Holographic thermalization at intermediate coupling

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Motivation, Goals & Strategy

Quark gluon plasma

- Produced in heavy collisions at RHIC and LHC
- Behaves as a strongly coupled liquid
- Thermalization process not well understood: $\tau < 1 fm/c$

Goals

- Gain insight into the thermalization process
- Modification of production rates of photons
- Modification of energy momentum tensor correlators
- Which modes thermalize first: top-down or bottom-up ?
- Dependence on coupling strength

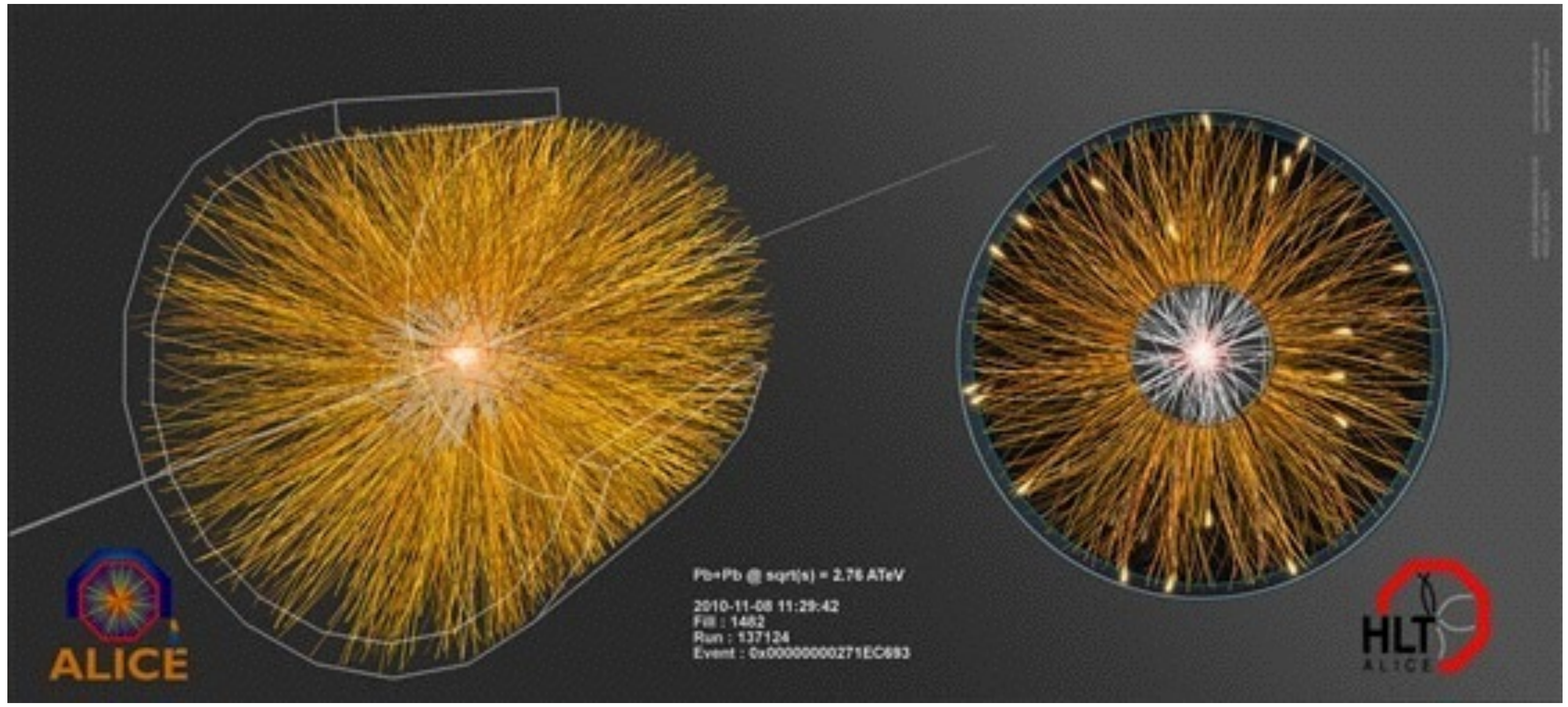
Strategy

- SYM where strong and weak coupling regimes are accessible

Outline

- **Early dynamics of a heavy ion collision**
- **Holographic Thermalization**
- **Results**

QGP in heavy ion experiments

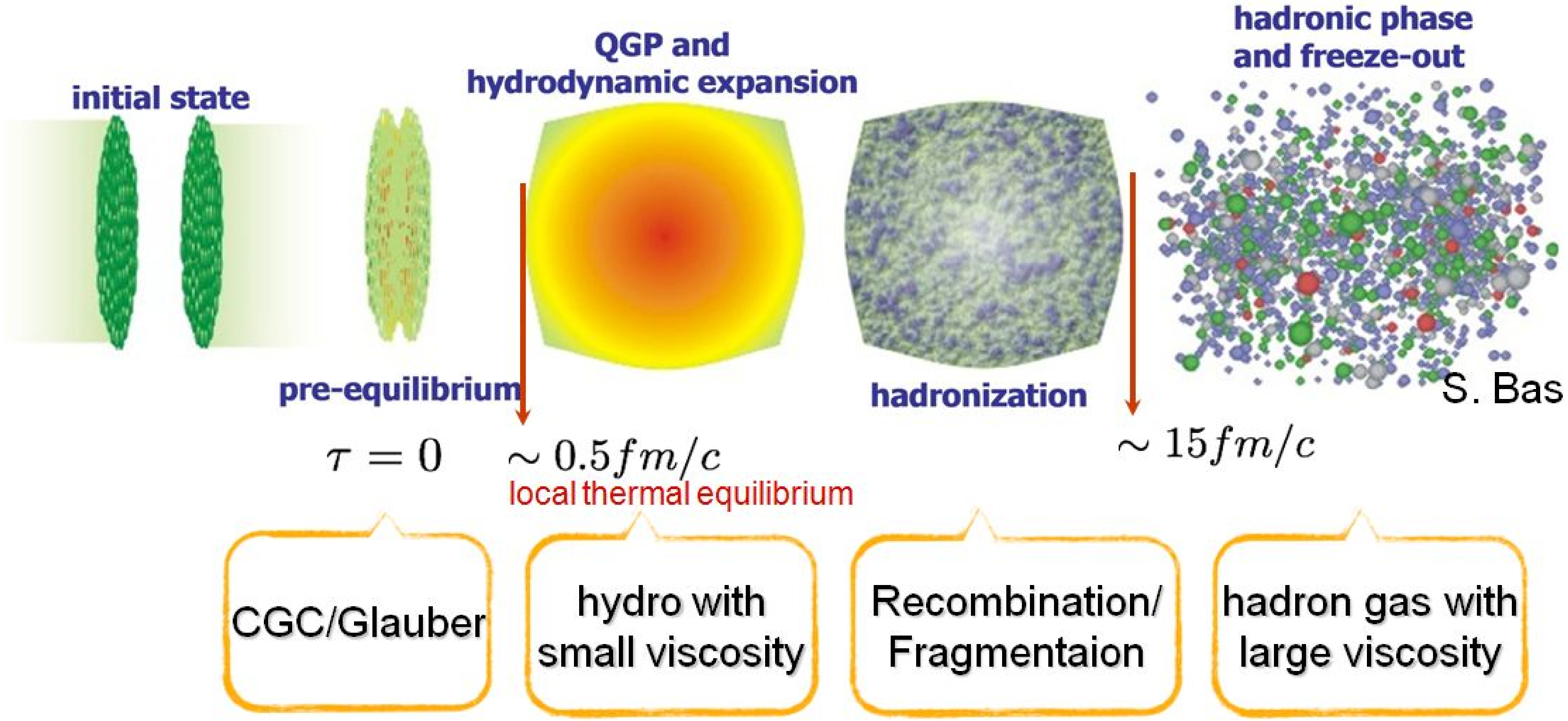


Creating Quark-Gluon Plasma in ultrarelativistic heavy ion collisions: Window into deconfined phase of QCD

- Allows to study fundamental properties of the theory: deconfinement transition and phase structure of the theory

- Theoretical and phenomenological description extremely challenging
  - Physical processes probe a vast range of scales
  - Strongly time dependent system: Heavy nuclei ⇒ (thermal) QGP ⇒ hadrons, photons, leptons

Stages of a heavy ion collision

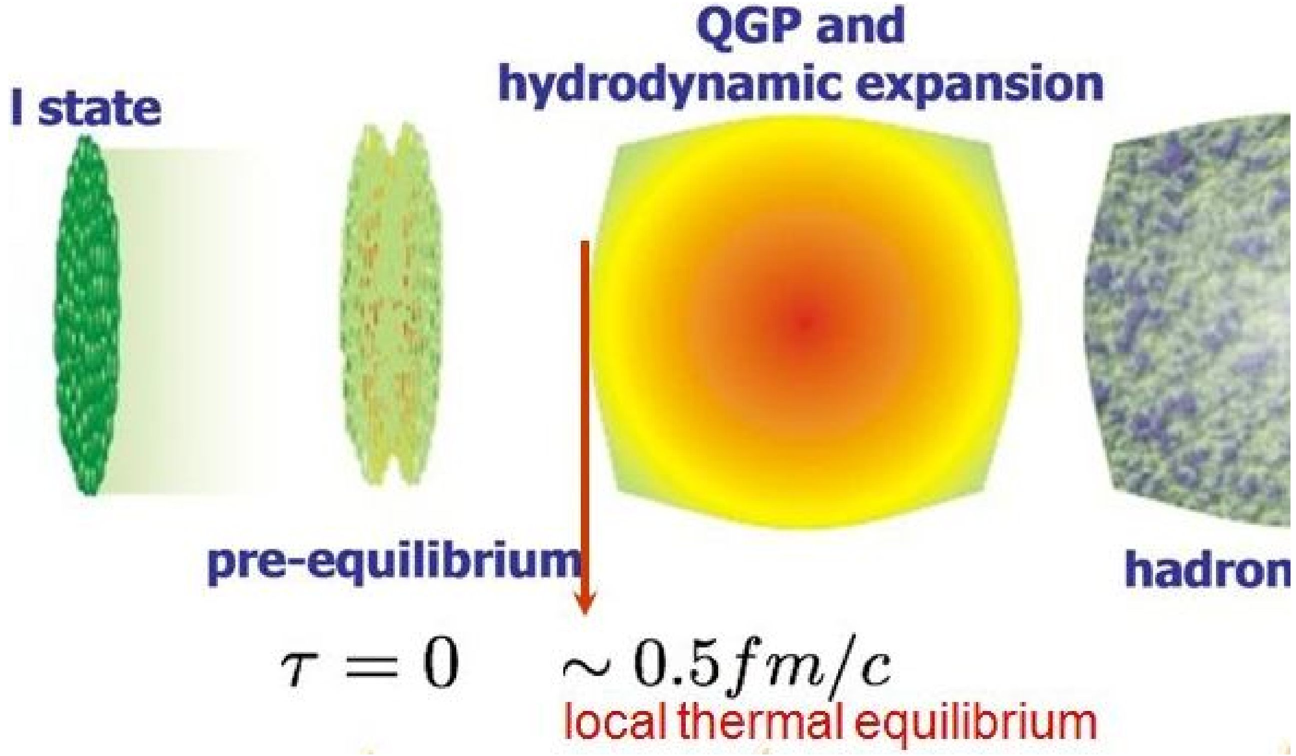


Nontrivial observation: hydro description of fireball evolution works extremely well

- Relatively easy: equation of state and freeze out
- Hard: Transport coefficients of the plasma
- Very hard: Initial conditions and dynamics of far from equilibrium situation

Surprise from RHIC/LHC: Extremely fast equilibration into almost ideal fluid behaviour — hard to explain via weakly coupled quasiparticle picture

