Cold holographic matter and a color group-breaking instability

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Take home message

⇒ Take (super)Yang–Mills theories at strong coupling and large N

⇒ Add fundamental matter (quarks) at finite charge density

⇒ Then...

...the simplest$^{(*)}$ ground states are described in the IR by a non-relativistic field theory with specific scaling properties...

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  ...and they are probably unstable towards color-superconducting phases

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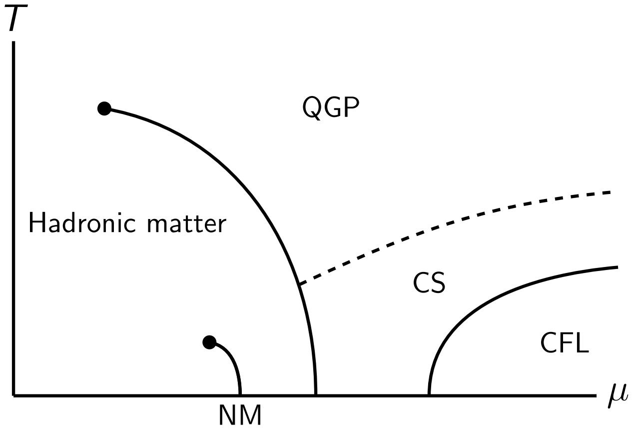
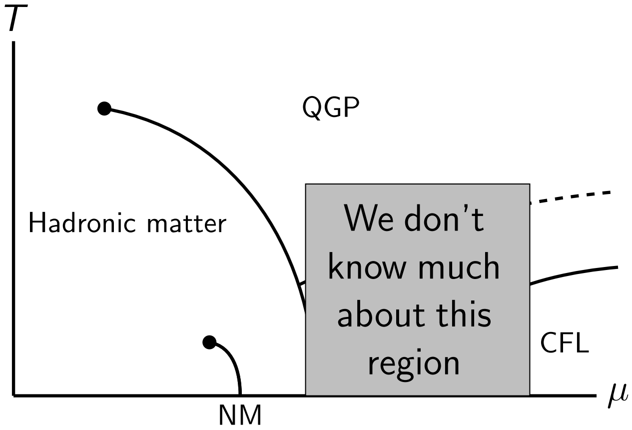
A cartoon of the QCD phase diagram



A cartoon of the QCD phase diagram



Interest and use of holography

⇒ Why care? Color-superconductivity phases and transitions; neutron stars...

⇒ Region of strong coupling suggests a holographic approach, but no QCD dual, we take $\mathcal{N} = 4$SYM as ballpark

⇒ Try to extract qualitative lessons of the effects of the chemical potential in strongly coupled systems

- What is the equation of state?
- How to observe color superconductivity?

Strings ho!

I describe results from top-down models, where we extremize type II SUGRA, DBI+WZ and NG actions.

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⇒ Take the geometry sourced by a set of Dp-branes on (flat) spacetime

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⇒ Take the geometry sourced by a set of Dp-branes on (flat) spacetime

⇒ Backreact Dq-branes with gauge field on the w.v. turned on

⇒ Then...

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Take home message

- $\Rightarrow$ Take the geometry sourced by a set of Dp-branes on (flat) spacetime
- $\Rightarrow$ Backreact Dq-branes with gauge field on the w.v. turned on
- $\Rightarrow$ Then...

  ...the simplest$^{(*)}$ supergravity solutions at $T = 0$ are described in the IR by a Hyperscaling-Violating Lifshitz spacetime...

  ...and they seem to be unstable towards Higgs-branch phases

In this talk I will have the gauge/gravity duality in mind

(non-)AdS spacetime	$\longleftrightarrow$	(non-)conformal theory
Domain wall solution	$\longleftrightarrow$	Renormalization group flow
Boundary	$\longleftrightarrow$	UV (high energies)
Origin	$\longleftrightarrow$	IR (low energies)

