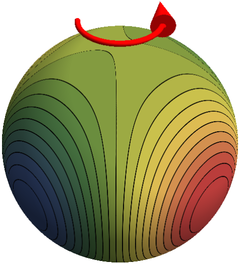
Black holes with a single Killing vector field

Jorge E. Santos

Cambridge University - DAMTP

Oxford - 23/02/2016



In collaboration with

Óscar J. C. Dias, Ben Niehoff and Benson Way

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6. Cosmic Censorship protect us from naked singularities.
7. If a gravitational system is linearly stable, it ought to be nonlinearly stable.

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In a longer talk, I would argue that all known SUSY black holes in AdS_5 are nonlinearly unstable.

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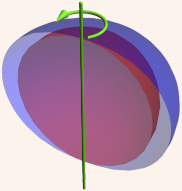

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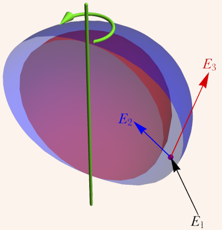

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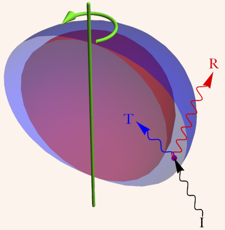

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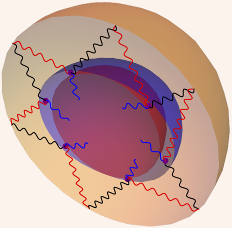

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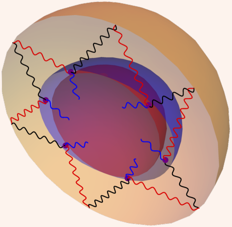

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$$ds^2 = -\frac{\Delta_r}{r^2+x^2}\left[\mathrm{d}t - (1-x^2)\mathrm{d}\phi\right]^2 + \frac{\Delta_x}{r^2+x^2}\left[\mathrm{d}t - (1+r^2)\mathrm{d}\phi\right]^2 + a^2(r^2+x^2)\left(\frac{\mathrm{d}r^2}{\Delta_r}+\frac{\mathrm{d}x^2}{\Delta_x}\right),$$

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$$\Delta_r = (1+r^2)\left(1+\frac{a^2}{L^2}r^2\right) - \frac{2M}{a}r\,,\quad \Delta_x = (1-x^2)\left(1-\frac{a^2}{L^2}x^2\right).$$

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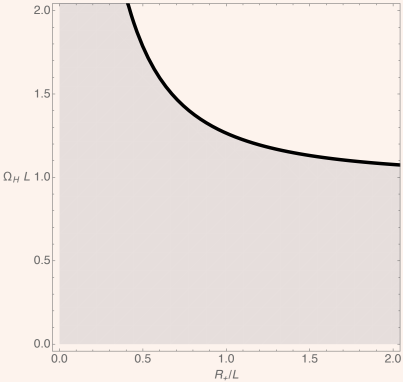
- ∂_t and ∂_ϕ are commuting Killing fields; decompose perturbations in Fourier modes: $e^{-i\omega t+im\phi}$.
- Unstable if quasi-normal modes with $\mathrm{Im}(\omega)>0$ exist.

Superradiance Instability - 3/3:



Phase Diagram for Kerr-AdS black holes

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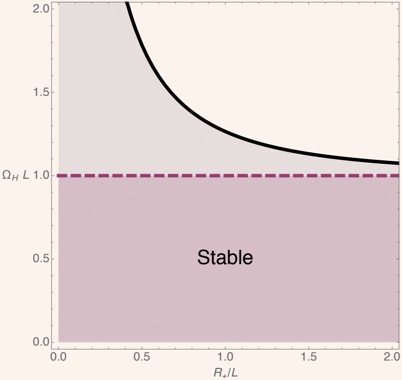

Kerr-AdS with $|\Omega_H L| \leq 1$:
likely to be stable - Hawking and Reall '00.

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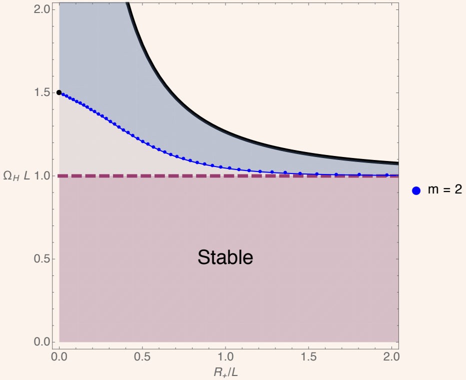

Perturbations with $m \neq 0$ are unstable if $\mathrm{Re}(\omega) \leq m\Omega_H$:
onset saturates inequality - Cardoso et al. '14.

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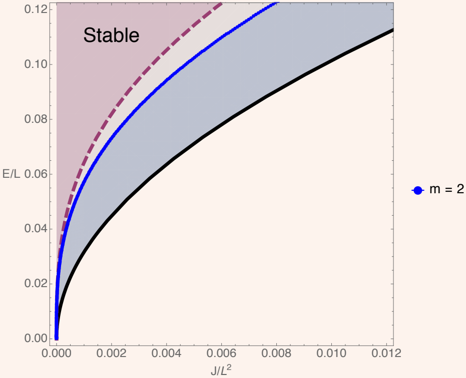

In the microcanonical ensemble:
natural variables are (J, E).