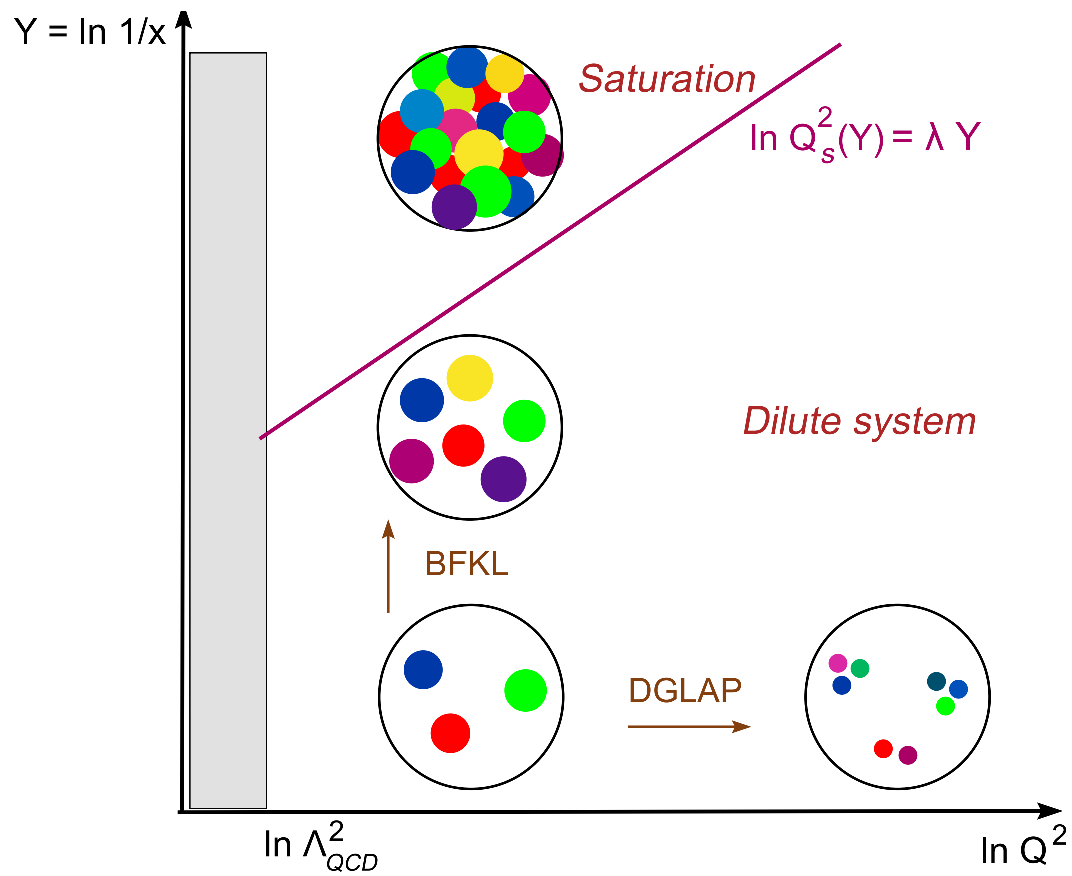
Thermalization puzzle



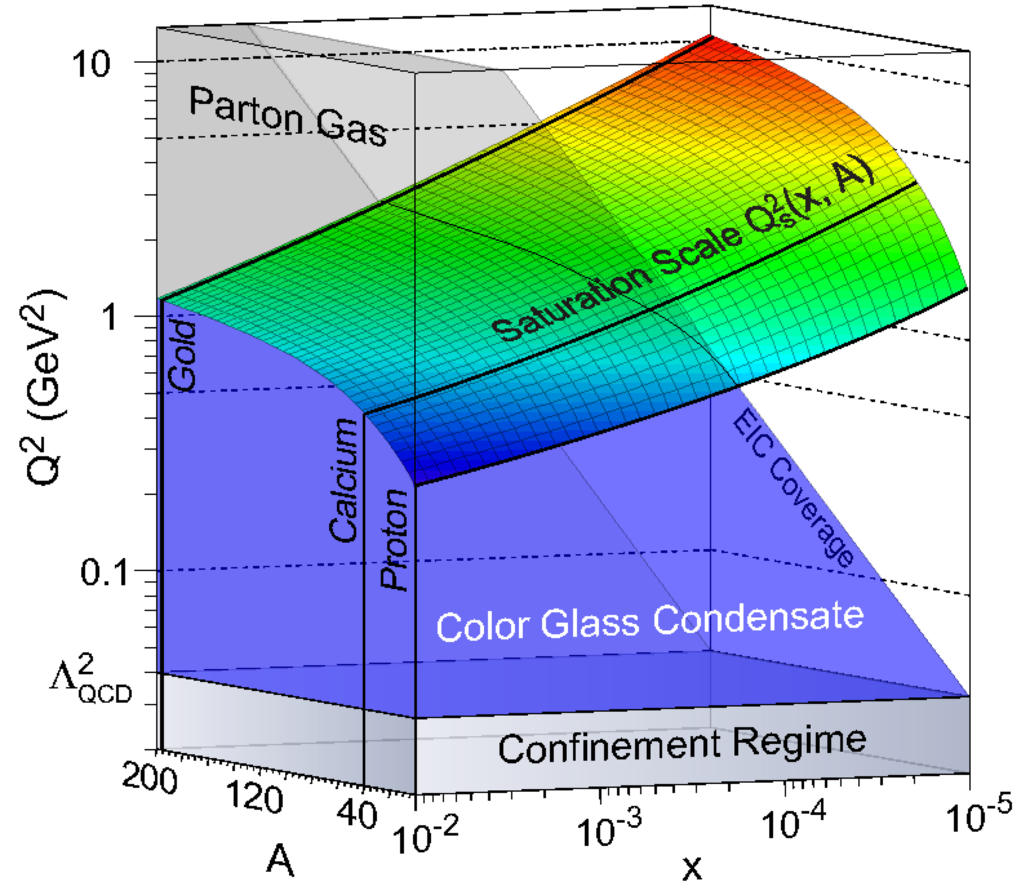
Major challenge: Understand the fast dynamics that take the system from complicated far-from-equilibrium initial state to near-thermal "hydrodynamized" plasma

Problem: Characteristic energy scales and nature of the plasma evolve fast (running coupling) $\Rightarrow$ Need to combine perturbative and nonperturbative machinery

Early dynamics of a heavy ion collision



(D.N. Triantafyllopoulos)



Initial state: Color Glass condensate characterized by

- One hard scale: Saturation momentum $Q_s \gg \Lambda_{QCD}$
- Overoccupation of gluons: $f \sim 1/\alpha$
- High anisotropy: $q_L \ll q_T$

Early dynamics of a heavy ion collision

Describing early dynamics one needs to take into account

- Longitudinal expansion
- Elastic and inelastic scatterings
- Plasma instabilities

Traditional field theory tools

- Classical (bosonic) lattice simulations — work as long occupation number is large *(Berges et al.)*
- Effective kinetic theory — works for smaller occupancies but breaks down in the IR *(Kurkela & Moore)*
- Parametric weak coupling estimates *(Baier et al., Kurkela & Moore)*

Thermalization at weak coupling

Questions one wants to answer

- Parametric weak coupling estimate: How does the therm time depend on the coupling constant

$$t_{equ} \sim \frac{\alpha^n}{Q_s}$$

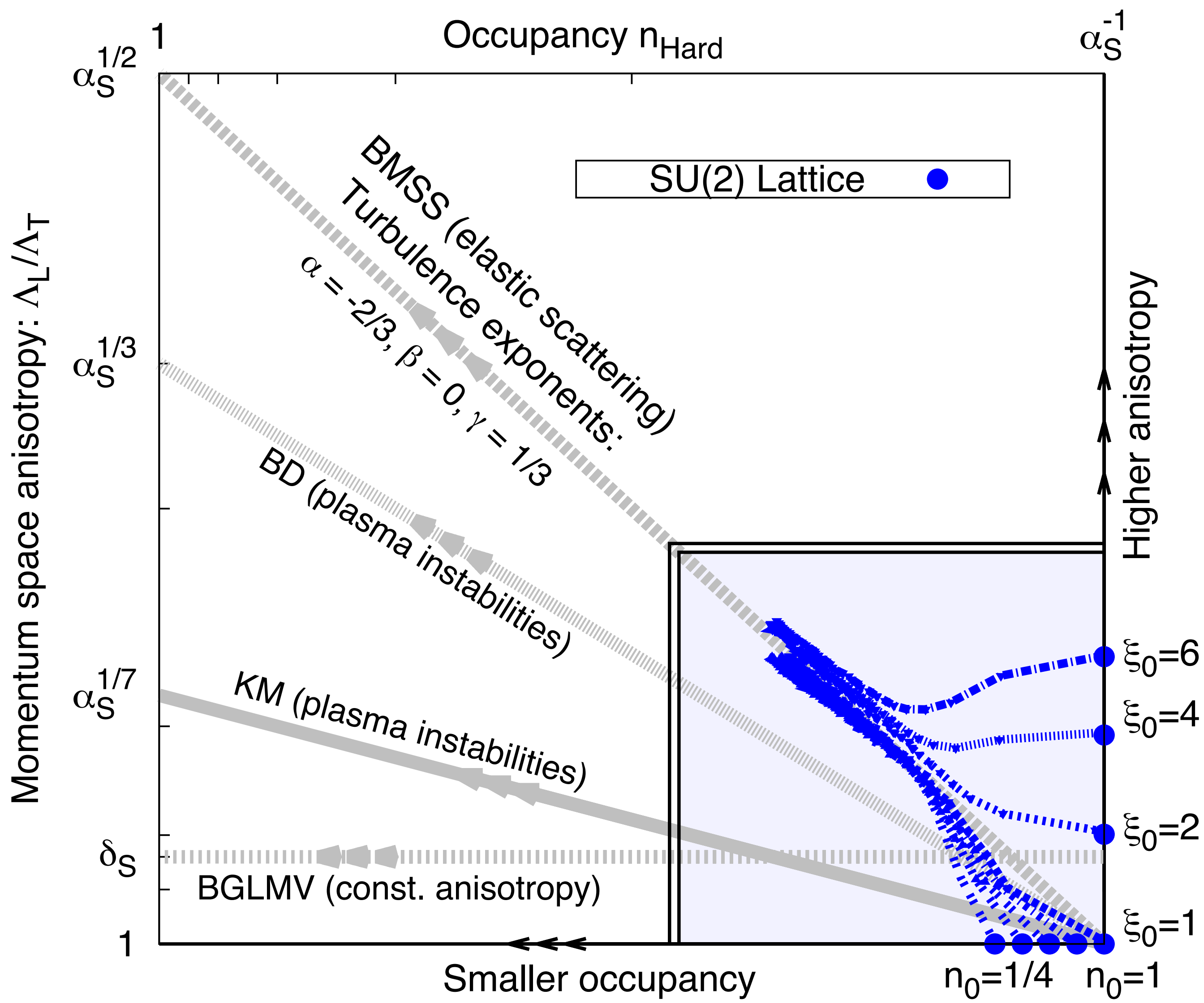
- what are the dominant processes?

Bottom-up thermalization *(Baier et al (2001))*

- Scattering processes
  - In the early stages many soft gluons are emitted which then thermalize the system (*Baier et al (2001)*): $n_{BMSS} \sim -13/5$
- Driven by instabilities
  - Instabilities isotropize the momentum distributions more rapidly than scattering processes (*Kurkela, Moore (2011)*): $n_{KM} \sim -5/2$

Thermalization at weak coupling

Classical (bosonic) lattice simulation



(Berges et al.)

Numerical evolution of expanding SU(2) YM plasma seen to show attractor behaviour and always lead to Baier-Mueller-Schiff-Son type scaling at late times

Thermalization beyond weak coupling

Impressive progress so far but problematic to apply to the full thermalization process

- Dynamics assumed classical in lattice simulations — works only for the earliest times
- System clearly not asymptotically weakly coupled $\Rightarrow$ Parametric scaling of the coupling constant of limited use

Need for additional tools to access the strongly coupled window of a heavy ion collision

Use AdS/CFT to study strongly coupled thermalization

Holography

Approach: Take different expansion point

- N=4 super Yang Mills theory
- Large 't Hooft coupling
- N_c taken to infinity

Accessible via the AdS/CFT correspondence

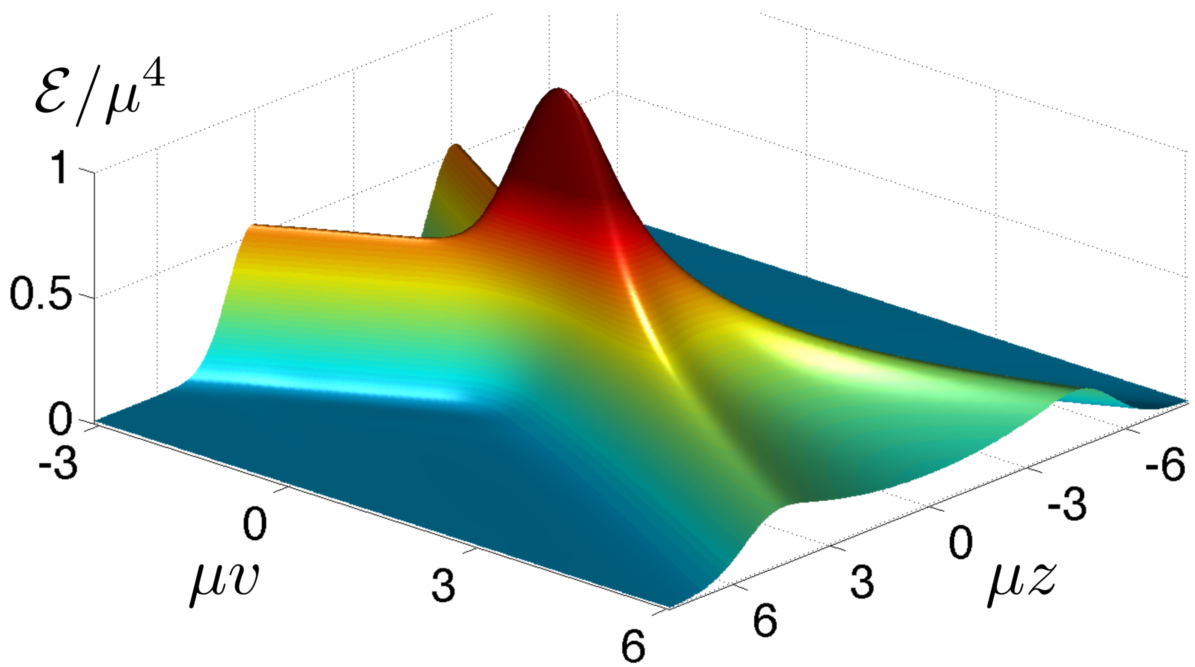
- IIB string theory in $AdS_5 \times S_5$ dual to N=4 SYM theory living on the 4d boundary of the AdS space
- strongly coupled SYM dual to classical supergravity

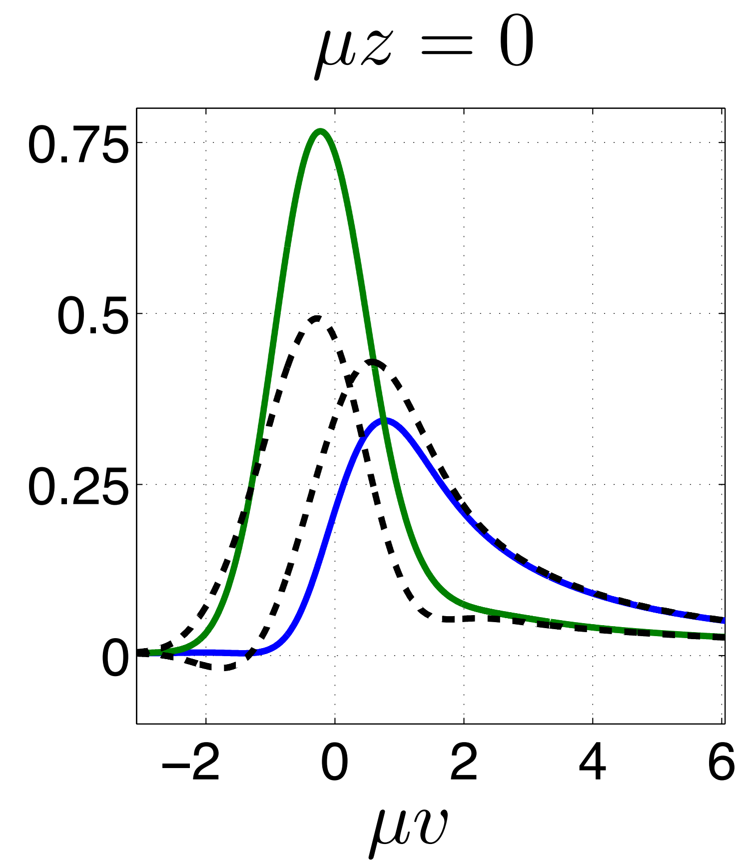
N=4 SYM very different from QCD at T=0 but similar at finite temperature

- Finite T breaks supersymmetry and conformal invariance
- describes deconfined plasma with Debey screening and finite static screening length

Thermalization at strong coupling

Thermalization process of strongly coupled N=4 SYM is mapped to black hole formation in asymptotically AdS space





Lessons from gauge/gravity duality

- Thermalization time naturally short $t_{eq} \sim 1/T$
- Hydrodynamization ≠ thermalization, isotropization
- Thermalization always top down (causal argument)

