric Effect

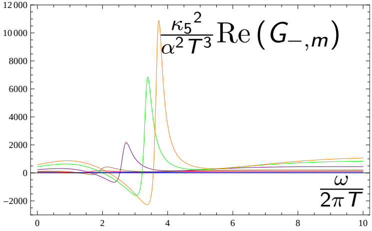


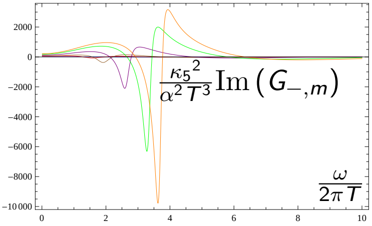

- diagonal strain leads to electric field
- electric field leads to diagonal stress
- our system: coupling between flavour fields and diagonal strain

# $G_{-,m}$ for $\alpha = 0.316$

$$G_{\text{hel0}} = \begin{pmatrix} G_{+,+} & G_{+,-} & G_{+,m} \\ G_{-,+} & G_{-,-} & G_{-,m} \\ G_{m,+} & G_{m,-} & G_{m,m} \end{pmatrix}$$

# $G_{-,m}$ for $\alpha = 0.316$




colour coding:

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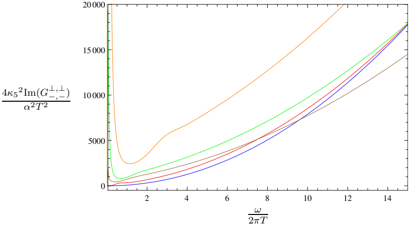
# Flavour Currents

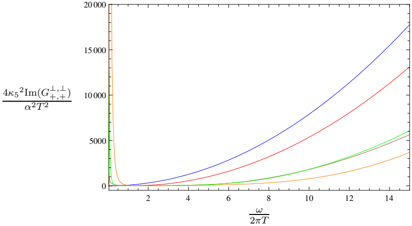
- helicity 1: coupling between $a_\perp^+$ and $a_\perp^-$
- helicity 0: coupling between $\Phi_+$ and $\Phi_-$

# $\mathrm{Im}(G^{\perp,\perp}_{\mp,\mp})$ for $\alpha = 0.316$

$$\begin{pmatrix} G^{\perp,\perp}_{+,+} & G^{\perp,\perp}_{+,-} & G^{\perp\,x\perp}_{+} \\ G^{\perp,\perp}_{-,+} & G^{\perp,\perp}_{-,-} & G^{\perp\,x\perp}_{-} \\ G^{x\perp\,\perp}_{+} & G^{x\perp\,\perp}_{-} & -\langle T_{xx}\rangle - \mathrm{i}\omega\eta_{x\perp} \end{pmatrix}$$

# $\mathrm{Im}(G^{\perp,\perp}_{\mp,\mp})$ for $\alpha = 0.316$

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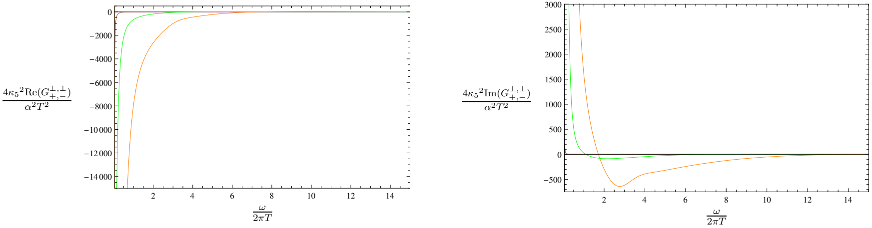
$T = 1.00\, T_c$ > $T = 0.88\, T_c$ > $T = 0.50\, T_c$

# $G_{+,-}^{\perp,\perp}$ for $\alpha = 0.316$

$$\begin{pmatrix} G_{+,+}^{\perp,\perp} & G_{+,-}^{\perp,\perp} & G_{+}^{\perp}{}^{x\perp} \\ G_{-,+}^{\perp,\perp} & G_{-,-}^{\perp,\perp} & G_{-}^{\perp}{}^{x\perp} \\ G^{x\perp}{}_{+}^{\perp} & G^{x\perp}{}_{-}^{\perp} & -\langle T_{xx}\rangle - \mathrm{i}\omega\eta_{x\perp} \end{pmatrix}$$

# $G_{+,-}^{\perp,\perp}$ for $\alpha = 0.316$

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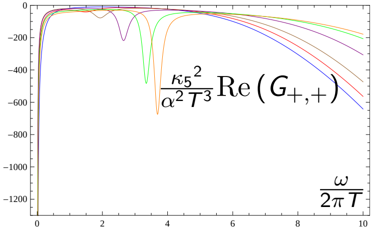
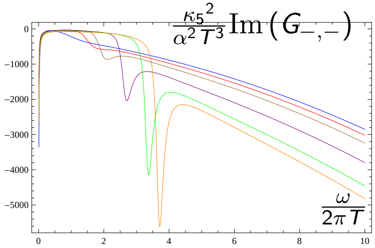
# $\mathrm{Im}(G_{\mp,\mp})$ for $\alpha = 0.316$

$$\begin{pmatrix} G_{+,+} & G_{+,-} & G_{+,m} \\ G_{-,+} & G_{-,-} & G_{-,m} \\ G_{m,+} & G_{m,-} & G_{m,m} \end{pmatrix}$$

# $\mathrm{Im}(G_{\mp,\mp})$ for $\alpha = 0.316$

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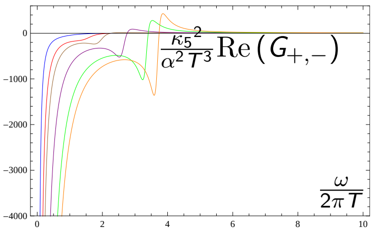
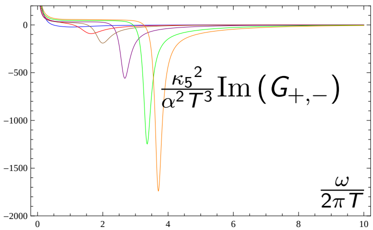
$T = 0.50\,T_c > T = 0.46\,T_c$

# $G_{+,-}$ for $\alpha = 0.316$

$$\begin{pmatrix} G_{+,+} & G_{+,-} & G_{+,m} \\ G_{-,+} & G_{-,-} & G_{-,m} \\ G_{m,+} & G_{m,-} & G_{m,m} \end{pmatrix}$$

# $G_{+,-}$ for $\alpha = 0.316$

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