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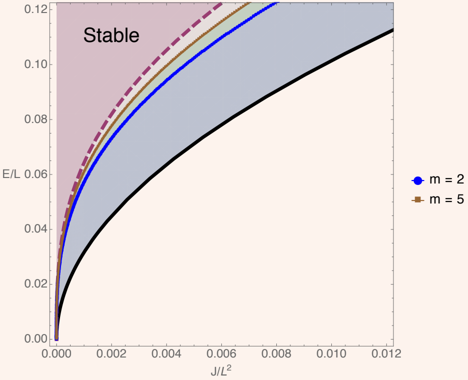

Higher m modes appear closer to $\Omega_H L = 1$:
$\Omega_H L = 1$ is reached $m \to +\infty$ - Kunduri et. al. '06.

The nonlinear stability of AdS

The stability problem for spacetimes in general relativity

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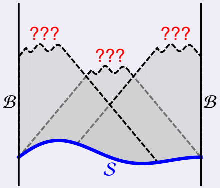

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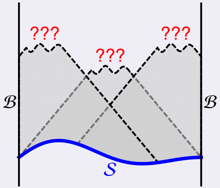

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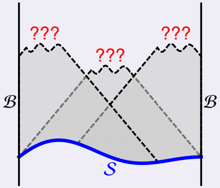

In particular, if a geodesically complete spacetime is perturbed, does it remain "complete"?

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- The energy cascades from low to high frequency modes in a manner reminiscent of the onset of turbulence.

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    - These solutions are special since they are exactly periodic in time and invariant under a single continuous symmetry.
    - Geons are analogous to nonlinear gravitational plane waves.
- This Heuristic argument has been observed numerically for certain types of initial data, but fails for other types.

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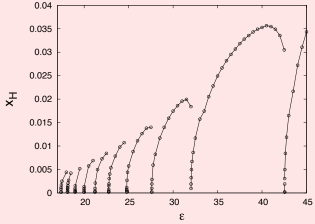

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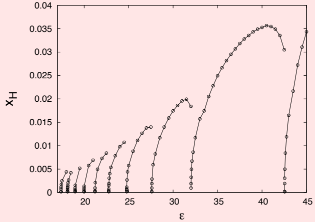

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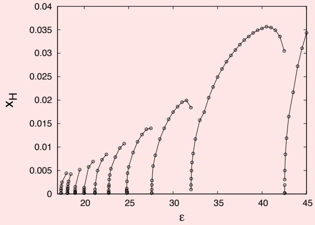

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- Certain types of initial data do not do this: do not seem to form black holes at late times! - Balasubramanian et. al.

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- Spherical scalar field collapse in AdS - Bizon and Rostworowski.
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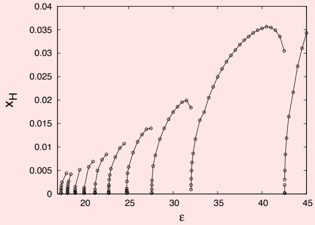

- Black holes form: $\Delta t \propto \varepsilon^{-2}$, matches naïve KAM intuition and $3^{\rm rd}$ order calculation - Dias, Horowitz and JES.
- Certain types of initial data do not do this: do not seem to form black holes at late times! - Balasubramanian et. al.
- Understand why special fine tuned solutions - Geons - exist.

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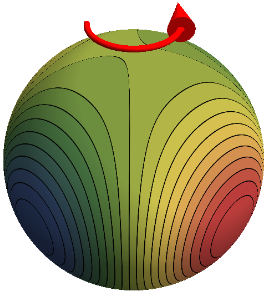

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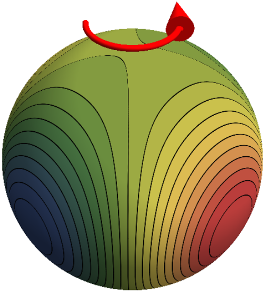

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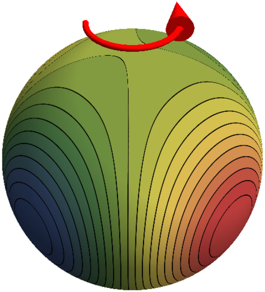

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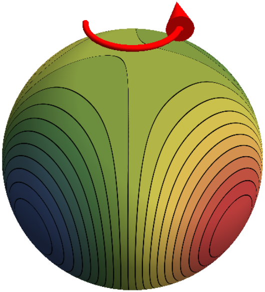

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Unclear if they can have the same energy, *i.e.* coexist, with large AdS black holes!

One black hole to interpolate them all and in the darkness bind them

Surfing the Geon:

- Since Geons rotate rigidly, one can ask whether small black holes can surf the Geon!