System action

$$
\begin{aligned}
S = {} & \frac{1}{2\kappa^2}\int e^{-2\phi}\left(R * 1 - \frac{1}{2}H \wedge *H - \frac{1}{2}\mathrm{d}\phi \wedge *\mathrm{d}\phi\right) \\
& - \frac{1}{2\kappa^2}\int \frac{1}{2}F_1 \wedge *F_1 + \frac{1}{2}F_3 \wedge *F_3 + \frac{1}{4}F_5 \wedge *F_5 \\
& - \frac{1}{2\kappa^2}\int \frac{1}{2}C_4 \wedge H \wedge F_3 \\
& - N_f\, T_7 \int \mathrm{d}^8 x\, e^{-\phi}\sqrt{-|\hat{G} + \mathrm{d}A + \hat{B}|} + N_f\, T_7 \int e^{\mathrm{d}A + \hat{B}}\, \hat{C}_q
\end{aligned}
$$

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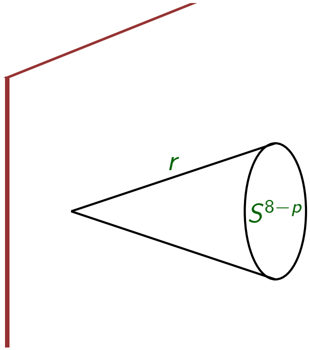
SYM theories from type II SUGRA



$SU(N)$ SYM with 16 supercharges

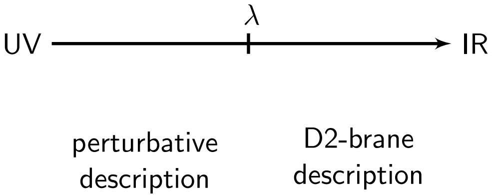
$$\mathrm{d}s^2 = h^{-1/2}\,\mathrm{d}x_{1,p}^2 + h^{1/2}\,\left(\mathrm{d}r^2 + r^2\mathrm{d}\Omega_{8-p}^2\right)$$

$$\int_{S^{8-p}} *F_{8-p} \sim N$$

SYM theories from type II SUGRA [Itzhaki et al '98]

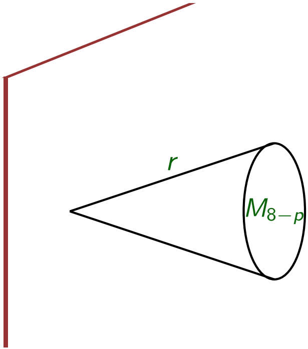
$\Rightarrow$ For D3-branes one has $\mathcal{N} = 4$ SYM, conformal

$\Rightarrow$ For D2-branes λ is dimensionful and there is a running



$\Rightarrow$ The holographic radius is related to the energy scale $r = E\,\ell_S^2$.

SYM theories from type II SUGRA



SYM with other gauge group
and less supercharges

$$\mathrm{d}s^2 = h^{-1/2}\,\mathrm{d}x_{1,p}^2 + h^{1/2}\,\left(\mathrm{d}r^2 + r^2\mathrm{d}\Sigma_{8-p}^2\right)$$

$$\int_{M_{8-p}} *F_{8-p} \sim N$$

SYM theories from type II SUGRA

To preserve $\mathcal{N} = 1$ supersymmetry the internal manifold must admit one Killing spinor. This constrains the possible choices. From now on

	dim	base	cone
$D3$-branes	5	Sasaki-Einstein	Calabi-Yau
$D2$-branes	6	nearly Kähler	G_2-cone

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Fundamental matter [Karch and Katz '03]

	x^μ	r	$\theta^{1,2,3}$	$\theta^{4,\cdots,8-p}$
Dp	$\times$	$-$	$-$	$-$
D(p+4)	$\times$	$\times$	$\times$	$-$

$\Rightarrow$ The action as sum of two parts: type II SUGRA and

$$S_{flavor} = -T_{p+4} \int \mathrm{d}^{p+1}x\, \mathrm{d}^4 y\, e^{-\phi} \sqrt{\hat{G}} + T_{p+4} \int \hat{C}_{5+p}$$

Backreaction of the flavor branes

⇒ Consider the RR form the flavor brane sources

$$S_{IIB+D7} \supset \frac{1}{2}\int \mathrm{d}C_8 \wedge *\mathrm{d}C_8 + \int C_8 \wedge \underbrace{(\delta(f_1)\delta(f_2)\mathrm{d}f_1 \wedge \mathrm{d}f_2)}_{\Xi_2}$$

which implies the Bianchi identity for a sourced RR form

$$\mathrm{d}F_1 = -\Xi_2$$

⇒ The number of flavor branes is given by Gauss law

$$\int F_1 \sim N_f$$

Backreaction with smearing (D3/D7) [Benini et al '06]

