AdS COLLAPSE AND RELAXATION IN CLOSED QUANTUM SYSTEMS

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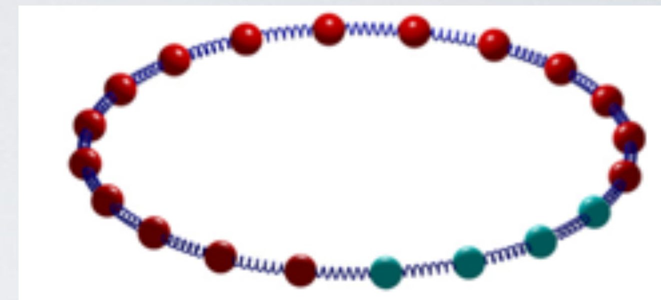
arXiv:1403.2632 and 1412.6002

MOTIVATION

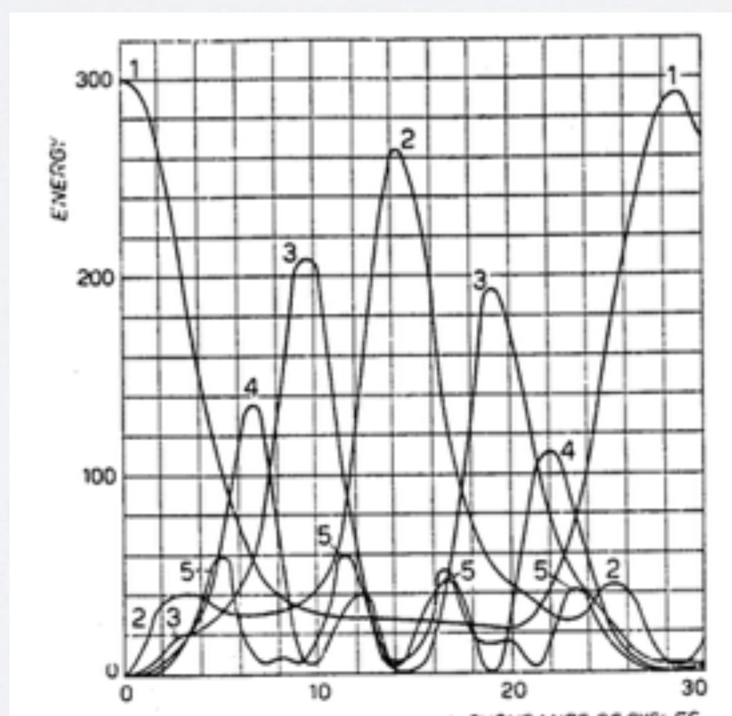
Classical Dynamics: one expects non-linear dynamics should lead **on its own** to thermalisation:

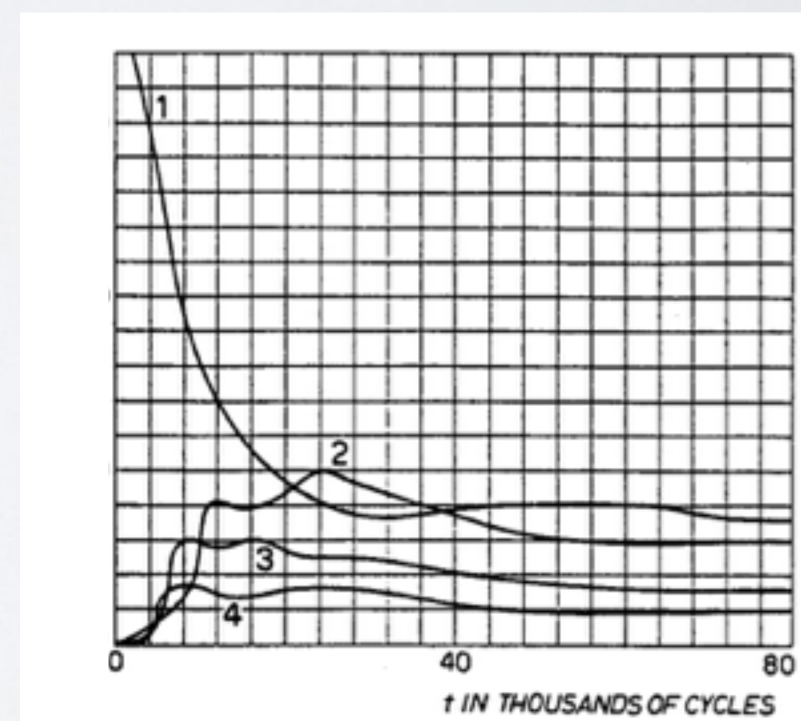
STUDIES OF NON LINEAR PROBLEMS

E. Fermi, J. Pasta, and S. Ulam
Document LA-1940 (May 1955).



$$\ddot{x}_i = (x_{i+1} + x_{i-1} - 2\,x_i) + \alpha\,[(x_{i+1} - x_i)^2 - (x_i - x_{i-1})^2]$$
$$(i = 1\,,\,2\,,\cdots,\,64),$$





MOTIVATION

¿How do closed quantum systems thermalise?

- no dynamical chaos since time evolution is linear
- discrete spectrum
- how conserved quantities constraint relaxation

Fundamental questions: for $\langle \hat{\mathcal{O}}_i(t) \rangle$

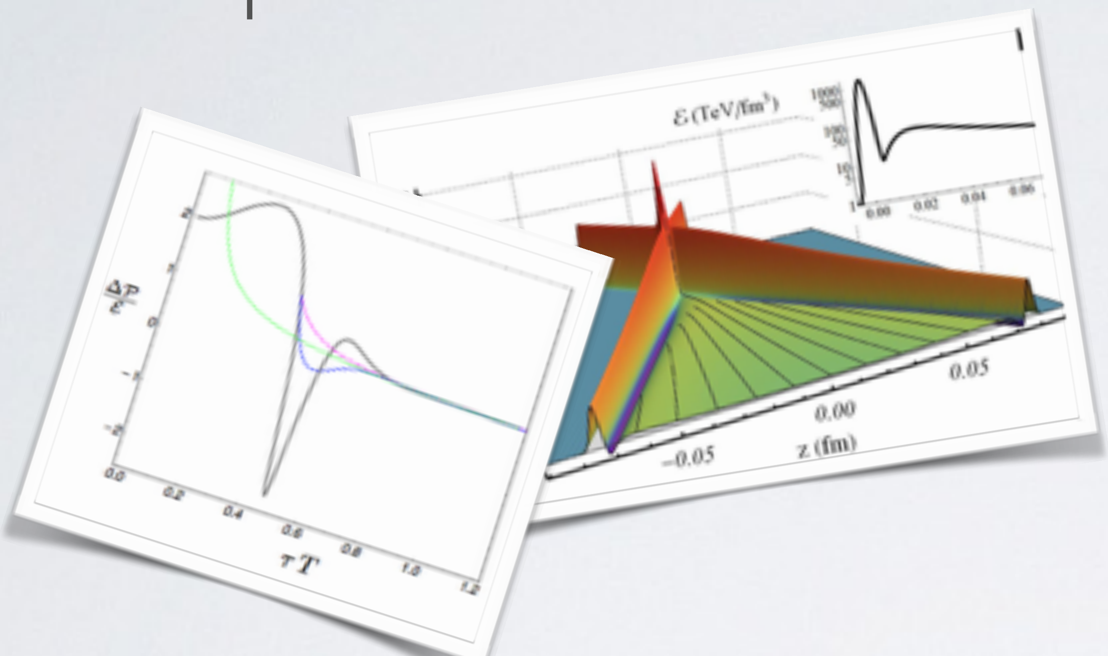
- is there a stationary state being reached?
- can it be described by a Gibbs ensemble?
- are initial conditions erased?

Addressable with :

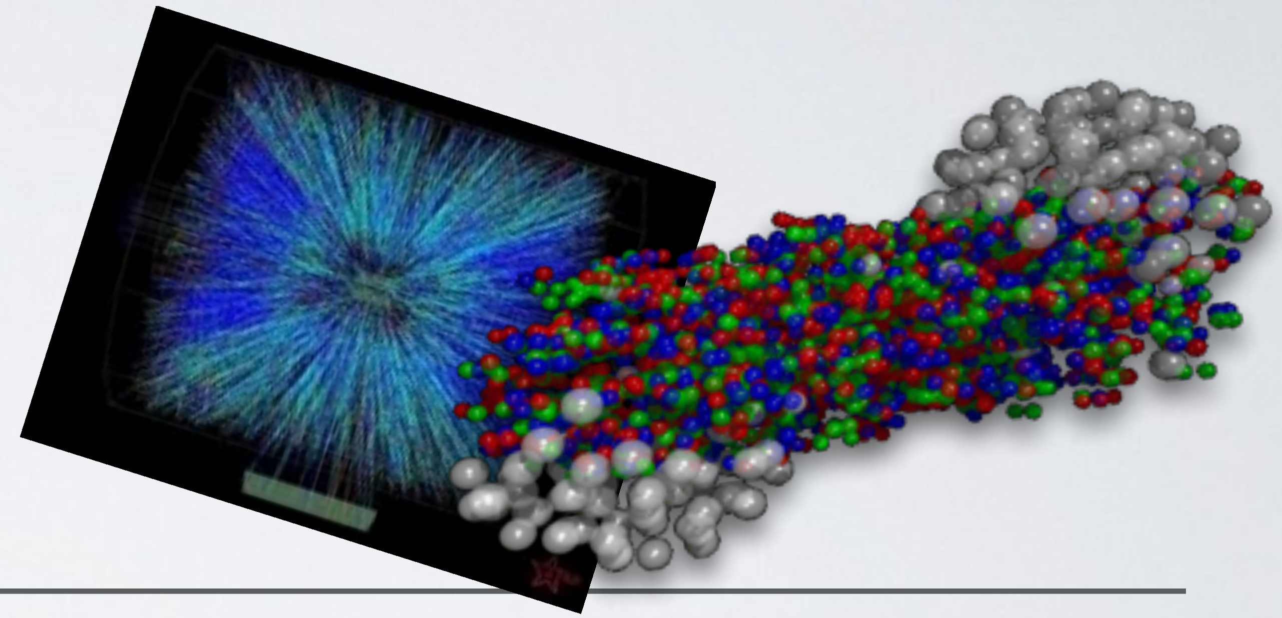
- recent advances in ultracold atom systems
- exact results in integrable chains and CFT
- AdS/CFT

