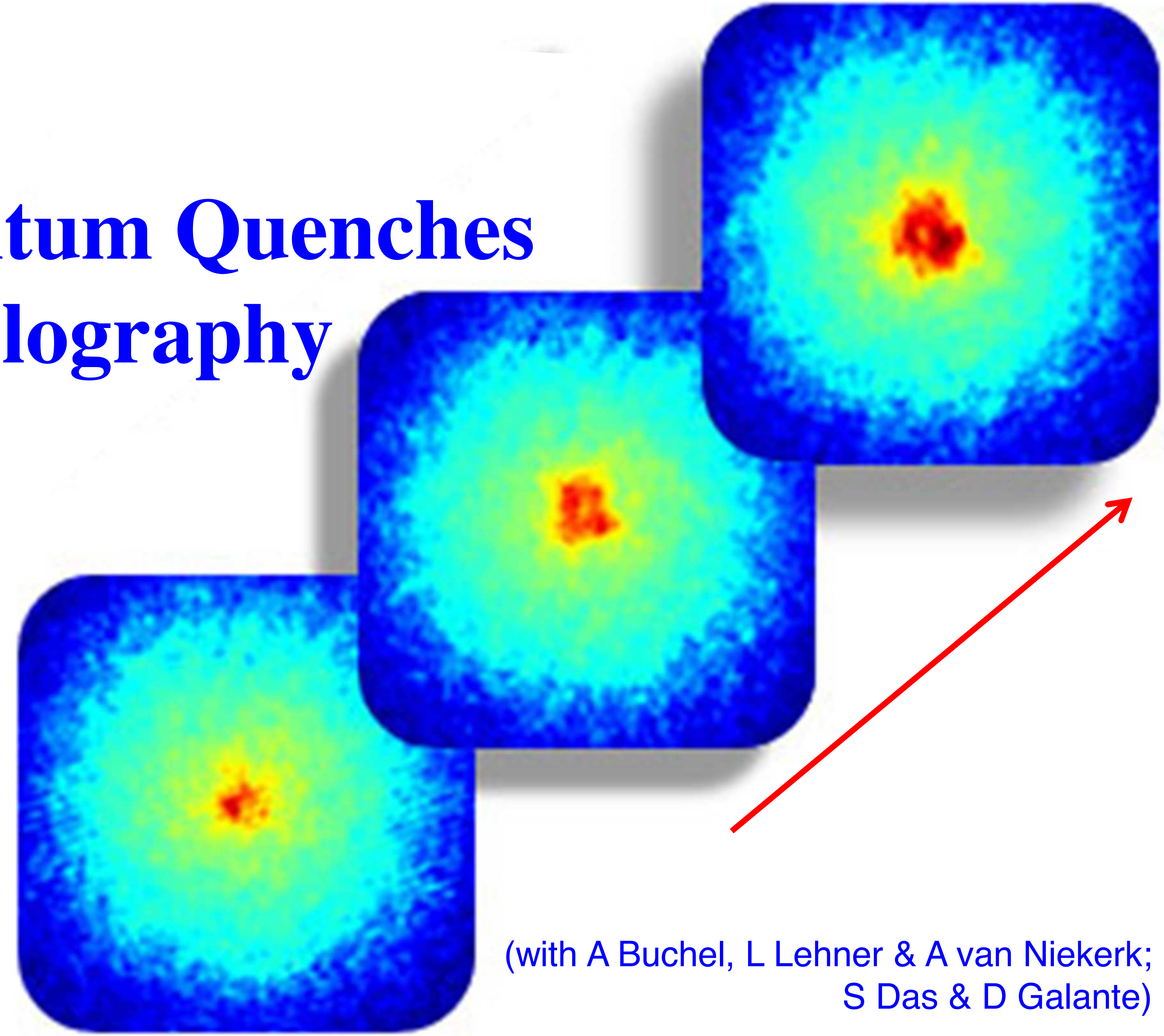
Quantum Quenches & Holography



(with A Buchel, L Lehner & A van Niekerk;
S Das & D Galante)

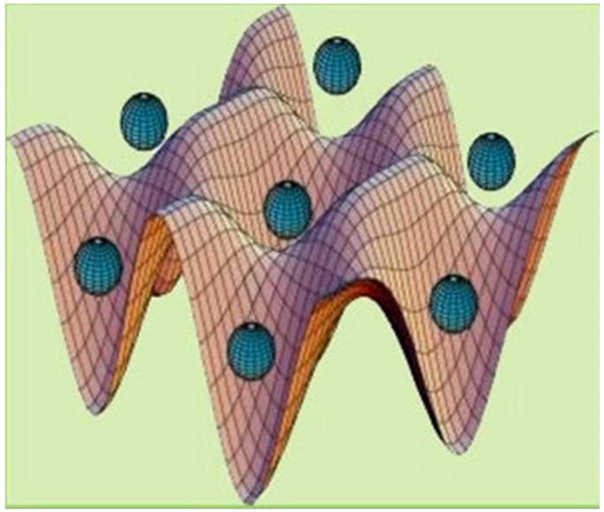
PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Quantum Quenches:

- consider quantum system with Hamiltonian:

$$H = H_0 + \lambda(t)\,\delta H$$

- prepare system in eigenstate $|\psi_0\rangle$ of Hamiltonian H_0
- abruptly turn on λ ; system evolves *unitarily* according to H
- Question: How do observables, eg, expectation values and correlation functions, evolve in time?
- for most systems, coupling to environment is unavoidable $\longrightarrow$ decoherence, dissipation
- effects minimized for, eg, cold atoms in optical lattice



$\longrightarrow$ is there "universal" behaviour?

Quantum Quenches & Holography:

→ is there “universal” behaviour?

what are organizing principles for out-of-equilibrium systems?

- theoretical progress made for variety systems: d=2 CFT, (nearly) free fields, integrable models, ...
- still seeking broadly applicable and efficient techniques
- what can AdS/CFT correspondence offer?
  - → strongly coupled field theories
  - → real-time analysis
  - → finite temperature (if desired)
  - → general spacetime dimension
- perhaps re-organization of problem will lead to new insights

Quantum Quenches & Holography:

- AdS/CFT lends itself to the study quantum quenches for a new class of strongly coupled field theories

- there has been a great deal of interest in the past few years

  Chesler, Yaffe; Das, Nishioka, Takayanagi, Basu; Bhattacharyya, Minwalla; Abajo-Arrastia, Aparicio, Lopez; Albash, Johnson; Ebrahim, Headrick; Balasubramanian, Bernamonti, de Boer, Copland, Craps, Keski-Vakkuri, Mueller, Schafer, Shigemori, Staessens, Galli; Allias, Tonni; Keranen, Keski-Vakkuri, Thorlacius; Galante, Schvellinger; Carceres, Kundu; Wu; Garfinkle, Pando Zayas, Reichmann; Bhaseen, Gauntlett, Simons, Sonner, Wiseman; Auzzi, Elitzur, Gudnason, Rabinovici; . . . . . .

- much of work aimed at "thermalization" (eg, quark-gluon plasma)

- AdS/CFT connects far-from-equilibrium physics is naturally leads to studying highly dynamical situations in gravity

  $\longrightarrow$ new dialogue with "numerical relativity"

Quantum Quenches & Holography:

- AdS/CFT allows us to study quantum quenches for strongly coupled field theories in any number of dimensions

Where are control parameters in AdS/CFT framework?

AdS/CFT dictionary:

$$\text{gravity fields} \longleftrightarrow \text{boundary operators}$$

$$\Phi \qquad\qquad \delta H$$

eg, consider some scalar field in AdS:

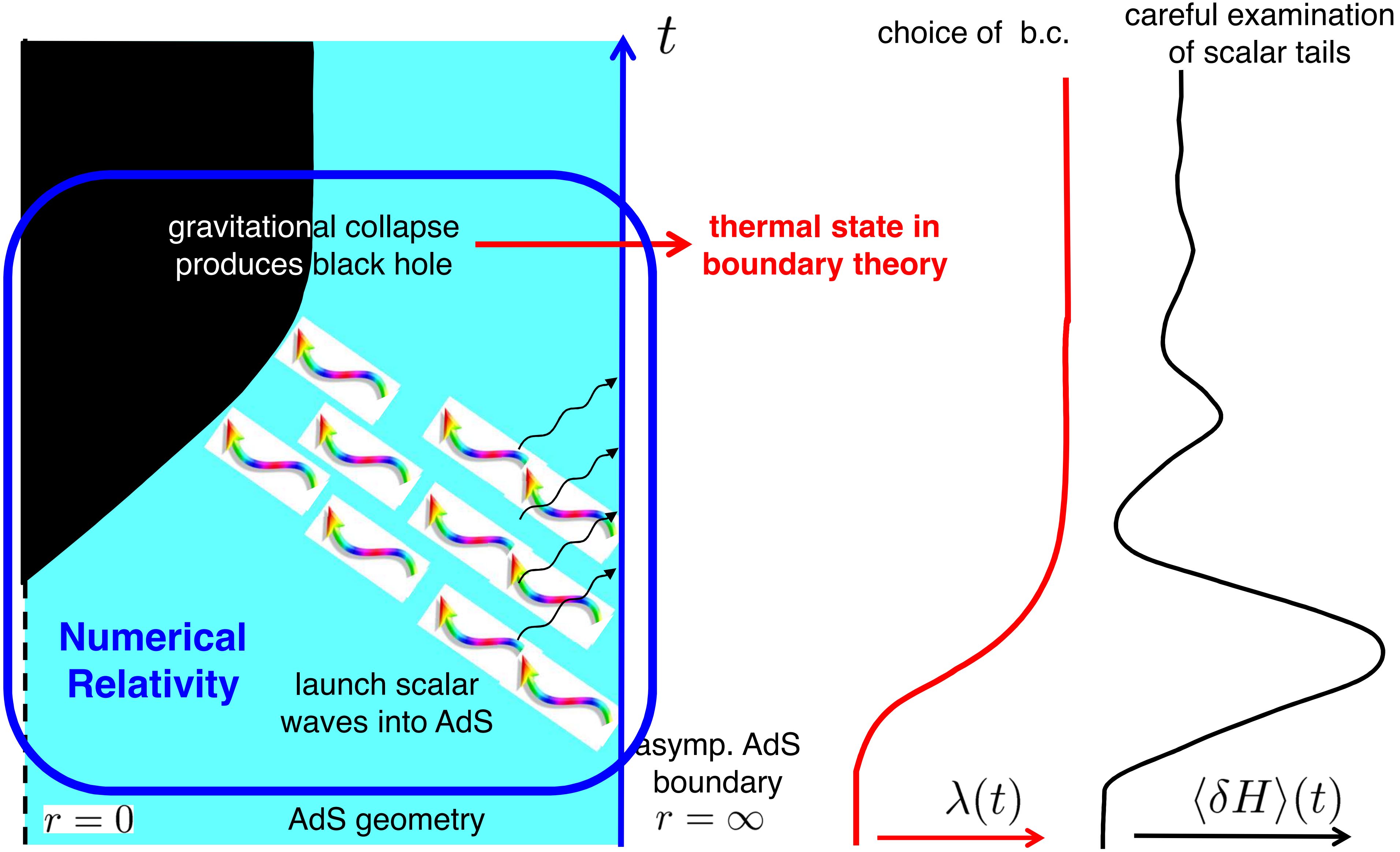
equation of motion: $(\nabla^2 - m^2)\Phi + \cdots = 0$

asymptotic solutions: $\Phi \sim \dfrac{\lambda}{r^{d-\Delta}} + \dfrac{\langle \delta H \rangle}{r^{\Delta}} + \cdots$