⇒ Two things I have shown in previous slides

1. For $D3$-branes the compact manifold is a SE
2. $S_{D7} = T_7 \int \left(-\mathrm{d}^8 x\, e^{-\phi} \sqrt{-G} + C_8 \right) \wedge \Xi_2$ with Ξ_2 exact

⇒ SE manifolds can be expressed as U(1) fibrations over KE manifolds and are equiped with an SU(2)-structure

$$\mathrm{d}\eta_{KE} = 2 J_{KE} \ , \qquad vol(SE) = \frac{1}{2} J_{KE} \wedge J_{KE} \wedge \eta_{KE}$$

⇒ Idea: to identify $\Xi_2 \sim J_{KE}$ and use the SU(2)-structure to write a *consistent radial ansatz* for the IIB+sources action

$$F_1 \sim N_f\, \eta_{KE} \quad \Rightarrow \quad \mathrm{d}F_1 \sim N_f\, J_{KE}$$

A taste of the smeared solution (D3/D7) [Benini et al '06]

⇒ With a simple ansatz

$$\mathrm{d}s^2 = g_1(r)\,\mathrm{d}x_{1,3}^2 + g_2(r)\,\mathrm{d}r^2 + g_3(r)\,\mathrm{d}s_{KE}^2 + g_4(r)\,\eta_{KE}^2 \ ,$$

with dilaton and RR forms

$$F_5 \sim N\,(1+*)J_{KE} \wedge J_{KE} \wedge \eta_{KE} \ , \qquad F_1 \sim N_f\,\eta_{KE} \ ,$$

⇒ A SUSY solution exists

$$\phi' = N_f\,e^{\phi} \quad \Rightarrow \quad e^{\phi} = \frac{1}{N_f\,(r_{LP} - r)}$$

When is backreaction needed? (D3/D7)

$\Rightarrow$ When can we omit backreaction (probe approximation) and when is it necessary?

$\Rightarrow$ Compare energies (effect on metric)

$$\frac{|F_1|}{|F_5|} \sim \lambda \frac{N_f}{N}$$

and one conclues (wrongly) that for $\lambda \frac{N_f}{N} \ll 1$ probe approx. is enough at all scales.

Backreaction and smearing (D2/D6) [Faedo et al '15]

For the D2/D6 case a similar situation holds

⇒ Start with a NK (6d) manifold and
$S_{D6} = T_6 \int \left(-\mathrm{d}^7 x\, e^{-\phi}\sqrt{-G} + C_7\right) \wedge \Xi_3$ with Ξ_3 exact

⇒ There is a SU(3)-structure

$$\mathrm{d}\mathcal{J} = 3\mathrm{Im}\Omega\ , \quad \mathrm{d}\mathrm{Re}\Omega = 2\mathcal{J}\wedge\mathcal{J}\ , \quad vol(NK) = \frac{1}{6}\mathcal{J}^3$$

⇒ In this case

$$F_2 \sim N_f\,\mathcal{J} \quad \Rightarrow \quad F_2 \sim N_f\,\mathrm{Im}\Omega \sim \Xi_3$$

When is backreaction needed? (D2/D6)

$\Rightarrow$ Backreaction matters in the IR

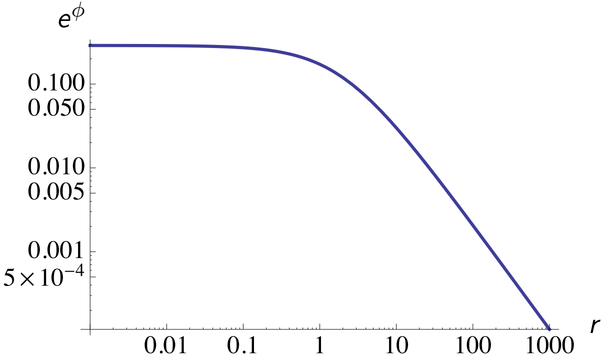
$$\frac{|F_2|}{|F_6|} \sim \frac{\lambda \frac{N_f}{N}}{E} \equiv \frac{E_{flavor}}{E}$$

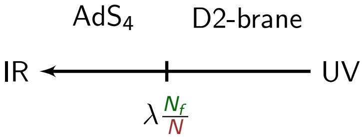
and there is a change in the dynamics of the theory with a crossover at $E \sim E_{flavor}$.

