Dressing the electron star in a holographic superconductor

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November 12th, 2013

F. Nitti, G. Policastro, T.V., *JHEP* 1310 (2013) 019, arXiv:1307:4558

Outline

We will present new asymptotically AdS_4 solutions of Einstein-Maxwell gravity at zero temperature and finite chemical potential.

These solutions exhibit both fermionic and bosonic charged degrees of freedom.

- The fermionic degrees of freedom will consist in an **electron star**.
- The bosonic degrees of freedom are simply a **charged scalar field**.
- It will be shown that these solutions are **favoured** when they exist.

Plan

1. Systems of fermions : the strong coupling issue
   - Conventional weak coupling analysis
   - Landau's Fermi liquid theory
   - Non-Fermi liquids
   - High-Tc superconductivity
2. Holography and charged matter in *AdS*
   - The gauge-gravity correspondence
   - Charged asymptotically AdS_4 solutions
   - Extremal Reissner-Nordstrom-AdS black hole (ERN)
   - Holographic superconductor (HSC)
   - Electron star (ES)
   - Competition between fermionic and bosonic matter ?
   - Instability of the electron star to the formation of a condensate ?
3. Model, new solutions and phase diagrams
   - The model
   - The new solutions
   - Phase diagrams
   - Similar system : potential for the scalar field
4. Conclusions and perspectives

Conventional weak coupling analysis

Compressible systems :

Consider a translationally-invariant system of fermions with a global $U(1)$ symmetry.

Put this system at finite density :

$$H \longrightarrow H - \mu Q$$

μ is the chemical potential and Q the conserved $U(1)$ charge (the electron density).

Q varies smoothly as a function of the parameter μ.

The features of the ground state of most fermionic systems decompose into :

- Phases with unbroken symmetry: described by Landau's Fermi liquid theory, with the important concept of **quasiparticles**
- Phases with broken symmetry: leading to superconductivity/superfluidity

  Landau-Ginzburg paradigm : the symmetry is broken in one of the two phases and the phase transition is described by an order parameter

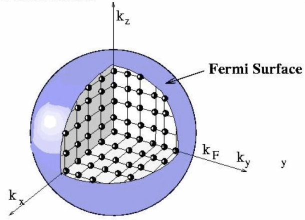
Landau's Fermi liquid theory

Description of low energy physics of gapless systems of fermions at low temperature when the $U(1)$ is unbroken.

The system is adiabatically connected to a gas of free fermions.

Low energy states are filled by the electrons.

The highest energy state filled defines the **Fermi surface**.



Around the Fermi surface, the low energy excitations

- are **gapless**
- have a long lifetime : **quasiparticles** (weakly-coupled dressed electrons)

Resistivity : $\rho = \rho_0 + AT^2$

BCS theory for superconductors is based on Landau's Fermi liquid theory.

Non-Fermi liquids

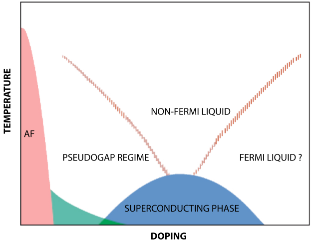
However, in many systems, **strong coupling** between fermions does not allow to apply the standard weak coupling methods (high Tc superconductors, heavy fermions materials. . .).

When the $U(1)$ symmetry is unbroken, these materials are **not described by Fermi liquid theory** : the system is **not** adiabatically connected to the free gas, there is a phase transition between the two phases.