$\longrightarrow$ integration constants become coupling and expectation value

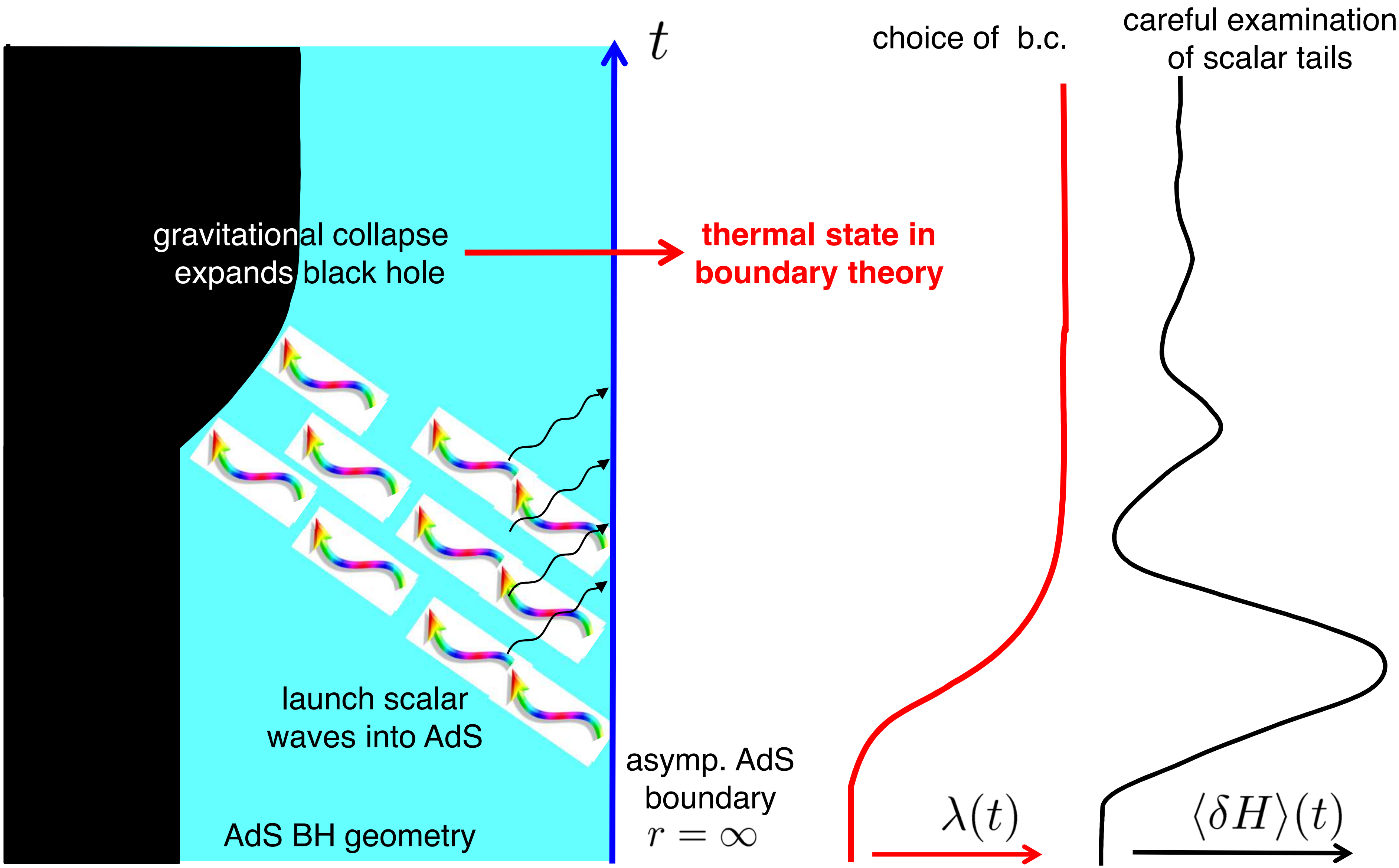
recall conformal dimension: $\Delta = \dfrac{d}{2} + \sqrt{\dfrac{d^2}{4} + m^2 L^2}$

Holographic Quantum Quench (cartoon):



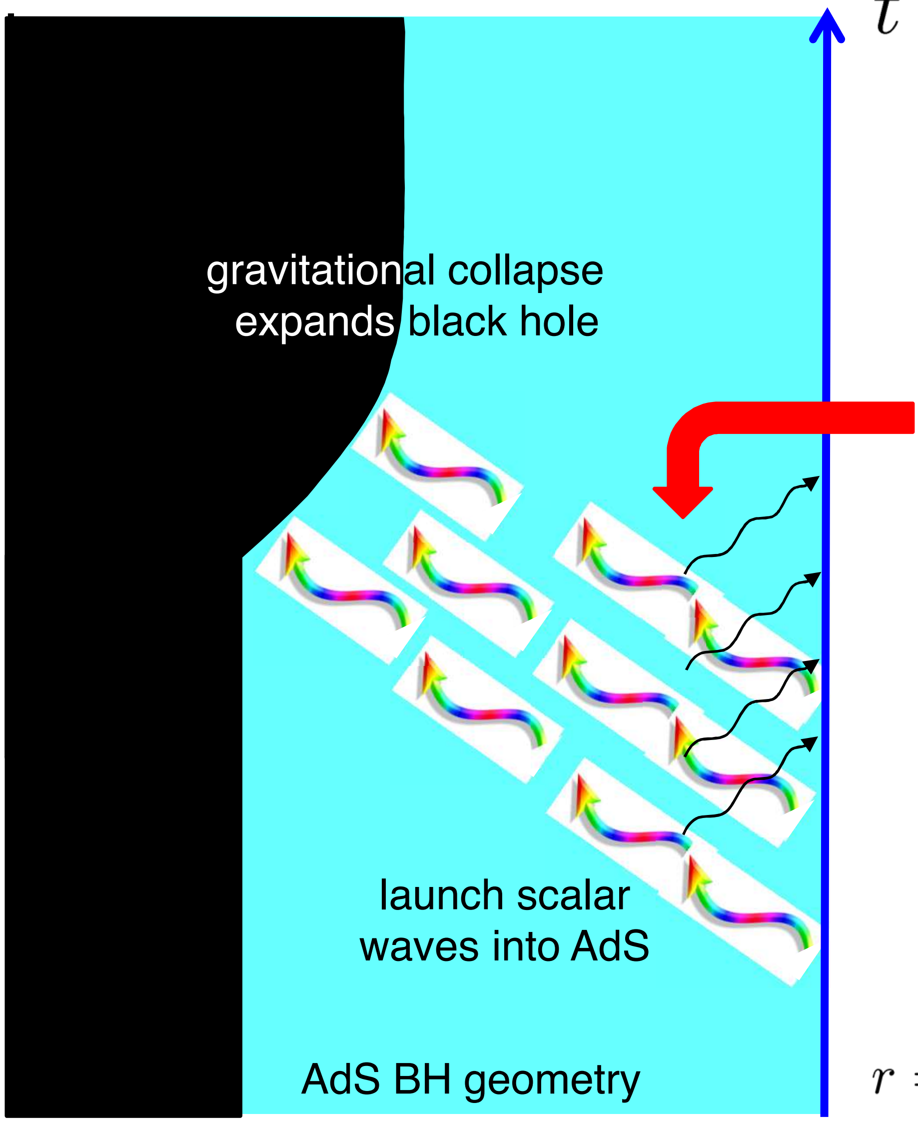
• quench → thermal state, in expectation with previous analyses

Holographic Quantum Quench (cartoon):



• **"thermal quench"**: quantum quench at finite temperature

Holographic Thermal Quench:



- fix boundary dimension: $d = 4$
- choose conformal dimension, $2 \leq \Delta \leq 4$ and profile

$$\lambda(t) = \frac{\Delta\lambda}{2}\left[1 + \tanh(t/\Delta t)\right]$$

- solve linearized scalar eom in fixed BH geometry

  $\longrightarrow$ determines $\langle \mathcal{O}_\Delta \rangle(t)$

- determine "BH mass" $\mathcal{E}(t)$ with diffeomorphism Ward identity*:

$$\partial^i \langle T_{ij} \rangle \;=\; \langle \mathcal{O}_\Delta \rangle\, \partial_j \lambda$$

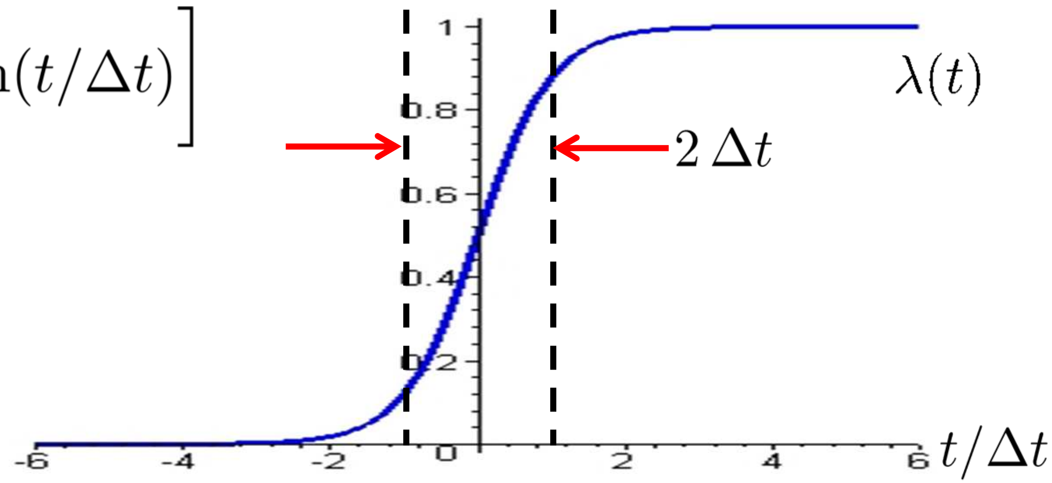
$\longrightarrow$ integrate for $\mathcal{E}(t)$, ie,

$$\partial_t \mathcal{E} = -\langle \mathcal{O}_\Delta \rangle\, \partial_t \lambda$$

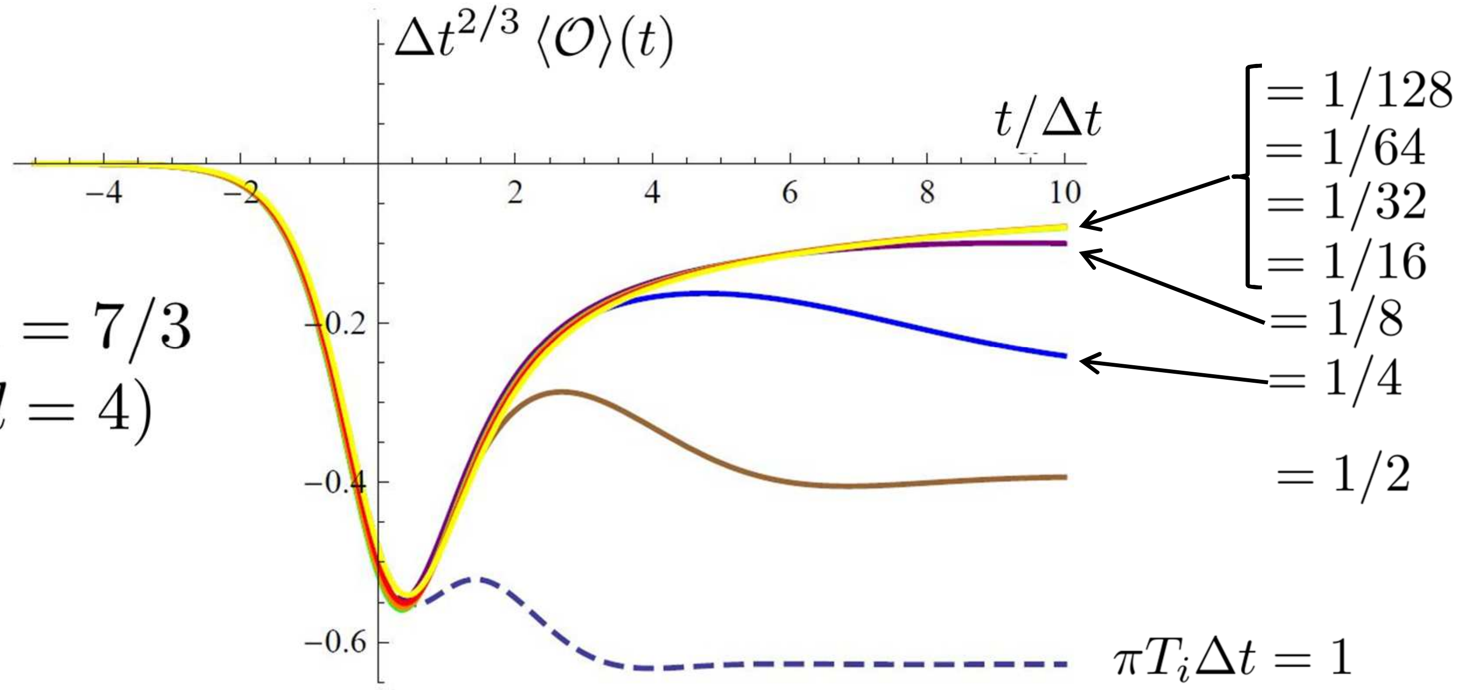
* boundary constraint from Einstein eq's