$\Rightarrow$ UV boundary conditions

A taste of the smeared solution (D2/D6) [Faedo et al '15]

The D2/D6 solution





Including charge in the setup

$\Rightarrow$ My motivation was to add a 'quark' density to these setups, but keeping vanishing temperature.

$\Rightarrow$ Dissolved strings in the flavor branes

$$U(1) \text{ global current} \quad \overset{\text{dual}}{\longleftrightarrow} \quad U(1) \text{ gauge field}$$

$\Rightarrow$ In particular a charge density corresponds to $A = A_t(r)\mathrm{d}t$

$$S_{D7} = -T_7 \int \mathrm{d}^8x\, e^{-\phi} \sqrt{-|G+F+B|} + T_7 \int C_8 - C_6 \wedge (F+B) + \cdots$$

When does the charge density matter [Chen et al '09] [Bigazzi et al '11]

⇒ Take the e.o.m. for the NS form (and set $B = 0$)

$$\mathrm{d}(e^{-2\phi} * H) = 0 = F_3 \wedge F_5 + F_1 \wedge *F_3 + (DBI) * \mathrm{d}t \wedge \mathrm{d}r$$

⇒ From the second term in RHS we deduce a component

$$F_3 \supset \mathcal{C}'(r)\mathrm{d}t \wedge \mathrm{d}r \wedge \eta_{KE} + \cdots$$

⇒ From the first term in RHS we deduce a constant (density) term

$$F_3 \supset N_q\, \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3$$

When does the charge density matter [Faedo et al '14]

$\Rightarrow$ Same game as before: When is the effect of charge comparable to the effect of color physics?

$$\frac{|F_3|}{|F_5|} = \left(\frac{\lambda^{2/3} (N_q / N^2)^{1/3}}{E} \right)^3 \equiv \left(\frac{E_{charge}}{E} \right)^3$$

so charge becomes important in IR for $E < E_{charge}$.

$\Rightarrow$ Similarly, we can show from $|F_1|/|F_3|$ that the charge dominates *always* in the IR.

When does the charge density matter [Faedo et al '14]

⇒ The same argument also works for the D2/D6 system

$$\frac{|F_2|}{|F_6|} = \left(\frac{\lambda^{1/2}(N_q/N^2)^{1/4}}{E}\right)^4 \equiv \left(\frac{E_{charge}}{E}\right)^4$$

so charge becomes important in IR for $E < E_{charge}$.

⇒ Similarly, we can show from $|F_2^{flavor}|/|F_2^{charge}|$ that the charge dominates *always* in the IR.

How does the charge density affect dynamics [Kumar '12] [Faedo et al '14]

⇒ To see charge effects we can take a limit in which we discard distracting flavor effects.

⇒ The IR turns out to be a non-relativistic theory (Lifshitz HV metric)

$$t \to \xi^z t \ , \qquad \vec{x} \to \xi \vec{x}$$

with hyperscaling-violation

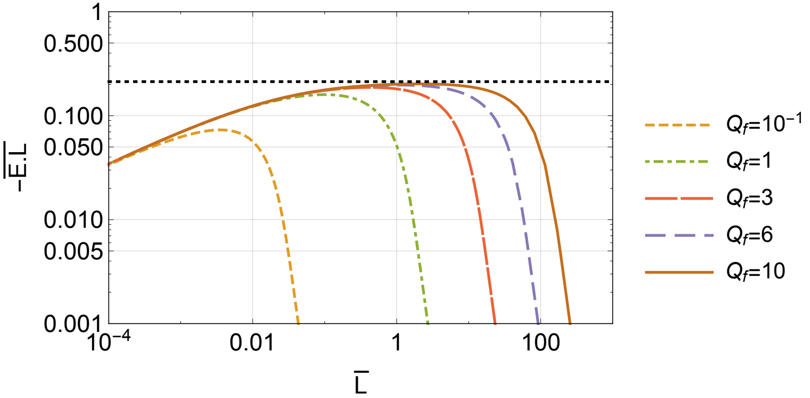
$$\mathrm{d}s^2 \to \xi^{\frac{\theta}{d-1}} \mathrm{d}s^2 \quad \Rightarrow \quad F \sim T^{\frac{d+z-1-\theta}{z}}$$