A **non-Fermi liquid** :

- admits a Fermi surface (*i.e.* gapless excitations)
- BUT the gapless excitations are strongly coupled $\Rightarrow$ **no quasiparticles**

High-Tc superconductivity



- Quantum critical region above the superconducting phase : non-Fermi liquid

$$\rho = \rho_0 + AT$$

- The pseudogap regime is not connected adiabatically to the Fermi liquid phase
- The mechanism leading to superconductivity is not well understood (no Cooper pairs as in the weak-coupled BCS theory)

Holographic methods may be able to describe the quantum critical region

The gauge-gravity correspondence

Large N strongly-coupled gauge theory in d dimensions	$\Longleftrightarrow$	Classical gravitational theory on asymptotic AdS_{d+1} space

- Strong-weak duality
- Gravity as a **tool** to compute observables in strongly-coupled field theories
- Gravitational theory is a geometric realization of the RG flow of the field theory
- **Global** symmetries in field theory map to **local** symmetries in the bulk

Field/operator correspondence and correlation functions

- Field/operator correspondence :

Field theory		**Gravity**
Gauge-invariant operator with spin s and conformal dimension Δ	$\Longleftrightarrow$	Classical field with spin s and mass $m(\Delta)$

For a scalar field : $\Delta = \frac{d}{2}(1+\sqrt{1+4{m_s}^2/d^2})$

$$\psi \sim \psi_-\, r^{d-\Delta} + \psi_+\, r^{\Delta}\,, \qquad r \to 0$$

$$\psi_- \propto J\,, \qquad \psi_+ \propto \langle \mathcal{O} \rangle$$

- Equality of partition functions :

$$\boxed{Z_{FT}[J] = Z_{Gravity}[\psi_-]|_{on-shell}}$$

For classical gravity, saddle-point approximation :

$$Z_{Gravity}[\psi_-] = \mathrm{e}^{i\, S_{\mathrm{cl.}}[\psi_-]}$$

Correlation functions are obtained by taking derivatives of the classical gravity action.

Charged asymptotically AdS_4 solutions

We consider solutions of d=4 gravity that

- are asymptotically AdS_4
- have finite electric charge

The action has the form

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{ab} F^{ab} + \mathcal{L}_{\mathrm{matter}} \right] + S_{\mathrm{bdry}}$$

We focus on the homogeneous and isotropic ansatz

$$\mathrm{d}s^2 = L^2 \left[-f(r)\mathrm{d}t^2 + g(r)\mathrm{d}r^2 + \frac{1}{r^2} \left(\mathrm{d}x^2 + \mathrm{d}y^2 \right) \right]$$
$$A = \frac{e}{\kappa} h(r) \mathrm{d}t$$

with the UV asymptotics

$$h \sim \mu - Qr \,, \qquad f \sim \frac{1}{r^2} \,, \qquad g \sim \frac{1}{r^2} \,, \qquad \text{for} \quad r \sim 0$$

Extremal Reissner-Nordstrom-AdS black hole (ERN)

When the matter fields are trivial, the dual field theory is described by RN-AdS black hole, which carries the full charge of the system.

In particular, at zero temperature :

- The metric and gauge field are exactly given by :

$$f(r) = \frac{1}{r^2}\left(1 - Mr^3 + \frac{Q^2}{2}r^4\right), \qquad g(r) = \frac{1}{r^4 f(r)}, \qquad h(r) = \mu - Qr$$

  The solution is fully determined by the charge Q of the black hole :

$$M = \frac{4}{r_+^3}, \qquad Q = \frac{\sqrt{6}}{r_+^2} \qquad \mu = \frac{\sqrt{6}}{r_+}$$

  where r_+ is the event horizon.

- Close to the horizon, the geometry is $AdS_2 \times \mathbb{R}^2$.

This solution has finite entropy, and the dual ground state is **degenerate**.