Holographic Thermal Quench:

- profile: $\lambda(t) = \frac{\Delta\lambda}{2}\left[1 + \tanh(t/\Delta t)\right]$



$\Delta = 7/3$
$(d = 4)$



Holographic Thermal Quench:

- lessons learned:

  1. Renormalization of (strongly coupled) boundary QFT with time-dependent couplings works in a straightforward way

- holography gives well-defined approach to renormalize bdry QFT
- bdry theory has new divergences: (Λ = UV cut-off scale)

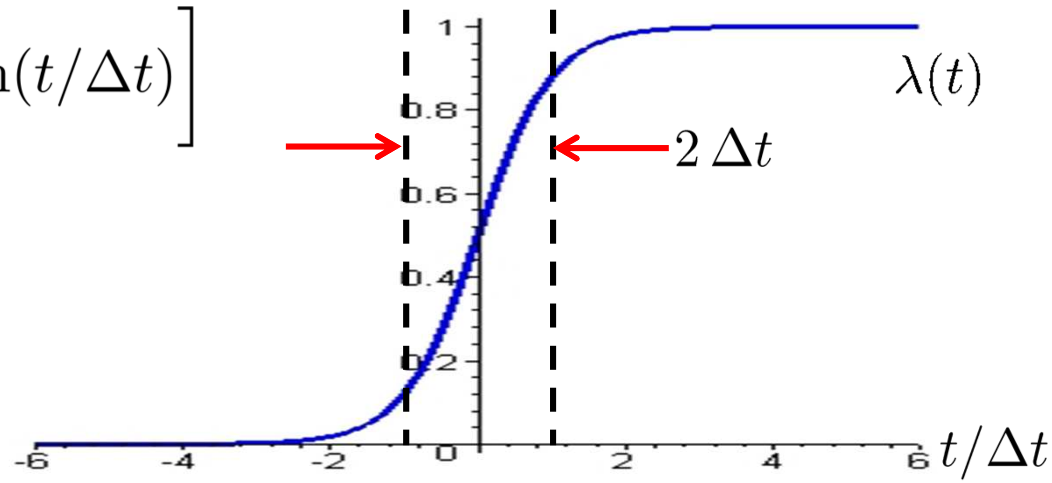
$$I_{ct} \simeq \int d^4x\sqrt{-g}\Big(\Lambda^4 + \Lambda^{2\Delta-4}\,\lambda^2(t) + \cdots \\ + \Lambda^{2\Delta-6} g^{ij}\,\partial_i\lambda\,\partial_j\lambda + \Lambda^{2\Delta-6}\mathcal{R}(g)\,\lambda^2 + \cdots \quad \Big)$$

- familiar in the context of QFT in curved backgrounds
- new log divergences lead to new scheme dependent ambiguities

(Bianchi, Freedman & Skenderis; Aharony, Buchel & Yarom; Petkou & Skenderis; Emparan, Johnson & Myers; . . .)

Holographic Thermal Quench:

- profile: $$\lambda(t) = \frac{\Delta\lambda}{2}\left[1 + \tanh(t/\Delta t)\right]$$



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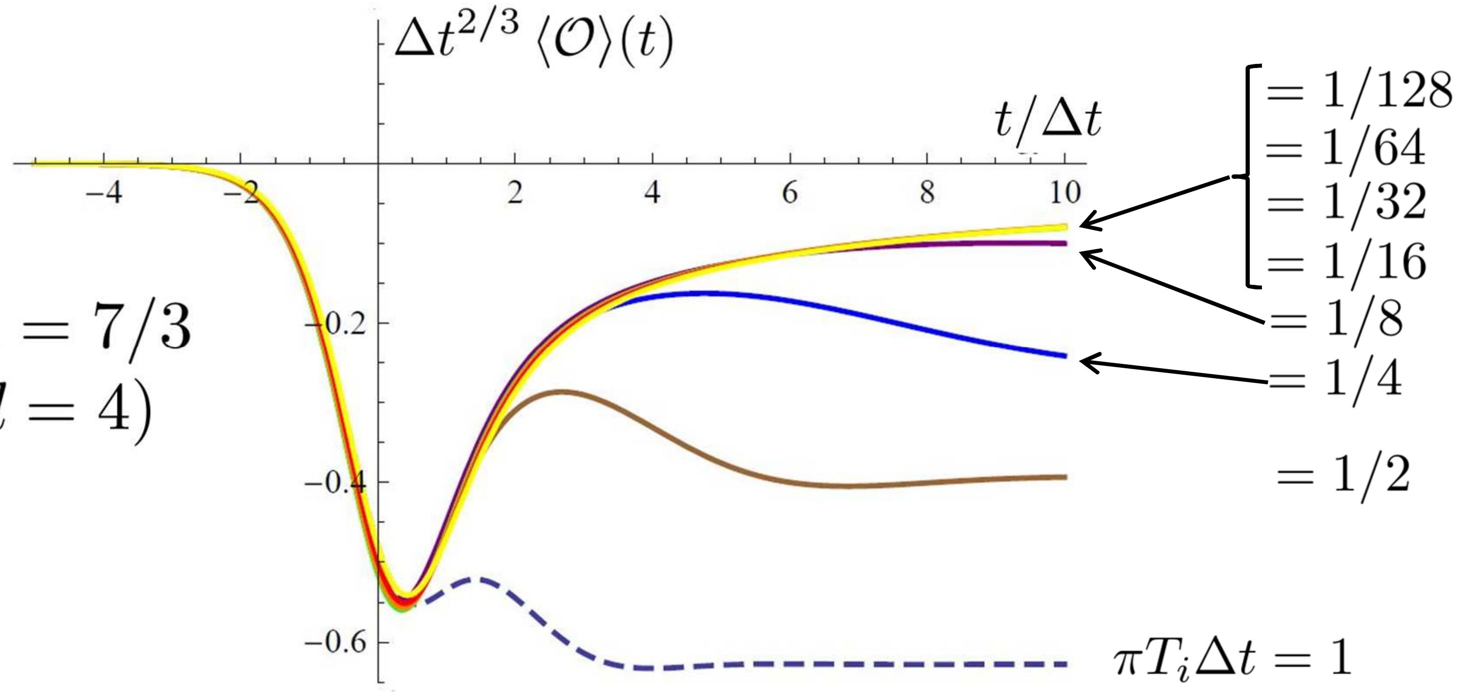


Holographic Thermal Quench:

- lessons learned:

  2. Response to “fast” quenches exhibits universal scaling

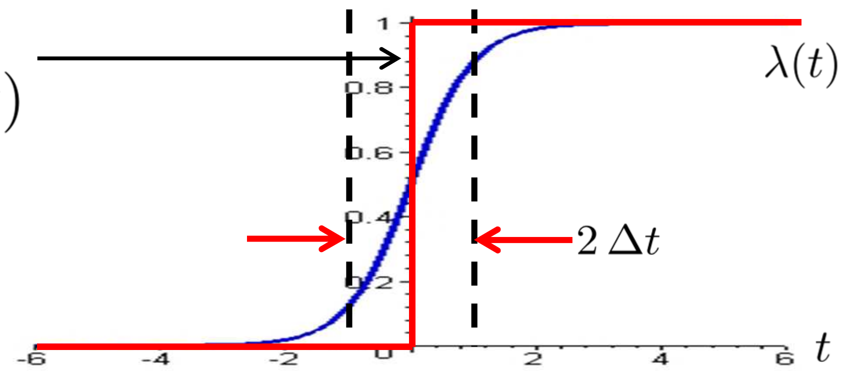
- for example:
  $(d=4)$

$$\left.\begin{aligned} \max\langle \mathcal{O}_\Delta \rangle &\sim \frac{\Delta\lambda}{(\Delta t)^{2\Delta-4}} \\ \Delta\mathcal{E} &\sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta-4}} \end{aligned}\right\} \quad \Delta t \to 0$$

yields physical divergence!!

- seems to indicate instantaneous quench is problematic

with $\Delta t = 0$, get
abrupt jump in $\lambda(t)$



Holographic Thermal Quench:

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yields physical divergence!!

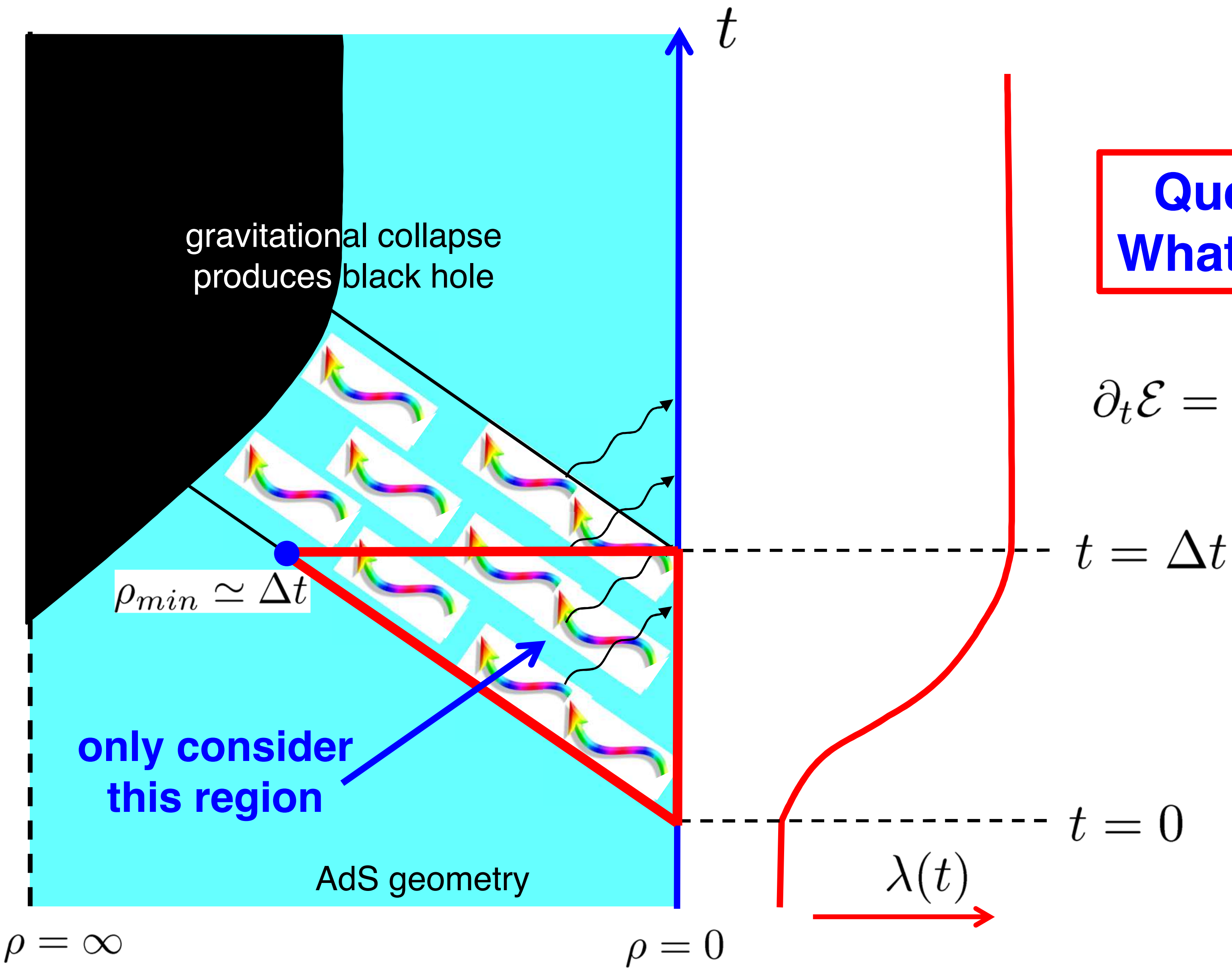
- seems to indicate instantaneous quench is problematic
- compare to seminal work of, eg, Calabrese & Cardy

$\longrightarrow$ "instantaneous quench" is basic starting point

► identified a physical problem?

