Semi-holography for heavy-ion collisions

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Semi-holographic models

Semi-holography: dynamical boundary theory coupled to a strongly coupled conformal sector with gravity dual

oxymoron coined by Faulkner & Polchinski, JHEP 1106 (2011) 012 [arXiv:1001.5049] in study of holographic non-Fermi-liquid models

- retains only the universal low energy properties, which are most likely to be relevant to the realistic systems
- allows more flexible model-building

further developed for NFLs in:
A. Mukhopadhyay, G. Policastro, PRL 111 (2013) 221602 [arXiv:1306.3941]

Semi-holographic model for heavy-ion collisions

Aim: hybrid strong/weak coupling model of quark-gluon plasma formation
(QCD: strongly coupled in IR, weakly coupled in UV)

(different) successful example:
J. Casalderrey-Solana et al., JHEP 1410 (2014) 19 and JHEP 1603 (2016) 053

Idea of semi-holographic model by
E. Iancu, A. Mukhopadhyay, JHEP 1506 (2015) 003 [arXiv:1410.6448]:
combine pQCD (Color-Glass-Condensate) description of initial stage of HIC through overoccupied gluons with AdS/CFT description of thermalization

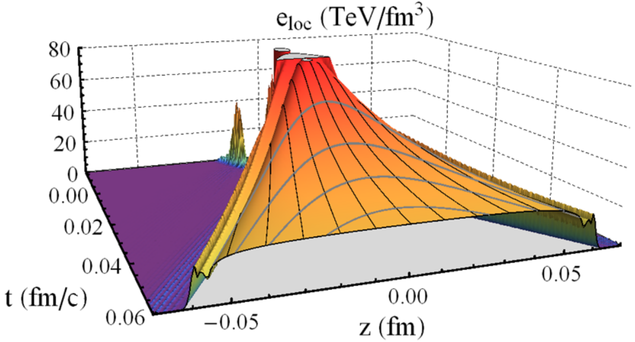
modified and extended recently in
A. Mukhopadhyay, F. Preis, A.R., S. Stricker, arXiv:1512.06445

- s.t. $\exists$ conserved local energy-momentum tensor for combined system
- verified in (too) simple test case

Gravity dual of heavy-ion collisions

pioneered and developed in particular by P. Chesler & L. Yaffe [JHEP 1407 (2014) 086]

most recent attempt towards quantitative analysis along these lines:
Wilke van der Schee, Björn Schenke, 1507.08195



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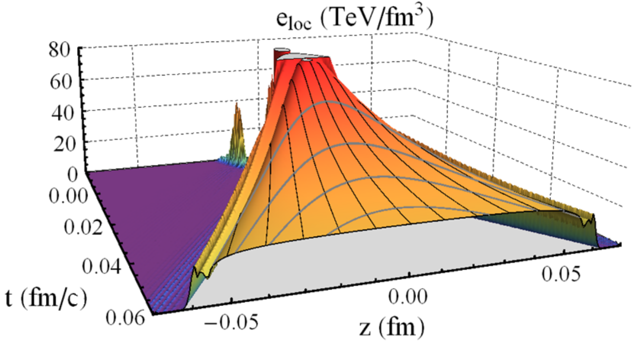


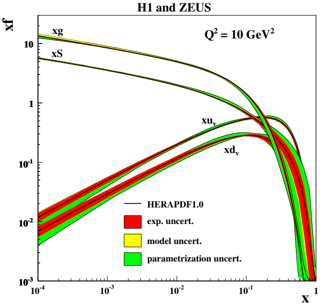
had to scale down energy density by a factor of 20 (6) for the top LHC (RHIC) energies

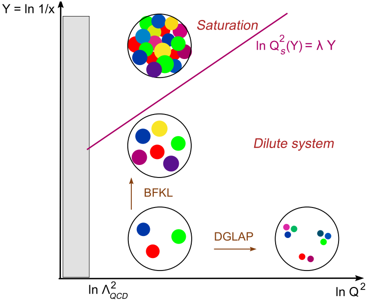
perhaps improved by involving pQCD for (semi-)hard degrees of freedom?