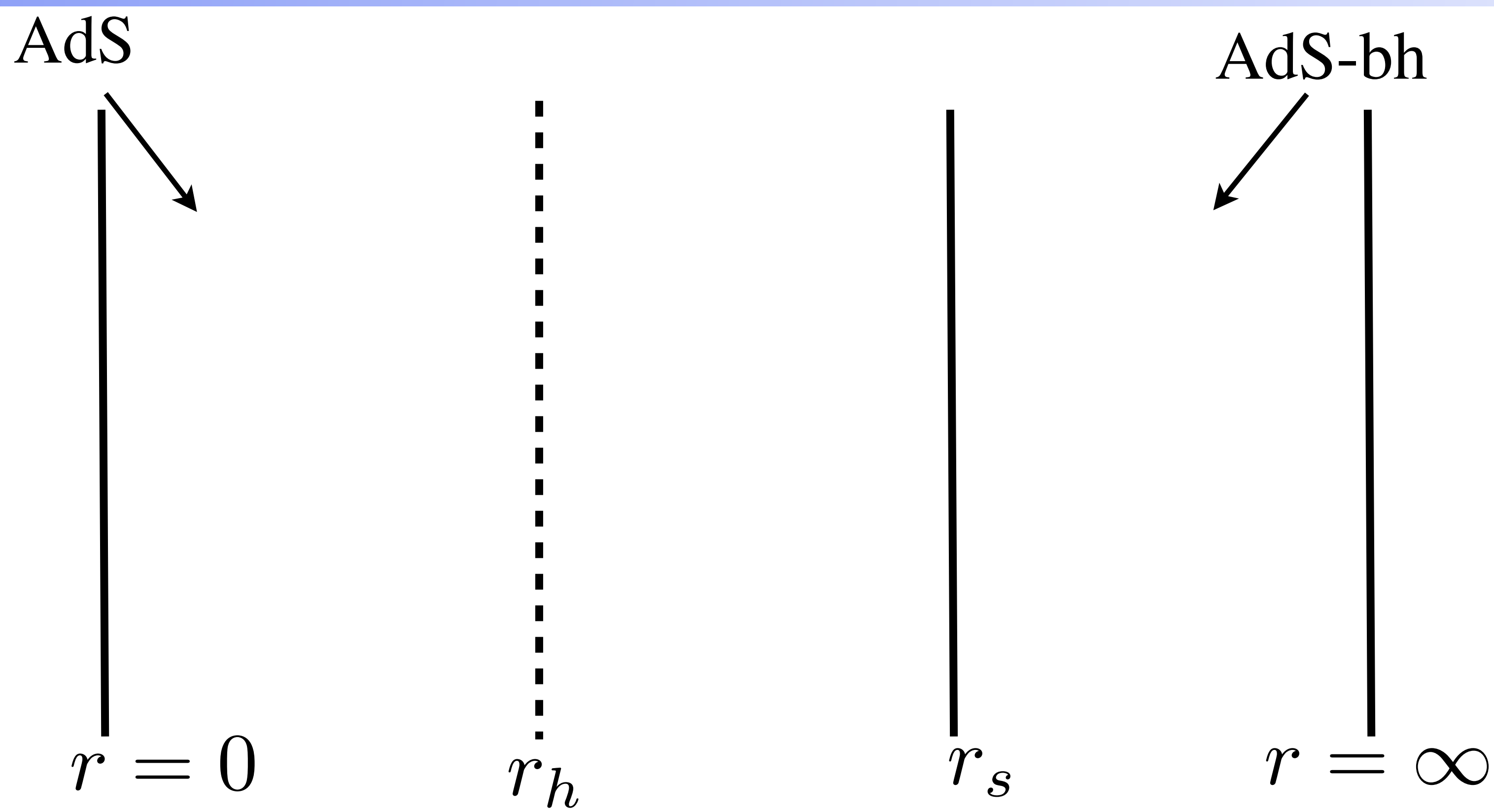
Bridging the gap

Rest of the talk: try to relax the infinite coupling limit and bring the two limiting cases closer together

Holographic thermalization

- **Collapsing shell model**
- **Greens functions as probe of thermalization**
- **Finite coupling corrections**

The falling shell setup



Danielsson, Keski-Vakkuri, Kruczenski (1999)

Outside and inside spacetime

- metric: $$ds^2 = \frac{(\pi T L)^2}{u}\left(f(u)dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{L^2}{4u^2 f(u)}du^2 \qquad u = \frac{r_h^2}{r^2}$$

$$f(u) = \begin{cases} f_+(u) = 1 - u^2\,, & \text{for } u > 1 \\ f_-(u) = 1\,, & \text{for } u < 1 \end{cases}\,,$$

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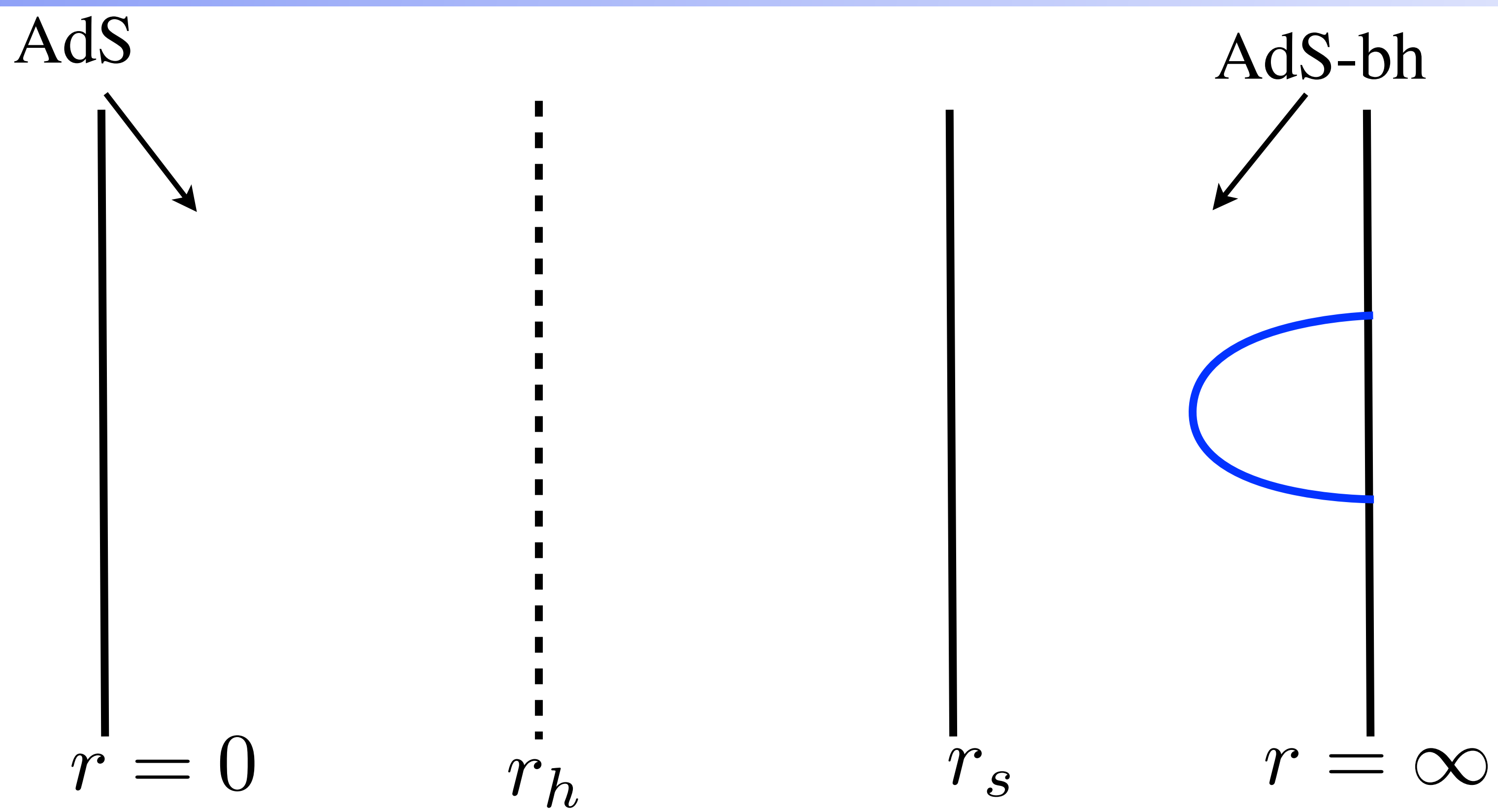

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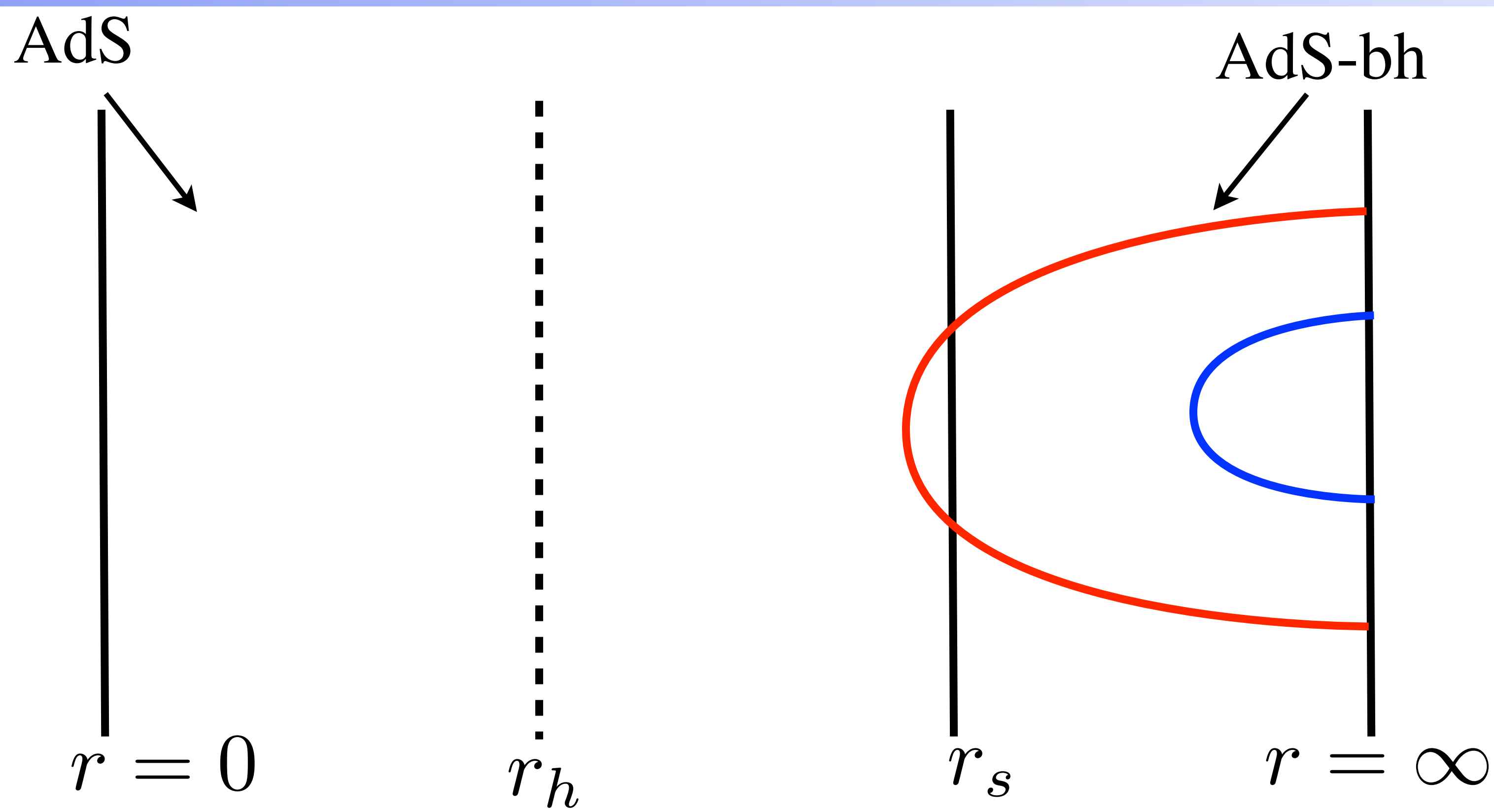
Thermalization from geometric probes:

- Entanglement entropy and Wilson loop: always top down thermalization

The falling shell setup



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Thermalization from geometric probes:

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Falling shell set up

Dynamics of the shell:

- Israel matching conditions: $[K_{ij} - \gamma_{ij} K] = -8\pi g_5 S_{ij}$

Quasistatic approximation:

- motion of the shell is slow compared to other scales of interest
- Huge advantage: Greens functions available with minor modification to the standard holographic recipe

Field theory side

- Rapid, spatially homogenous injection of energy at all scales
- Shell can be realized by briefly turning on a spatially homogenous scalar source coupled to a marginal operator

Holographic Green's function

In- and off-equilibrium correlators offer useful tool for studying thermalization

- Poles of retarded thermal Green's functions give dispersion relation of field excitations: Quasiparticle / quasinormal mode spectrum
    - describe response of the system to infinitesimal perturbation
- Time dependent off-equilibrium Greens functions probe how fast different energy (length) scales equilibrate

Two examples

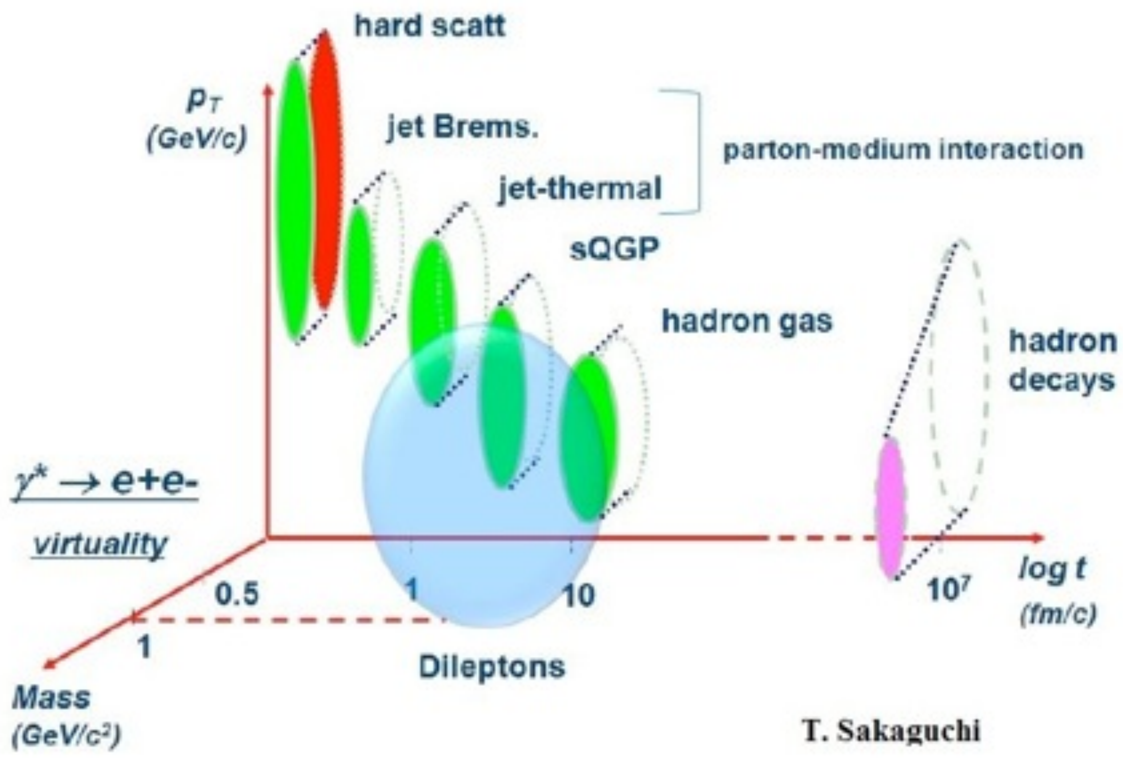
Energy momentum tensor correlators

- linearized perturbations of $g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$
- construct gauge invariants from symmetry channels *(Kovtun, Starinets)*
  - scalar channel: h_{xy}
  - shear channel: $h_{tx},\ h_{zx}$
  - sound channel: $h_{tt},\ h_{tz},\ h_{zz},\ h$

