# Effective actions for fluids from holography and the membrane paradigm

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Fluid behavior is ubiquitous in physics

**Fluid dynamics** is the low energy **effective description** of a system valid when fluctuations around thermal equilibrium are sufficiently long-wavelength $L \gg l_{mfp}$

**Conventional description:**

[Landau et al.]

- $\nabla_\mu T^{\mu\nu} = 0$
- constitutive relations
- constraints on the transport coefficients coming from $\nabla_\mu J^\mu \geq 0$

**Effective field theory description:**

[Nicolis, Son et al. 2006]

- based on an action principle
- more economic and natural
- no systematic inclusion of dissipation so far

What can holography teach us?

Holographic gravity
$=$
QFT $+$ its renormalization group flow

- one should be able to **derive the low energy effective action** of the dual field theory from holography

- In gravity **dissipation is naturally encoded** in the one way nature of the **event horizon** [Nickel and Son 2010]

  $\Rightarrow$ one should be able to characterize easier the nature of dissipation

**Membrane paradigm:**

[Damour; Thorne, MacDonald and Price 80´s]

**approximation scheme** in which near horizon details of a black hole are neglected and one retains only the ingoing behavior property of the horizon

- In order to include dissipation one can rely on such membrane paradigm approximation
- It is important to understand what are the limits of validity of such approximation

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3) Is the membrane paradigm a good approximation?

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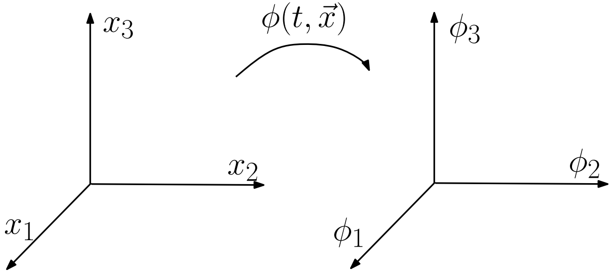
Consider for simplicity an uncharged fluid

**Fluid degrees of freedom:**

$\phi^I = \phi^I(t, \vec{x})$ as a map between Eulerian and Lagrangian coordinates



# Effective actions for fluids -The symmetries

**Spacetime symmetries:**

- Poincaré invariance

**Internal symmetries:**

- shifts $\phi^I \to \phi^I + c^I$
- rotations $\phi^I \to R^I_J \phi^J$
- for ideal fluids volume preserving diffs invariance

$$\phi^I \to \xi^I(\phi);\ \det\left(\frac{\partial \xi^I}{\partial \phi^J}\right) = 1$$

Goldstones break the global subgroup of the internal and spacetime symmetries down to a diagonal subgroup

$$\vec{\phi} = \vec{x} + \vec{\pi}(t, \vec{x})$$

# Effective actions for fluids - The leading order

Based on those symmetries construct the most general effective action in a derivative expansion

$$S^{(0)} + S^{(1)} + S^{(2)} + \dots$$

**The leading order effective action:**

$$S^{(0)} = \int d^d x \sqrt{-g} F(s)$$

where s is unique volume preserving diffs invariant object

$$s = \sqrt{\det(\partial_\mu \phi^I \, \partial_\nu \phi^J \, g^{\mu\nu})}$$

# Effective actions for fluids - The stress-energy tensor

- The conserved stress-energy tensor is the ideal fluid stress tensor

$$T^{(0)}_{\mu\nu} = p\,(g_{\mu\nu} + u_\mu u_\nu) + \rho\, u_\mu u_\nu$$

- provided that

$$\rho = -F, \quad p = -F's + F, \quad T = -F'$$

$$J^\mu = {}^*(d\phi^1 \wedge d\phi^2 \wedge \dots)$$

$$s = \sqrt{-J_\mu J^\mu}, \quad J^\mu = s\, u^\mu$$

- $\nabla_\mu J^\mu = 0$ identically, hence this construction is dissipationless

# Effective actions for fluids - The linearized expansion

- Divide into longitudinal and transverse modes $\vec{\phi} = \vec{x} + \vec{\pi}$

$$\pi = \pi^T + \pi^L, \quad \text{such that} \quad \nabla \wedge \pi^L = 0, \quad \nabla \cdot \pi^T = 0$$