The holographic superconductor (HSC) (Hartnoll, Herzog and Horowitz, 2008)

Consider Einstein-Maxwell gravity minimally-coupled to a charged free scalar field :

$$S_{\mathrm{matter}} = -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[|\partial_a\psi - iqA_a\psi|^2 + m_s^2|\psi|^2\right]$$

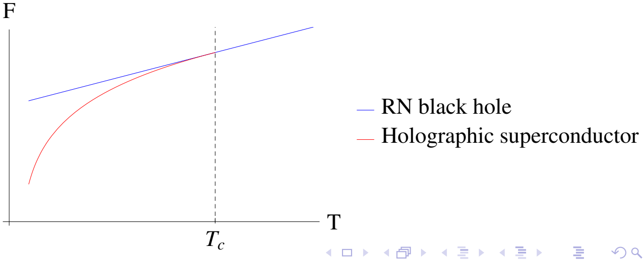
where $m^2_{BF,AdS_4} < m_s^2 < 0$, corresponding to a relevant operator in the dual field theory.

In the probe approximation, ERN is **unstable to the creation of bosonic matter** when the effective scalar mass in the IR is lower than the AdS_2 BF bound : $\frac{m_s^2-q^2}{6} < -\frac{1}{4}$

$\Longrightarrow$ formation of a **scalar hair** for $\boxed{q > q_{st}}$

Backreacted solution :

$\langle\mathcal{O}\rangle \neq 0$ for $T < T_c$



The zero temperature limit (Horowitz and Roberts, 2009)

In particular, at zero temperature, the hairy black hole area shrinks to zero.

The full charge of the system is carried by bulk matter.

- **IR asymptotics** ($r \sim \infty$) :

$$f(r) \sim \frac{1}{r^2} \qquad g(r) \sim -\frac{3}{2m_s^2}\frac{1}{r^2 \log r}$$

$$h(r) \sim h_0\, r^\delta\, (\log r)^{1/2} \qquad \psi(r) \sim 2\,(\log r)^{1/2}$$

This solution exists for $\delta \equiv \frac{1}{2} - \frac{1}{2}\left(1 - \frac{24q^2}{m_s^2}\right)^{1/2} < -1 \iff \boxed{q^2 > -\frac{m_s^2}{3}}$

- **UV asymptotics** ($r \sim 0$) :

The geometry is AdS_4. The IR parameter h_0 is chosen so that the source ψ_- of the dual scalar operator is set to zero, so

$$\psi \sim \psi_+ r^\Delta$$

Electron star

RN-AdS black hole is also **unstable to the creation of fermionic matter**.

Thomas-Fermi approximation : number of fermions is large and the level spacing is zero.

$\Longrightarrow$ Treat the fermions as being in flat space, and only interacting with the gauge field.

Consider Einstein-Maxwell gravity coupled to a perfect fluid of charged fermions :

- Stress-tensor of the fluid : $T_{ab}^{\text{fluid}} = (\rho + p) u_a u_b + p\, g_{ab}$
- Current : $J_a^{\text{fluid}} = \sigma u_a$

Fluid quantities are :

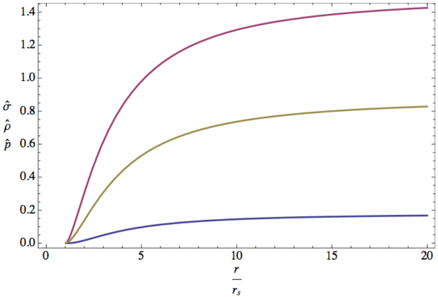
- Local chemical potential : $\mu_l = A_a u^a$
- Energy density : $\rho = \beta \int_{m_f}^{\mu_l} \mathrm{d}\epsilon\, \epsilon^2 \sqrt{\epsilon^2 - m_f^2}$
- Charge density : $\sigma = \beta \int_{m_f}^{\mu_l} \mathrm{d}\epsilon\, \epsilon \sqrt{\epsilon^2 - m_f^2}$
- Pressure : $p = -\rho + \mu_l \sigma$

μ_l can be derived from the coupling of the fluid to the gauge field : $\mathcal{L}_c = A_a \sigma u^a = \mu_l \sigma$

Below a critical temperature, the favoured solution is an electron star located at finite distance from the black hole. A fraction of the total charge is carried by the star (Hartnoll and Petrov, 2010).