pQCD and Color-Glass-Condensate framework

recap: [e.g. F. Gelis et al., arXiv:1002.033]

gluon distribution $xG(x, Q^2)$ in a proton rises very fast with decreasing longitudinal momentum fraction x at large, fixed Q^2





HIC: high gluon density $\sim \alpha_s^{-1}$ at "semi-hard" scale Q_s ($\sim$ few GeV)

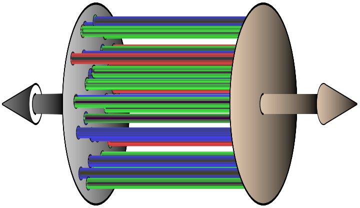
weak coupling $\alpha_s(Q_s) \ll 1$ but highly nonlinear because of large occupation numbers

description in terms of classical YM fields as long as gluon density nonperturbatively high

Color-Glass-Condensate evolution of HIC at LO

effective degrees of freedom in this framework:

1. color sources ρ at large x (frozen on the natural time scales of the strong interactions and distributed randomly from event to event)
2. gauge fields A^μ at small x
   (saturated gluons with large occupation numbers $\sim 1/\alpha_s$, with typical momenta peaked about $k_\perp Q_s$)

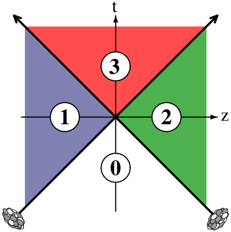


glasma: non-equilibrium matter, with high occupation numbers $\sim 1/\alpha_s$

initially longitudinal chromo-electric and chromo-magnetic fields that are screened at distances $1/Q_s$ in the transverse plane of the collision

Color-Glass-Condensate evolution of HIC at LO

colliding nuclei as shock waves with frozen color distribution



classical YM field equations

$$D_\mu F^{\mu\nu}(x) = \delta^{\nu +}\rho_{(1)}(x^-, \mathbf{x}_\perp) + \delta^{\nu -}\rho_{(2)}(x^+, \mathbf{x}_\perp)$$

in Schwinger gauge $A^\tau = (x^+ A^- + x^- A^+)/\tau = 0$

with ρ from random distribution (varying event-by-event)

outside the forward light-cone (3):

(causally disconnected from the collision)

pure-gauge configurations

$$A^+ = A^- = 0$$

$$A^i(x) = \theta(-x^+)\theta(x^-)A^i_{(1)}(\mathbf{x}_\perp) + \theta(-x^-)\theta(x^+)A^i_{(2)}(\mathbf{x}_\perp)$$

$$A^i_{(1,2)}(\mathbf{x}_\perp) = \frac{\mathrm{i}}{g} U_{(1,2)}(\mathbf{x}_\perp)\partial_i U^\dagger_{(1,2)}(\mathbf{x}_\perp)$$

$$U_{(1,2)}(\mathbf{x}_\perp) = \mathrm{P}\ \exp\left(-\mathrm{i}g \int \mathrm{d}x^\mp \frac{1}{\nabla^2_\perp}\rho_{(1,2)}(x^\mp, \mathbf{x}_\perp)\right)$$

inside forward light-cone:

numerical solution with initial conditions at $\tau = 0$:

$$A^i = A^i_{(1)} + A^1_{(2)}, \qquad A^\eta = \frac{\mathrm{i}g}{2}\left[A^i_{(1)}, A^i_{(2)}\right], \qquad \partial_\tau A^i = \partial_\tau A^\eta = 0$$

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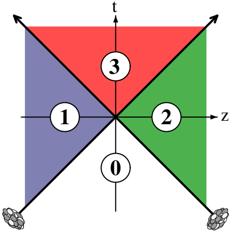

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Aim of semi-holographic model: include bottom-up thermalization from relatively soft gluons with higher α_s and their backreaction when they build up thermal bath