EM current correlators — photon production

- Obtained by adding a U(1) vector field coupled to a conserved current corresponding to a subgroup of the $SU(4)_R$

Photon emission in heavy ion collisions



Photons are emitted at all stages of the collision

- Initial hard scattering processes: quark anti-quark annihilation:
  - on-shell photon or virtual photon → dilepton pair
- Strongly coupled out of equilibrium phase: no quasiparticle picture
- Additional (uninteresting) emissions from charged hadron decays

Probing the plasma

Probing the plasma

- Once produced photons stream through the plasma almost unaltered
- Provide observational window in the thermalization process of the plasma

Quantity of interest

- Spectral density : $\chi^{\mu}_{\mu} = -2\mathrm{Im}(\Pi^{\mathrm{ret}})^{\mu}_{\mu}(k_0)$
- Number of emitted photons

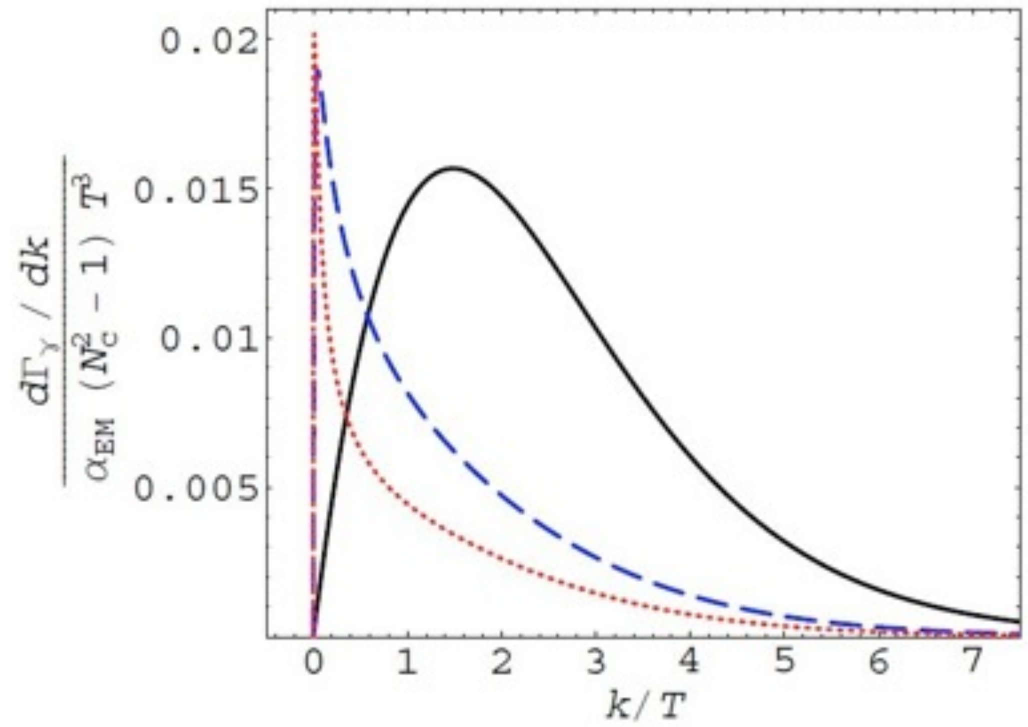
Fluctuation dissipation theorem

$$\eta^{\mu\nu}\Pi^{<}_{\mu\nu}(\omega) = -2n_B(\omega)\mathrm{Im}(\Pi^{ret})^{\mu}_{\mu}(\omega) = n_B(\omega)\chi(\omega)$$

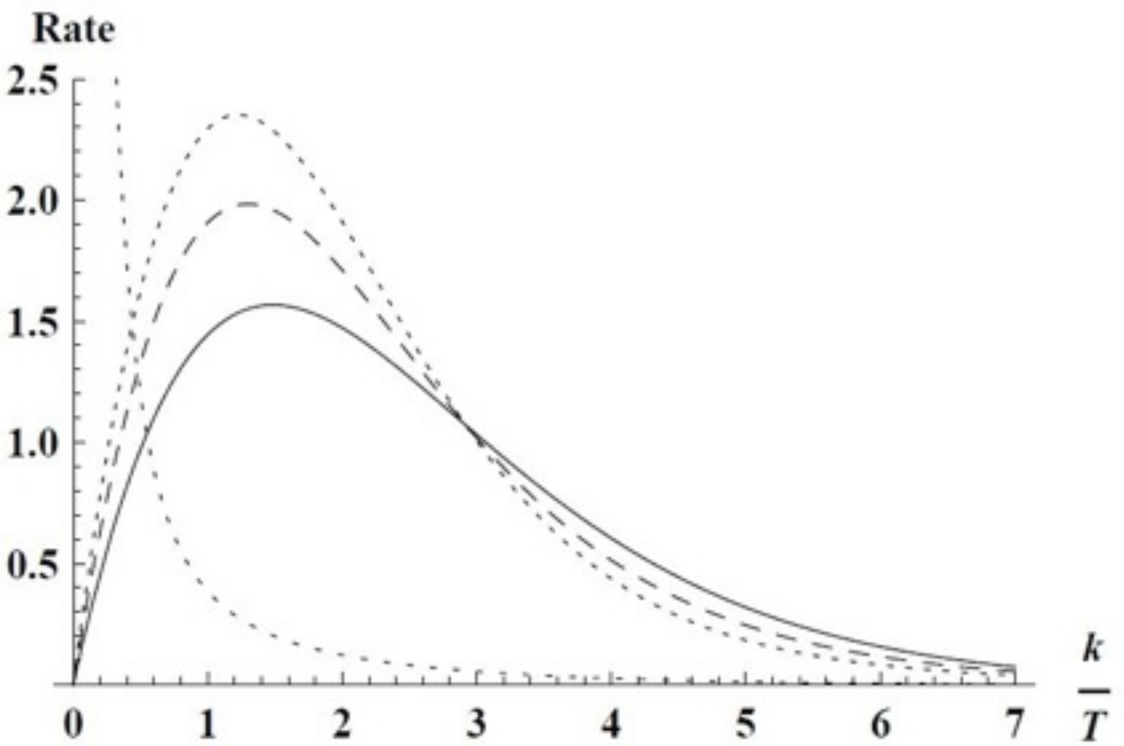
Production rate

$$k^0\frac{d\Gamma_\gamma}{d^3k} = \frac{\alpha}{4\pi^2}\eta^{\mu\nu}\Pi^{<}_{\mu\nu}(\omega = k^0)$$

Photon emission in equilibrium SYM plasma



Huot et al (2006)



Hassanain, Schvellinger (2012)

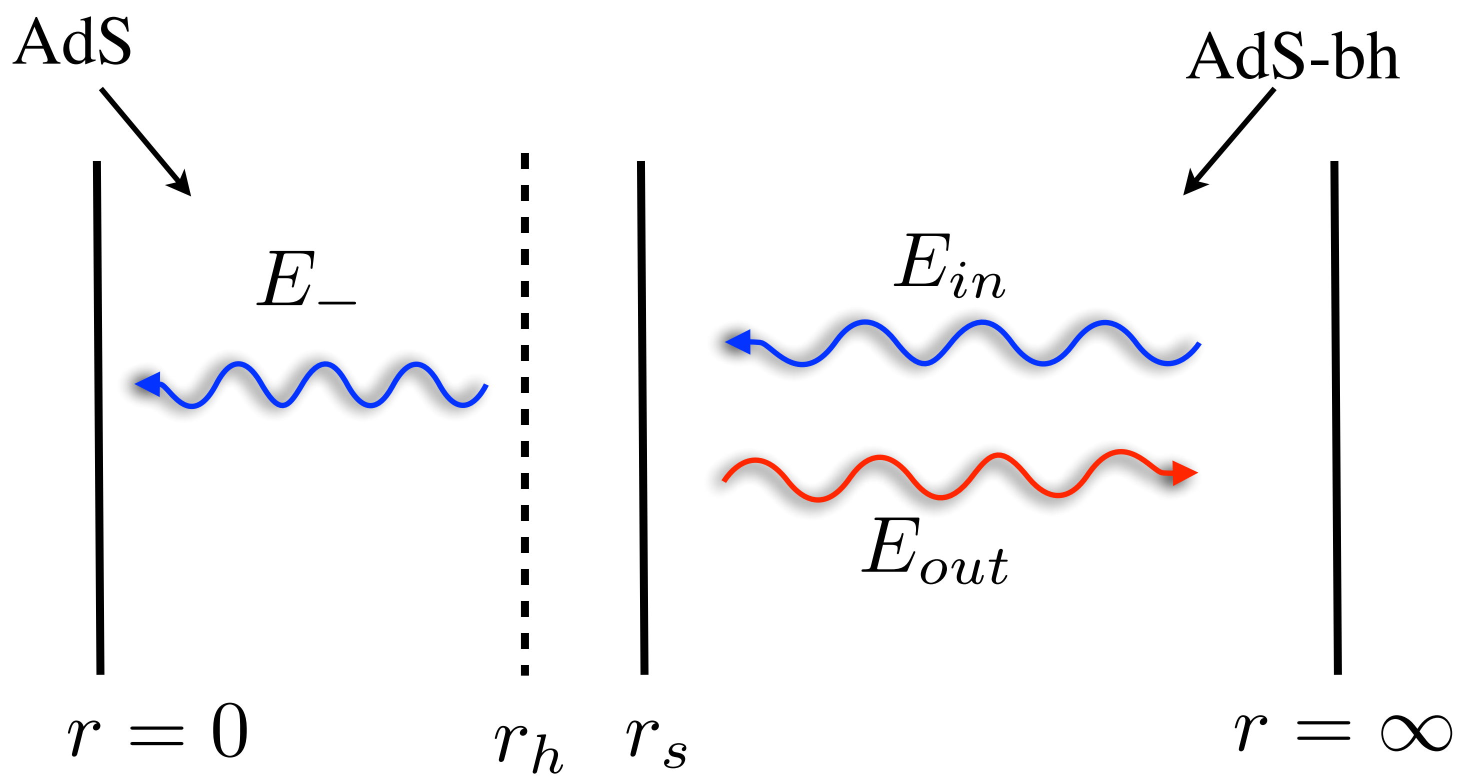
Perturbative result

- Increasing the coupling: slope at k=0 decreases, hydro peak broadens and moves right

Strong coupling result

- Decreasing coupling from $\lambda = \infty$: peak sharpens and moves left

Recipe for retarded correlators



Danielsson, Keski-Vakkuri, Kruczenski (1999)

difference to equilibrium situation

- outside solution is a linear combination of ingoing and outgoing modes

$$\phi_a^+ = c_+ \phi_{a,in} + c_- \phi_{a,out} \qquad \phi_a = E, Z_i$$

Holographic Green's functions

Some computational details

- Solve classical EoM for the relevant bulk field inside and outside the shell
- Match solutions at the shell using Israel junction conditions
  - Quasistic limit: Ignore time derivatives
- Use conventional methods to obtain retarded correlator

$$\Pi(\omega,\mathbf{q}) = -\frac{N_c^2T^2}{8}\lim_{u\to 0}\frac{E'(u,Q)}{E(u,Q)} = -\frac{N_c^2T^2}{8}\Pi_{therm}\frac{1+\frac{c_-}{c_+}\frac{E'_{out}}{E'_{in}}}{1+\frac{c_-}{c_+}\frac{E_{out}}{E_{in}}}$$

$$\Pi_{Z_i}(\omega,q) = \lim_{u\to 0}\frac{N_c^2T^4}{2}\frac{Z''_{i,+}(u,Q)}{Z_{i,+}(u,Q)}$$

- Behaviour of c_-/c_+ crucial for out of equilibrium dynamics

Finite coupling corrections

Key relation in AdS/CFT: $(L/l_s)^4 = L^4/\alpha'^2 = \lambda$

- Go beyond $\lambda = \infty$: add α' terms to SUGRA action, i.e. first non trivial terms in a small curvature expansion
- Leading order corrections: $\mathcal{O}(\alpha'^3) = \mathcal{O}(\lambda^{-3/2})$

Gubser et al; Pawelczyk, Theisen (1998)

Improved type IIB SUGRA action:

$$S^0_{IIB} = \frac{1}{2\kappa_{10}} \int d^{10}x\sqrt{-g}\left(R_{10} - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4.5!}(F_5)^2 + \gamma e^{\frac{-3}{2}\phi}(C+\mathcal{T})^4\right)$$

$$\mathcal{T}_{abcdef} = i\nabla_a F^+_{bcdef} + \frac{1}{16}\left(F^+_{abcmn}F^+_{def}{}^{mn} - 3F^+_{abfmn}F^+_{dec}{}^{mn}\right), \quad \gamma \equiv \frac{1}{8}\zeta(3)\lambda^{-\frac{3}{2}}$$

Paulos (2008)

- Leads to γ-corrected metric and EoMs for the different fields

Results

- **Quasinormal modes**
- **Photon production**
- **Thermalization of the spectral density**
- **Analysis of results**

Quasinormal modes infinite coupling

- Structure of retarded thermal Greens functions $\Rightarrow$ Dispersion relation of field excitations