time dependent AdS/CFT

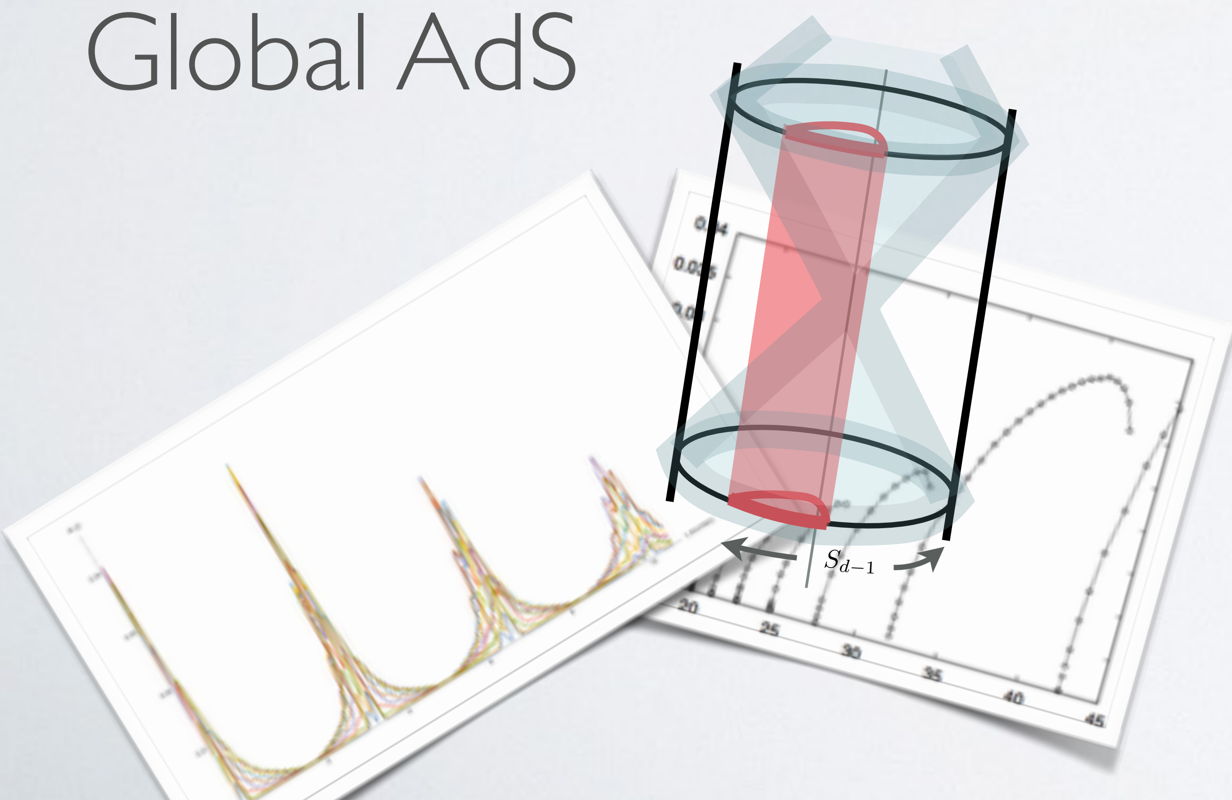
Poincaré patch



Heavy Ion Collisions



Global AdS



PLAN

- Relaxation in Closed Quantum Systems
- Dynamics of Entanglement Entropy
- Holographic Revivals

Relaxation in Quantum Closed Systems


nature

LETTERS

A quantum Newton's cradle

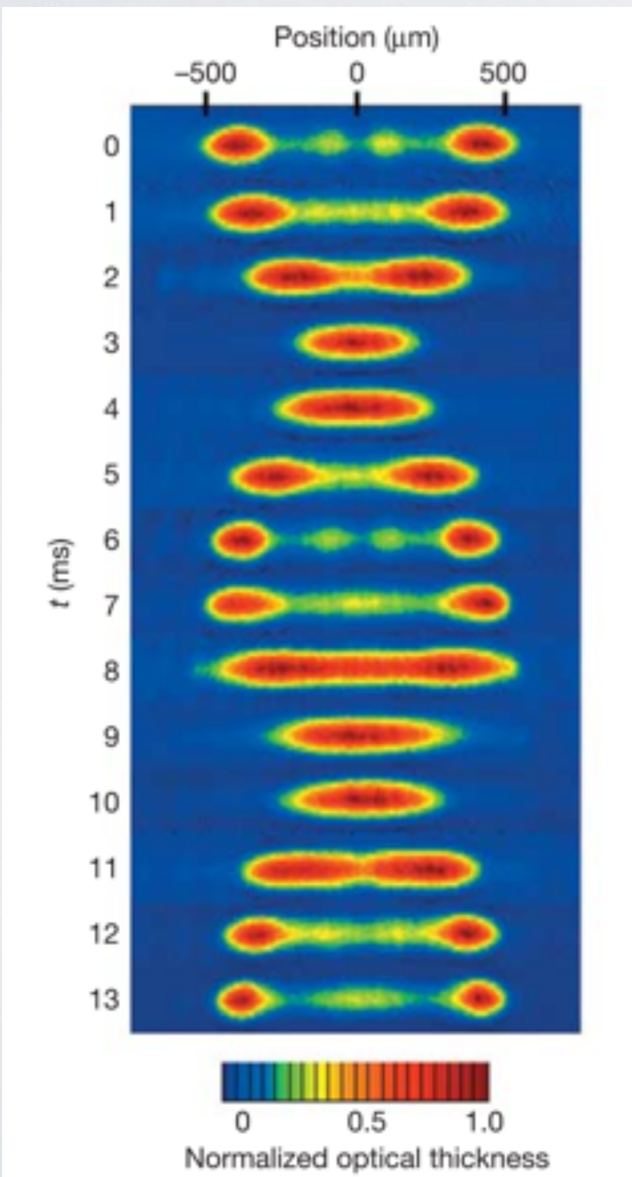
Toshiya Kinoshita[1], Trevor Wenger[1] & David S. Weiss[1]