Semi-holographic glasma evolution

[E. Iancu, A. Mukhopadhyay, JHEP 1506 (2015) 003]
[A. Mukhopadhyay, F. Preis, AR, S. Stricker, arXiv:1512.06445]
UV-theory=classical Yang-Mills theory for overoccupied gluon modes with $k \sim Q_s$
IR-CFT=effective theory of strongly coupled soft gluon modes $k \ll Q_s$,
modelled by N=4 SYM gravity dual marginally deformed by: boundary metric $g^{(b)}_{\mu\nu}$, dilaton $\phi^{(b)}$, and axion $\chi^{(b)}$ which are functions of A_μ

$$S = S_{\rm YM}[A] + W_{\rm CFT}\left[g^{(b)}_{\mu\nu}[A], \phi^{(b)}[A], \chi^{(b)}[A]\right]$$

$W_{\rm CFT}$: generating functional of the IR-CFT (on-shell action of its gravity dual)
minimalistic coupling through gauge-invariant dimension-4 operators
IR-CFT energy-momentum tensor $\frac{1}{2\sqrt{-g^{(b)}}}\frac{\delta W_{\rm CFT}}{\delta g^{(b)}_{\mu\nu}} = \mathcal{T}^{\mu\nu}$ coupled to energy-momentum tensor $t_{\mu\nu}$ of YM (glasma) fields through

$$g^{(b)}_{\mu\nu} = \eta_{\mu\nu} + \frac{\gamma}{Q_s^4} t_{\mu\nu}, \quad t_{\mu\nu}(x) = \frac{1}{N_c}\mathrm{Tr}\Big(F_{\mu\alpha}F_\nu{}^\alpha - \frac{1}{4}\eta_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\Big);$$

$$\phi^{(b)} = \tfrac{\beta}{Q_s^4}h, \quad h(x) = \tfrac{1}{4N_c}\mathrm{Tr}(F_{\alpha\beta}F^{\alpha\beta}); \quad \chi^{(b)} = \tfrac{\alpha}{Q_s^4}a, \quad a(x) = \tfrac{1}{4N_c}\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

α, β, γ dimensionless and $O(1/N_c^2)$

Semi-holographic glasma evolution

IR-CFT: marginally deformed AdS/CFT

in Fefferman-Graham coordinates:

$$\begin{aligned}
\chi(z,x) &= \frac{\alpha}{Q_s^4}a(x)+\cdots+z^4\frac{4\pi G_5}{l^3}\mathcal{A}(x)+\mathcal{O}(z^6),\\
\phi(z,x) &= \frac{\beta}{Q_s^4}h(x)+\cdots+z^4\frac{4\pi G_5}{l^3}\mathcal{H}(x)+\mathcal{O}(z^6),\\
G_{rr}(z,x) &= \frac{l^2}{z^2},\\
G_{r\mu}(z,x) &= 0,\\
G_{\mu\nu}(z,x) &= \frac{l^2}{z^2}\Big(\underbrace{\eta_{\mu\nu}+\frac{\gamma}{Q_s^4}t_{\mu\nu}(x)}_{g^{(\mathrm{b})}_{\mu\nu}=g_{(0)\mu\nu}}+\cdots+z^4\big(\underbrace{\frac{4\pi G_5}{l^3}}_{2\pi^2/N_c^2}\mathcal{T}_{\mu\nu}(x)+P_{\mu\nu}(x)\big)\\
&\qquad +\mathcal{O}(z^4\ln z)\Big),
\end{aligned}$$

with $P_{\mu\nu}=\frac{1}{8}g_{(0)\mu\nu}\left(\left(\mathrm{Tr}\,g_{(2)}\right)^2-\mathrm{Tr}\,g_{(2)}^2\right)+\frac{1}{2}(g_{(2)}^2)_{\mu\nu}-\frac{1}{4}g_{(2)\mu\nu}\mathrm{Tr}\,g_{(2)}$

[de Haro, Solodukhin, Skenderis, CMP 217 (2001) 595]