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- We have constructed these solutions: ten coupled 3D nonlinear partial differential equations of Elliptic type.

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\begin{aligned}
\mathrm{d}s^2 = \frac{L^2}{(1-y^2)^2}\Bigg[& -y^2 A \Delta_y \left(\mathrm{d}T + y\,\chi_1 \mathrm{d}y\right)^2 + \frac{4y_+^2\, B \mathrm{d}y^2}{\Delta_y} \\
& + \frac{4y_+^2 S_1}{2-x^2}\left(\mathrm{d}x + yx\sqrt{2-x^2}\chi_3 \mathrm{d}y + y^2 x\sqrt{2-x^2}\chi_2 \mathrm{d}T\right)^2 \\
& + (1-x^2)^2 y_+^2 S_2 \left(\mathrm{d}\Psi + y^2 \Omega \mathrm{d}T + \frac{x\sqrt{2-x^2}\chi_4 \mathrm{d}x}{1-x^2} + y\,\chi_5 \mathrm{d}y\right)^2 \Bigg]
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- 2D moduli space:

$$T \equiv \frac{1+3y_+^2}{4\pi y_+} \quad \text{and} \quad \varepsilon \equiv \int_0^\pi \mathrm{d}\phi\chi_4(0,1,\phi)\sin(m\,\phi)\,.$$

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- Bifurcating Killing sphere - Killing horizon generated by ∂_T.

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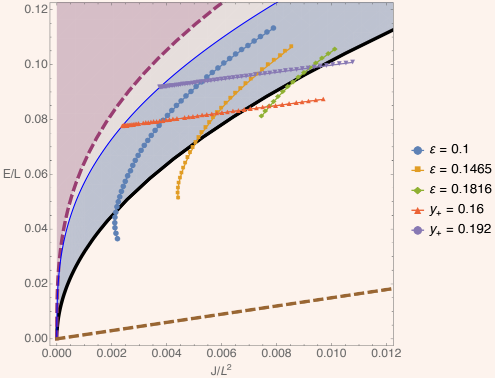

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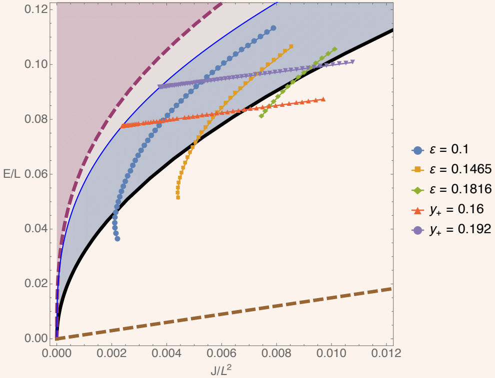

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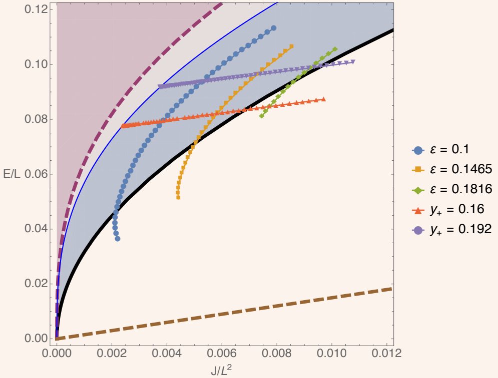

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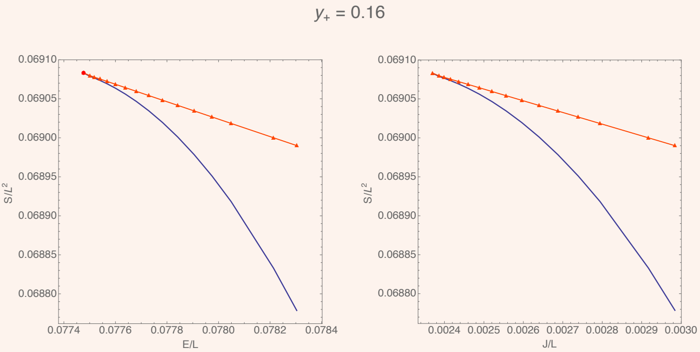

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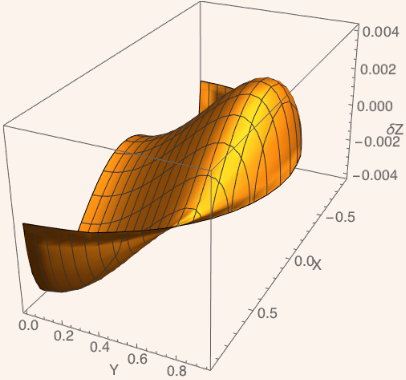

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- Their horizon is deformed along the ϕ direction along which they rotate - embedding in 3D spacetime - $\delta Z \equiv Z - \bar{Z}$.

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Conjecture: there is no endpoint -
Dias, Horowitz and JES '11

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Short answer:

No!

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Outlook:

- What is the field theory interpretation of this phenomenon?
- Can we make a connection with glassy physics?
- ...

Thank You!