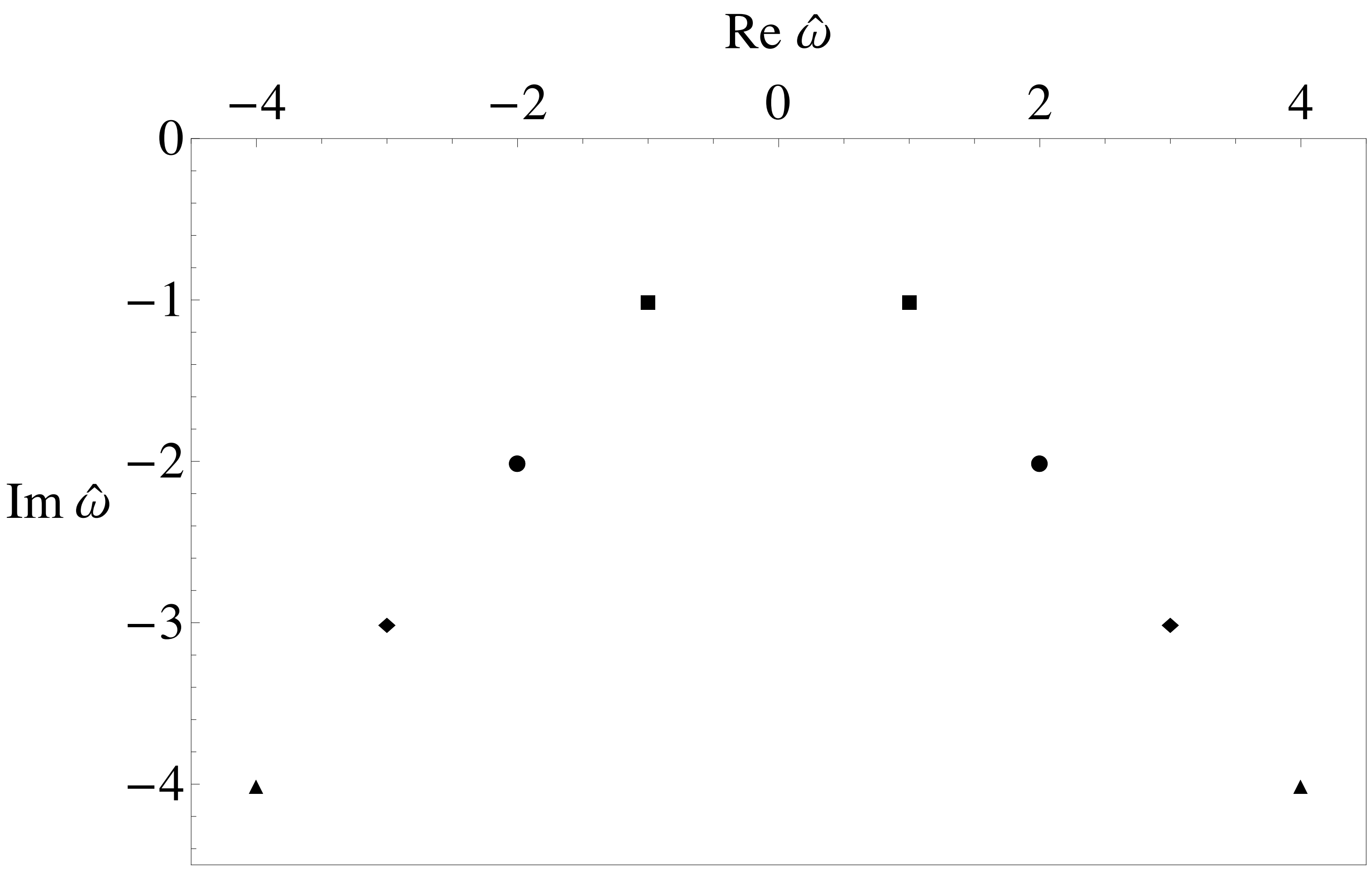
$$\omega_n(q) = M_n(q) - i\Gamma_n(q),$$

- Reveal striking difference between weakly and strongly coupled systems
  - At weak coupling long lived quasiparticles
  - At strong coupling infinity tower of modes

$$\omega_n|_{q=0} = n(\pm 1 - i)$$

- Magnitude of Γ_n related to thermalization pattern: At strong coupling highest energy modes decay fastest — top down thermalization

QNM at infinite coupling: Photons

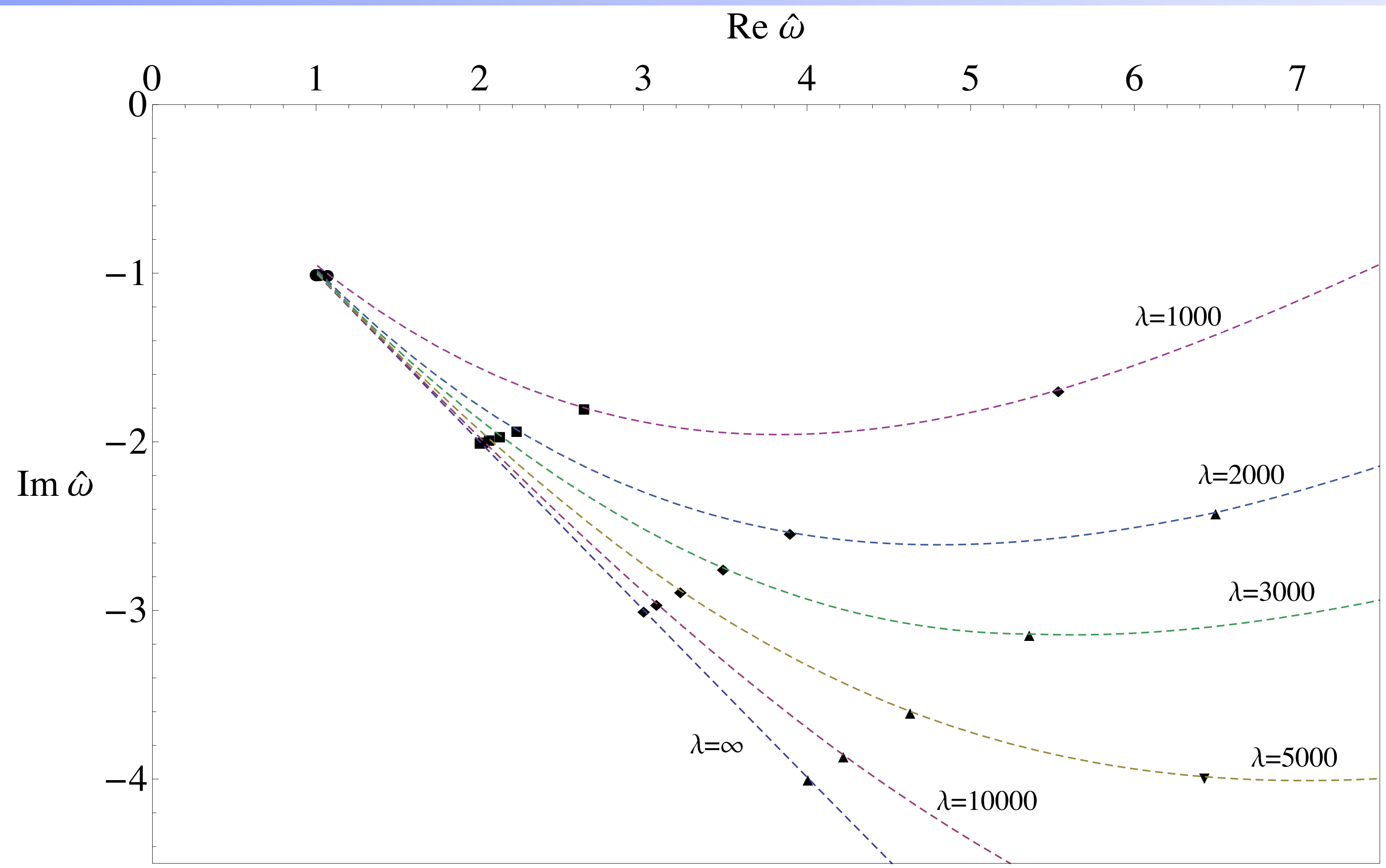


$$\omega_n|_{q=0} = n(\pm 1 - i)$$

(Kovtun, Starinets)

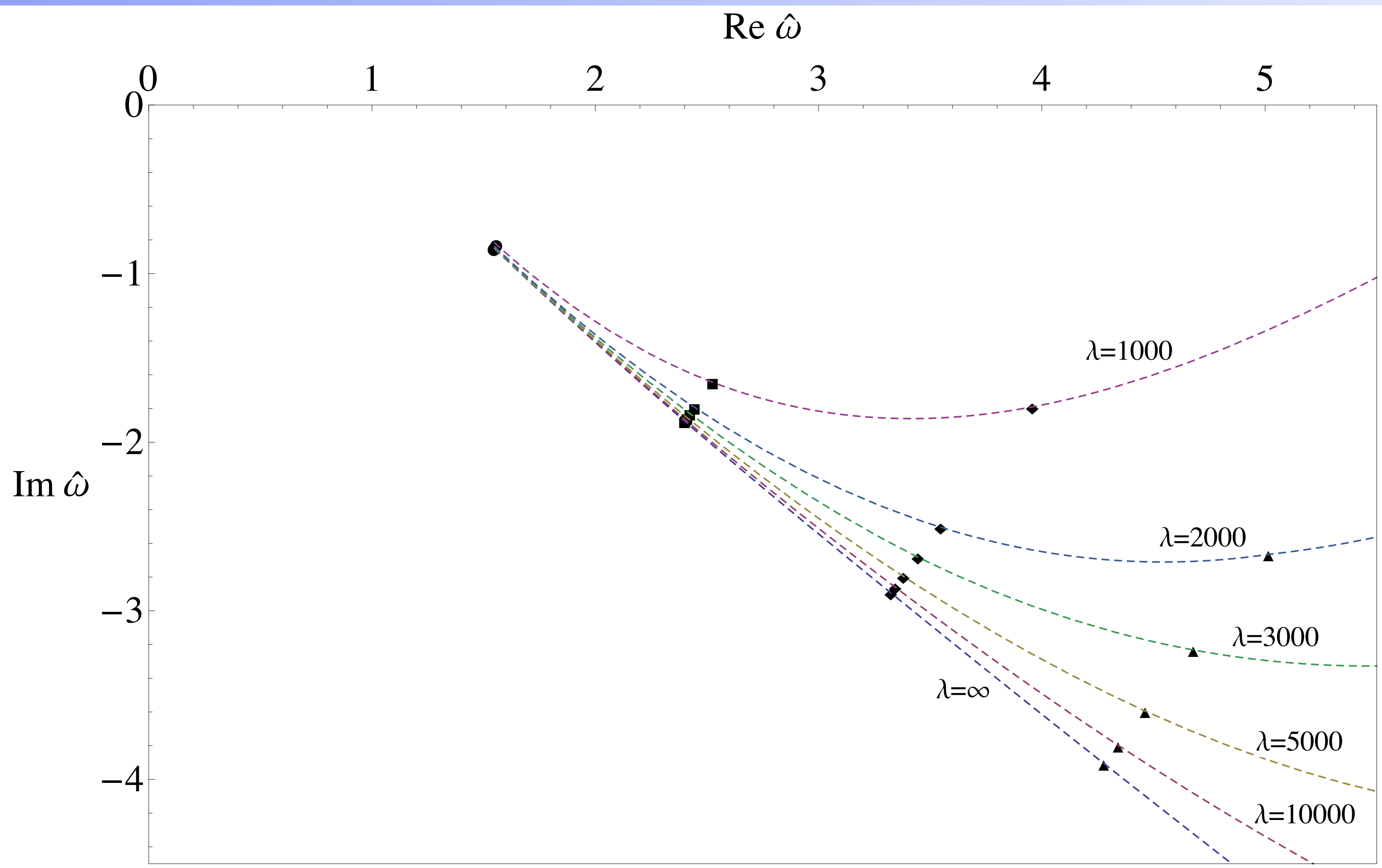
- Pole structure of EM current-current correlator displays usual quasinormal mode spectrum at infinite coupling
- How does the QNM spectrum get modified at finite coupling?

QNM at finite coupling: Photons



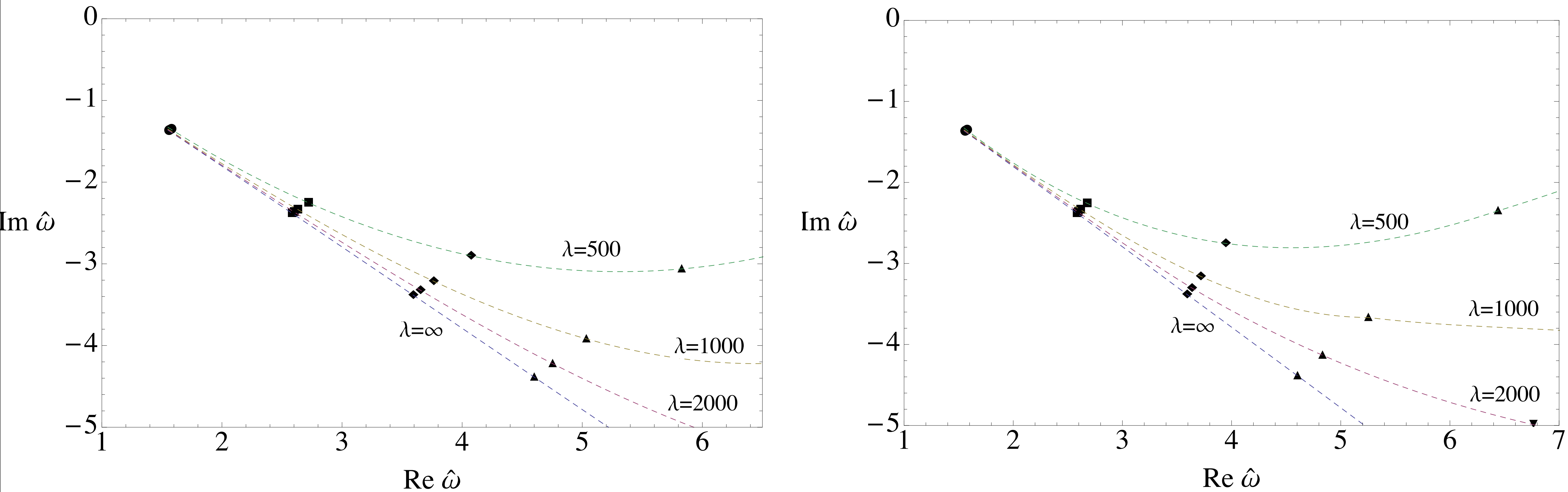
- Effect of decreasing coupling: Width of excitations consistently decrease $\Rightarrow$ modes become longer - lived
- Larger impact on higher energetic modes
- Convergence of strong coupling expansion not guaranteed when shift is of $\mathcal{O}(1)$

QNM at finite coupling: Photons



- similar shift at nonzero three momentum: q=2πT

QNM at finite coupling: $T_{\mu\nu}$ correlators



Same effect for the shear (left) and sound (right) channel (here k=0)

- Outside the infinite coupling, the response of a strongly coupled plasma appears to change, with the QNM mode spectrum moving towards a quasiparticle one
- What happens if we the take the system further away from equilibrium by using the collapsing shell model?