Semi-holographic glasma evolution

Modified YM (glasma) field equations

$$\frac{\delta S}{\delta A_\mu(x)} = \frac{\delta S_{\rm YM}}{\delta A_\mu(x)} + \int \mathrm{d}^4 y \left(\frac{\delta W_{\rm CFT}}{\delta g^{(\rm b)}_{\alpha\beta}(y)} \frac{\delta g^{(\rm b)}_{\alpha\beta}(y)}{\delta A_\mu(x)} + \frac{\delta W_{\rm CFT}}{\delta \phi^{(\rm b)}(y)} \frac{\delta \phi^{(\rm b)}(y)}{\delta A_\mu(x)} + \frac{\delta W_{\rm CFT}}{\delta \chi^{(\rm b)}(y)} \frac{\delta \chi^{(\rm b)}(y)}{\delta A_\mu(x)} \right)$$

gives

$$D_\mu F^{\mu\nu} = \frac{\gamma}{Q_s^4} D_\mu \left(\hat{\mathcal{T}}^{\mu\alpha} F_\alpha{}^\nu - \hat{\mathcal{T}}^{\nu\alpha} F_\alpha{}^\mu - \frac{1}{2} \hat{\mathcal{T}}^\alpha_\alpha F^{\mu\nu} \right) + \frac{\beta}{Q_s^4} D_\mu \left(\hat{\mathcal{H}} F^{\mu\nu} \right) + \frac{\alpha}{Q_s^4} \left(\partial_\mu \hat{\mathcal{A}} \right) \tilde{F}^{\mu\nu}$$

with $\hat{\mathcal{T}}^{\alpha\beta} = \frac{\delta W_{\rm CFT}}{\delta g^{(\rm b)}_{\alpha\beta}} = \sqrt{-g^{(b)}} \mathcal{T}^{\alpha\beta}$, $\quad \hat{\mathcal{H}} = \frac{\delta W_{\rm CFT}}{\delta \phi^{(\rm b)}} = \sqrt{-g^{(b)}} \mathcal{H}$, $\quad \hat{\mathcal{A}} = \frac{\delta W_{\rm CFT}}{\delta \chi^{(\rm b)}} = \sqrt{-g^{(b)}} \mathcal{A}$

Total energy-momentum tensor of combined system

IR-CFT, like glasma EFT, interpreted as living in Minkowski space
instead of covariantly conserved energy-momentum tensor

$$\nabla_\mu \mathcal{T}^{\mu\nu}(x) = -\frac{\beta}{Q_s^4}\,\mathcal{H}(x)\nabla^\nu h(x),$$

with metric $g^{(\mathrm{b})}_{\mu\nu}(x) = \eta_{\mu\nu} + \frac{\gamma}{Q_s^4} t_{\mu\nu}(x)$

nonconservation in Minkowski space, with driving forces derived from UV $t_{\mu\nu}[A]$

$$\partial_\mu \mathcal{T}^{\mu\nu} = -\frac{\beta}{Q_s^4}\,\mathcal{H}\, g^{\mu\nu}_{(\mathrm{b})}[t]\,\partial_\mu h - \mathcal{T}^{\alpha\nu}\Gamma^\gamma_{\alpha\gamma}[t] - \mathcal{T}^{\alpha\beta}\Gamma^\nu_{\alpha\beta}[t]$$

with $\Gamma^\mu_{\nu\rho}[t] = \frac{\gamma}{2Q_s^4}\Big(\partial_\nu t^\mu{}_\rho + \partial_\rho t^\mu{}_\nu - \partial^\mu t_{\nu\rho}\Big) + \mathcal{O}\big(t^2\big)$

total conserved energy-momentum tensor in Minkowski space ($\partial_\mu T^{\mu\nu} = 0$):

$$T^{\mu\nu} = t^{\mu\nu} + \mathcal{T}^{\mu\nu} + \text{hard-soft interaction terms}$$

(but $\mathcal{T}^{\mu\nu}$ not purely soft, contains also some hard-soft pieces through $g^{\mu\nu}_{(\mathrm{b})}[t]$)