Electron star

At zero temperature, the charge is fully carried by the star (Hartnoll and Tavanfar, 2010).



- **IR asymptotics** : Lifshitz geometry (for $\boxed{0 \leq m_f < 1}$ and $\boxed{z \geq \frac{1}{1-m_f^2} \geq 1}$)

$$f(r) \sim \frac{1}{r^{2z}} \qquad g(r) \sim \frac{g_\infty(m_f,\beta)}{r^2} \qquad h(r) \sim \frac{h_\infty(m_f,\beta)}{r^z} \qquad z = z(m_f,\beta)$$

- **Outside the star** : RN-AdS geometry

The electron star is dual to a finite density of fermions interacting with emergent critical bosonic modes in the vicinity of a quantum critical point.

Competition between fermionic and bosonic matter ?

(F. Nitti, G. Policastro, T.V., 2013)

(Y. Liu, K. Schalm, Y.-W. Sun, J. Zaanen, 2013)

- RN-AdS black hole is unstable to the formation of both fermionic and bosonic matter
- So far, only bosonic **or** fermionic charged degrees of freedom have been studied separately in AdS.

Competition between fermionic and bosonic phases ?

The ground state is not always occupied by bosons only :

> For relativistic massless fermions and massive bosons, the ground state is occupied by the fermions for sufficiently small chemical potential.

Instability of the electron star to the formation of a condensate ?

(F. Nitti, G. Policastro, T.V., 2013)

Consider a probe charged free scalar field with $m_s^2 < 0$ above the BF bound on top of the $T = 0$ electron star.

The scalar field behaves in the IR as

$$\psi \sim A_- \, r^{\Delta_{IR}} + A_+ \, r^{(z+2)-\Delta_{IR}} \qquad r \sim \infty$$

$$\Delta_{IR} = \frac{1}{2}\left[(z+2) - 2\sqrt{g_\infty}\sqrt{m_s^2 - m_c^2}\right]$$

The electron star is **unstable to the formation of scalar hair** for large enough charge q :

$$m_s^2 < m_c^2 \,, \qquad m_c^2 \equiv -\frac{(z+2)^2}{4g_\infty} + h_\infty^2 q^2$$

The model

We propose a model involving both fermionic and bosonic charged matter at zero temperature :

$$S_{\mathrm{matter}} = -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[|\partial_a\psi - iqA_a\psi|^2 + m_s^2|\psi|^2\right] + S_{\mathrm{fluid}}$$

The perfect fluid of fermionic particles contributes to Einstein and Maxwell equations through its stress tensor and electromagnetic current, respectively.

Parameters of the theory : scalar: (m_s^2, q) fluid: (m_f, β)

We restrict the analysis to the case :

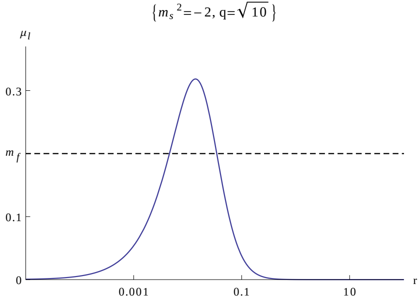
$$q^2 > -\frac{m_s^2}{3}\,, \qquad 0 \leq m_f < 1\,, \qquad z(\beta, m_f) \geq \frac{1}{1-m_f^2} \geq 1\,, \qquad \beta = 1$$

The model admits as solutions :

- **ERN** : the extremal Reissner-Nordstrom black hole
- **HSC** : the HR holographic superconductor
- **ES** : the electron star
- *A new solution* : the **compact star** (**CS**)

Constructing the new solutions

In the HSC solution, μ_l vanishes asymptotically both in the UV and the IR, and has a maximum value $\mu_{\max}(m_s, q)$.