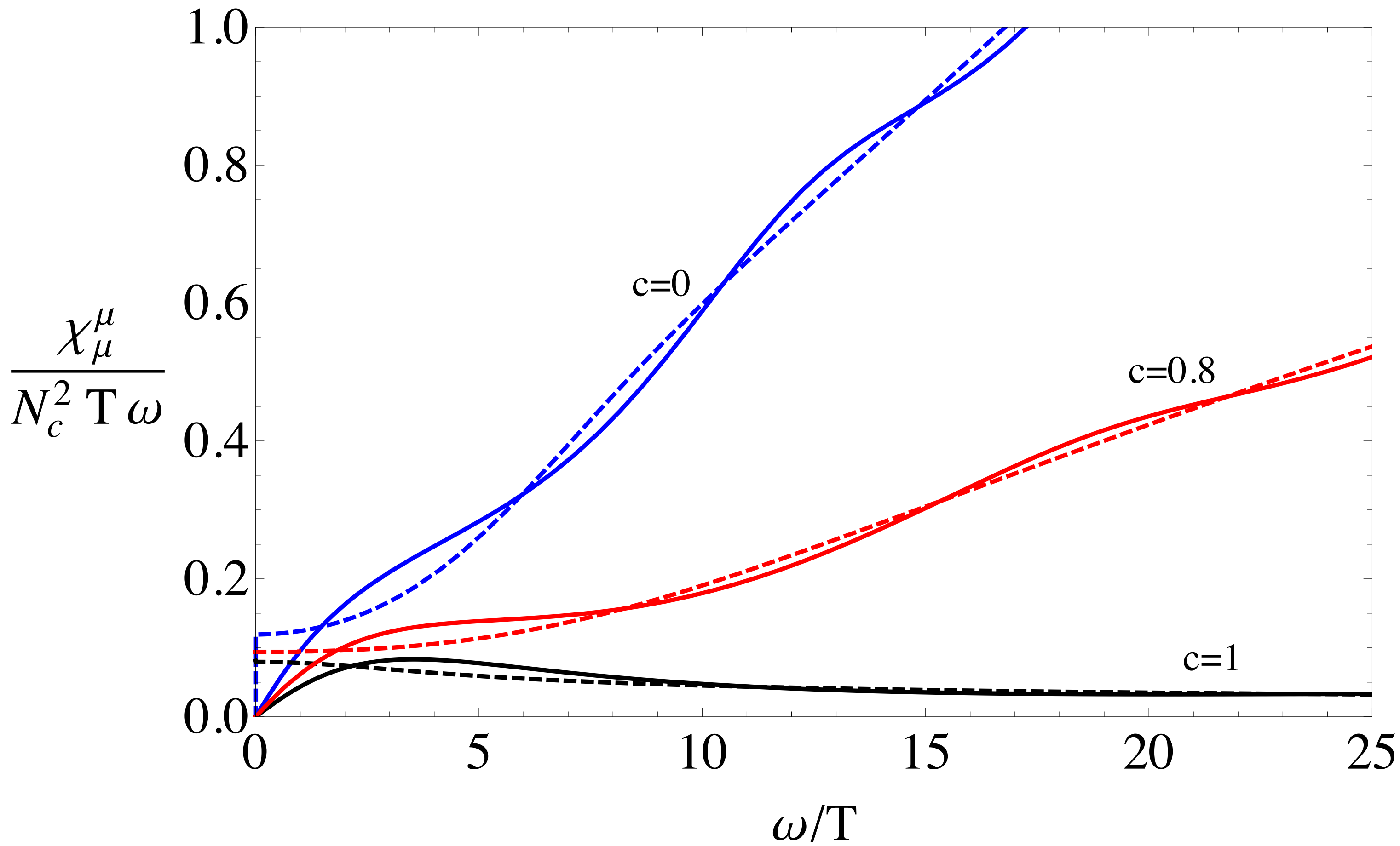
Photon spectral density

natural quantity to study: spectral density: $\chi^{\mu}_{\mu} = -2\mathrm{Im}(\Pi^{\mathrm{ret}})^{\mu}_{\mu}(k_0)$

- virtuality

$$v = \frac{\hat{\omega}^2 - \hat{q}^2}{\hat{\omega}^2}$$

- parametrize $\quad q = c\,\hat{\omega}$



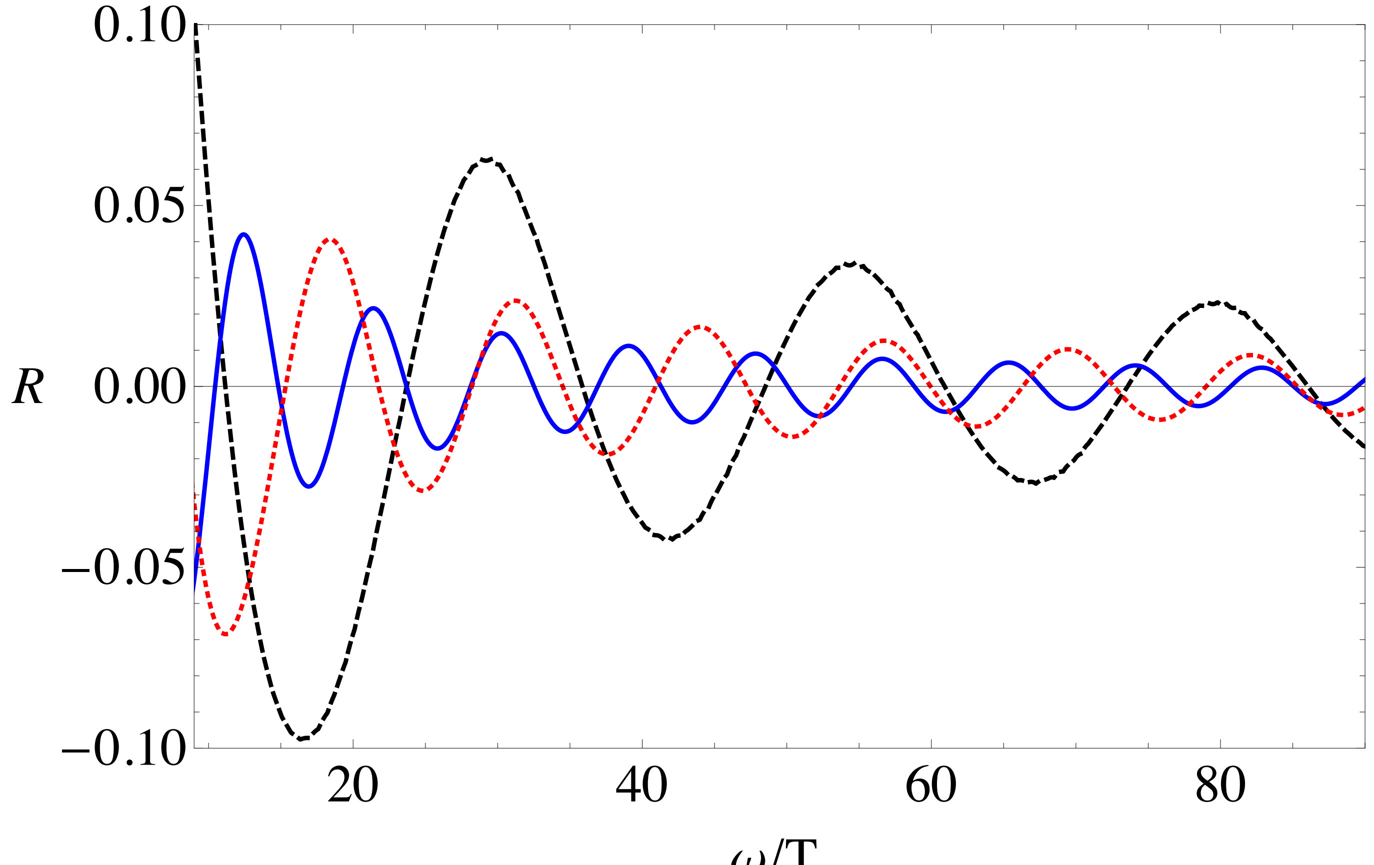
spectral density for $r_s/r_h = 1.1$ for different virtualities

- Out of equilibrium effect: oscillations around thermal value
- As the shell approaches the horizon equilibrium is reached

Relative deviation of spectral density

- Usefull measure of out-of equilibriumness: Relative deviation of spectral density from thermal limit

$$R(\hat{\omega}) = \frac{\chi(\hat{\omega}) - \chi_{th}(\hat{\omega})}{\chi_{th}(\hat{\omega})}$$

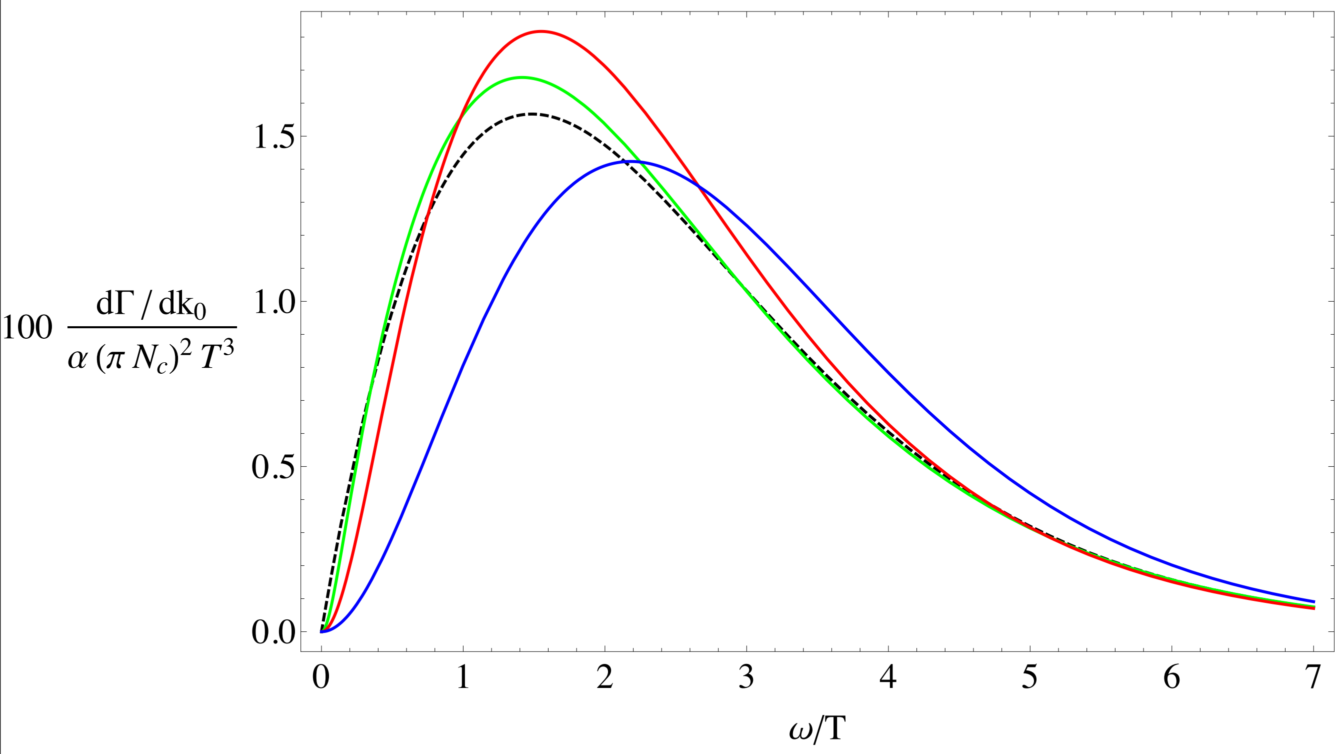


relative deviation R for r_s=1.1 and c=1, 0.8, 0

- Top down thermalization: highly energetic modes are closer to equ. value
- Highly virtual field modes thermalize first

$$\chi(\hat{\omega}) \approx \hat{\omega}^{\frac{2}{3}} \left(1 + \frac{f_1(u_s)}{\hat{\omega}}\right), \qquad R \approx \frac{1}{\hat{\omega}}$$

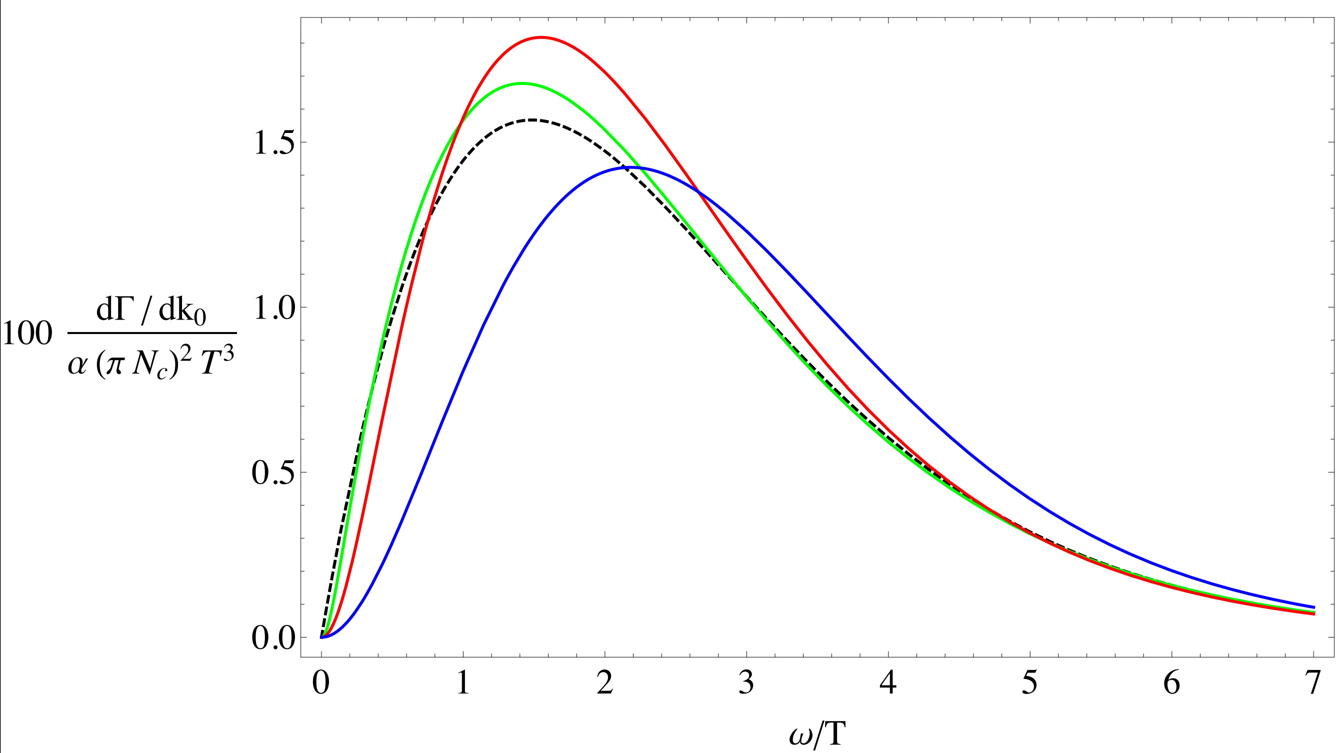
Photon production rate at infinite coupling