- The action up to quadratic order in an amplitude expansion

$$S^{(0)} \sim \int d^d x \left( (\dot{\pi}^T)^2 + (\dot{\pi}^L)^2 - c_s^2 (\nabla \cdot \pi^L)^2 \right) + \ldots$$

- The dispersion relations for the Goldstones are then

$$\begin{aligned} \pi^T : &\quad \omega_T = 0 \\ \pi^L : &\quad \omega_L = c_s k \end{aligned}$$

- $\pi^T$ is non propagating, reflecting the volume preserving diffs invariance

# Effective actions for fluids - Comments

- Proceed at higher orders: $S^{(1)}, \quad S^{(2)}, \ldots$
- Application to superfluids, solids, inflationary models, quantum Hall effect etc. [Nicolis et al, Rangamani et al, Son et al]
- Dissipative effects?
- Is volume preserving symmetry a necessary fundamental symmetry? [Rangamani et al 2012]

# Outline

Effective actions for fluids

Motivation

1) Effective actions for fluids from holography

2) Including dissipation

3) Is the membrane paradigm a good approximation?

Conclusions & Outlook

# Motivation

1) Is it possible to **derive** an effective action for conformal fluids from holography?

$\Rightarrow$ **Yes!** This provides an explicit example of such effective actions constructions

2) Is it possible to **decouple** the dissipative from the dissipationless sector?

$\Rightarrow$ **No!** Certain divergent terms are only removed when dissipation is taken into account

3) What are the **limits of validity** of the membrane paradigm as an approximation scheme?

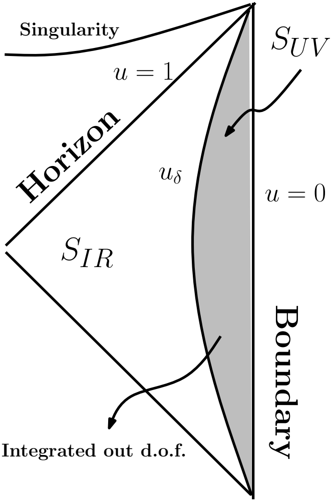
$\Rightarrow$ OK for hydrodynamic quasi normal modes*!
KO for massive quasi normal modes!

# Outline

# 1) Effective actions from holography - The set-up

Integrating out high energy d.o.f. $\Leftrightarrow$ ¨Integrating out¨ shells of geometry

[Faulkner, Liu and Rangamani; Heemskerk and Polchinski, 2010]



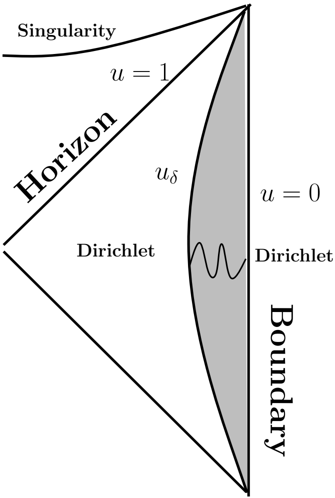

- Divide spacetime in UV and IR with a finite cutoff $u_\delta$

$$S^{\text{tot.}} = S_{IR} + S_{UV}$$

- Solve a double-Dirichlet problem for gravitational perturbations in UV

**The effective action in holography:**
is the (partially) on-shell UV action
$+$ near horizon limit $u_\delta \to 1$

# 1) Effective actions from holography - The Goldstones




**The Goldstones:**

correspond to a spontaneous symmetry breaking by the classical solution with double-Dirichlet boundary conditions

Poincaré $\times$ Poincaré $\rightarrow$ Diag (Rot. + Transl.)

- On a finite cutoff: $\vec{\pi}, \quad \pi^t$
- On the horizon: $\vec{\pi}$

- Goldstones correspond to diffeomorphisms $x^\alpha \rightarrow x^\alpha + \xi^\alpha(x)$ from general gauge to radial gauge where $\xi^\alpha(x) = \pi^\alpha(x)$

# 1) Effective actions from holography - Ex: linearized pert. in AdS$_5$

- Low energy behavior of thermal $\mathcal{N}=4$ SYM (conformal fluid)

- The action is

$$S = \frac{1}{2k_5^2}\int d^5x\sqrt{-g}(R-2\Lambda)$$

- Black-brane geometry background in AdS

$$ds^2 = -\frac{(\pi T L)^2}{u}(-f(u)dt^2 + d\vec{x}^2) + \frac{L^2}{4u^2 f(u)}du^2, \quad f = 1-u^2$$

