Holographic superconductors and spatial modulation

Christiana Pantelidou
Imperial College London

work with A.Donos and J.P.Gauntlett, 1310.5741[hep-th]

Introduction: AdS/CMT

Use the gauge/gravity correspondence as a tool to investigate the dynamics of strongly coupled CFTs at finite temperature and charge density and/or placed in an external magnetic field.

- Attempt to understand universal features of strongly coupled condensed matter systems found in the vicinity of 'quantum critical points'.
- Explore black hole physics: construct and study novel charged black hole solutions that asymptote to AdS.

Introduction: Top-Down Vs Bottom-Up

Top-Down Approach:

- Consider theories obtained by consistent truncations of the D=10,11 supergravities.
- Difficult to obtain CFTs of interest; involved calculations.
- CFT guaranteed to be well defined.

Bottom-Up Approach:

- Consider phenomenological gravity theories with only few additional d.o.f. that are dual to CFTs with the desirable features.
- No guarantee to have a string embedding.
- Simple calculations.

Introduction: Questions

Specific questions that can be addressed:

- What type of phases are possible and what are the transport properties of each phase?
- What kind of ground states are possible? classification of IR geometries, investigation of new emergent IR scaling behaviours?
- How do these phases compete? What is the dual phase diagram?

Holographic superconductors

High T_c-superconductors consitute one the most challenging problems in condensed matter physics. Remain mysterious due to their strongly coulped nature.

Minimal ingredients:

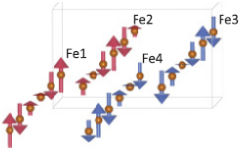
- finite temperature $\rightarrow$ black holes in the bulk
- finite charge density $\rightarrow$ U(1) gauge field
- an "order parameter" that spontaneously acquires an expectation value, e.g. a charged scalar for s-wave superconductors.

Superconducting instabilities: As the temperature is reduced, a new branch of black holes supported by non-vanishing hair emerges at some critical temperature. The $U(1)$ symmetry is now spontaneously broken.

Spatially modulated phases

In condensed matter, phases with *spontaneously* broken translational invariance are very common. Realised in various configurations, e.g. stripes, checkerboards and helices.

- Spin Density Wave
- Charge Density Wave
- Current density wave

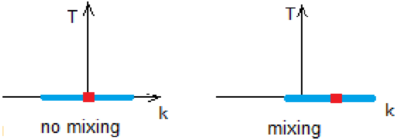


The modulation is fixed by an order parameter with non-zero momentum and it's not related to the underlying lattice of the material.

Holographically, SM phases dual to BHs with broken translational invariance.

Key point to get SM phases = mixing modes

- Consider a perturbations of a charged scalar in $AdS_2 \times R^2$.The e.o.m. becomes $\Box_{Ads2}\phi - M^2\phi = 0$, where $M^2 = m^2 - ce^2 + k^2$. Unstable region is centered around $k = 0$.
- Finding the temprature at which the instability sets in, one gets a "bell curve" with the max. at $k = 0$.
- *Mixing of modes* would introduce off-diagonal terms in the mass matrix that drive the most unstable mode off $k = 0$, shifting of the "bell curve" to $k \neq 0$.



Examples, at finite charge density:

- in D=5[Nakamura,Ooguri,Park] and in D=4 [Donos,Gauntlett] with the mixing introduced by CS terms and axions respectively.
- PT does not necessarily need to be broken [Donos,Gauntlett].

Examples, in magnetic field:

- in D=4,5 with mixing term $\phi * F \wedge G$ [Donos,Gauntlett,CP]
- in $U(1)^3$ and $U(1)^4$ sugra with metric components mixing as well [Donos,Gauntlett,CP]. Interesting interplay with susy solutions.

$\rightarrow$ Spatially modulated phases are more the rule, than the exception.

Plan for this talk

Question:
Is it possible to have spatially modulates superconducting states in holography?[Donos,Gauntlett]

- The FFLO state describes an s-wave superconductor in which the cooper pair has non-vanishing momentum.
- This was conjectured to exist in the '60s and possibly has been seen experimentally in heavy fermion materials and some organic superconductors, eg $CeCoIn_5$.

The model of interest

Aim: Study p-wave superconductors in D=5. The order parameter can be either an $SU(2)$ vector or a two-form. Both cases give similar results; we focus on the second case.

Consider a theory of gravity in D=5 coupled to a U(1) gauge field and a complex two-form

$$\mathcal{L} = (R + 12) * 1 - \frac{1}{2} * F \wedge F - \frac{1}{2} * C \wedge \bar{C} - \frac{i}{2m} C \wedge \bar{H}$$

where $F = dA$ and $H = dC + ieA \wedge C$.