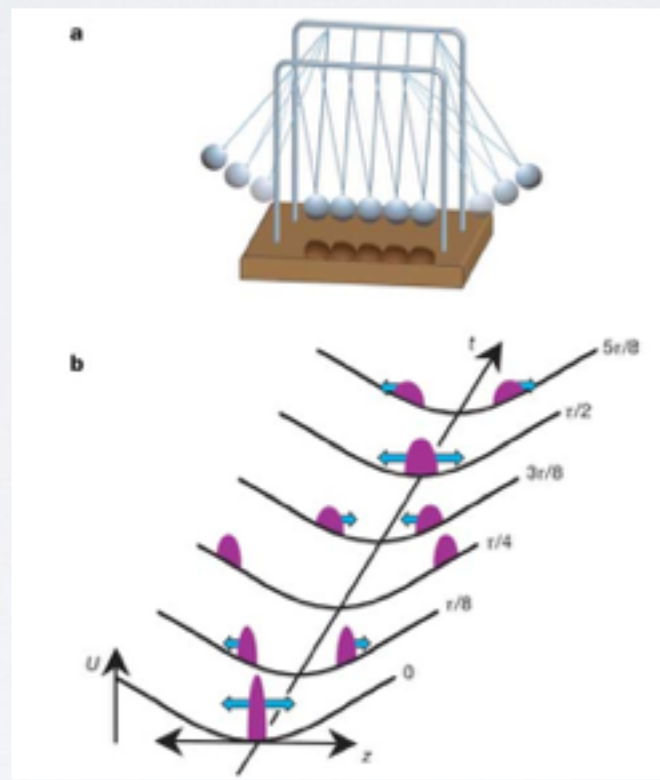
2006

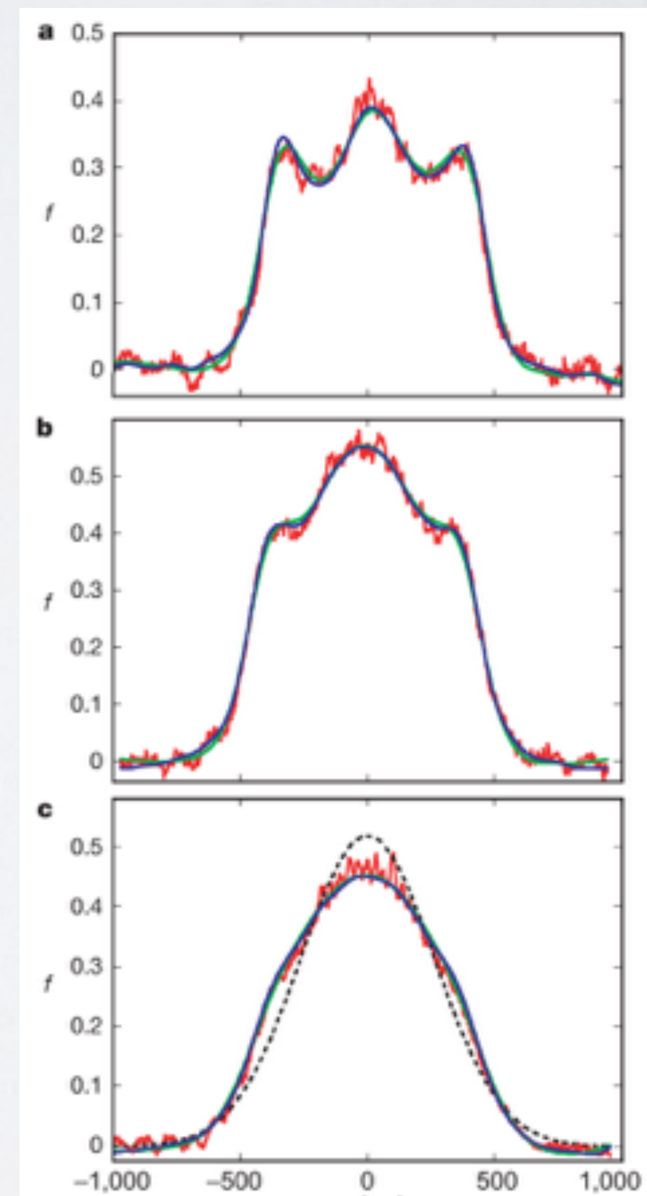
When addition conserved charges

prethermalization, described by Generalized Gibbs Ensemble

no erasure of initial condition







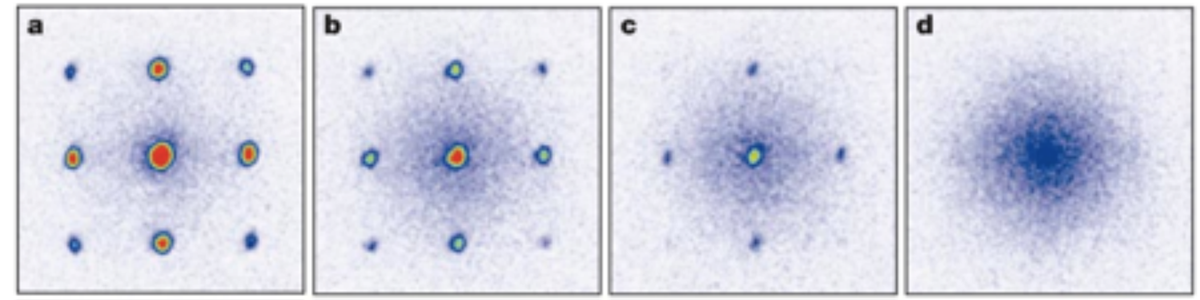
$f(p_x)$

Relaxation in Quantum Closed Systems

Collapse and revival of the matter wave field of a Bose–Einstein condensate

Markus Greiner, Olaf Mandel, Theodor W. Hänsch & Immanuel Bloch

2002



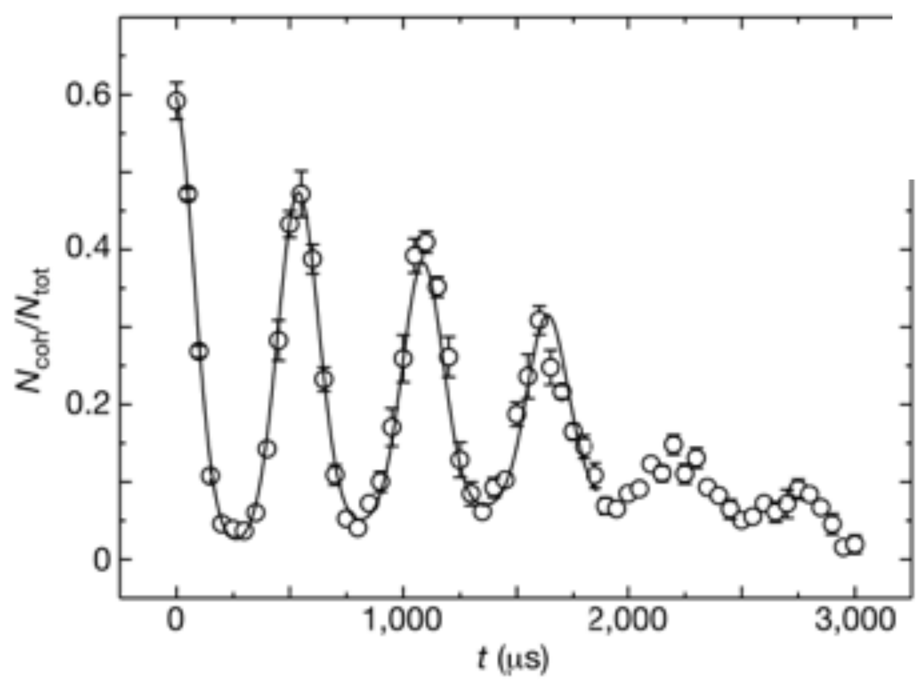
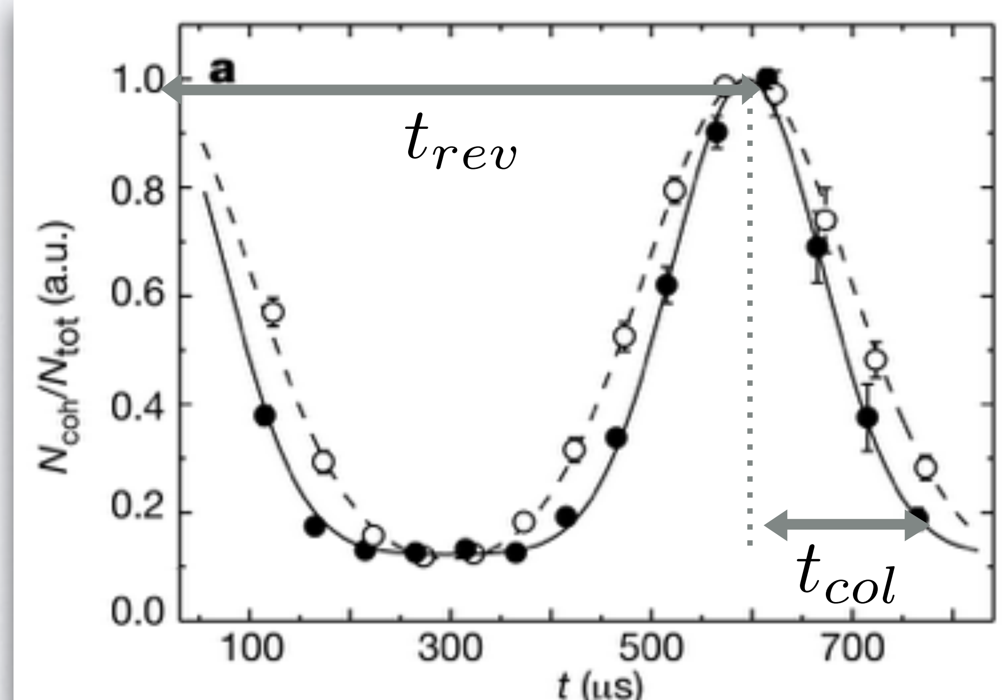


Figure 3 Number of coherent atoms relative to the total number of atoms monitored



$$\hat{H} = \frac{U}{2}\hat{n}(\hat{n}-1)$$

$$\psi(t) = \sqrt{\bar{n}}\, e^{-\bar{n}(1-\cos Ut)} e^{i\bar{n}\sin Ut}$$

$$t_{rev} = \frac{2\pi}{U} \qquad t_{col} = \frac{1}{\bar{n}U}$$

Relaxation in Quantum Closed Systems

The Finite Group Velocity of Quantum Spin Systems

Elliott H. Lieb*
Dept. of Mathematics, Massachusetts Institute of Technology
Cambridge, Massachusetts, USA

Derek W. Robinson**
Dept. of Physics, Univ. Aix-Marseille II, Marseille-Luminy, France

1972

Light-cone-like spreading of correlations in a quantum many-body system

Marc Cheneau, Peter Barmettler, Dario Poletti, Manuel Endres, Peter Schauß, Takeshi Fukuhara, Christian Gross, Immanuel Bloch, Corinna Kollath & Stefan Kuhr