- Linearized perturbations: $\delta h_{\mu\nu}(t,x,u) = \int \frac{d\omega dk}{(2\pi)^2}\delta h_{\mu\nu}(\omega,k,u)e^{-i\omega t + ikx}$

transverse sector: $\delta h_{t\alpha}, \quad \delta h_{x\alpha}, \quad \delta h_{\alpha u}$ with $\alpha = y, z$

longitudinal sector: $\delta h_{tt}, \ \delta h_{tx}, \ \delta h_{\vec{x}\vec{x}}, \ \delta h_{tu}, \ \delta h_{xu}, \ \delta h_{uu}$

# 1) Effective actions from holography - Ex: transverse sector

Transverse sector: $\delta h_{t\alpha}, \quad \delta h_{x\alpha}, \quad \delta h_{\alpha u}$ with $\alpha = y, z$

- E.o.m: 2 second order + 1 first oder constraint
- Since we want to solve a double-Dirichlet problem we leave the constraints unsolved
- The (hydrodynamic) solution is not unique since it depends on the arbitrary gauge choice encoded in the fields $\delta h_{u\alpha}$
- Goldstones are the Wilson line-like objects

$$\pi^\alpha \sim \int_0^{u_\delta} \delta h_{u\alpha} du$$

- Imposing radial gauge $\delta h_{u\alpha} = 0$, the Goldstones are non trivial boundary conditions to be imposed on the second boundary

# 1) Effective actions from holography - Ex: transverse sector

- Imposing vanishing Dirichlet boundary conditions the (partially) on-shell UV action

$$S_T^{(0)} \sim \int d^4x \left( (\dot{\pi}^\alpha)^2 - c_T^2 (\nabla \wedge \pi^\alpha)^2 \right) + \ldots$$

- The dispersion relation is non vanishing on a generic cutoff (Volume preserving diffs breaking?)

$$\omega_T = \pm\, c_T\, k + \mathcal{O}(k^2)$$

where

$$c_T = \frac{u_\delta}{\sqrt{-\log(1-u_\delta^2)}} \to \mathcal{O}(1-u_\delta)$$

# 1) Effective actions from holography - Ex: Longitudinal sector

Longitudinal sector: $\delta h_{tt}, \quad \delta h_{tx}, \quad \delta h_{\vec{x}\vec{x}}, \quad \delta h_{tu}, \quad \delta h_{xu}, \quad \delta h_{uu}$

- The Goldstone bosons are

$$\pi^t \sim \int_0^{u_\delta} \delta h_{tu}\, du, \quad \pi^x \sim \int_0^{u_\delta} \delta h_{xu}\, du,$$

- $\delta h_{uu}$ parametrizes the position of the cutoff. Can be integrated out from the effective action

$$\begin{aligned} S_L^{(0)} \quad &\sim \quad \int d^4x \Big( f_\delta ((\dot{\pi}^t)^2 - c_t^2 (\partial_x \pi^t)^2) + f_\delta\, \dot{\pi}^t\, \partial_x \pi^x + \\ &\qquad + (\dot{\pi}^x)^2 - c_s^2 (\partial_x \pi^x)^2 \Big) + \dots \\ &\rightarrow \quad \int d^4x \Big( (\dot{\pi}^x)^2 - c_s^2 (\partial_x \pi^x)^2 \Big) + \mathcal{O}(1 - u_\delta) \end{aligned}$$

# 1) Effective actions from holography - Longitudinal sector

- The dispersion relation after the near horizon limit $u_\delta \to 1$

$$\pi^x : \quad \omega_L = \pm\frac{1}{\sqrt{3}}k + \mathcal{O}(1-u_\delta) + \mathcal{O}(k^2)$$

$$\pi^t : \quad \omega = \pm\sqrt{\frac{2}{3}}k + \mathcal{O}(1-u_\delta) + \mathcal{O}(k^2)$$

- However we see that the timelike Goldstone decouples from the effective action when $u_\delta \to 1$ and one has to discard such mode on the horizon
- At higher order in hydro expansion the dispersion relation is divergent!