- For particular values of (e, m), this theory can be obtained as a consistent truncation from $D = 10, 11$; corresponds to a subsector of the $D = 5$ Romans theory.

- This theory admits a unit radius AdS_5 vacuum solution with $A = C = 0$ which is dual to a d=4 CFT with a conserved $U(1)$ current and a tensor operator with charge, e, and scaling dimension $\Delta = 2 + m$.

- Another solution of the theory is the electrically charged AdS-RN black hole.

$$ds^2 = -gdt^2 + g^{-1}dr^2 + r^2(dx_i^2)\,, \qquad A = \mu(1 - \frac{r_+^2}{r^2})dt\,.$$

This corresponds to the high temperature, spatially homogeneous and isotropic phase of the dual CFTs when held at finite chemical potential μ.

Step 1: Instabilities in the NHL

Study perturbations around the near-horizon limit of the electric AdS-RN: $AdS_2 \times R^{D-2}$.

- Modes tend to be more unstable in this region.
- Use the AdS_d BF bound criterion to check for instabilities: if the bound is violated, the theory is unstable (converse not always true)

$$L^2 M^2 \geq -\frac{(d-1)^2}{4}.$$

In our model:
In the NHL, peturbations of the two-form, δC, decouple; consider the ansatz [Donos,Gauntlett]

$$\delta C = \cdots + dx_1 \wedge \left(u_3 dx^3 + v_3 dx^2\right),$$

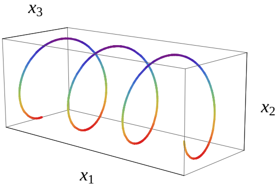
where $u_3 = d_3 cos(kx_1), v_3 = d_3 sin(kx_1)$ $\rightarrow$ p-wave
<u>or</u> $u_3 = d_3 e^{ikx_1}, v_3 = id_3 e^{ikx_1}$ $\rightarrow$ p+ip-wave.

- For both cases, if $e^2 > \frac{m^2}{2}$, the BF bound is violated for a certain k. Most unstable mode has $k \neq 0$; when heated up, we expect the preferred branch to be modulated.

p-wave: $\delta C = \cdots + d_3(r)\, dx_1 \wedge [\sin(kx_1)\ dx_2 + \cos(kx_1)\ dx_3]$

$k = 0$: order parameter pointing in $-dx_2$
3 translations and rotations in (x_1, x_3).

$k \neq 0$: dualise to see the helical structure, pitch is $2\pi/k$.
x_2, x_3 translations, x_1 translation combined with a rotation, (x_2, x_3) rotation (Bianchi VII_0)



(p+ip)-wave: $\delta C = \cdots + e^{-ikx_1} i d_3(r) dx_1 \wedge (dx_2 - i dx_3)$

$k = 0$: order parameter pointing in $dx_2 + i dx_3$
3 translations and (x_2, x_3) rotations upto const gauge transf.
$k \neq 0$: same as before, but x_1 translations are compensated by a gauge transf.

No helical structure, no symmetry reduction.

Step 2: Perturbations around AdS_5-RN

Consider linearised perturbations around the full AdS-RN black hole. [Donos,Gauntlett]

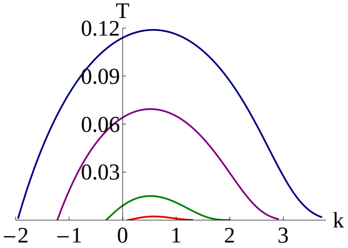
- Specify the critical temperature at which the instability sets in.
- Allows to search for instabilities localised far from the horizon.

In our model:

- Two-form perturbations, δC, decouple again. Consider the same ansatz as before.
- All the action is included in the last term $\sim d_3(r)$: *regular* at the horizon and *spontaneously* breaking the $U(1)$.

$$d_3 = d_{3+} + \mathcal{O}(r - r_+), \qquad d_3 = c_{d_3} r^{-|m|} + \cdots .$$

- Obtain an one-parameter family of solutions as expected. Plot the critical temperatures T_c versus k for the existence of normalisable static perturbations of the two-form for fixed (m, e).



- For fixed m, T_c increases as e increases. For fixed e, T_c decreases as m increases.
- Depending on (e, m) this plot may not cross the $k = 0$ axis.
- p- and (p+ip)-wave set in at the same temperature.

Step 3: Backreacted solutions

Construct the backreacted BHs to get information about the thermodynamics: is the new branch of black holes preferred?