Charge density of the fluid :

$$\sigma = \beta \int_{m_f}^{\mu_l} \mathrm{d}\epsilon\, \epsilon \sqrt{\epsilon^2 - m_f^2}$$

Starting from HR asymptotic solution in the IR, one can start filling the energy levels of the fluid star when $\mu_l(r) = m_f$.

A compact star (CS) may form for $\mu_{\max}(r) > m_f$.

Above a critical charge q_c, the star cannot form.

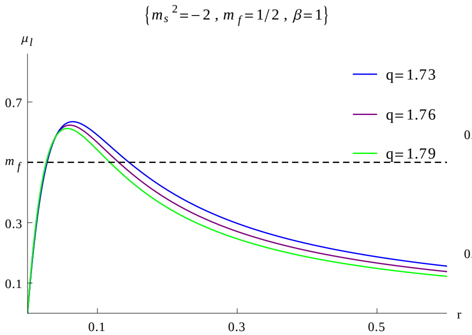
Profile of the new solutions

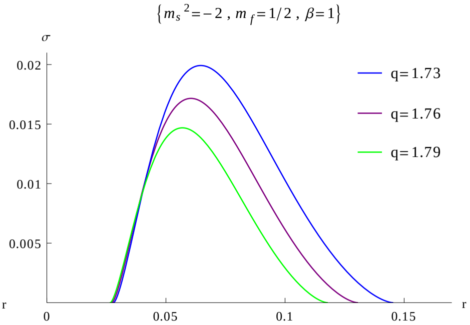
We shoot from the HR solution in the IR :

$$f(r) \sim \frac{1}{r^2} \qquad g(r) \sim -\frac{3}{2m_s^2}\frac{1}{r^2 \log r}$$

$$h(r) \sim h_0\, r^\delta\, (\log r)^{1/2} \qquad \psi(r) \sim 2\,(\log r)^{1/2}$$

to the UV, and fix the free parameter h_0 such that $\psi_- = 0$.

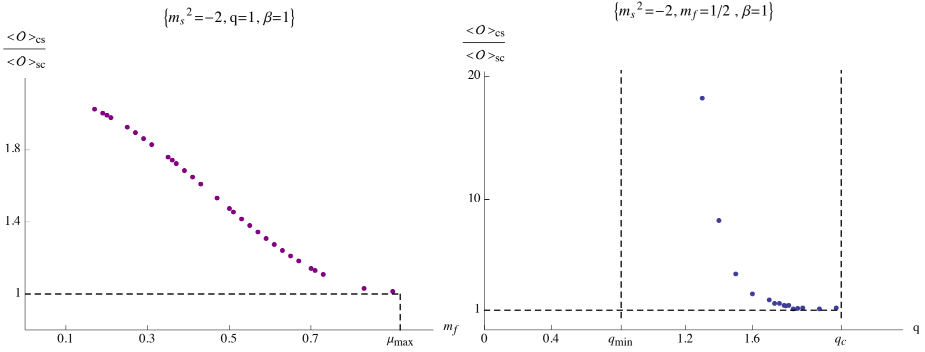




Charge and condensate

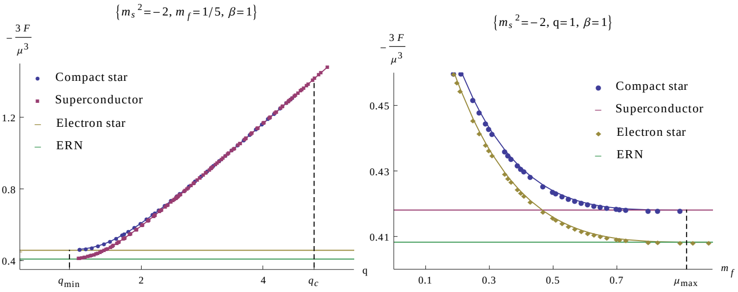
In these solutions, the electric flux vanishes at infinity in the IR.
The charge is then shared between the fluid and the scalar field.