⇒ For $p = 3$ one gets $\quad z = 7 \ , \quad \theta = 0$

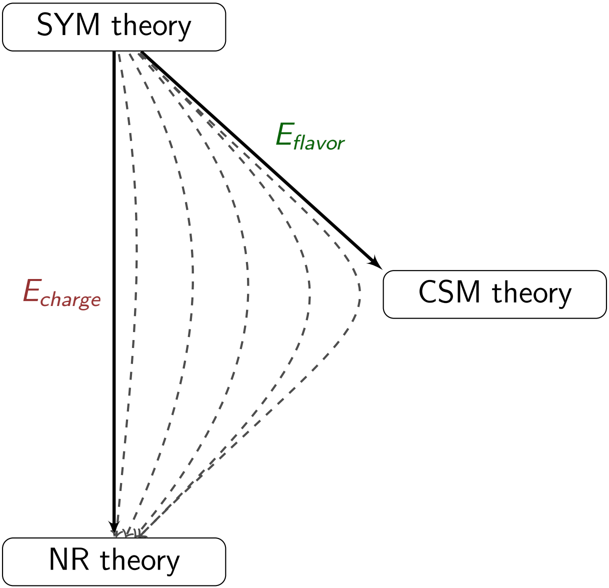
⇒ For $p = 2$ instead $\quad z = 5 \ , \quad \theta = 1$

3d SYM theory with quark density [Faedo et al '15]

A glimpse of the D2/D6 solution



3d SYM theory with quark density [Faedo et al '15]



4d SYM theory with quark density work in progress

- $\Rightarrow$ In 4d SYM there is only one *classical* scale
- $\Rightarrow$ Presence of the Landau pole complicates numerical analysis
- $\Rightarrow$ However the IR analysis showing the existence of a Lifshitz solution in the IR still holds

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Potential instabilities

If the IR we have proposed is realized one has to wonder about its stability

⇒ Thermodynamically it is stable

$$\lim_{T \to 0} S = 0 \text{ and } T\frac{\partial S}{\partial T} > 0$$

⇒ One possible instability would be towards striped (or other inhomogenous) phase

⇒ Or maybe there is a dynamic instability of some field that wants to condense. The BF bound in Lifshitz is

$$m^2 \geq -\frac{(p+z-\theta)^2}{4} = -25$$

An instability of the solution

⇒ In particular, take the U(1) BI vector field with one component in the internal directions

$$A_\mu \mathrm{d}x^\mu \supset \Psi(r)\, \eta_{KE}$$

⇒ In $\mathcal{N} = 4$ SYM the scalar is dual to

$$\mathcal{O}^I \sim Q^\dagger\, \sigma^I\, Q$$

with mass squared on the BF bound

⇒ In Lifshitz spacetime this scalar has mass below the BF bound, so we expect it to condense if the Lifshitz region is large

An instability of the solution

⇒ Backreaction of the mode in the supergravity fields affects the Gauss law for the D3-branes

$$\int_{S^5} F_5 \sim N + \frac{1}{2} N_f \Psi(r)^2$$

the color branes are separated: $U(N)$. Since color symmetry is broken we have *superconductivity*

⇒ In fact this is the only field in the smeared setup with this property: any color superconductor in our setup must have non vanishing Ψ

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- $\Rightarrow$ We have identified a IR phase of cold YM theories with charge density, given in the gravity side by a HV-Lifshitz metric
- $\Rightarrow$ We have worked out the numerical solution for 3d SYM, and this is work in progress for the 4d version.
- $\Rightarrow$ The IR appears to be unstable towards condensation of $\mathcal{O}^I \sim Q^\dagger \, \sigma^I \, Q$.
- $\Rightarrow$ Condensation of the dual scalar field gives rise to a color superconductor phase with order parameter given by $\langle \mathcal{O}^I \rangle$.

Thank you