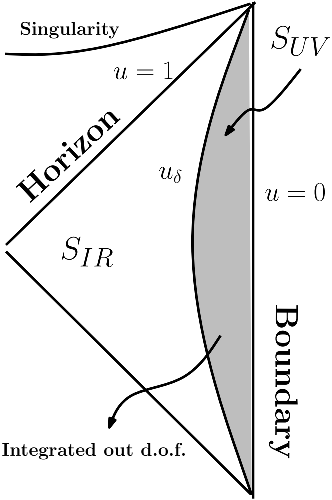
$$\pi^x : \quad \omega_L = \pm\frac{1}{\sqrt{3}}k + \left(\text{finite} + \#\log(1-u_\delta)\right)k^3 + \mathcal{O}(1-u_\delta) + \mathcal{O}(k^4)$$

# 1) Effective actions from holography - Comments

- Explicit example of derivation of a dissipationless effective action for fluids from the microscopic theory
- Perfect agreement with field theory prediction at lowest order in hydro expansion and quadratic order in linearized expansion
- Subleading order in hydro expansion has issues
- On a finite cutoff the transverse mode is dynamical. Is volume preserving diffs slightly broken?
- Be careful to the timelike mode!
- How to see volume preserving diffs symmetry from first principles geometrically?

# Outline

# 2) Including dissipation - The set-up




- Horizon is a dissipative membrane
- Couple the UV to the dynamical IR sector via a dynamical metric $G$ on the cutoff [Nikel and Son, 2010]

$$\frac{\delta S_{IR}}{\delta G} + \frac{\delta S_{UV}}{\delta G} = 0$$

**Approximation:**

replace the dynamical IR sector with a simple boundary condition on the cutoff: the **membrane paradigm type boundary condition**!

# 2) Including dissipation - Membrane paradigm for a scalar field

- Near horizon expansion of a scalar field

$$\phi = e^{-i\omega t + i\vec{k}\cdot\vec{x}}\Big(C_{\text{out}}(1-u)^{\frac{i\tilde{\omega}}{2}}(1+\dots) + \\ + C_{\text{in}}(1-u)^{-\frac{i\tilde{\omega}}{2}}(1+\beta(1-u)+\dots)\Big)$$

- Ingoing boundary condition: $C_{\text{out}} = 0$
- Or equivalently

$$2(1-u)\frac{\partial_u \phi}{i\tilde{\omega}\phi}\Big|_{\text{Hor}} = 1$$

**The membrane paradigm approximation:**

Neglect the near horizon dynamical details and impose a boundary condition on a stretched horizon [Iqbal and Liu, 2008]

$$2(1-u)\frac{\partial_u \phi}{i\tilde{\omega}\phi}\Big|_{u_\delta} = \sigma \quad \text{with} \quad \sigma = \pm 1$$

# 2) Including dissipation - Membrane paradigm for a gravitational field

- For gravitational perturbations use the scalar gauge invariant combinations *e.g.*

$$Z_T \sim \tilde{k}\,\delta h_{t\alpha} + \tilde{\omega}\,\delta h_{x\alpha}$$

- Solve the double-Dirichlet problem with non vanishing dynamical IR metric
- Use the solution and the constraint equations in

$$2(1-u)\frac{\partial_u Z}{i\tilde{\omega} Z}\Big|_{u_\delta} = \sigma \quad \text{with} \quad \sigma = \pm 1$$

- We are effectively replacing Dirichlet boundary conditions on $u_\delta$ with membrane paradigm type boundary conditions

# 2) Including dissipation - Ex: Sound and shear waves in $AdS_5$

- The new dispersion relations are

$$\tilde{\omega}_T = -\frac{i}{2}\sigma\tilde{k}^2 - \frac{i}{4}\sigma\Big(1 - \log 2 + (1-\sigma^2)(\# + \log(1-u_\delta))\Big)\tilde{k}^4$$
$$\tilde{\omega}_L = \pm\frac{1}{\sqrt{3}}\tilde{k} - \frac{i}{3}\sigma\tilde{k}^2 + \Big(\frac{1}{2\sqrt{3}} - \frac{\log 2}{3\sqrt{3}} + \\ +(1-\sigma^2)(\# + \log(1-u_\delta))\Big)\tilde{k}^3 + \dots$$

- Imposing decoupling from the membrane $\sigma = 0$ dissipative effects vanish
- Divergent terms can now be set to zero by coupling to membrane $\sigma = 1$

# 2) Including dissipation - Comments

- Dissipation by including the IR sector
- The divergent terms are cured if dissipation is included
- Dissipationless and dissipative sectors cannot be decoupled
- Improved dissipationless boundary conditions which cure the divergent terms?
- Dissipation at the level of an effective action?