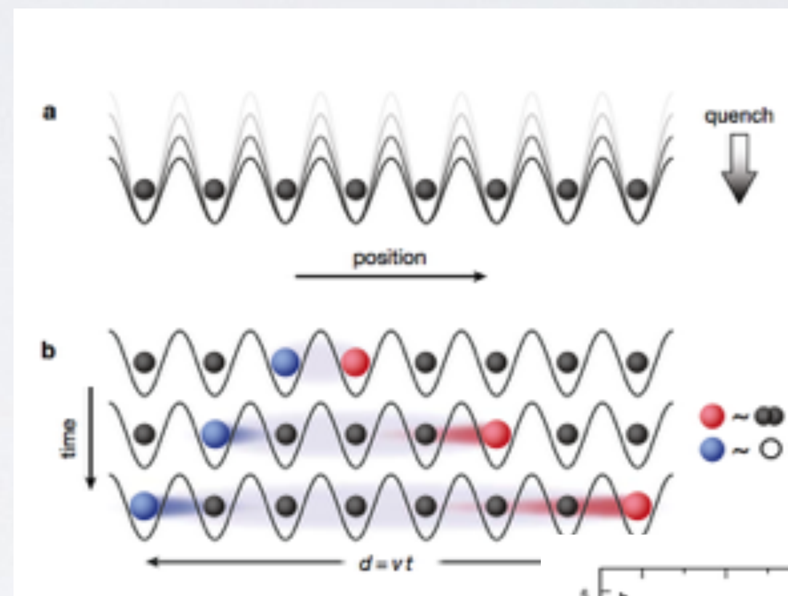
2012

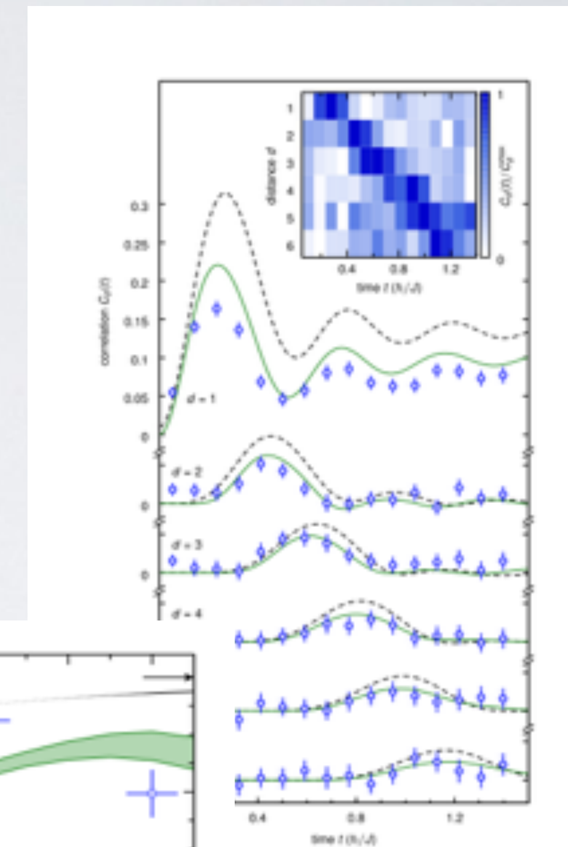
"Light-cone" dynamics after quantum quenches in spin chains

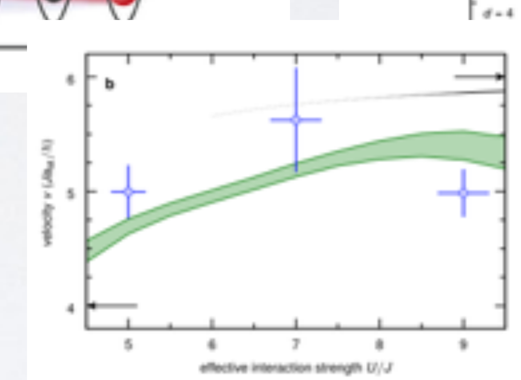
Lars Bonnes,[1,*] Fabian H. L. Essler,[2] and Andreas M. Läuchli[1]

2014

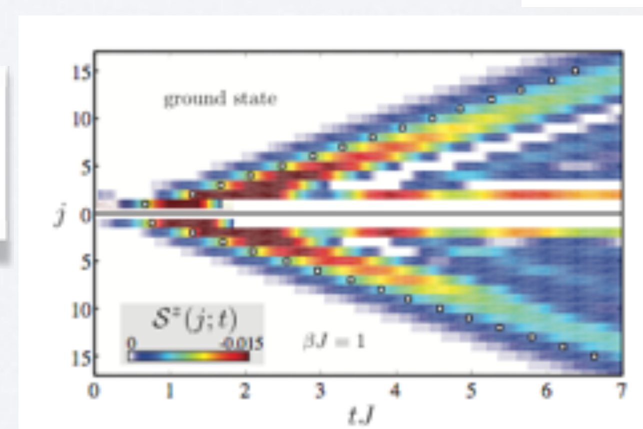
Quantum systems described by a sum of local Hamiltonians → emergent maximum speed v_{LR}







what is the speed ?



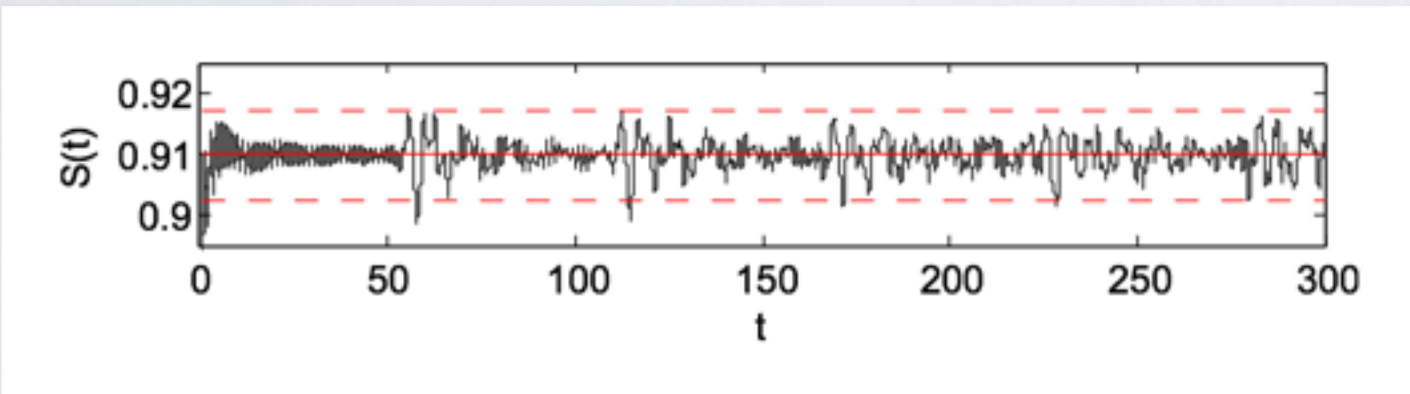
Relaxation in Quantum Closed Systems

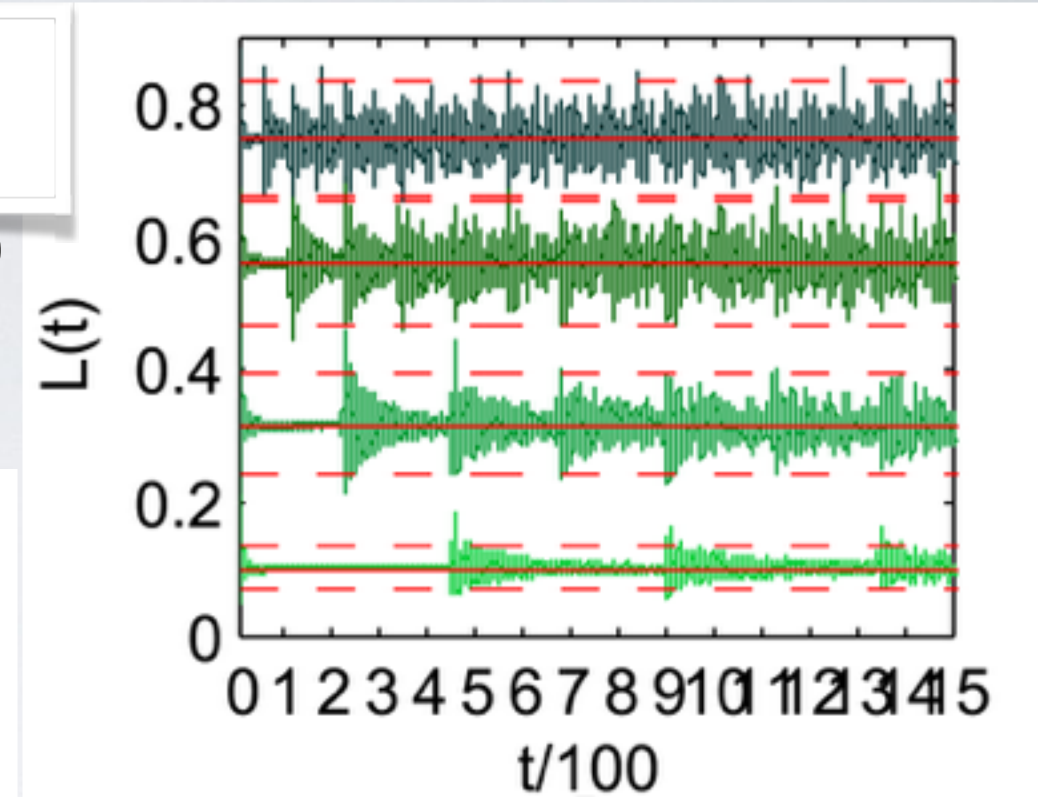
Revivals of a closed quantum system and Lieb-Robinson speed

Juho Häppölä[4],[1,2] Gábor B. Halász,[3,4] and Alioscia Hamma[4]