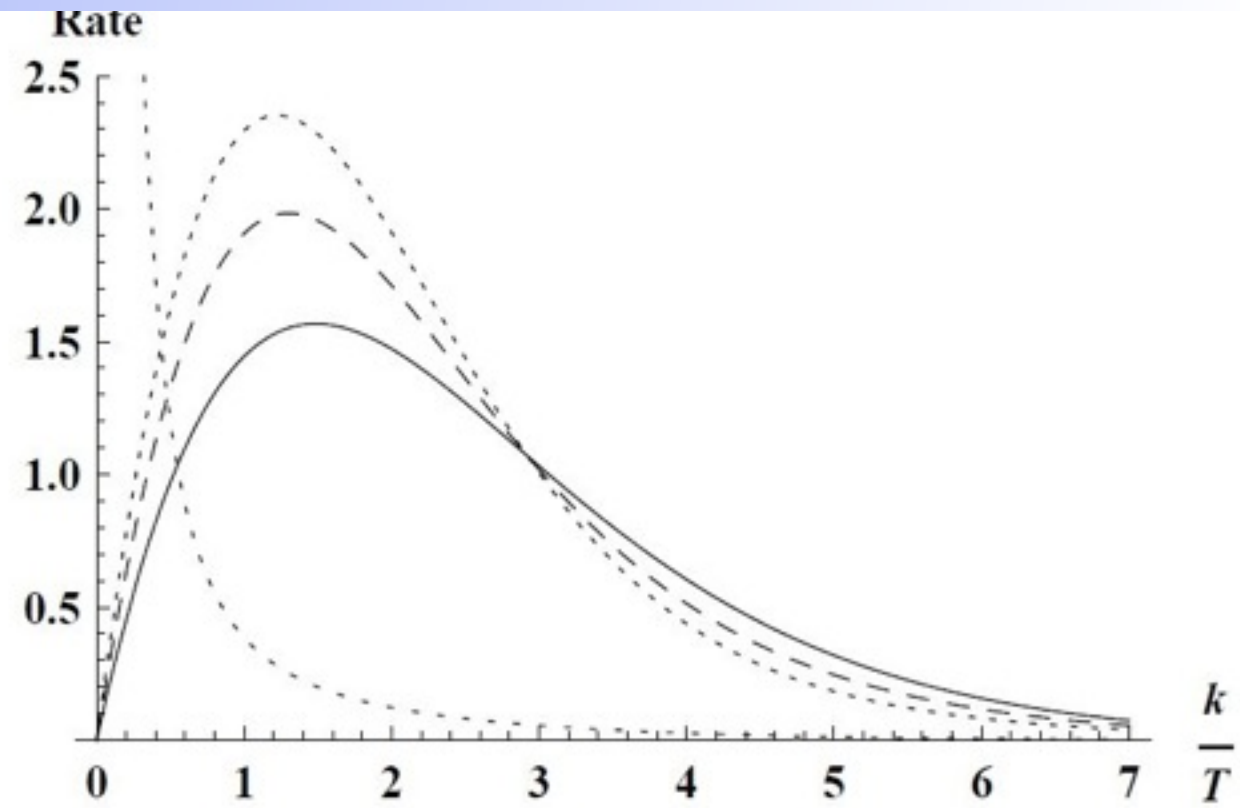


photon production rate for r_s/r_h=1.1, 1.01, 1.001

- Enhancement of production rate
- Hydro peak broadens and moves right
- Apparently no dramatic observable signature in off-equilibrium photon production

Photon production rate at infinite coupling

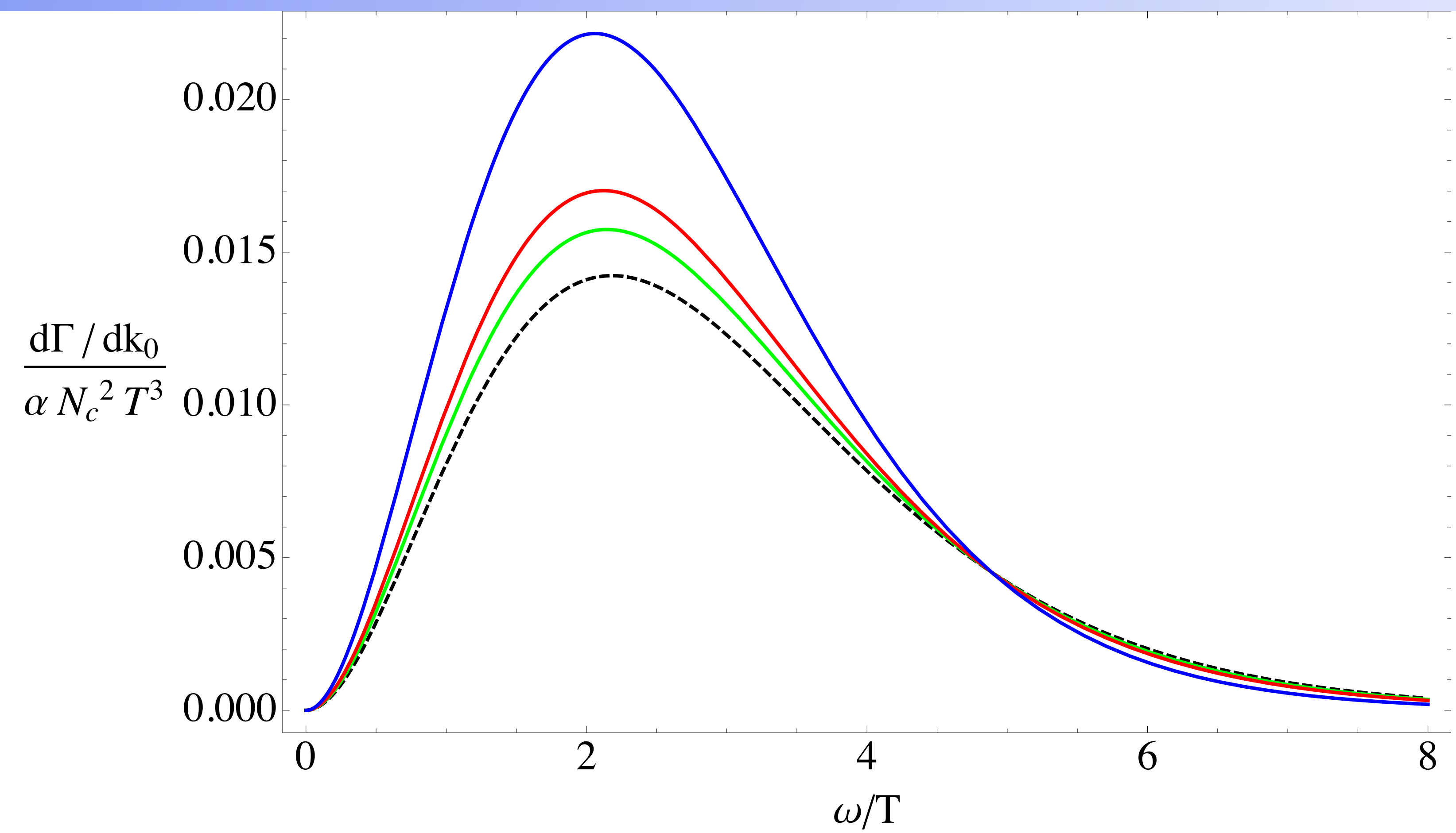




photon production rate for r_s/r_h=1.1, 1.01, 1.001

- Enhancement of production rate
- Hydro peak broadens and moves right
- Apparently no dramatic observable signature in off-equilibrium photon production
- Combining the two allows to study thermalization at finite coupling!

Photon production rate at intermediate coupling

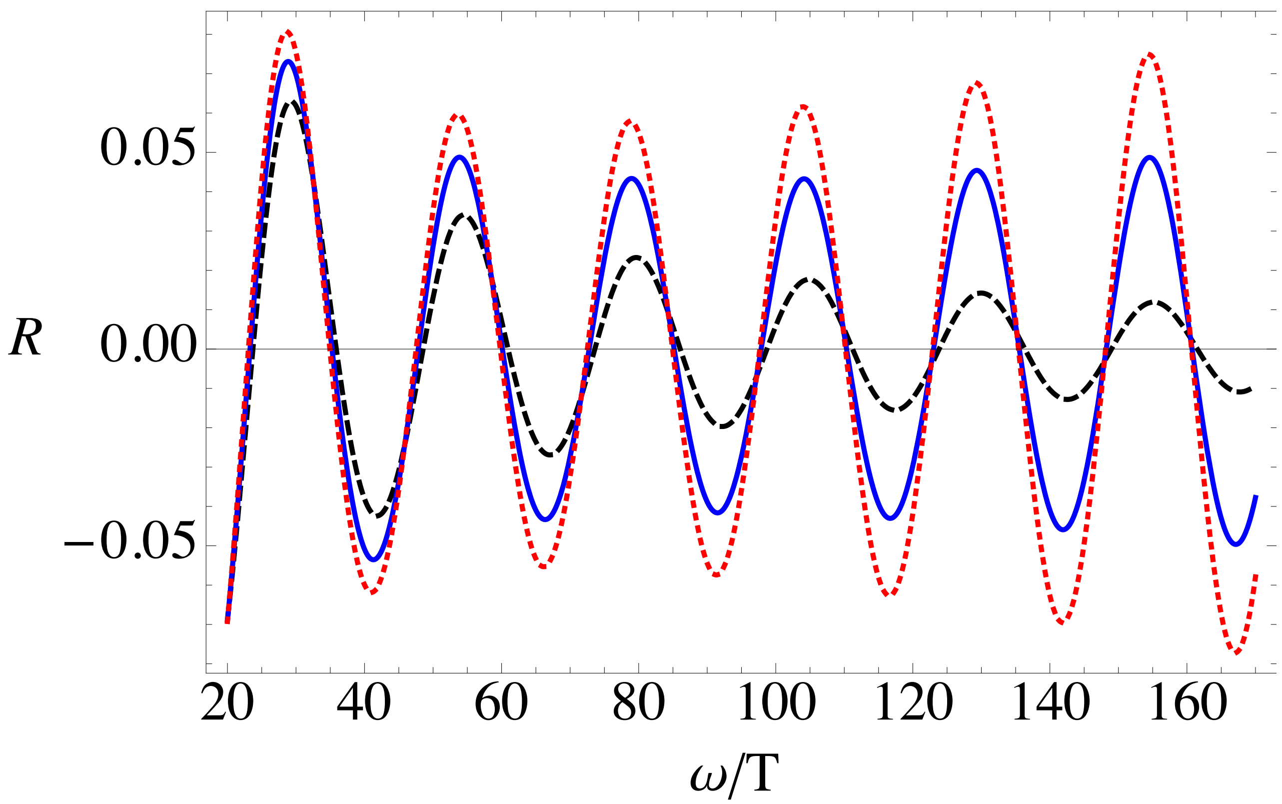


emission rate for photons r_s/r_h=1.01 and $\lambda = \infty, 120, 80, 40$

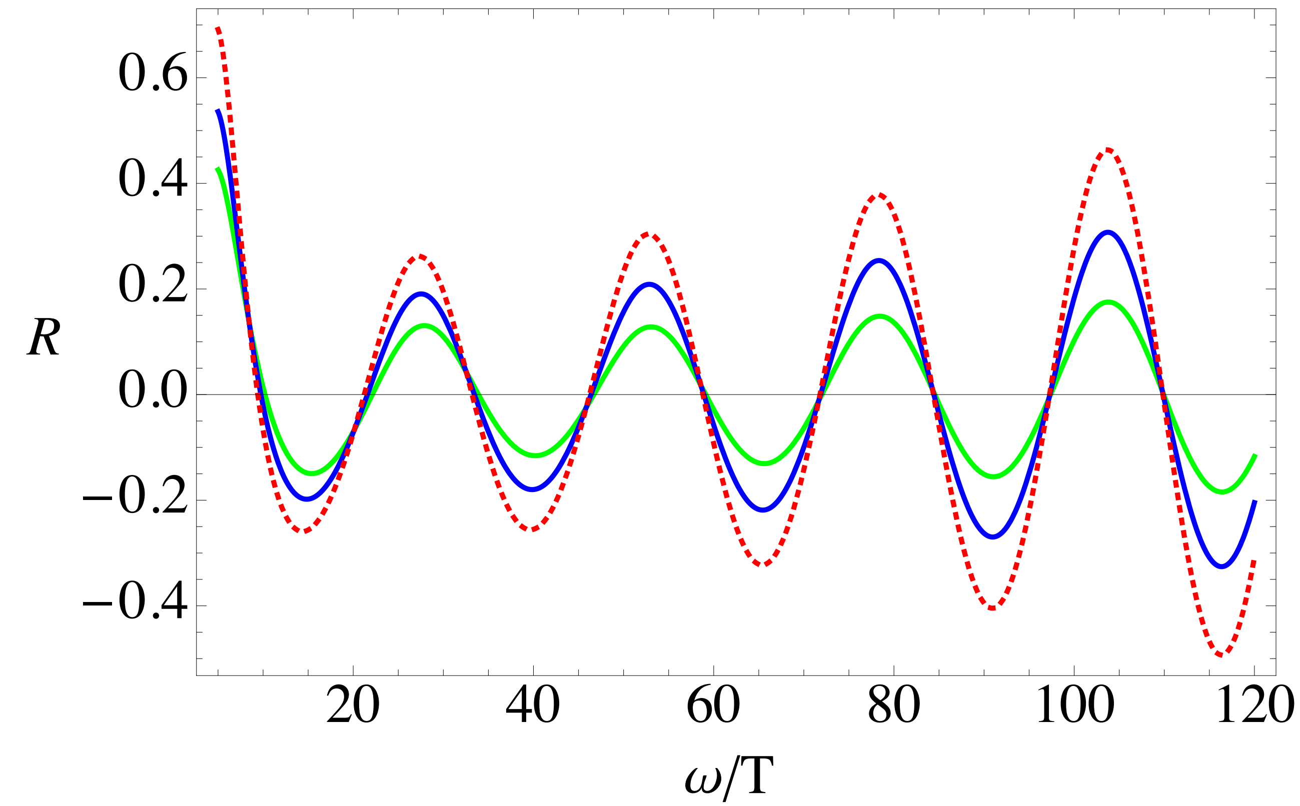
- Behaviour qualitatively similar to equilibrium case: in particular the result is much less sensitive to finite coupling corrections than QNM spectrum