► simply an issue with perturbative expansion?

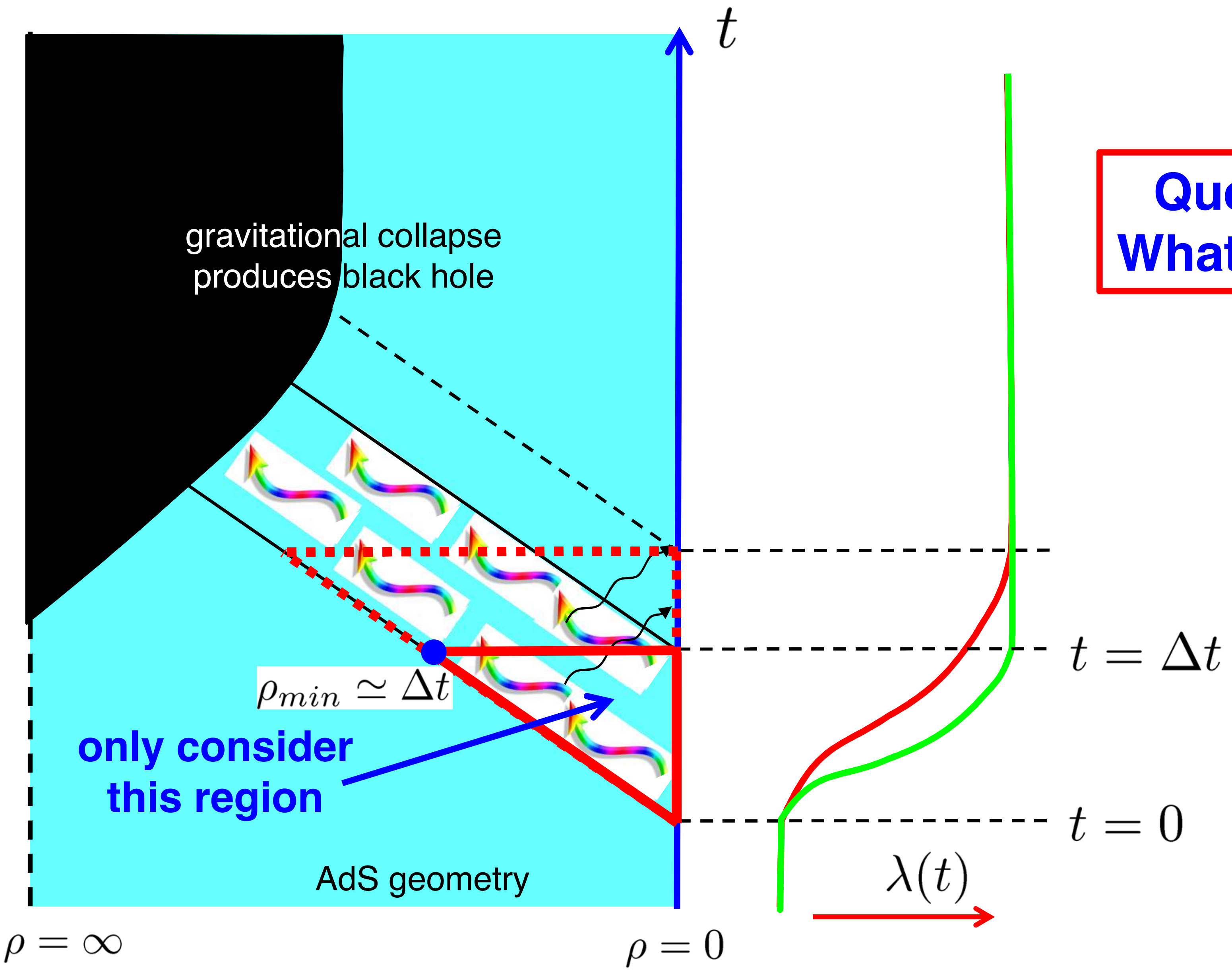
Holographic Quantum Quench (cartoon):



Question:
What is $\Delta\mathcal{E}$?

$$\partial_t \mathcal{E} = -\langle \mathcal{O}_\Delta \rangle \; \partial_t \lambda$$

Holographic Quantum Quench (cartoon):



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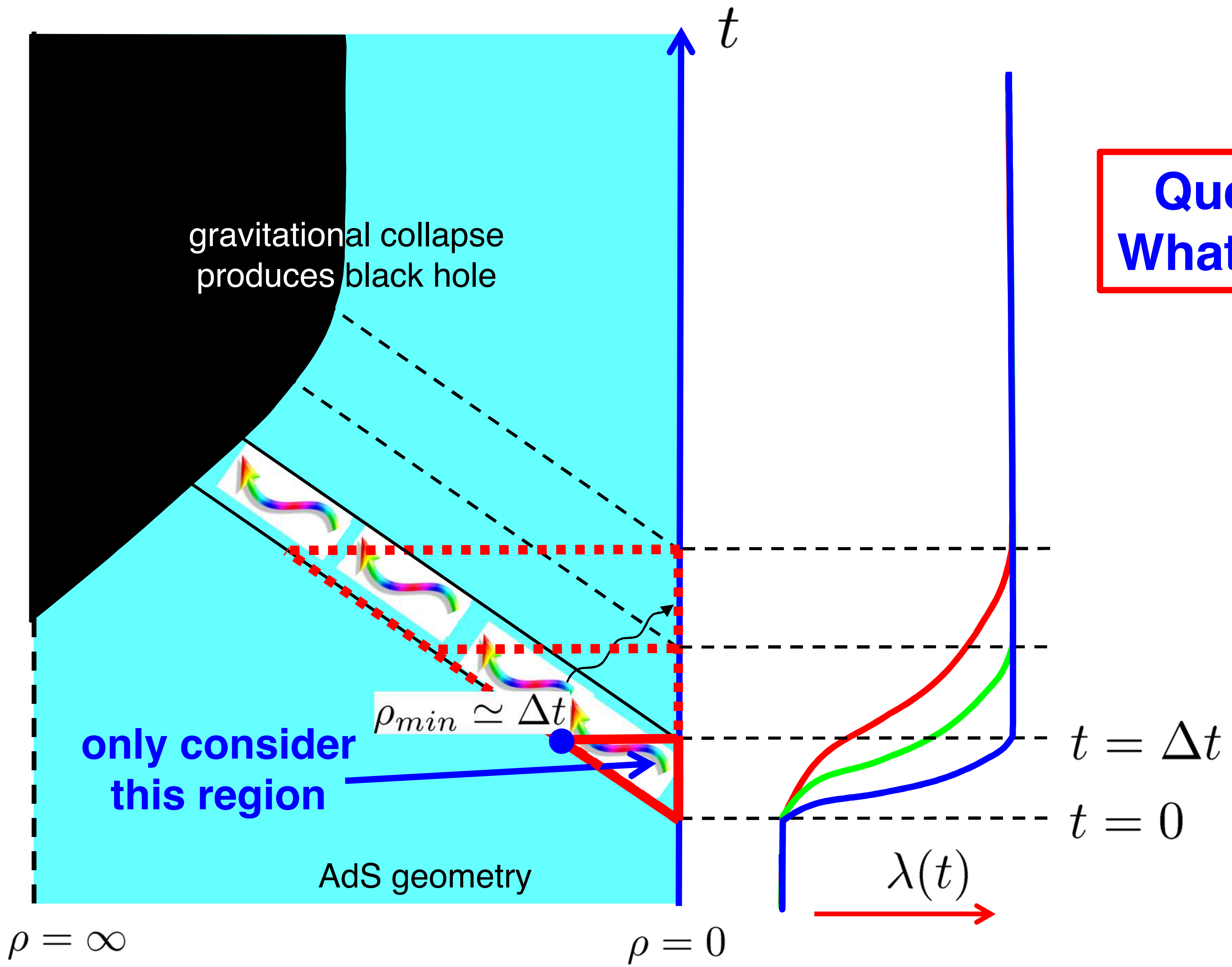


Question:
What is $\Delta\mathcal{E}$?

Generalizing “Fast” Quenches:

$\longrightarrow$ **Question: What is $\Delta\mathcal{E}$?**

- focus: full details of evolution, eg, approach to final state, are not determined but allows us to understand scaling behaviour
- as we scale $\Delta t \to 0$, only “tiny” region of solution in asymptotic AdS relevant for this question

$\longrightarrow$ certainly full numerical simulations are not needed

$\longrightarrow$ solvable with purely analytic approach!!

Generalizing "Fast" Quenches:

$\longrightarrow$ **Question: What is $\Delta\mathcal{E}$?**

- solve full bulk equations of motion perturbatively for $\rho \ll 1$

$$
\begin{aligned}
0 &= -\frac{2(d-3)}{(d-1)A}u(\phi) + \frac{2d(d-3)}{A} + \rho^4\left(\phi'\right)^2 - \frac{d-3}{(d-1)A}m^2\phi^2 - \left(\frac{\dot\phi}{A}\right)^2 \\
&\quad +2(d-2)(d-1)\left[\left(\frac{\dot\Sigma}{A\Sigma}\right)^2 - \left(\frac{\rho^2\Sigma'}{\Sigma}\right)^2\right] + \frac{2\rho^2\left(\rho^2 A'\right)'}{A} - 4\left(\frac{\dot A}{A^2}\right)^2 + 2\frac{\ddot A}{A^3} \\
0 &= d - \frac{u(\phi)}{(d-1)} - \frac{m^2\phi^2}{2(d-1)} + \frac{\rho^4 A}{2(d-1)}\left(\phi'\right)^2 + \frac{\dot\phi^2}{A} - \rho^4\frac{A'\Sigma'}{\Sigma} - (d-2)\rho^4 A\frac{(\Sigma')^2}{\Sigma^2} \\
&\quad + \frac{2\ddot\Sigma}{A\Sigma} - \frac{\dot A\dot\Sigma}{A^2\Sigma} + (d-2)\frac{\dot\Sigma^2}{A\Sigma^2} \\
0 &= \frac{\left(\phi'\right)^2}{2(d-1)} + \frac{1}{2(d-1)}\left(\frac{\dot\phi}{\rho^2 A}\right)^2 + \frac{\Sigma''}{\Sigma} + \frac{2\Sigma'}{\rho\Sigma} + \frac{\ddot\Sigma}{\rho^4 A^2\Sigma} \\
0 &= \frac{\phi'\dot\phi}{d-1} + \frac{\dot A\Sigma'}{A\Sigma} - \frac{A'\dot\Sigma}{A\Sigma} + 2\frac{\dot\Sigma'}{\Sigma} \\
0 &= -\frac{\delta u(\phi)}{\delta\,\phi} - m^2\phi + \rho^4 A\phi'' + 2\rho^3 A\phi' + \rho^4 A'\phi' + \frac{(d-1)\rho^4 A\Sigma'\phi'}{\Sigma} + \frac{\dot A\dot\phi}{A^2} - \frac{(d-1)\dot\Sigma\dot\phi}{A\Sigma} - \frac{\ddot\phi}{A}
\end{aligned}
$$

Generalizing “Fast” Quenches:

- key: asymptotic fields in AdS decay in precise manner (ie, Fefferman-Graham expansion)

$$\phi \;=\; \rho^{d-\Delta}\left(p_0(t) + a_1\,\rho\,p_0'(t) + a_2\,\rho^2\,p_0''(t) + \cdots\right)$$

$$+\rho^{\Delta}\left(b_1 p_2(t) + b_2\,\rho\,p_2'(t) + b_3\,\rho^2\,p_2''(t) + \cdots\right)$$

linear scalar eq:

$$\rho^2\partial_\rho^2\phi - (d-1)\rho\partial_\rho\phi - \rho^2\partial_t^2\phi + \Delta\,(d-\Delta)\,\phi = 0$$

recall $\lambda(t) \sim p_0(t)$ and $\langle\mathcal{O}_\Delta\rangle(t) \sim p_2(t)$

Generalizing "Fast" Quenches:

- key: asymptotic fields in AdS decay in precise manner (ie, Fefferman-Graham expansion)

$$
\begin{aligned}
\phi \;=\; & \rho^{d-\Delta}\left(p_0(t) + a_1\,\rho\, p_0'(t) + a_2\,\rho^2\, p_0''(t) + \cdots\right) \\
& \quad +c_1\rho^{2(d-\Delta)} p_0^2(t) + c_2\rho^{2(d-\Delta)+1} p_0(t) p_0'(t) + \cdots \\
& \quad +d_1\rho^{3(d-\Delta)} p_0^3(t) + \cdots \\
& +\rho^{\Delta}\left(b_1 p_2(t) + b_2\,\rho\, p_2'(t) + b_3\,\rho^2\, p_2''(t) + \cdots\right) \\
& \quad +e_1\rho^{d} p_0(t) p_2(t) + e_2\rho^{d+1} p_0(t) p_2'(t) + \cdots \\
& \quad + \cdots
\end{aligned}
$$

nonlinearities in eom

- set $\Delta t = \alpha\,\widehat{\Delta t}$ and take limit $\alpha \to 0$ (while $p_0(t/\Delta t)$ kept fixed)

$\longrightarrow$ natural to scale coordinates: $t = \alpha\,\hat{t}\,, \quad \rho = \alpha\hat{\rho}$

eg, $p_0(t/\Delta t) = p_0(\hat{t}/\widehat{\Delta t})$

Generalizing "Fast" Quenches:

- key: asymptotic fields in AdS decay in precise manner (ie, Fefferman-Graham expansion) $\longrightarrow$ nonlinearities unimportant!

$$
\begin{aligned}
\phi \;=\;& \underline{\alpha^{d-\Delta}}\,\hat\rho^{d-\Delta}\left(p_0(\hat t) + a_1\,\hat\rho\, p_0'(\hat t) + a_2\,\hat\rho^2\, p_0''(\hat t) + \cdots\right)\\
& \cancel{+\underline{\alpha^{2(d-\Delta)}}\left(c_1\hat\rho^{2(d-\Delta)}\,p_0^2(\hat t) + c_2\hat\rho^{2(d-\Delta)+1}\,p_0(\hat t)\,p_0'(\hat t) + \cdots\right)}\\
& \cancel{+\underline{\alpha^{3(d-\Delta)}}\,d_1\hat\rho^{3(d-\Delta)}\,p_0^3(\hat t) + \cdots}\\
& +\underline{\alpha^{\Delta+\delta}}\,\hat\rho^{\Delta}\left(b_1\hat p_2(\hat t) + b_2\,\hat\rho\,\hat p_2'(\hat t) + b_3\,\hat\rho^2\,\hat p_2''(\hat t) + \cdots\right)\\
& \cancel{+\underline{\alpha^{d+\delta}}\left(e_1\hat\rho^{d}\,p_0(\hat t)\,\hat p_2(\hat t) + e_2\hat\rho^{d+1}\,p_0(\hat t)\,\hat p_2'(\hat t) + \cdots\right)}\\
& \cancel{+}
\end{aligned}
$$

- set $\Delta t = \alpha\,\widehat{\Delta t}$ and take limit $\alpha \to 0$ (while $p_0(t/\Delta t)$ kept fixed)

$\longrightarrow$ natural to scale coordinates: $t = \alpha\,\hat t\,, \quad \rho = \alpha\hat\rho$

$\longrightarrow$ add: $\underline{p_2(t) = \alpha^{\delta}\,\hat p_2(t)}$ $\longrightarrow$ "matching bc": $\delta = -(2\Delta - d)$

Generalizing "Fast" Quenches:

- key: asymptotic fields in AdS decay in precise manner (ie, Fefferman-Graham expansion) $\longrightarrow$ nonlinearities unimportant!

$$\begin{aligned}\phi &= \alpha^{d-\Delta}\hat{\rho}^{d-\Delta}\left(p_0(\hat{t}) + a_1\,\hat{\rho}\,p_0'(\hat{t}) + a_2\,\hat{\rho}^2\,p_0''(\hat{t}) + \cdots\right)\\ &\quad +\alpha^{d-\Delta}\hat{\rho}^{\Delta}\left(b_1\hat{p}_2(\hat{t}) + b_2\,\hat{\rho}\,\hat{p}_2'(\hat{t}) + b_3\,\hat{\rho}^2\,\hat{p}_2''(\hat{t}) + \cdots\right)\end{aligned}$$

- set $\Delta t = \alpha\,\widehat{\Delta t}$ and take limit $\alpha \to 0$ (while $p_0(t/\Delta t)$ kept fixed)

$\longrightarrow$ natural to scale coordinates: $t = \alpha\,\hat{t}\,,\quad \rho = \alpha\hat{\rho}$

$\longrightarrow$ need: $p_2(t) = \alpha^{-(2\Delta-d)}\,\hat{p}_2(t)$ $\longrightarrow$ $\phi \to \alpha^{d-\Delta}\phi$

- similar scaling arguments yield:

$$\Sigma = 1/\rho\,,\qquad \Sigma \to \Sigma/\alpha;\qquad A = 1/\rho^2\,,\qquad A \to A/\alpha^2$$

- **relevant solution = linearized scalar solution in AdS space!**

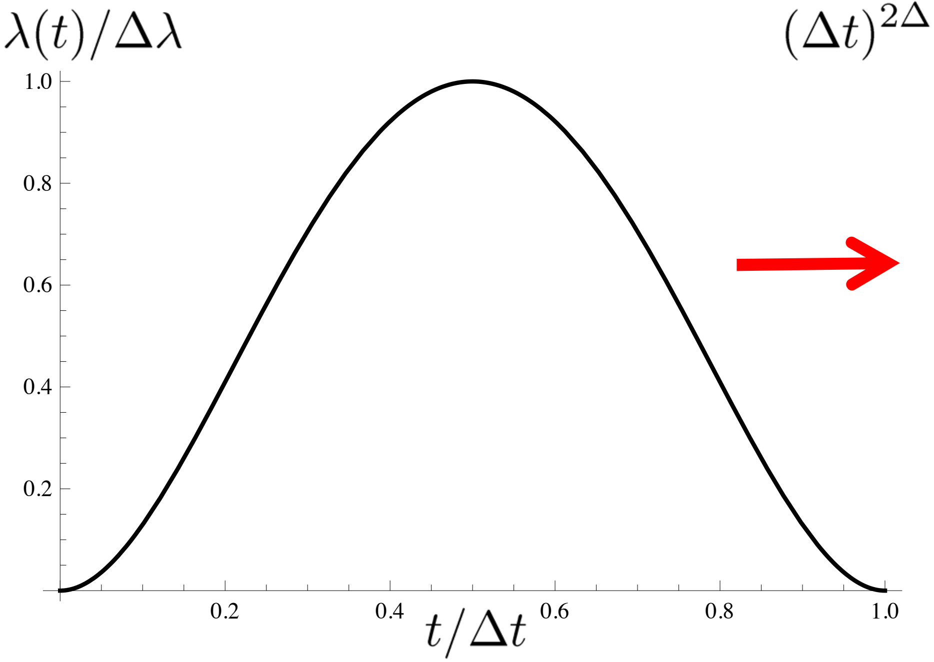
but solving $\Delta\mathcal{E}$ for full nonlinear problem!

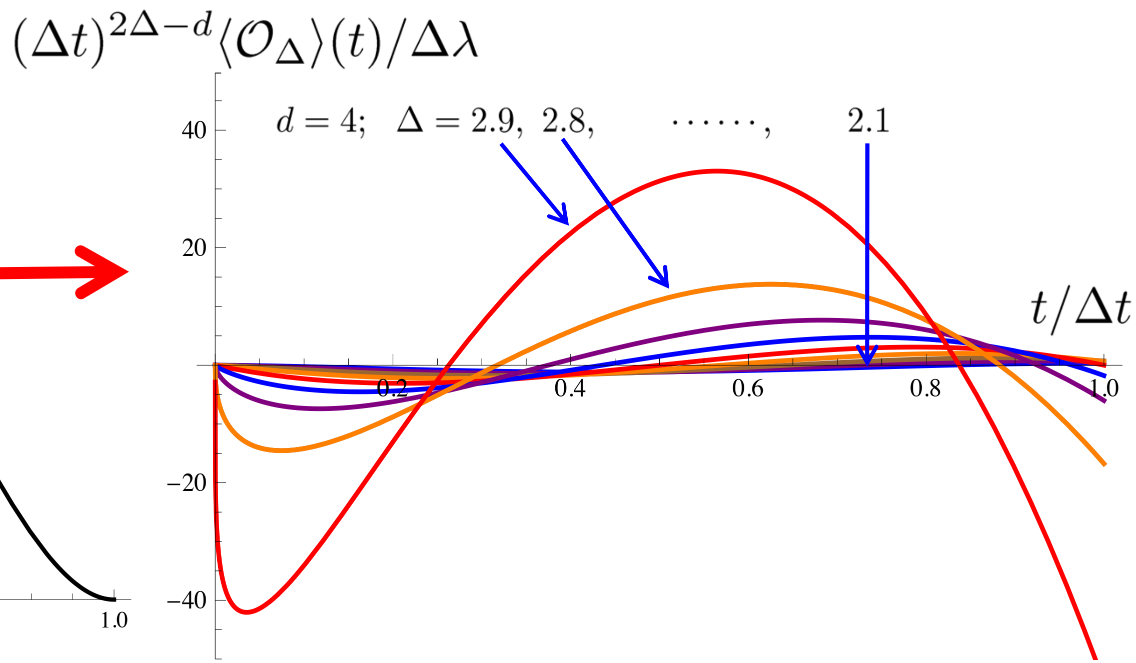
Generalizing "Fast" Quenches:

- analytic solutions, eg: $\lambda(t) = 16\Delta\lambda\big((t/\Delta t)^2 - 2(t/\Delta t)^3 + (t/\Delta t)^4\big)$

$$\langle\mathcal{O}_\Delta\rangle(t) = \frac{16\,\Delta\lambda}{(\Delta t)^{2\Delta-d}}\left(\frac{t}{\Delta t}\right)^{d-2\Delta}\left(b_2\left(\frac{t}{\Delta t}\right)^2 - 2b_3\left(\frac{t}{\Delta t}\right)^3 + b_4\left(\frac{t}{\Delta t}\right)^4\right)$$

where $$b_\kappa = -\frac{2^{d-2\Delta}\,\Gamma(\kappa+1)\,\Gamma(\frac{d+2}{2}-\Delta)}{\Gamma(d+1+\kappa-2\Delta)\,\Gamma(\Delta-\frac{d-2}{2})}$$





Generalizing "Fast" Quenches:

- as we scale $\Delta t \to 0$, only "tiny" region in asymptotic AdS relevant
- **relevant solution = linearized scalar solution in AdS space!**
- general scaling $\langle \mathcal{O}_\Delta \rangle \sim \Delta\lambda/(\Delta t)^{2\Delta-d}$ with holographic dictionary, ie, "energy conservation":

$$\Delta\mathcal{E} = -\int_0^{\Delta t} dt\, \langle \mathcal{O}_\Delta \rangle\, \partial_t \lambda$$

$$\Rightarrow \boxed{\langle \mathcal{O}_\Delta \rangle \sim \frac{\Delta\lambda}{(\Delta t)^{2\Delta-d}}\ ; \qquad \Delta\mathcal{E} \sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta-d}}}$$

- matches previous perturbative numerical calc's (for d=4)
- **result here applies for full nonlinear solution!!**
  - ▶ identified a physical ~~problem?~~ effect
  - ~~▶ simply an issue with perturbative expansion?~~

Generalizing "Fast" Quenches:

$$\boxed{\Delta\mathcal{E} \sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta-d}}}$$

- $\Delta t \to 0$ yields physical divergence for $\frac{d}{2} < \Delta < d$

  $\longrightarrow$ "instantaneous" quench seems problematic!?!