Total energy-momentum tensor of combined system

Temporarily replacing Minkowski metric $\eta_{\mu\nu}$ by $g^{\rm YM}_{\mu\nu}$:

$$T^{\mu\nu} = \frac{2}{\sqrt{-g^{\rm YM}}}\left[\frac{\delta S_{\rm YM}}{\delta g^{\rm YM}_{\mu\nu}(x)} + \int {\rm d}^4y \left(\frac{\delta W_{\rm CFT}}{\delta g^{(b)}_{\alpha\beta}(y)}\frac{\delta g^{(b)}_{\alpha\beta}(y)}{\delta g^{\rm YM}_{\mu\nu}(x)} + \frac{\delta W_{\rm CFT}}{\delta \phi^{(b)}(y)}\frac{\delta \phi^{(b)}(y)}{\delta g^{\rm YM}_{\mu\nu}(x)} + \frac{\delta W_{\rm CFT}}{\delta \chi^{(b)}(y)}\frac{\delta \chi^{(b)}(y)}{\delta g^{\rm YM}_{\mu\nu}(x)}\right)\right]$$

At $g^{\rm YM}_{\mu\nu} = \eta_{\mu\nu}$, this gives

$$\begin{aligned} T^{\mu\nu} &= t^{\mu\nu} + \hat{\mathcal{T}}^{\mu\nu} \\ &\quad - \frac{\gamma}{Q_s^4 N_c}\hat{\mathcal{T}}^{\alpha\beta}\left[{\rm Tr}(F_\alpha{}^\mu F_\beta{}^\nu) - \frac{1}{2}\eta_{\alpha\beta}{\rm Tr}(F^{\mu\rho}F^\nu{}_\rho) + \frac{1}{4}\delta^\mu_{(\alpha}\delta^\nu_{\beta)}{\rm Tr}(F^2)\right] \\ &\quad - \frac{\beta}{Q_s^4 N_c}\hat{\mathcal{H}}\,{\rm Tr}(F^{\mu\alpha}F^\nu{}_\alpha) - \frac{\alpha}{Q_s^4}\eta^{\mu\nu}\hat{\mathcal{A}}\,a \end{aligned}$$

Can indeed prove $\partial_\mu T^{\mu\nu} = 0$

Iterative solution

Practical implementation will have to be done presumably in iterative procedure

1. Solve LO glasma evolution with $\gamma = \beta = \alpha = 0$
2. solve gravity problem with boundary condition provided by glasma $t^{\mu\nu}(\tau)$,... to obtain $\mathcal{T}^{\mu\nu}(\tau), \ldots$
3. solve glasma evolution with $\gamma, \beta, \alpha \neq 0$ and given $\mathcal{T}^{\mu\nu}(\tau), \ldots$
4. goto 2) until convergence reached

Simple test case

First test with dimensionally reduced (spatially homogeneous) YM fields $A_\mu^a(t)$ which already have nontrivial (chaotic) dynamics

Simplest situation with $\beta = 0$ and $t^{\mu\nu}$ isotropic has closed-form gravity solution (constructed by F. Preis & S. Stricker)

only $\mathcal{E} + \mathcal{P}$ needed in semi-holographic glasma equations: function of $\tilde{p} = \frac{\gamma}{Q_s^4} p$

$$\frac{\bar{\mathcal{E}} + \bar{\mathcal{P}}}{N_c^2/2\pi^2} = (1-3\tilde{p})^{-3}(\tilde{p}+1)^{-4}\Big\{ c\left[1 - 27\tilde{p}^5 - 27\tilde{p}^4 + 18\tilde{p}^3 + 10\tilde{p}^2 - 7\tilde{p}\right] - \frac{9}{64}\tilde{p}\tilde{p}'^4 - \frac{1}{64}\tilde{p}'^4 + \frac{3}{32}\tilde{p}^2\tilde{p}'^2\tilde{p}'' + \frac{1}{16}\tilde{p}\tilde{p}'^2\tilde{p}'' - \frac{1}{32}\tilde{p}'^2\tilde{p}'' \Big\},$$

with c an integration constant that corresponds to having a black hole already at $t = 0$
(necessary because with homogeneity and isotropy and no dilaton/axion ($\beta = \alpha = 0$), there are no local degrees of freedom that could create black hole)
nevertheless nontrivial effects from boundary deformation on $\mathcal{T}^{\mu\nu}$