The presence of the star increases the consensate of the dual scalar operator :



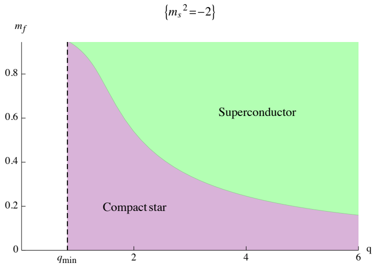
Phase diagrams

We plot the quantitiy $-\frac{3F}{\mu^3}$ because it is dimensionless.



- The compact star solution is favoured when it exists.
- Continuous phase transition between HSC and CS as functions of q and m_f
- At small q, our numerical analysis does not permit to say whether there is a phase transition or not.

Phase diagrams



The resulting phase diagram of the CS/HSC transition

Similar system : potential for the scalar field

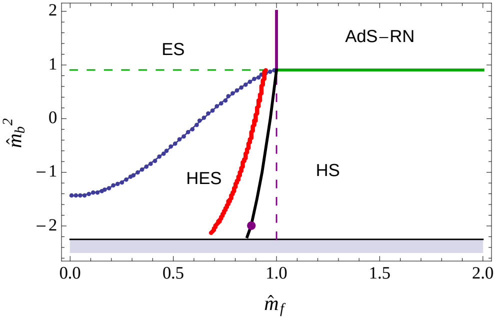
(Y. Liu, K. Schalm, Y.-W. Sun, J. Zaanen, 2013)

One can consider the same Einstein-Maxwell system coupled to a perfect fluid of charged fermions and a charged scalar field, but with a **quartic potential** for the scalar :

$$V(\psi) = \frac{u}{4L^2}\left(|\psi|^2 + \frac{m_s^2 L^2}{u}\right)^2 - \frac{m_b^4 L^2}{4u}$$

Five possible solutions :

- ERN
- ES
- Holographic superconductor
- Hairy ES with one edge
- Hairy ES with two edges



The four last solutions have Lifshitz geometry in the IR.

Conclusions and perspectives

- ERN-AdS black hole is unstable to the creation of bosonic and fermionic matter, and the presence of both is favoured when it is possible.
- The scalar field carries a fraction of the total charge.
  The total charge should not be given by the volume enclosed by the Fermi surface.
- The $U(1)$ symmetry is broken by the bosonic degrees of freedom.
  We should compute the Fermi surface. Gap in energy of the low energy excitations ?
- No known HSC solution with tunable parameter for $q < q_{min}$.
- Fermions and bosons may interact directly : to keep the fluid approximation, one can introduce a current-current coupling.

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Thank you !

Lagrangian description of a non-rotating fluid at zero temperature

$$\mathcal{L}_{\mathrm{fluid}} = -\rho(\sigma) + \sigma u^a(\partial_a \phi + A_a) + \lambda(u^a u_a + 1)$$

Equations of motion :

$$\delta\lambda : \; u^a u_a = 1$$

$$\delta\sigma : \; \rho'(\sigma) = u^a(\partial_a \phi + A_a)$$

$$\delta\phi : \; \nabla_a(\sigma u^a) = 0$$

$$\delta u^a : \; \sigma(\partial_a \phi + A_a) + 2\lambda u_a = 0$$

$$\delta g_{ab} : \; T^{\mathrm{fluid}}_{ab} = (\rho + p) u^a u^b + p g^{ab}$$

Define :

$$\mu_l(\sigma) \equiv \rho'(\sigma) = u^a(\partial_a \phi + A_a)$$

$$p(\sigma) \equiv -\rho(\sigma) + \mu_l(\sigma)\sigma$$

$$J^a \equiv \sigma u^a$$

We choose the gauge $\phi = 0$.

Fluid quantities are functions of the only parameter μ_l, itself given by the gauge field.

One can show that the on-shell action is $\mathcal{L}^{\mathrm{on-shell}}_{\mathrm{fluid}} = p$.