2010

$$t_{rev} = p\frac{L}{v_{LR}}$$

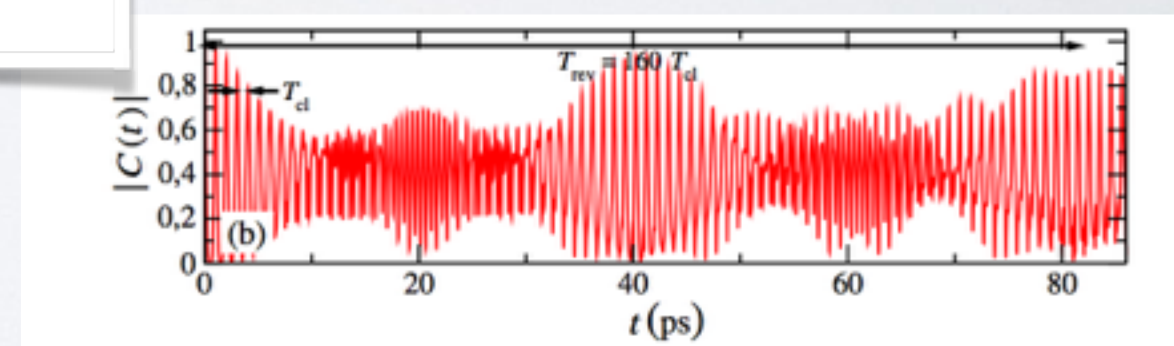




Revivals of quantum wave packets in graphene

Viktor Krueckl[1,3] and Tobias Kramer[1,2,3]

2009



R.W. Robinett, *"Quantum wave packet revivals"*, Physics Reports 392 (2004) 1-119

Quantum Field Theory

- are there revivals?

- at strong coupling ? at large c?

- what observables are nice to monitor

i) local: $\langle \hat{A} \rangle(t) = \langle \psi(t) | \hat{A} | \psi(t) \rangle$

ii) non-local:

- Wilson line $W_C = \mathrm{Tr}\left(\mathcal{P} \exp \frac{i}{\hbar} \int_C A \right)$
- **entanglement entropy**

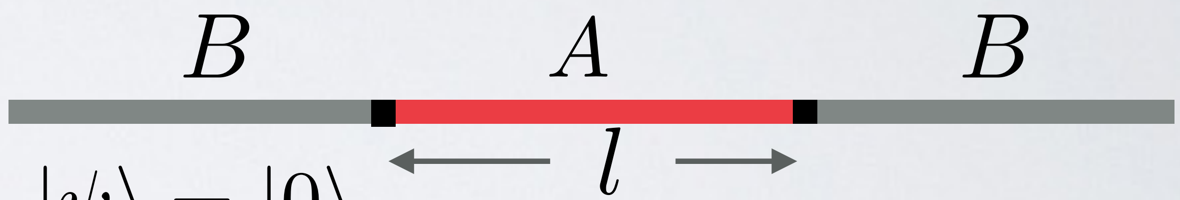
Quantum Field Theory

$$S_A = -\mathrm{Tr}\,\hat{\rho}_A \log \hat{\rho}_A \qquad \hat{\rho}_A = \mathrm{Tr}_B \hat{\rho}$$

Exact results for CFT in 1+1 dimensions

C. Holzey, F. Larsen, F. Wilczek 1994
P. Calabrese & J. Cardy 2003

$$T = 0 \qquad S_A = \frac{c}{3}\log\left(\frac{l}{\epsilon}\right)$$

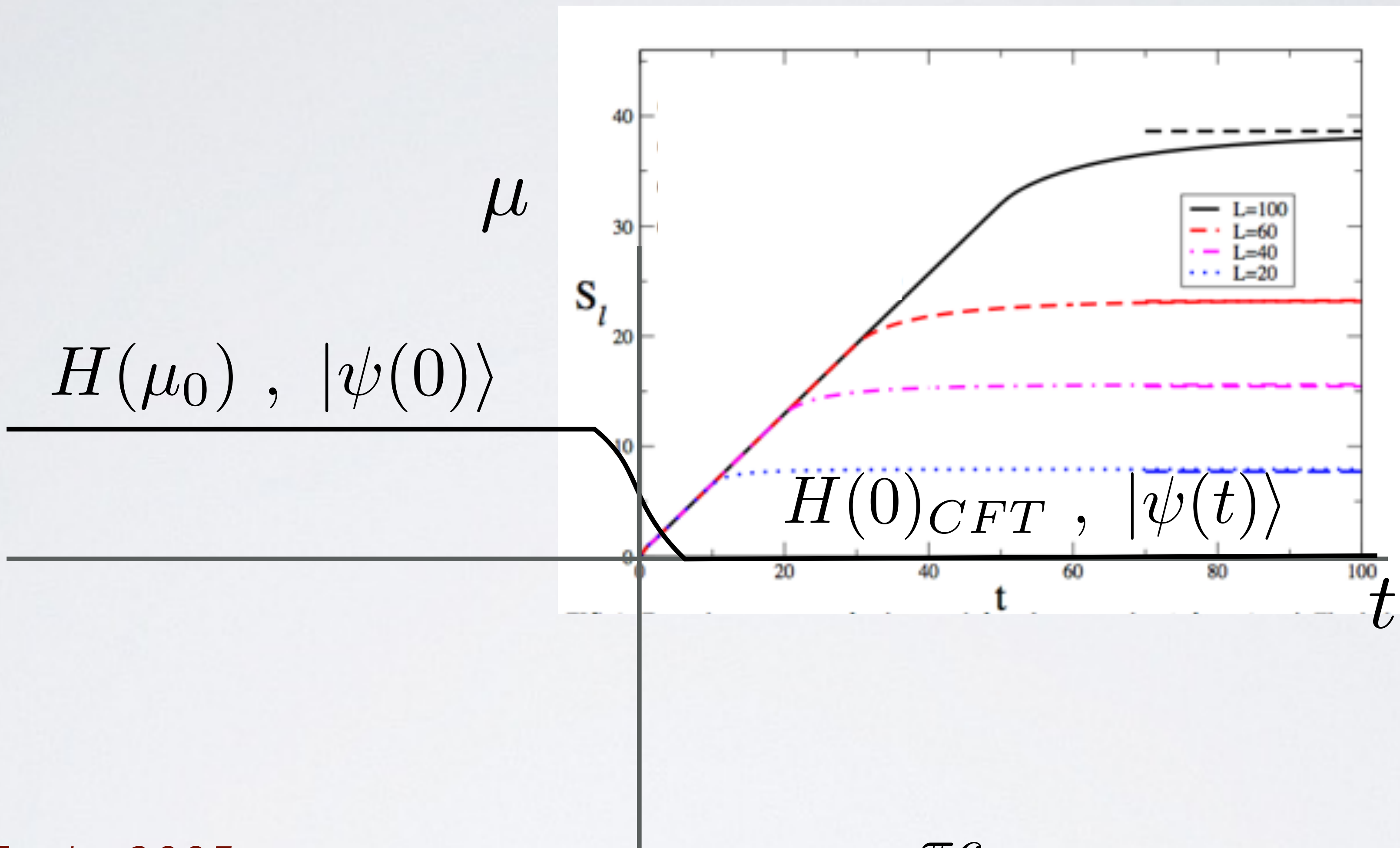


$$|\psi\rangle = |0\rangle$$

$$T = \frac{1}{\beta} \neq 0 \qquad S_A^\beta = \frac{c}{3}\log\left(\frac{\beta}{\pi\epsilon}\sinh\frac{\pi l}{\beta}\right) \longrightarrow \begin{cases} \xrightarrow{l \ll \beta} & \frac{c}{3}\log\frac{l}{\epsilon} \\ \xrightarrow{l \gg \beta} & \frac{c}{3}\log\left(\frac{\beta}{2\pi\epsilon}\right) + \frac{\pi c}{3\beta} l + \ldots \end{cases}$$

ENTANGLEMENT ENTROPY AFTER A QUENCH

At t = 0 the Hamiltonian changes abruptly, leaving an excited state. Watch it evolve.