Thermalization at finite coupling

Relative deviation from thermal limit for on shell photons



R for r_s/r_h=1.1 and $\lambda = \infty,\ 500,\ 300$

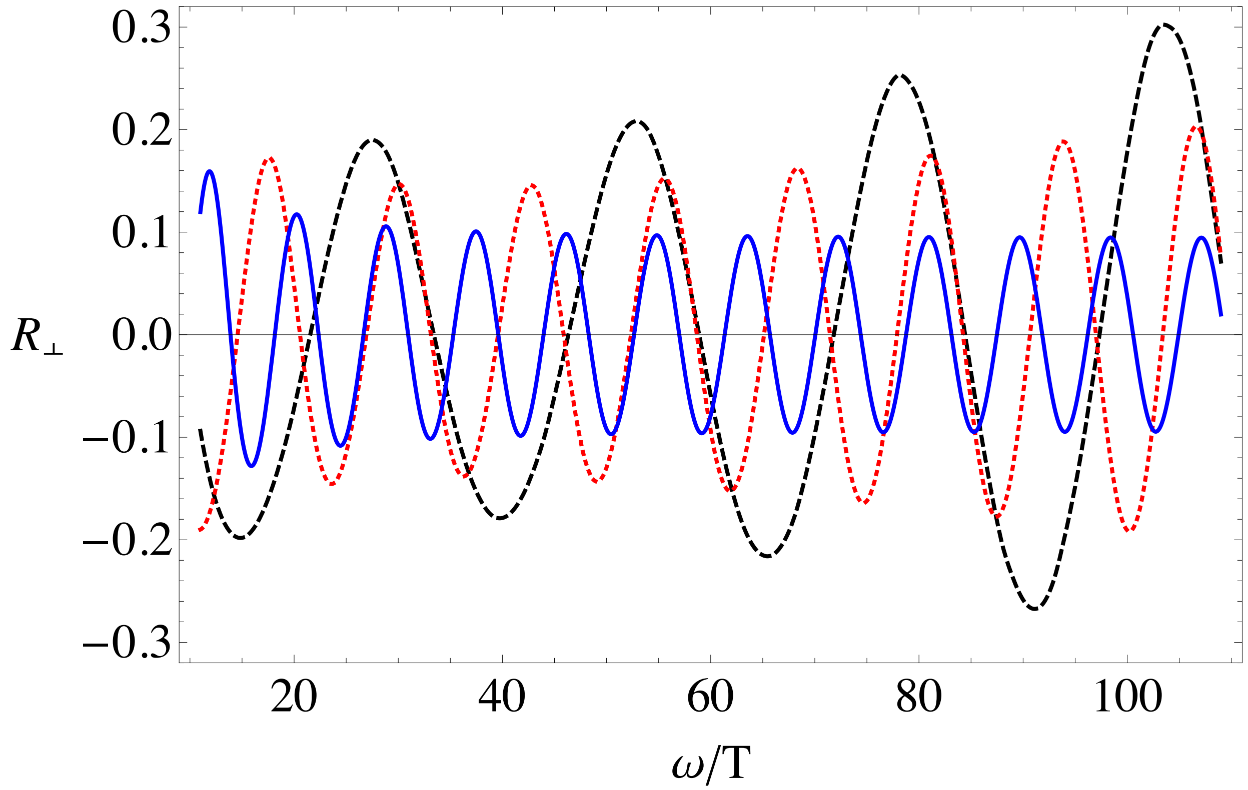


R for r_s/r_h=1.1 and $\lambda = 150,\ 100,\ 75$

- Behaviour of relative deviation changes at large frequency
- UV modes are no longer first to thermalize
- Decreasing the coupling: change happens at lower frequency

Thermalization at finite coupling

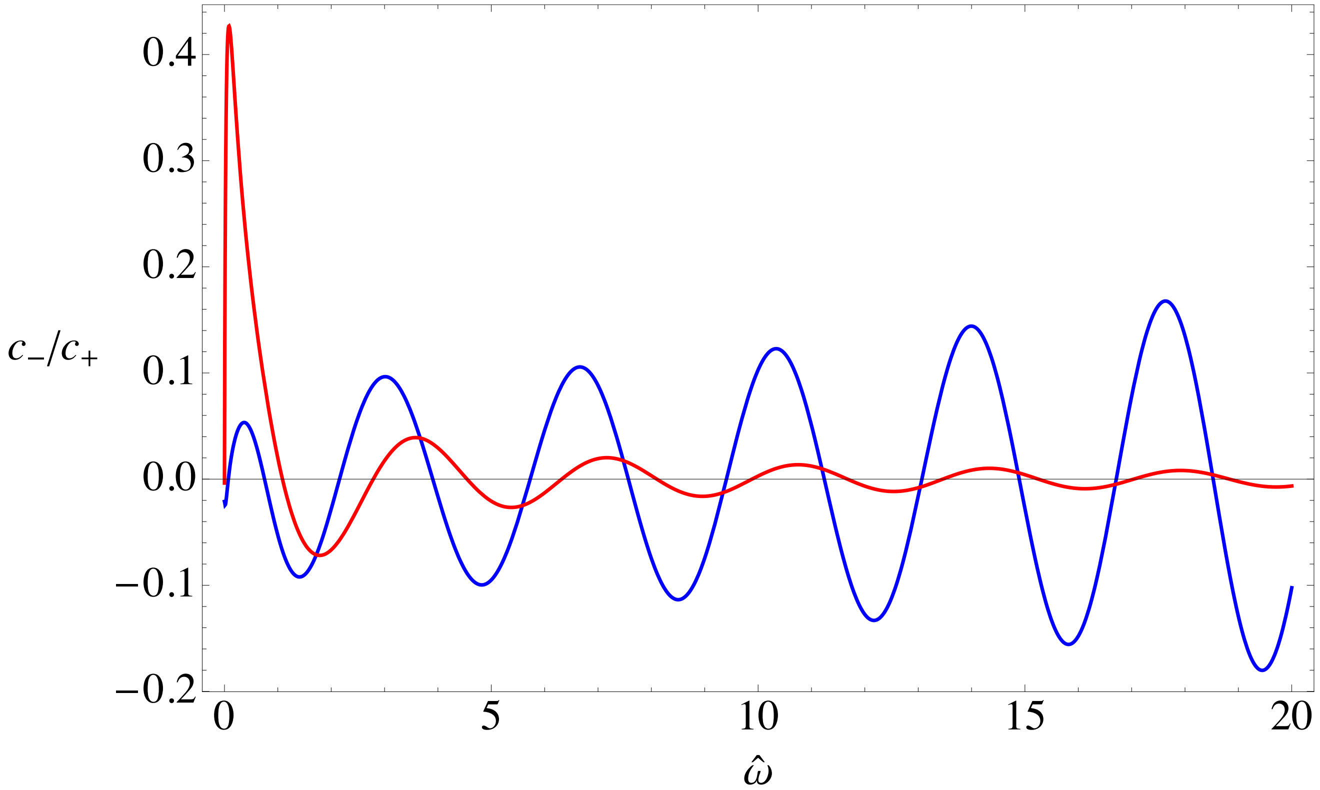
Virtuality dependence of the relative deviation



R for r_s/r_h=1.1 and c=1, 0.8, 0 for $\lambda = 100$

- For maximally virtual photons (c=0), R approaches a constant at $\omega \to \infty$
- For on-shell photons (c=1): amplitude of R rises linearly with ω
- Indication that thermalization pattern changes from top-down towards bottom-up

Thermalization at finite coupling



$$\Pi(\omega) \approx \Pi_{therm} \frac{1 + (C_0 + \gamma C_1)\frac{E'_{out}}{E'_{in}}}{1 + (C_0 + \gamma C_1)\frac{E_{out}}{E_{in}}}$$

$$\frac{c_-}{c_+} = C_0 + \gamma C_1$$

$$C_0 \approx \frac{1}{\omega}, \qquad C_1 \approx \omega$$

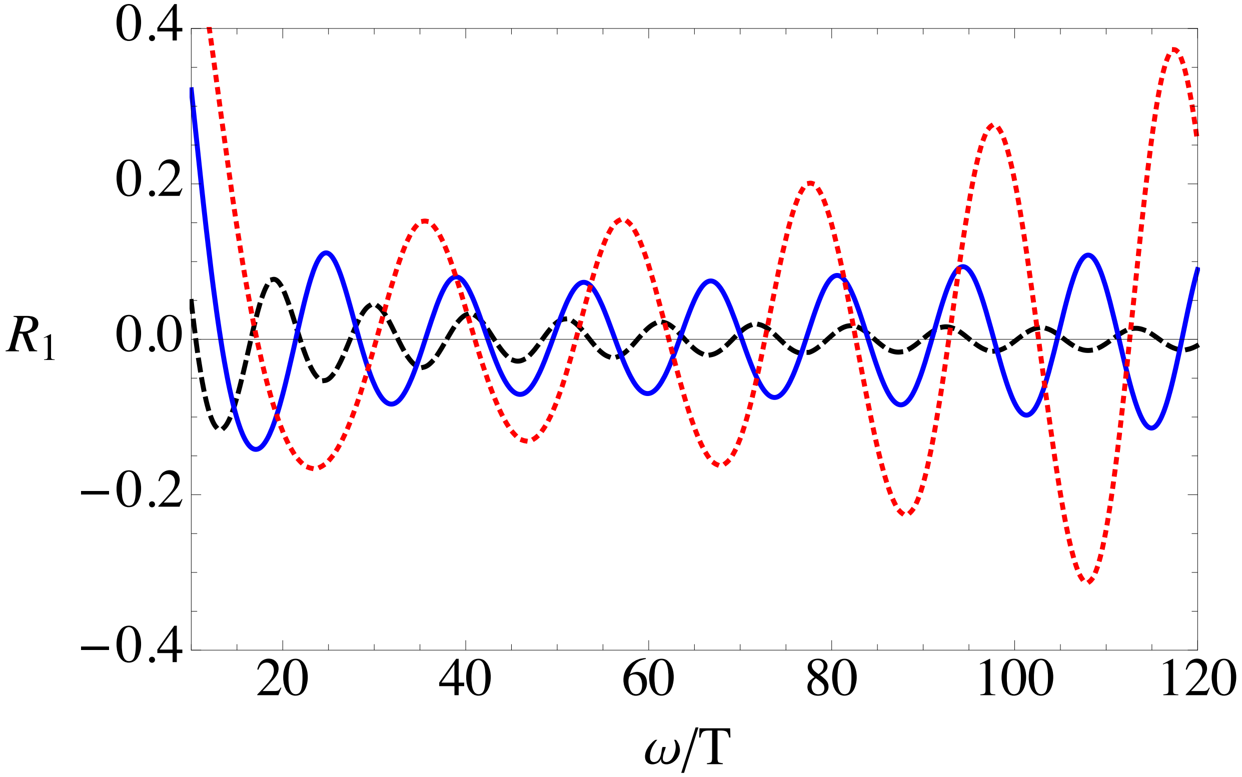
- behaviour of the fields near the horizon is crucial
- originates from the Schroedinger potential

WKB approximation

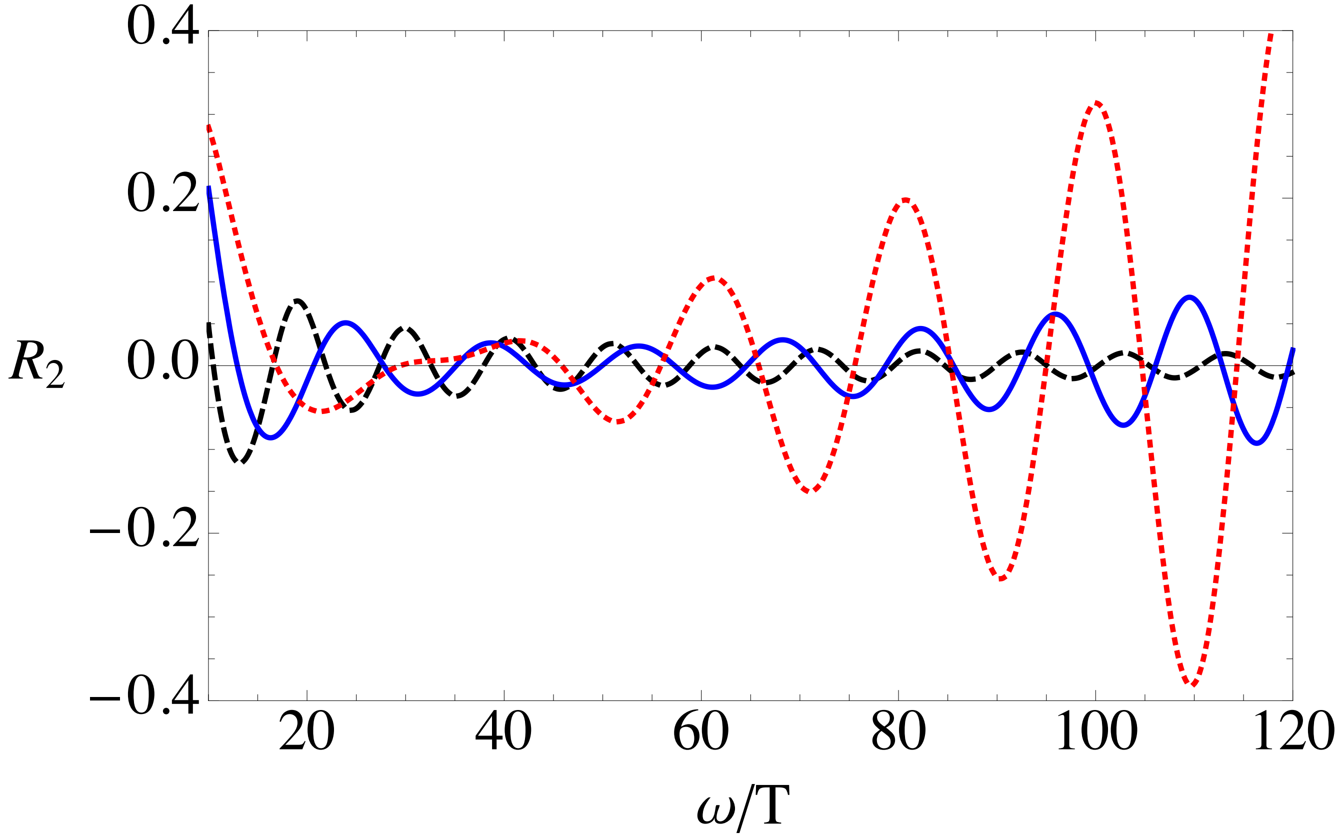
$$\chi(\hat{\omega}) \approx \hat{\omega}^{\frac{2}{3}} \left(1 + \frac{3\zeta(3)}{8\lambda^{\frac{3}{2}}} + \frac{f_1(u_s)}{\hat{\omega}} + \frac{f_2(u_s)\hat{\omega}}{\lambda^{\frac{3}{2}}} \right)$$

R at finite coupling: $T_{\mu\nu}$ correlators

Scalar channel



Shear channel



- Relative deviation for the scalar/shear channel for r_s/r_h=1.1, c=0, 6/9, 8/9 and $\lambda = 100$
- All three channels show the same behaviour
- Again similar to photons with the same dependence on c
- shift from top-down towards bottom-up

Reliability of results

What to make of all this? Evidence for the holographic plasma starting to behave like a system of weakly coupled quasiparticles, or simply

- Breakdown due to some approximation
  - Quasistatic limit Ok as long $\omega/T \gg 1$
  - Strong coupling expansion applied with care (NLO-LO)/LO $\lesssim$ $\mathcal{O}(1/10)$
- A peculiarity of the channels considered
  - EM current and $T_{\mu\nu}$ correlators probe systems in different ways
  - Purely geometric probes show different behaviour *(Galante, Schvellinger)*
- A sign of the unphysical nature of the collapsing shell model
  - Difficult to rule out, however QNM result universal

Conclusions

- Holographic (thermalization) calculations at finite coupling are possible and potentially a very fruitful exercise
- Indications that a holographic systems obtains weakly coupled characteristic within the realm of a strong coupling expansion
  - QNM modes: flow towards quasiparticle picture, independent of the thermalization model
  - Top-down thermalization pattern weakens and moves towards bottom-up
- As always: more work needed
  - in particular go beyond the quasistatic approximation and study full dynamical problem