- In principle, one needs to solve PDEs to study spatial modulation.
- Here, we can get away with solving ODEs: the three-dimensional Euclidean group breaks down to Bianchi VII_0.
- Use the left-invariant one-form of this Lie algebra when constructing the ansatz.

$$\begin{aligned} \omega_1 &= dx_1 \,, \\ \omega_2 &= \cos(kx_1)\; dx_2 - \sin(kx_1)\; dx_3 \,, \\ \omega_3 &= \sin(kx_1)\; dx_2 + \cos(kx_1)\; dx_3 \,. \end{aligned}$$

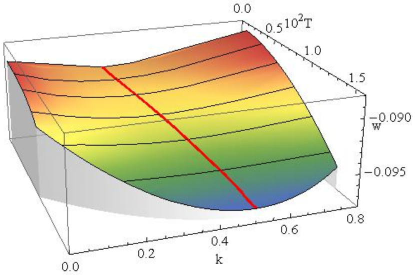
p-wave helical superconductors

Consider the following ansatz:

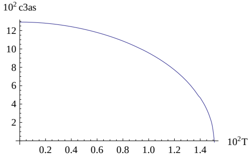
$$ds^2 = -g\, f^2\, dt^2 + g^{-1} dr^2 + h^2\, \omega_1^2 + r^2\, \left(e^{2\alpha}\, \omega_2^2 + e^{-2\alpha}\, \omega_3^2\right)\,,$$
$$A = a\, dt\,,$$
$$C = (i\, c_1\, dt + c_2 dr) \wedge \omega_2 + c_3\, \omega_1 \wedge \omega_3\,,$$

- E.o.m. boil down to a set of ODEs which is solved subject to boundary conditions: regularity at the horizon and AdS_5 asymptotics compatible with spontaneous symmetry breaking - no sources.
- Obtain a two parameter family of solutions, labeled by (k, T), consistent with the bell curves of step 2.

- All the solutions have a lower free energy than AdS-RN: the CFT undergoes a phase transition to a helical superconducting phase.
- The preferred ones lie along the red locus: as the temperature is lowered, the pitch is increasing. $(e, m) = (2, 2)$



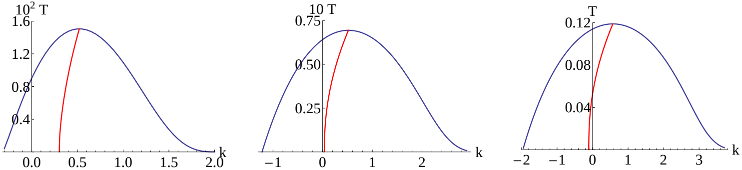
- Interestingly, preferred solutions satisfy $c_h = 0$. This is understood by varying the free energy with respect to k: $k\partial_k w = 8c_h = 0$.
- Plot the condensate along the preferred branch. The phase transition is second order: near T_c, we have a mean field behaviour.



- On the $k = 0$ branch, $c_h = c_\alpha = 0$, signaling the existence of a reduction of the ansatz: $h = re^{-\alpha}$. Consequence of enhanced symmetry.
- Boundary stress tensor is inhomogeneous and exhibits anisotropic nature of p-wave superconductors: (also, traceless and conserved)

$$\begin{aligned}
\langle T_{tt} \rangle &= 3M + 8c_h \,, \\
\langle T_{x_1x_1} \rangle &= M + 8c_h \,, \\
\langle T_{x_2x_2} \rangle &= M + 8c_\alpha \cos(2kx_1) \,, \\
\langle T_{x_3x_3} \rangle &= M - 8c_\alpha \cos(2kx_1) \,, \\
\langle T_{x_2x_3} \rangle &= -8c_\alpha \sin(2kx_1) \,.
\end{aligned}$$

- For large enough e, the preferred locus exhibit inversion of the pitch of the helix - this point is completely regular. This phenomenon was seen experimentally in e.g. helimagnets.



- Solutions with $k = 0$ are only relevant at the inversion point. Finite k solutions can not be ignored.

Ground States

Scaling solutions:

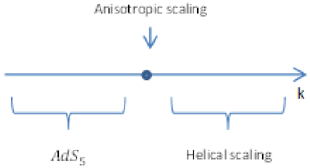
The system admits the following scaling solutions in the IR:

- If $k = 0$, the IR fixed point is invariant under the anisotropic scaling: similar to [Taylor]
  $r \to \lambda^{-1} r, \quad t \to \lambda^{z} t, \quad x_{1,3} \to \lambda^{1-\gamma} x_{1,3}, \quad x_2 \to \lambda^{1+\gamma} x_2 \,.$
  The entropy density scales like $S \sim T^{(3-\gamma)/z}$.
- If $k \neq 0$, there is a helical scaling symmetry: similar to [Kachru et al.]
  $r \to \lambda^{-1} r, \quad t \to \lambda^{z} t, \quad x_{2,3} \to \lambda x_{2,3}, \quad x_1 \to x_1 \,.$
  The entropy density scales like $S \sim T^{2/z}$.