P. Calabrese & J. Cardy 2005

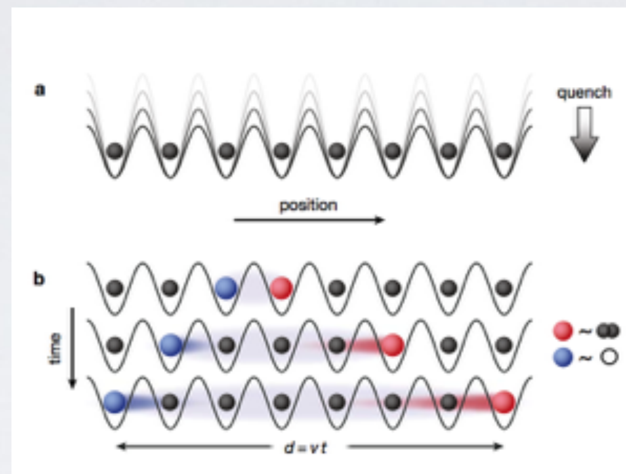
for $\quad l, t \gg \tau_0 \sim 1/m$

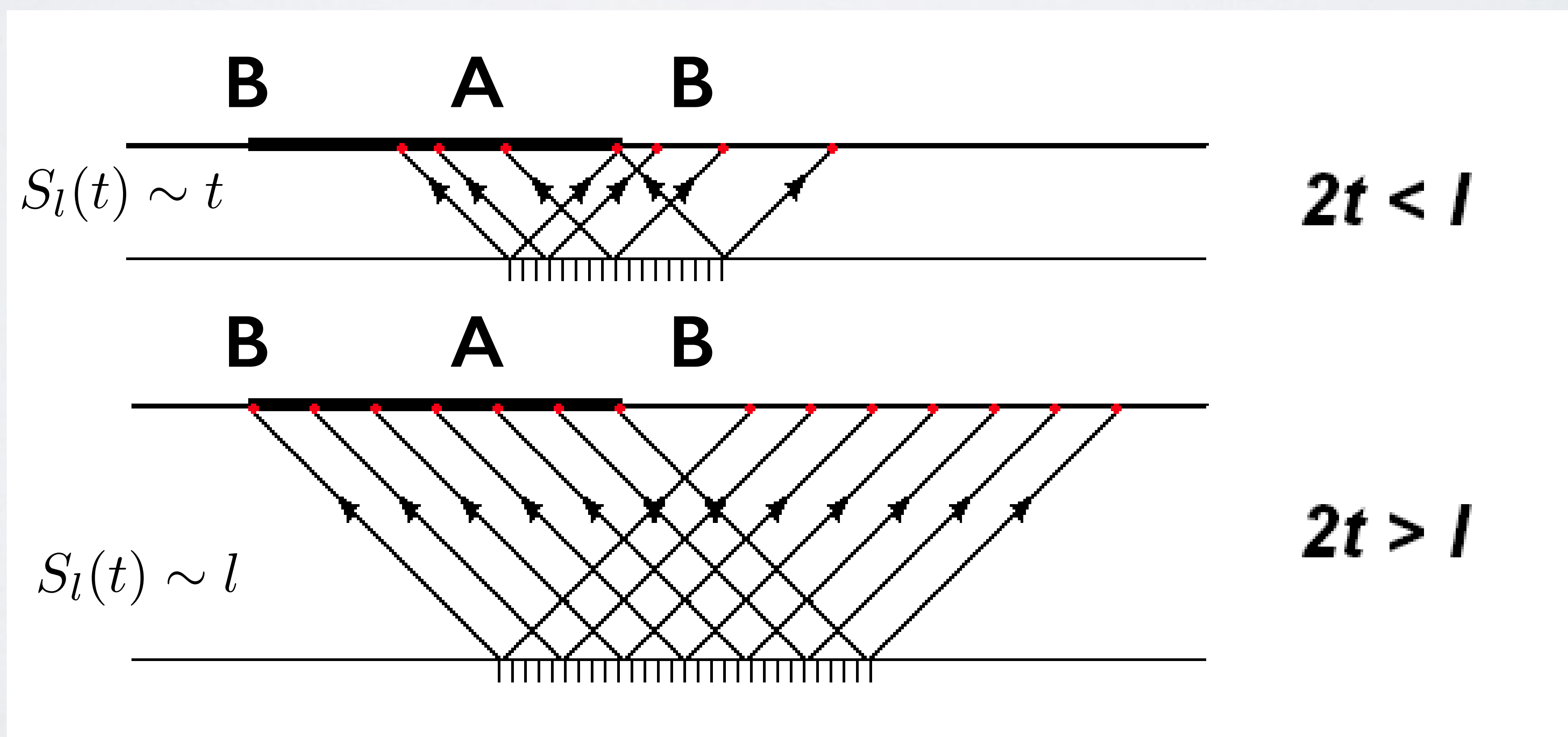
$$S_l(t) \sim \begin{cases} \dfrac{\pi c}{6\tau_0}\, t & t < l/2, \\ \dfrac{\pi c}{12\tau_0}\, l & l/2 < t \end{cases}$$

reaches an extensive EE with a $\quad T \sim 1/4\tau_0$

ENTANGLEMENT ENTROPY AFTER A QUENCH

kinematical argument: $t = 0$ quench leaves an excited sea of quasiparticle pairs
$0 \leq t \leq l/2$ entangled pairs fly apart at the speed of light

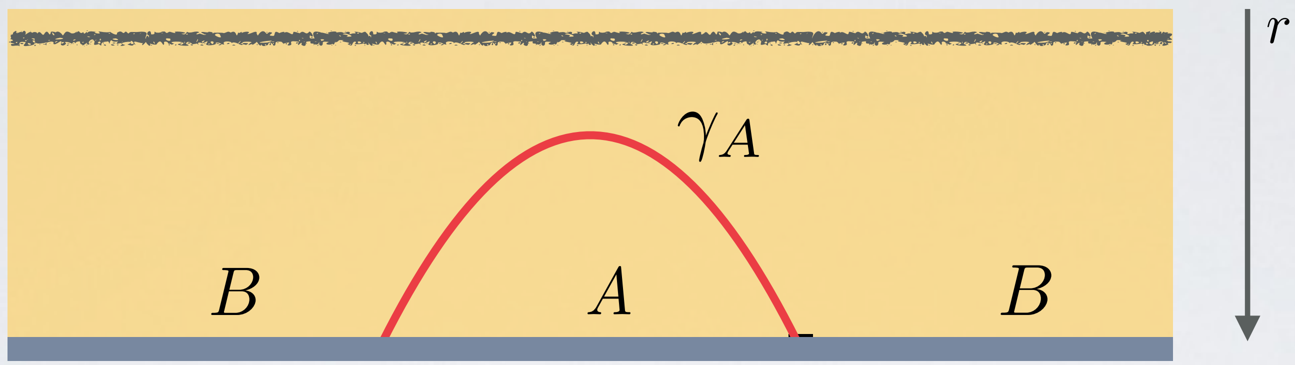




HOLOGRAPHIC ENTANGLEMENT ENTROPY

entanglement entropy ⟷ minimal surface homologous to A

$$S(\theta) = \frac{\mathrm{Area}(\gamma_A)}{4G_{d+1}}$$



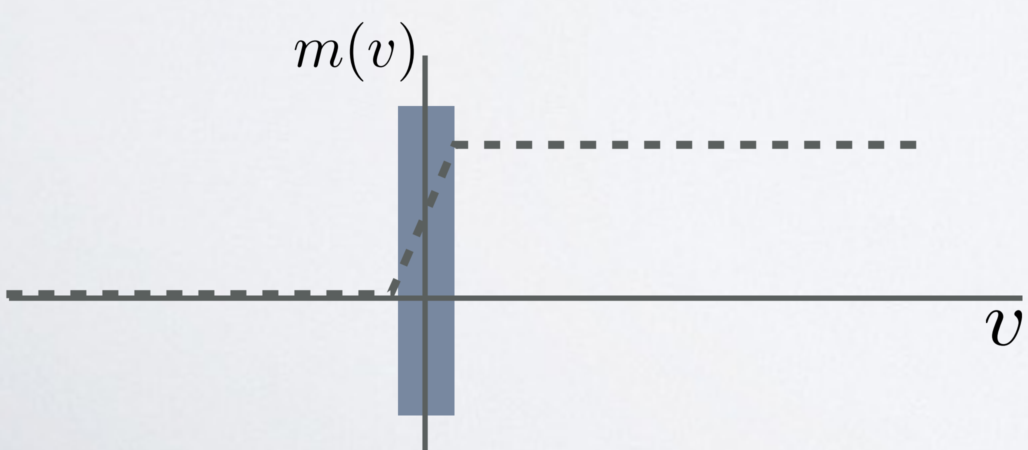
S. Ryu & T. Takayanagi 2006

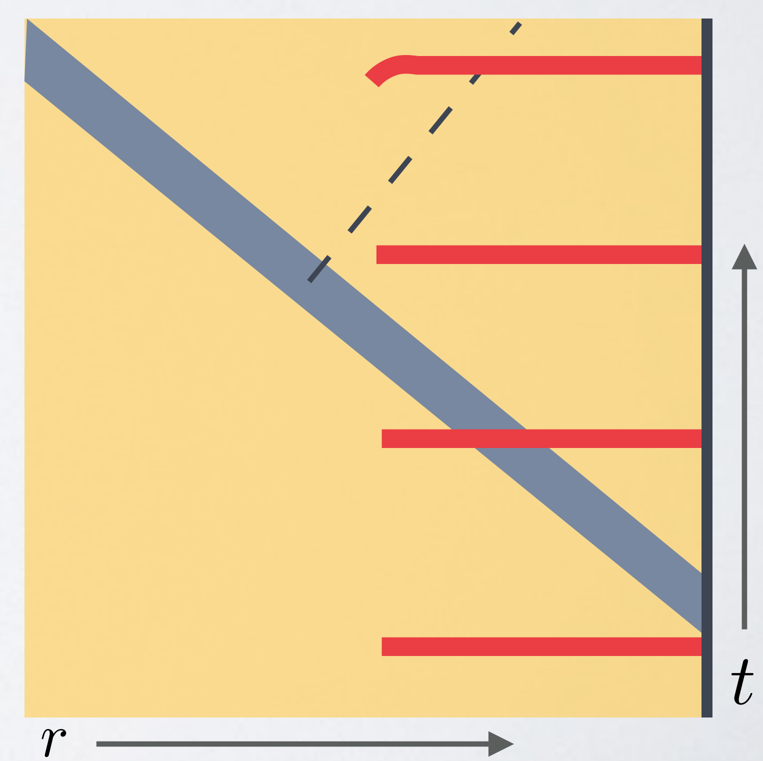
quench in CFT_d ⟷ shell collapse in AdS_{d+1}

analytic case: AdS_{d+2} -Vaidya: collapse of radiation shell

$$ds^2 = -\left(r^2 - \frac{m(v)}{r^{d-1}}\right) dv^2 + 2dr dv + r^2 \sum_{i=1}^{d} dx_i^2$$