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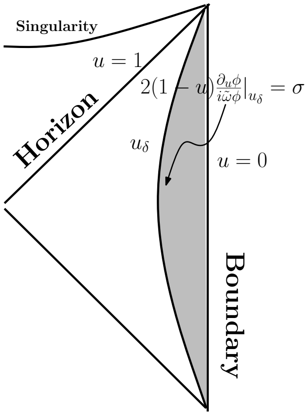
Conclusions & Outlook

# 3) Is the membrane paradigm a good approximation? -

## General argument

Check that the membrane paradigm boundary condition does not spoil the good ingoing behavior at the horizon:
**ingoing wave $\gg$ outgoing wave**




- Use the near horizon expansion of the scalar field

$$\phi \sim C_{\text{out}}\, e^{\frac{i\tilde{\omega}}{2}} + C_{\text{in}}\, e^{-\frac{i\tilde{\omega}}{2}}$$

in the membrane boundary condition to get

$$\frac{C_{\text{out}}}{C_{\text{in}}} = (1-u_\delta)^{1-i\tilde{\omega}}\ \frac{i\beta}{\tilde{\omega}} + \ldots$$

**General validity condition:**

$C_{\text{out}}/C_{\text{in}} \ll 1$ when $u_\delta \to 1$

$\Leftrightarrow \mathcal{I}m(\tilde{\omega}) > -1$

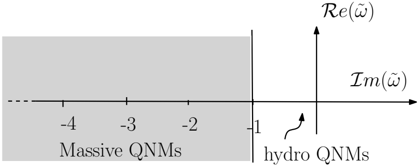
# 3) Is the membrane paradigm a good approximation? -

## OK for hydro QNMs* and KO for massive QNMs

For $\mathcal{I}m(\tilde{\omega}) < -1$ the membrane paradigm is not a good approximation

- Hydrodynamic modes are reproduced as long as we take good care of the additional timelike Goldstone

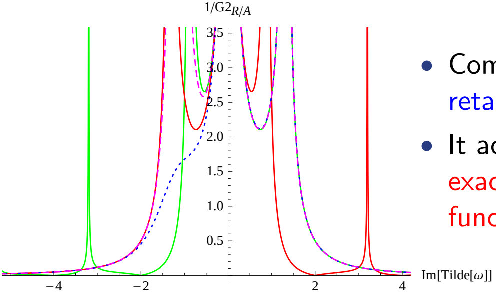
- In general massive quasinormal modes are then not reproduced except possibly for the lowest lying ones



# 3) Is the membrane paradigm a good approximation? -

## Ex: Massive QNMs in $BTZ_3$

Explicit example illustrating the validity of the argument




- Compute the approximated retarded Green's function
- It actually approximates the exact advanced Green's function for $\mathcal{I}m(\tilde{\omega}) < -1$!

- No way to see the poles $\omega = -2\,i\,n$, $k = 0$, with $n = 1, 2, \ldots$ as from the exact retarded Green's function!

# Outline

# Take home messages

1) Is it possible to derive an effective action for conformal fluids from holography?

We provided an explicit example of derivation of the dissipationless effective action for conformal fluids at **linearized level**. Perfect **agreement** with **leading order** effective action in literature. **Subleading order** has **issues**.

2) Is it possible to decouple the dissipative from the dissipationless sector?

**No!** Certain divergent terms at subleading hydro expansion are only removed when dissipation is taken into account.

3) What are the limits of validity of the membrane paradigm as an approximation scheme? [de Boer, Heller, NPF hep-th:1405.4243]

OK for hydrodynamic QNMs$^*$!
KO for massive QNMs!

# Outlook

- How to see volume preserving diffs symmetry from first principles geometrically?
- Improved dissipationless boundary conditions to resolve the divergences at subleading order?
- What is the interpretation of the system on a finite cutoff and the additional Goldstone? Is there Volume diffs invariance breaking?
- How to include dissipation at the level of the action?
- Generalize the technology non linearly and to non relativistic systems

Thank you!