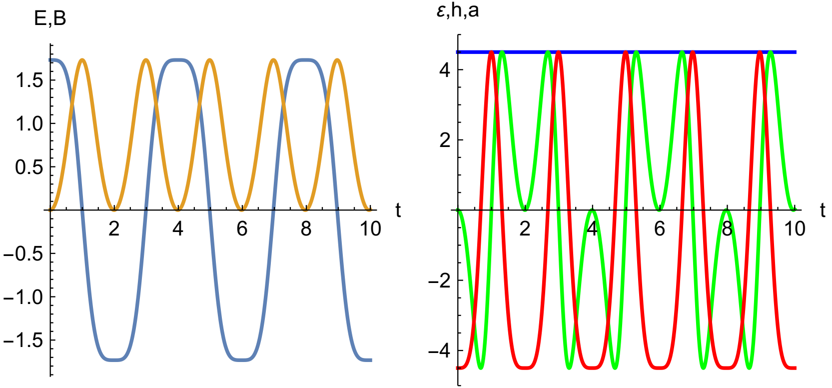
Simple test case

YM fields:

$\exists$ a solution with homogeneous isotropic energy-momentum tensor ($p = \varepsilon/3$) by homogeneous color-spin locked oscillations $A_0^a = 0, \quad A_i^a = \delta_i^a f(t)$

$f(t) = C\,\mathrm{sn}(C(t-t_0)|-1)$ (Jacobi elliptic function sn)

$E_i^a = \delta_i^a f', \quad B_i^a = \delta_i^a f^2 \qquad \varepsilon = const.,\ h = -\frac{1}{2}(\mathbf{E}^a \cdot \mathbf{E}^a - \mathbf{B}^a \cdot \mathbf{B}^a),\ a = -\mathbf{E}^a \cdot \mathbf{B}^a$



Convergence of iterations

Coupled glasma equation of test case is 4th order nonlinear ODE
— no reasonable solutions found directly —

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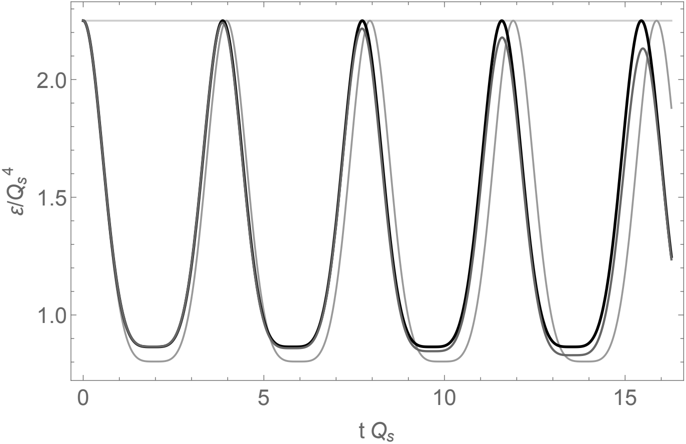
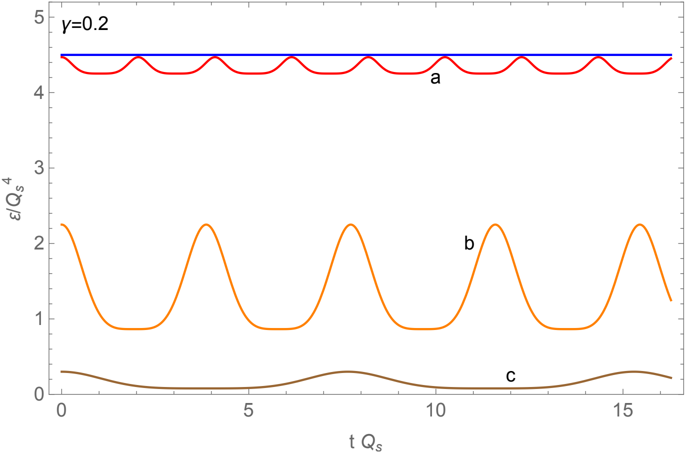
Coupled glasma equation of test case is 4th order nonlinear ODE
— no reasonable solutions found directly —
but iterative solution converges very quickly:



(UV not able to give off energy to IR permanently because of isotropy and homogeneity: gravity dual does not have propagating degrees of freedom!)

Solutions with different IR entropy

initial conditions with
little (a) - medium (b) - large (c)
thermal (IR) contribution to total (conserved) energy



Solutions with different IR entropy

initial conditions with
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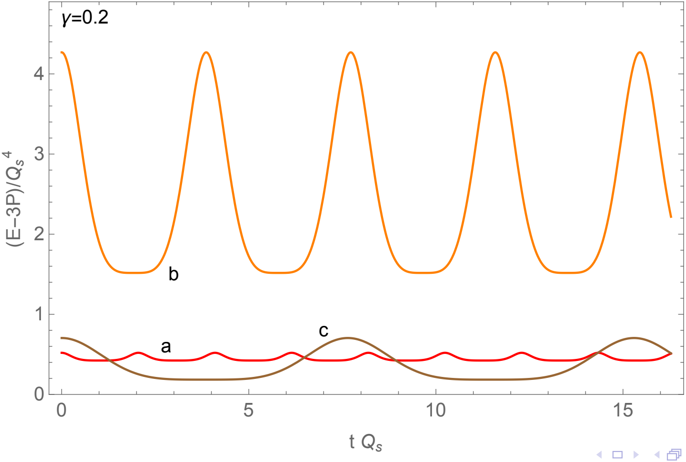
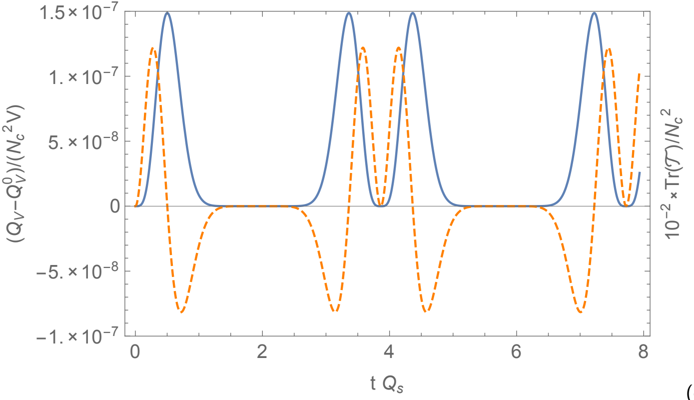
trace of the full energy-momentum tensor ("interaction measure"):



Energy exchanges but no thermalization

No test of thermalization yet:
entropy (area of the black hole) is conserved,
canonical charge of the black hole changes only according to trace anomaly:



(case b)

blue: canonical charge (thermal energy) returns to same value at stationary points (at different extrema of $\varepsilon^{\mathrm{YM}}$)
orange: $\mathrm{Tr}\,\mathcal{T}$

Conclusions and outlook

- Pure gauge-gravity thermalization likely too strong
- Semi-holographic framework of Iancu and Mukhopadhyay proposes to combine LO glasma evolution with thermalization of soft degrees of freedom in AdS/CFT
- First tests suggest that proposed iterative scheme can be numerically stable and convergent
- New scheme has conserved total energy-momentum tensor (in Minkowski space)
  formal proof + numerical verification in simple test case
- Next: anisotropic homogeneous toy models and/or dynamical scalar d.o.f. in bulk
- Also ongoing: Semi-holographic setup for coupling Yang-Mills fluctuations to late-time hydro evolution (with Y. Hidaka, A. Mukhopadhyay, F. Preis, A. Soloviev, D.-L. Yang)