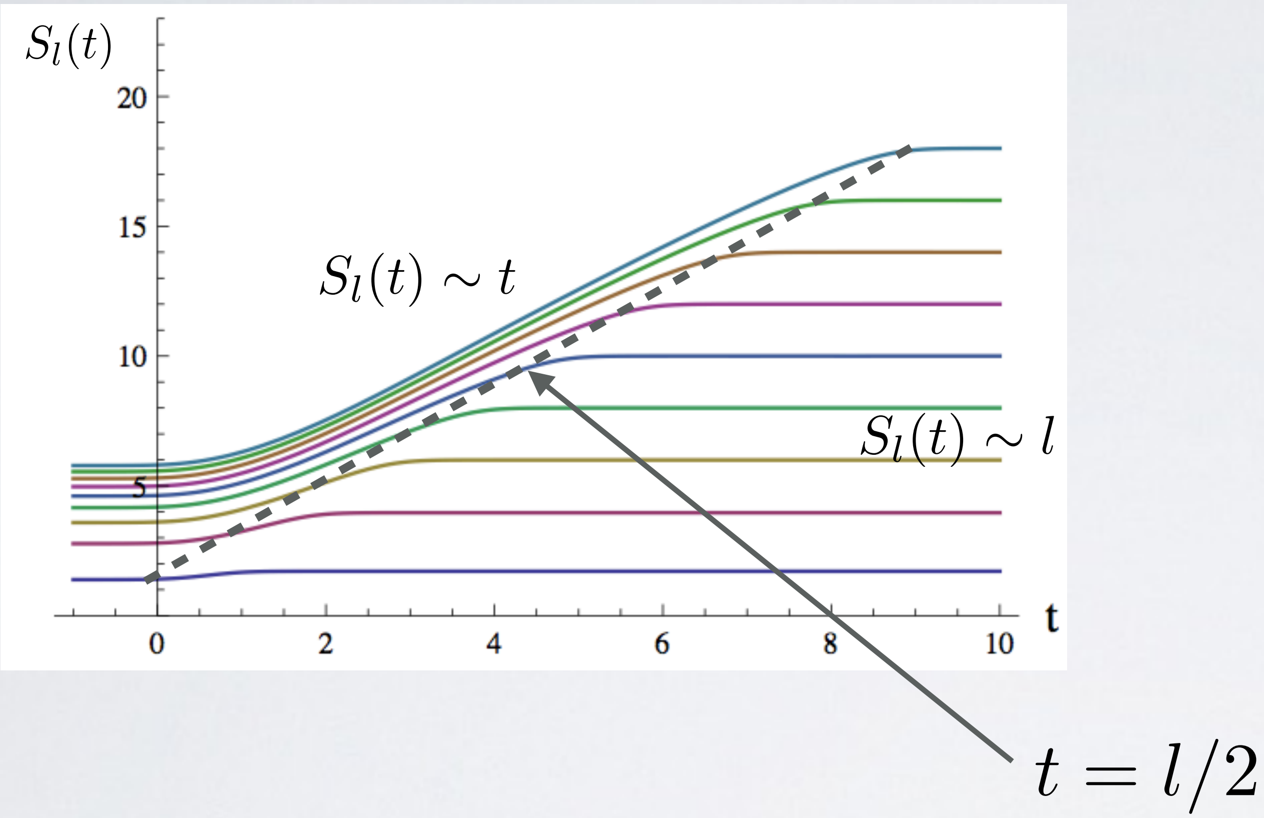
$$T_{vv} = d\frac{m'(v)}{2r^d}$$

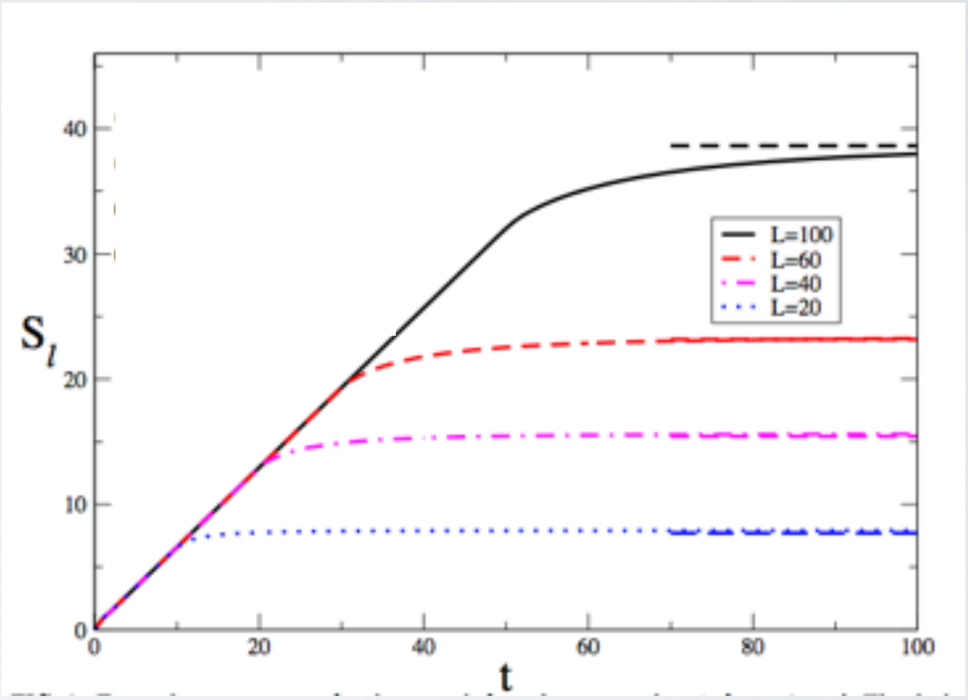




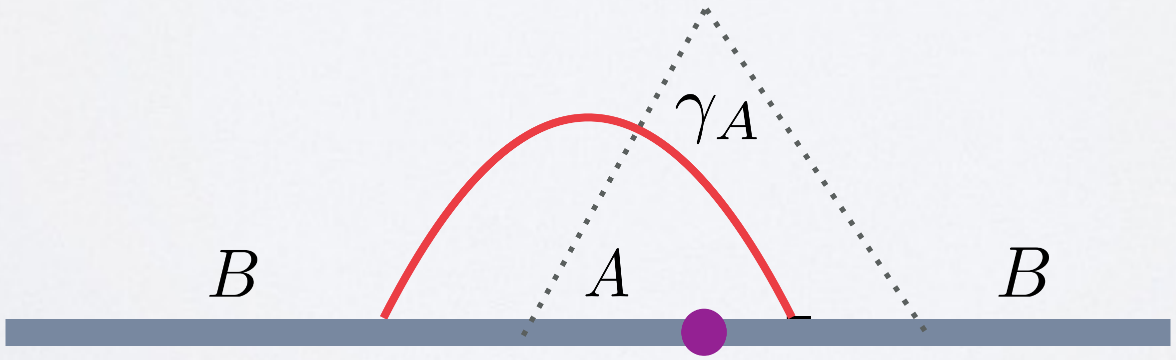
HOLOGRAPHIC ENTANGLEMENT ENTROPY

J. Abajo-Arrastia, J. Aparicio & E. López 2010
T. Albash & C.V. Johnson 2011



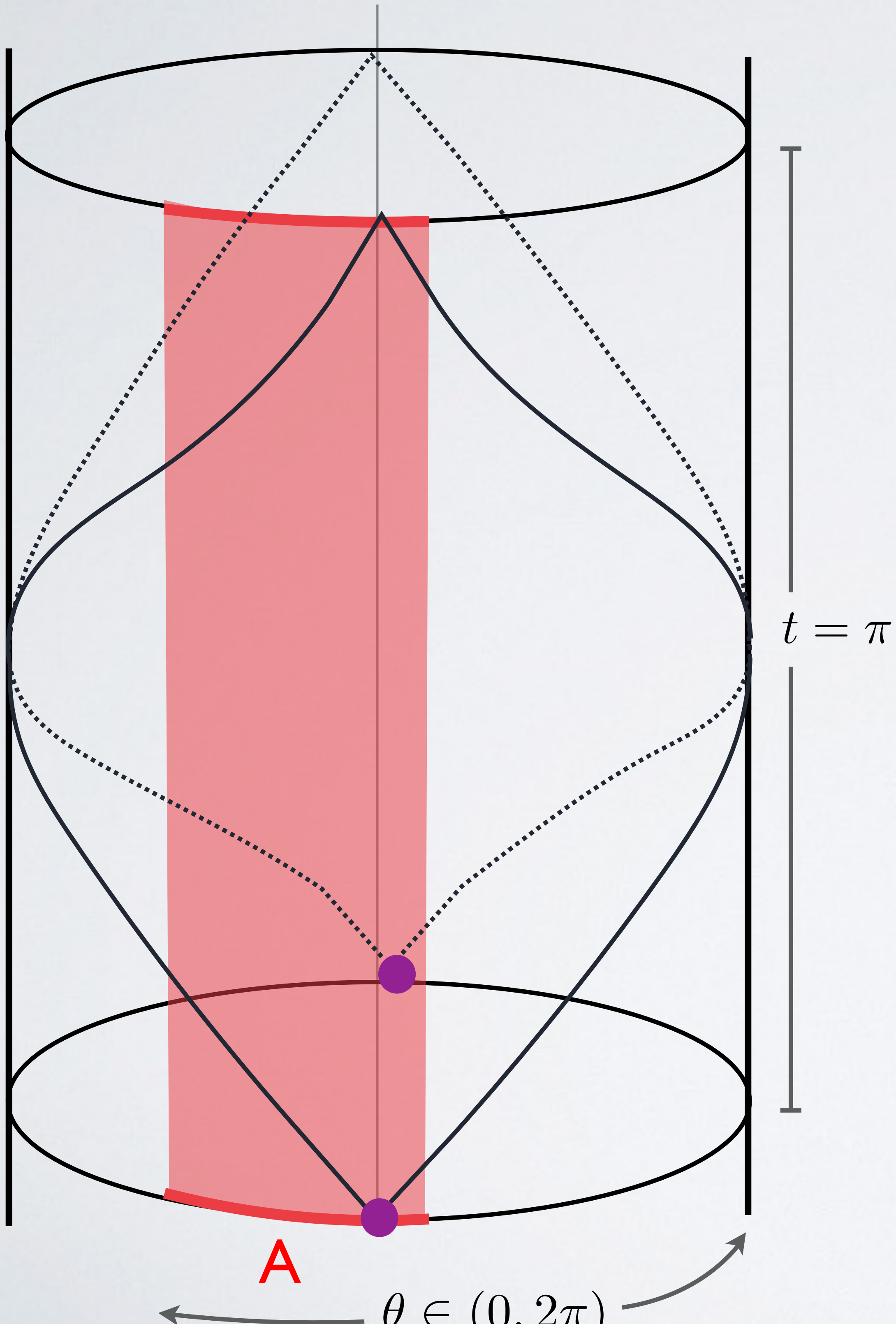


- **are we seeing a light-cone-like effect for entangled quasiparticles?**



- **is the radial position of the shell dual to the entangled pair separation?**