- can consider various scaling limits:

  $\longrightarrow$ $\Delta t = \alpha \Delta t_0\,;\ \Delta\lambda = \alpha^{\Delta - d/2}\Delta\lambda_0$

  as $\alpha \to 0$, $\Delta\mathcal{E}$ finite but $\langle \mathcal{O}_\Delta \rangle$ divergent

  $\longrightarrow$ $\Delta t = \alpha \Delta t_0\,;\ \Delta\lambda = \alpha^{2\Delta - d}\Delta\lambda_0$

  as $\alpha \to 0$, $\langle \mathcal{O}_\Delta \rangle$ finite but $\Delta\mathcal{E}$ vanishes

  but would not be "standard" protocol
- operators in range $\frac{d}{2} - 1 \leq \Delta < \frac{d}{2}$ seem to be okay
- note UV fixed point, ie, CFT, is source of divergence
- strongly coupled holographic QFT versus free fields???

Generalizing "Fast" Quenches:

- compare directly to C&C, ie, quench mass of a free scalar:

$$\lambda = m^2 \; ; \qquad\qquad \mathcal{O}_\Delta = \phi^2 \; ; \qquad\qquad \Delta = d - 2$$

- quench with finite Δt and examine limit $\Delta t \to 0$

eq. of motion: $\left[\nabla^2 - \frac{m^2}{2}\,(1 + \tanh(t/\Delta t))\right] \phi = 0$

- example in: Birrell & Davies, "*Quantum Fields in Curved Space*"

eg, "in" modes:

$$\begin{aligned} f_k(t) \quad &= \quad \frac{1}{\sqrt{4\pi k}} \exp\left[-i(\omega_+ + \omega_- \Delta t \log(2\cosh(t/\Delta t))\right] \\ &\qquad \times {}_2F_1\left(1 + i\omega_-\Delta t, i\omega_-\Delta t, 1 - ik\Delta t; \frac{1}{2}(1 + \tanh(t/\Delta t)\right) \end{aligned}$$

with $\omega_\pm = \frac{1}{2}\left(\pm k + \sqrt{k^2 + m^2}\right)$

Generalizing “Fast” Quenches:

- compare directly to C&C, ie, quench mass of a free scalar:

$$\lambda(t) = m^2(t) = \frac{m^2}{2}\left[1 + \tanh(t/\Delta t)\right] \;;$$

$$\mathcal{O}_\Delta = \phi^2 \;; \qquad\qquad \Delta = d - 2$$

- given individual modes, consider two point correlator

$$G_k(t_1, t_2) = {}_{in}\langle 0|\,\phi_k(t_1)\,\phi_{-k}(t_2)\,|0\rangle_{in}$$

- yields simple result in the limit $\Delta t \to 0$:

$$G_k(t_1,t_2) \quad\longrightarrow\quad \frac{1}{4\pi\omega}\,e^{-i\omega(t_1-t_2)} + \frac{(\omega-\omega_0)^2}{8\pi\omega_0\omega}\,\cos\omega(t_1-t_2)$$
$$+\frac{\omega^2-\omega_0^2}{8\pi\omega_0\omega}\,\cos\omega(t_1+t_2)$$

$$\left(\omega = \sqrt{k^2+m^2}\,,\ \ \omega_0 = k\,\right)$$

$\longrightarrow$ recover the “sudden quench” results of C&C!!

Generalizing "Fast" Quenches:

- consider response: $\langle\phi^2\rangle \simeq \int_0^{\underline{k_{max}}} dk\, k^{d-3}\, |{}_2F_1|^2$
- following holographic example, UV divergences are removed by adding appropriate counterterms in effective action
- "wherever you see terms with k_{max}, subtract them off"
- UV divergences: eg, consider a constant mass

$$\langle\phi^2\rangle \simeq \int_0^{k_{max}} dk\, \frac{k^{d-2}}{\sqrt{k^2+m^2}} \;=\; \int_0^{k_{max}} dk\, \left[k^{d-3} - \frac{1}{2}m^2 k^{d-5} + \cdots\right]$$
$$= \frac{1}{d-2}k_{max}^{d-2} - \frac{m^2}{2(d-4)}k_{max}^{d-4} + \cdots$$

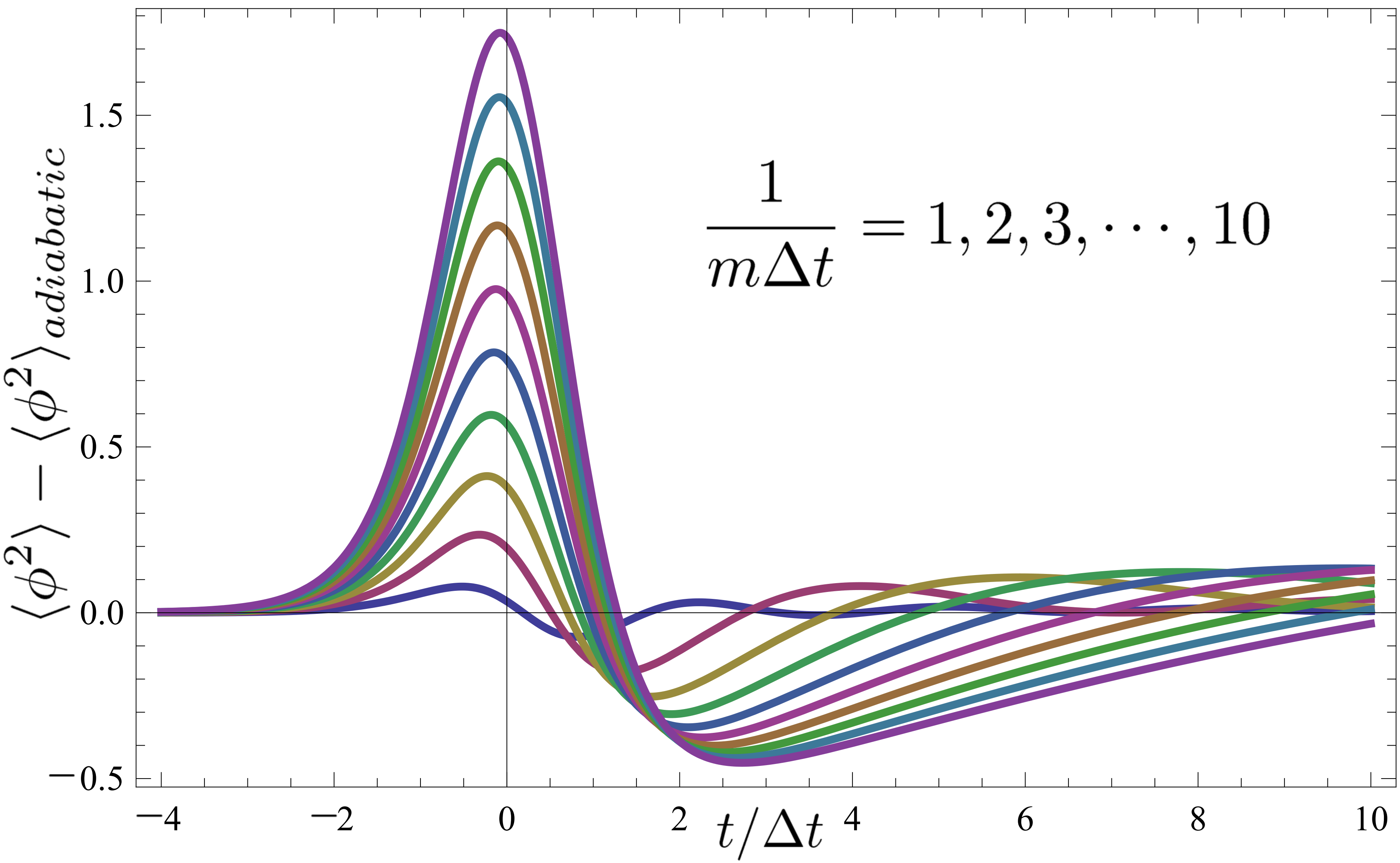
- regulated response (d=5):

$$\langle\phi^2\rangle \simeq \int_0^{\infty} dk\, \left[k^2\, |{}_2F_1|^2 - k^2 + \frac{1}{2}m^2(t)\right]$$

where $\quad m^2(t) \;=\; \frac{m^2}{2}\left[1+\tanh(t/\Delta t)\right]$

Generalizing "Fast" Quenches:

- regulated response (d=5): $\langle\phi^2\rangle \simeq \int_0^\infty dk\, \left[k^2\, |{}_2F_1|^2 - k^2 - \frac{1}{2}m^2(t)\right]$



Generalizing "Fast" Quenches:

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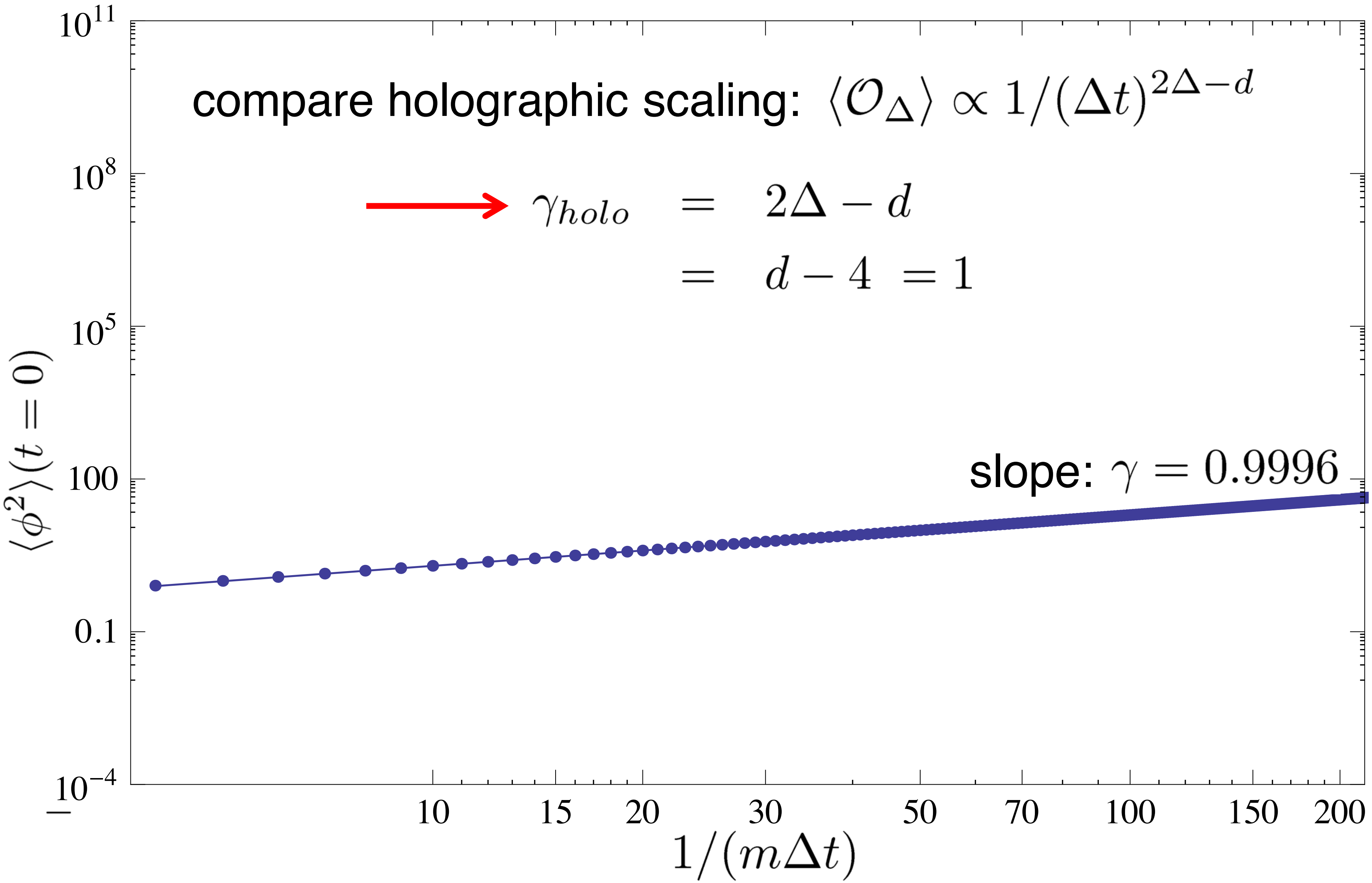
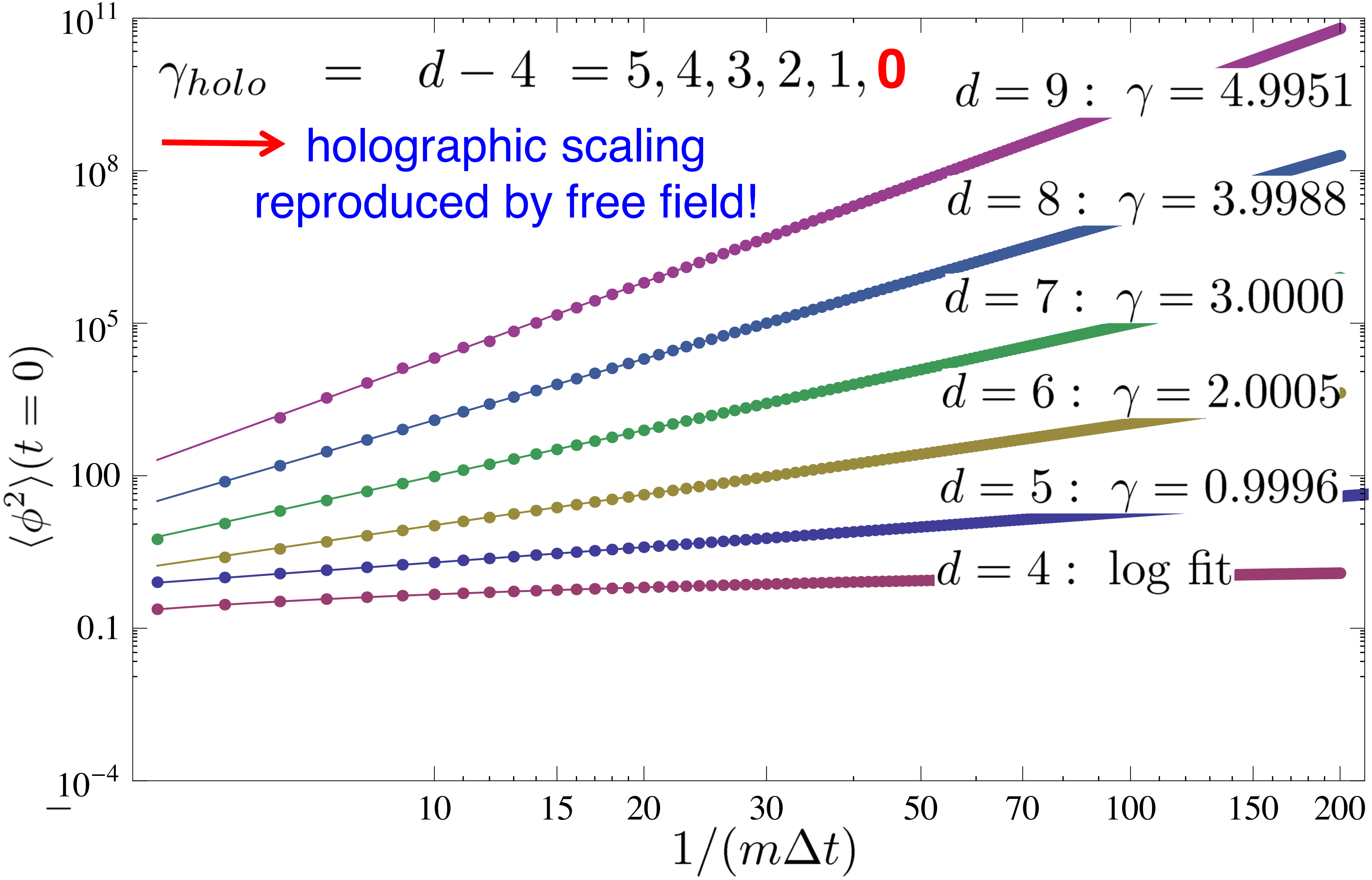


Generalizing "Fast" Quenches:

- regulated response (d=9,8,7,6,5,4):



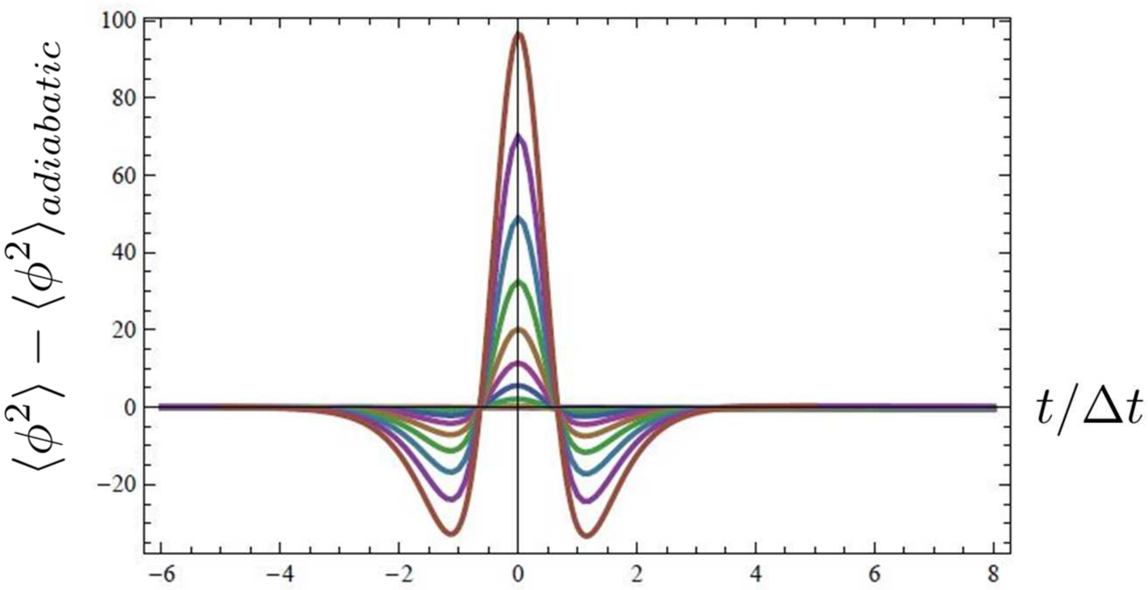
Generalizing "Fast" Quenches:

- can verify that Ward identity is satisified: $\partial_t \mathcal{E} = -\langle \mathcal{O}_\Delta \rangle \; \partial_t \lambda$
- evaluate $\mathcal{E}$ independently and compare above numerically

$$\mathcal{E} = T_{tt} = \partial_t\phi\partial_t\phi + \partial_i\phi\partial_i\phi + m^2(t)\phi^2$$

- some analytic progress: expand for small Δt , find

$$\langle \phi^2 \rangle_{fin} \simeq \frac{\pi}{2^{d-2}} \partial_t^{d-4} m^2(t/\Delta t) + O(1/\Delta t^{d-6}) \qquad \text{(odd d)}$$



Generalizing "Fast" Quenches:

- extend free field calculations to fermions:

$$\lambda = m \; ; \qquad\qquad \mathcal{O}_\Delta = \bar{\psi}\psi \; ; \qquad\qquad \Delta = d-1$$

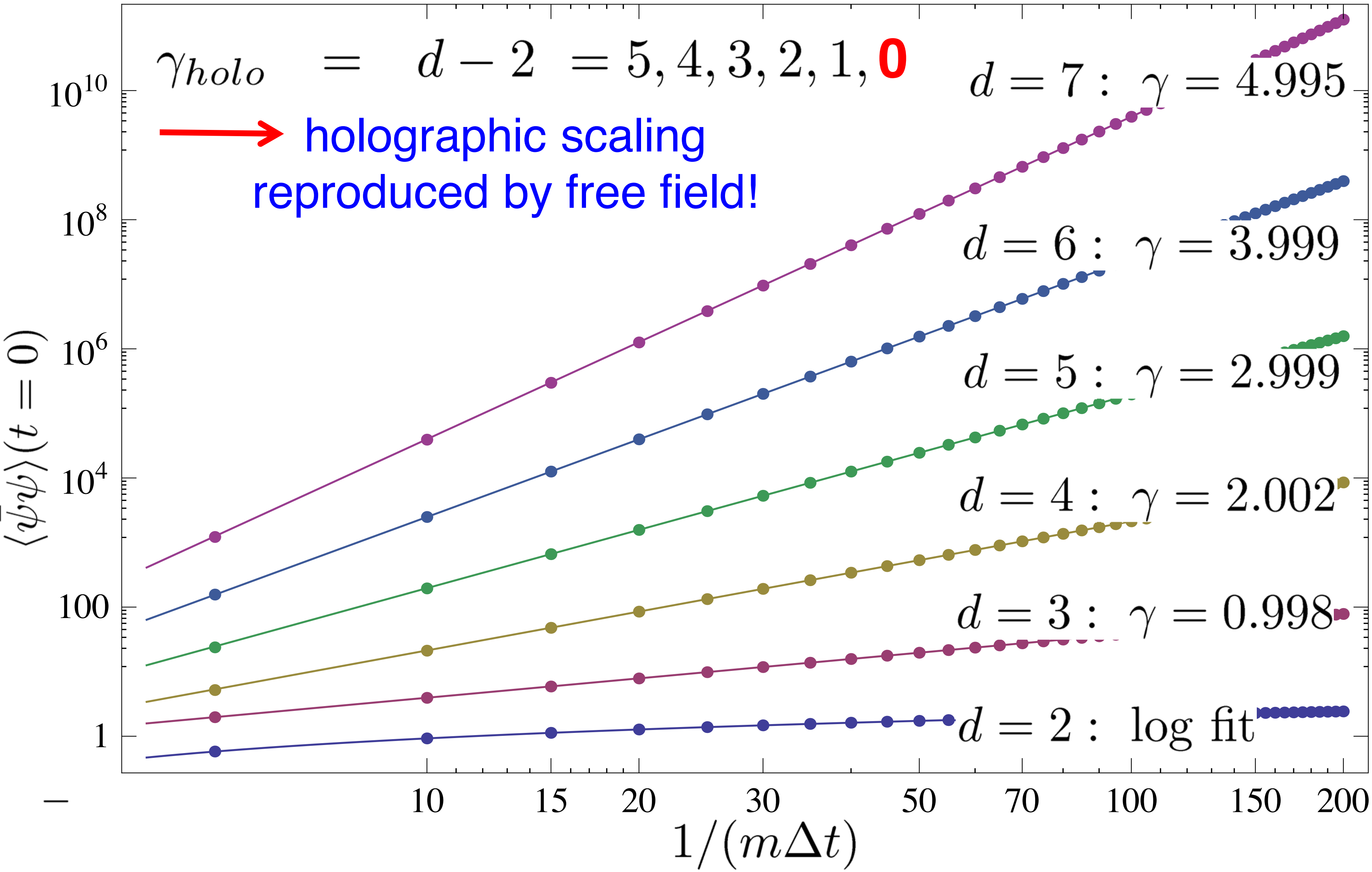
- quench with finite Δt and examine limit $\Delta t \to 0$

eq. of motion: $$\left[\gamma^\mu \partial_\mu - \frac{m}{2}\left(1+\tanh(t/\Delta t)\right)\right]\psi = 0$$

- again, related to problem of fermions in a cosmological bkgd

Generalizing "Fast" Quenches:

- regulated response (d=7,6,5,4,3,2):



Generalizing "Fast" Quenches:

→ why is holographic scaling reproduced by free field?!?!?