- If $k \neq 0$, possible to have the same AdS_5 in the IR as in the UV.
  - [Horowitz,Roberts]: s-wave superconductor. In the condensed phase, the gauge field is transformed to an irrelevant operator.
  -In our case, the condensate vanishes at $T = 0$ and we are left only with relevant modes. Puzzling!
  - [Sachdev et al.]: introducing a lattice. Exploid the k-dependence of the relevant modes.
  - k-dependence allows us to suppress relevant modes, e.g.

$$c_3(r) = c_3^0 \frac{e^{-kx/r}}{r^{1/2}} + \cdots$$

Black Holes at T=0:
The T=0 limit of our black holes approaches smooth domain walls that interpolate between AdS_5 in the UV and the following IR:



Question:
Both, the AdS_5 and the helical scaling, exist at $k \neq 0$ but emerge at $k < 0$ and $k > 0$ respectively. Is this related to thermodynamics or is there a problem with constructing the domain walls?

(p+ip)-wave superconductors

A very similar story for the (p+ip)-case:

$$ds^2 = -gf^2dt^2 + g^{-1}dr^2 + h^2(dx_1 + Qdt)^2 + r^2(dx_2^2 + dx_3^2)\,,$$
$$A = adt + wdx_1\,,$$
$$C = (ic_1dt + c_2dr + ic_3dx_1) \wedge (\omega_2 - i\omega_3)\,,$$

- The ansatz is now stationary, but not static.
- By solving a set of seven coupled ODEs subject to boundary conditions, we obtain a 2-parameter family of (p+ip)-wave superconductors. Label solutions by (k,T) as before.

- These solutions have lower free energy than the AdS-RN as well; p-wave and (p+ip)-wave phases compete.
- Apart from the standard smarr-type formula, all the solutions satisfy two additional contrains on the UV data [Donos,Gauntlett].

$$4c_h - \frac{k}{e}c_w = 0$$
$$2c_Q + c_w\mu = 0$$

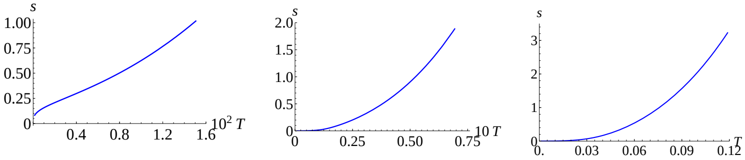
- The preferred locus of solutions satisfy $c_w = 0 \Rightarrow c_h = c_Q = 0$. Also understood by varying the action with respect to k.
- On the $k = 0$ branch, $c_h = 0$, but $c_w, c_Q \neq 0$: no symmetry enhancement.

- Boundary stress tensor is homogeneous and exhibits the isotropic nature of (p+ip)-wave superconductor:

$$
\begin{aligned}
\langle T_{tt} \rangle &= 3M + 8c_h \,, \\
\langle T_{tx_1} \rangle &= 4c_Q \,, \\
\langle T_{x_1x_1} \rangle &= M + 8c_h \,, \\
\langle T_{x_2x_2} \rangle &= M \,, \\
\langle T_{x_3x_3} \rangle &= M \,,
\end{aligned}
$$

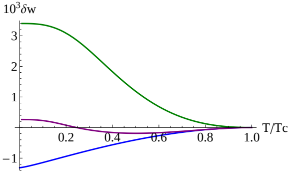
- For large e, k becomes slightly negative as the temperature is lowered.

- Have not yet been able to pin down the precise behaviour of the solution when $T \to 0$; scenarion of AdS_5 in the IR.
- The entropy of the ground states goes to zero. For small values of the charge, there is an interesting crossover in its behaviour.



Competition between p and p+ip

Both instabilities set in at the same T_c. Which is preferred?



- p-wave is preferred for small e and (p+ip)-wave for large e.
- For intermediate values of e, there is a first order transition between the two at T_*; the black holes have the same free energy, but do not intersect on field space.
  c.f. [Gubser,Pufu]

Further remarks

- The discussion of homogeneous p-wave superconductors has been generalised to include spatial modulation.
- Can you find two forms with e, m in top down setting, perhaps with addition of extra fields, that are unstable?
- Spatially modulated phases are more fundamental than expected.
- The field is moving towards solving PDEs [Donos],[Withers], [Rozali et al.]

Thank you!