Details of gravitational side of test case

Homogeneous isotropic ansatz in Eddington-Finkelstein coordinates

$$ds^2 = -A(r,v)dv^2 + 2dr\,dv + \Sigma(r,v)d\vec{x}^2$$

equations of motion take the compact form

$$\begin{aligned}
0 &= \Sigma(\dot{\Sigma})' + 2\Sigma'\dot{\Sigma} - 2\Sigma^2 \,, \\
0 &= A'' - 12\Sigma'\dot{\Sigma}/\Sigma^2 + 4 \\
0 &= 2\ddot{\Sigma} - A'\dot{\Sigma} \\
0 &= \Sigma''
\end{aligned}$$

Power series ansatz in r involves only finite number of terms, one integration constant c

$$\begin{aligned}
A(r,v) &= r^2\left(1 - \frac{c}{r^4\Sigma_0(v)}\right) - 2r\frac{\partial_v\Sigma_0(v)}{\Sigma_0(v)} \\
\Sigma(r,v) &= r\,\Sigma_0(v)
\end{aligned}$$

Details of gravitational side of test case

Transformation to Fefferman-Graham coordinates with

$$g_{(0)\mu\nu} = \mathrm{diag}\left(-1 + \frac{\gamma}{Q_s^4}3p, 1 + \frac{\gamma}{Q_s^4}p, 1 + \frac{\gamma}{Q_s^4}p, 1 + \underbrace{\frac{\gamma}{Q_s^4}p}_{\tilde{p}}\right)$$

$$T^\mu_\nu = \frac{N_c^2}{2\pi^2}\mathrm{diag}(-\mathcal{E}, P, P, P),$$

$$r^{-1} = u = u_1(t)z + O(z^2), \quad v = t + O(z)$$

$$\begin{aligned}
\mathcal{E} &= \frac{3}{4}\frac{c}{r^4\Sigma_0(v)}u_1^4 - \frac{2(u_1\Sigma_0^{(1)} - \Sigma_0 u_1^{(1)})^4}{16\Sigma_0^4}\,, \\
P &= \frac{1}{4}\frac{c}{r^4\Sigma_0(v)}u_1^4 \\
&\quad + \frac{1}{16\Sigma_0^4}\Big[\left(u_1\Sigma_0^{(1)} - \Sigma_0 u_1^{(1)}\right)^2\Big(-3\Sigma_0^2(u_1^{(1)})^2 + u_1^2\left((\Sigma_0^{(1)})^2 - 4\Sigma_0\Sigma_0^{(2)}\right) \\
&\qquad + 2\Sigma_0 u_1\left(\Sigma_0^{(1)}u_1^{(1)} + 2\Sigma_0 u_1^{(2)}\right)\Big)\Big]
\end{aligned}$$

with $u_1 = \frac{1}{\sqrt{1-3\tilde{p}}}$, $\Sigma_0 = \sqrt{\frac{1+\tilde{p}}{1-3\tilde{p}}}$

Details of gravitational side of test case

Solution in Eddington-Finkelstein coordinates

$$\begin{aligned} A(r,v) &= r^2\left(1-\frac{c}{r^4\Sigma_0(v)}\right)-2r\frac{\partial_v\Sigma_0(v)}{\Sigma_0(v)} \\ \Sigma(r,v) &= r\,\Sigma_0(v) \end{aligned}$$

is locally diffeomorphic to Schwarzschild solution

$$ds^2 = -R^2\left(1-\frac{c}{R^4}\right)dV^2 + 2dR\,dV + R^2 d\vec{x}^2$$

through coordinate transformation $R = r\Sigma_0(v)$ and $V = \int \frac{dv}{\Sigma_0(v)}$

Time dependence of boundary metric has however nontrivial effect on Brown-York stress tensor $\mathcal{T}^{\mu\nu}$