- consider $S = S_{CFT} + \int d^dx\, \lambda(t)\, \mathcal{O}_\Delta(x)$ with $\lambda(t) = \Delta\lambda\ f(t/\Delta t)$

- apply conformal perturbation theory

$$
\begin{aligned}
\langle \mathcal{O}_\Delta(0)\rangle &= \langle \mathcal{O}_\Delta(0)\, \exp[i\int d^dx \lambda(t)\mathcal{O}_\Delta(x)]\rangle_{\text{CFT}} \\
&= \langle \mathcal{O}_\Delta(0)\rangle_{\text{CFT}} + i\Delta\lambda\langle \mathcal{O}_\Delta(0) \int d^dx f(t/\Delta t)\mathcal{O}_\Delta(x)\rangle_{\text{CFT}} \\
&\quad -\frac{\Delta\lambda^2}{2}\langle \mathcal{O}_\Delta(0) \int d^dx f(t/\Delta t)\mathcal{O}_\Delta(x) \int d^dx' f(t'/\Delta t)\mathcal{O}_\Delta(x')\rangle_{\text{CFT}} + \cdots \\
&= b_1 \frac{\Delta\lambda}{(\Delta t)^{2\Delta-d}} + b_2 \frac{\Delta\lambda^2}{(\Delta t)^{3\Delta-2d}} + \cdots
\end{aligned}
$$

Generalizing "Fast" Quenches:

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&\quad -\frac{\Delta\lambda^2}{2}\langle \mathcal{O}_\Delta(0)\int d^d x f(t/\Delta t)\mathcal{O}_\Delta(x)\int d^d x' f(t'/\Delta t)\mathcal{O}_\Delta(x')\rangle_{\rm CFT} + \cdots \\
&= \frac{1}{(\Delta t)^\Delta}\left(b_1\, g + \ b_2\, g^2 + \ \cdots\right)
\end{aligned}
$$

- organized with dimensionless effective coupling: $g = \Delta\lambda\,(\Delta t)^{d-\Delta}$

- in limit $\Delta\lambda$ fixed and $\Delta t \to 0$: $g \to 0$ **!!**

→ leading term dominates: $\langle \mathcal{O}_\Delta(0)\rangle \simeq\ b_1\,\frac{\Delta\lambda}{(\Delta t)^{2\Delta-d}}$

Generalizing “Fast” Quenches:

- **holographic scaling should appear quite generally!!**

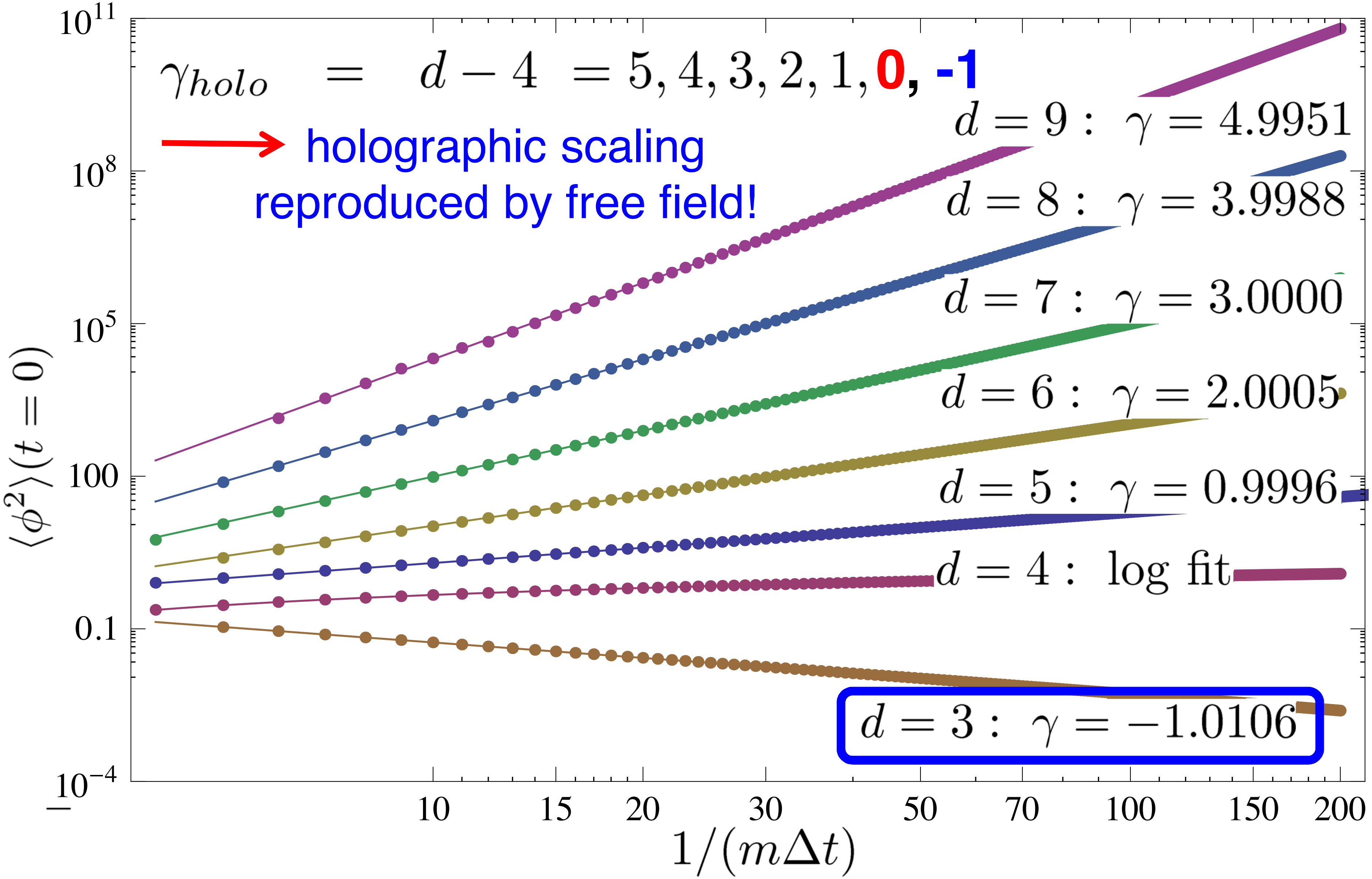
- for example:

$$\left.\begin{aligned} \langle \mathcal{O}_\Delta \rangle &\sim \frac{\Delta\lambda}{(\Delta t)^{2\Delta-4}} \\ \Delta\mathcal{E} &\sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta-4}} \end{aligned}\right\} \quad \Delta t \to 0$$

- **what about sudden quenches of C&C??**

Generalizing “Fast” Quenches:

- regulated response (d=9,8,7,6,5,4,**3**):



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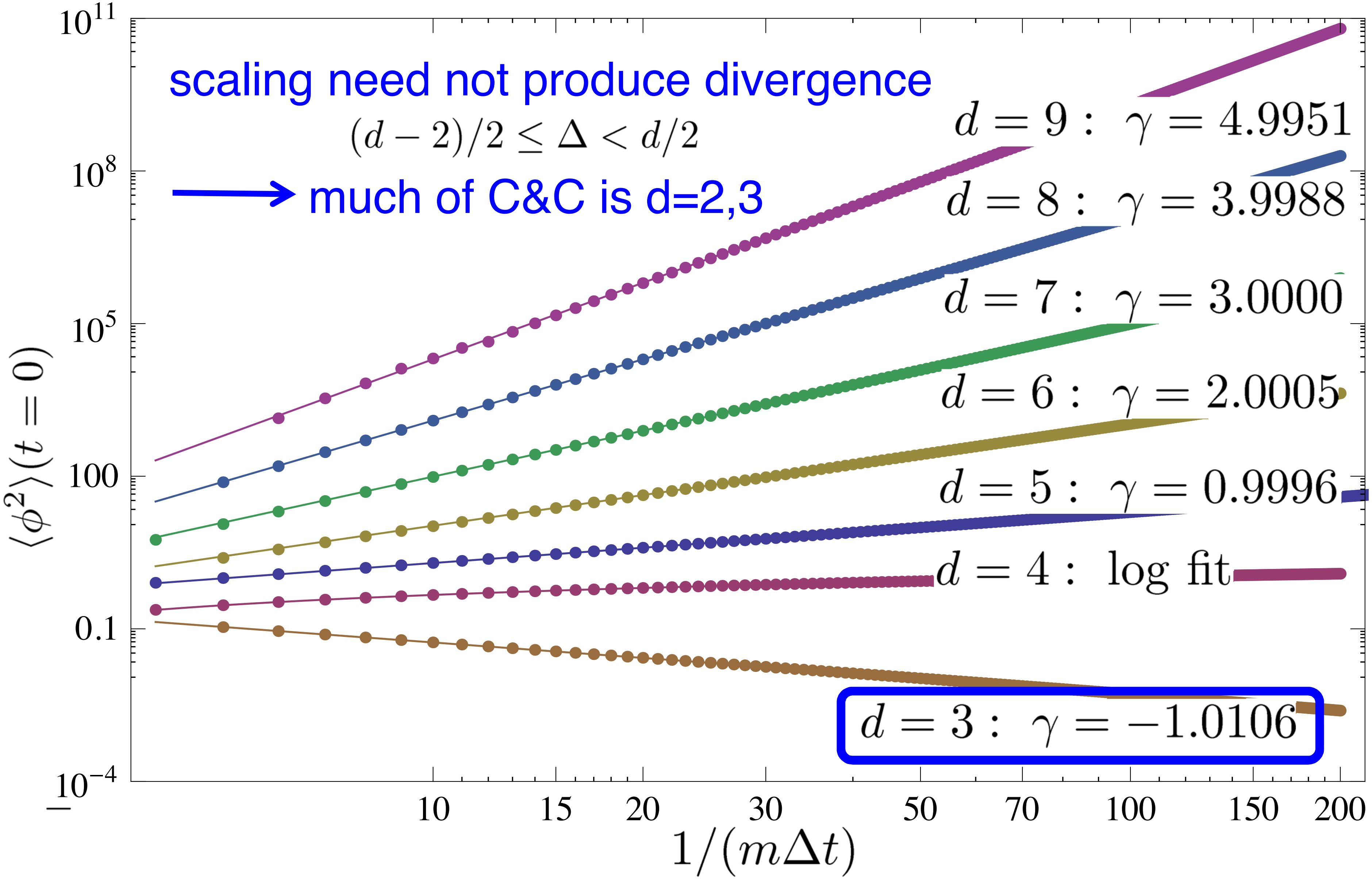

Generalizing "Fast" Quenches:

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$$\left.\begin{array}{r} \langle \mathcal{O}_\Delta \rangle \sim \dfrac{\Delta\lambda}{(\Delta t)^{2\Delta-4}} \\[2em] \Delta\mathcal{E} \sim \dfrac{\Delta\lambda^2}{(\Delta t)^{2\Delta-4}} \end{array}\right\} \quad \Delta t \to 0$$

- **what about sudden quenches of C&C??**

$d = 3: \;\; \gamma = -1.0106$ $\longrightarrow$ suggests $\Delta\mathcal{E} \to 0$???

- an order of limits???

Conclusions:

- quantum quenches: interesting arena for holographic study
- lessons learned:

1. Renormalization of (strongly coupled) boundary QFT with time-dependent couplings works in a straightforward way
2. Response to fast quenches exhibits universal scaling

► much of fast holographic quenches analytically accessible

► both lessons 1 & 2 apply beyond holographic arena!!

Lots to explore!