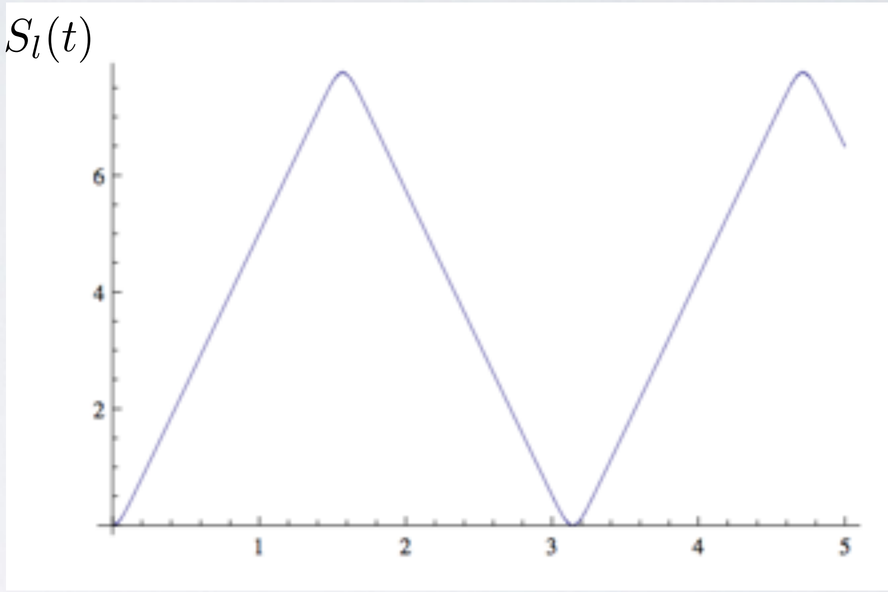
ENTANGLEMENT EVOLUTION IN COMPACT SPACE



Confirmed for:

CFT = 1+1 Free Fermion

T. Takayanagi & T. Ugajin 2010

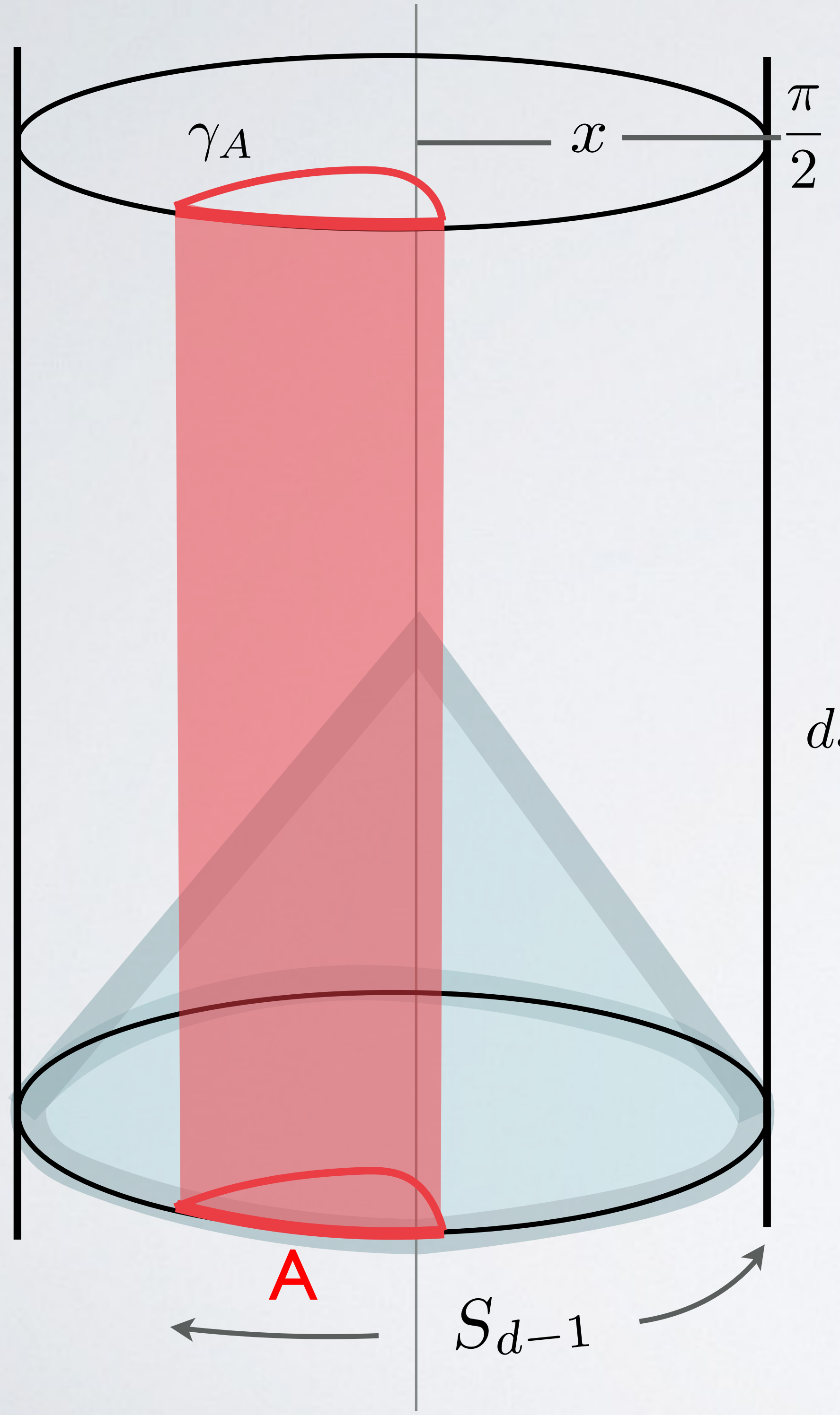


*R*CFT =minimal models

John Cardy 2014

ENTANGLEMENT EVOLUTION IN GLOBAL AdS

AdS-Vaydia leads to a direct black hole formation



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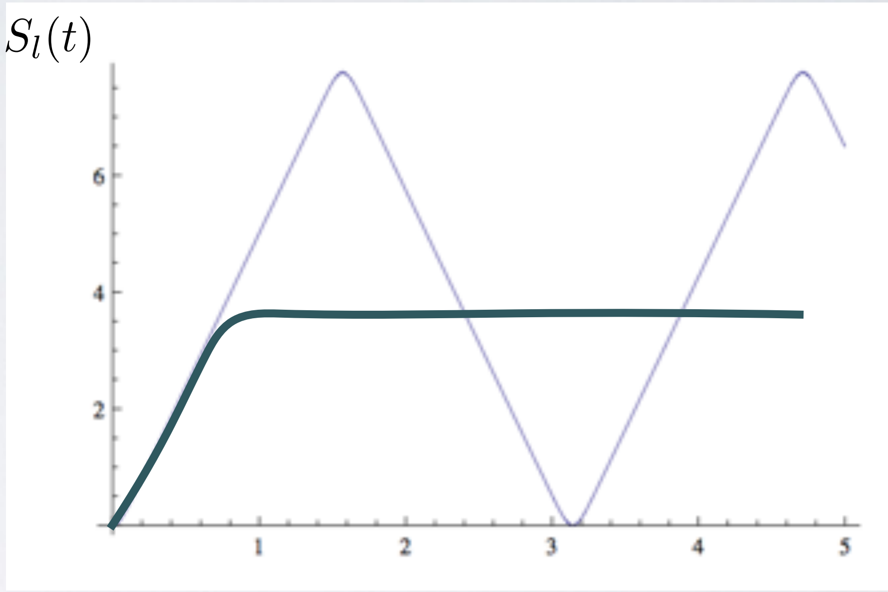
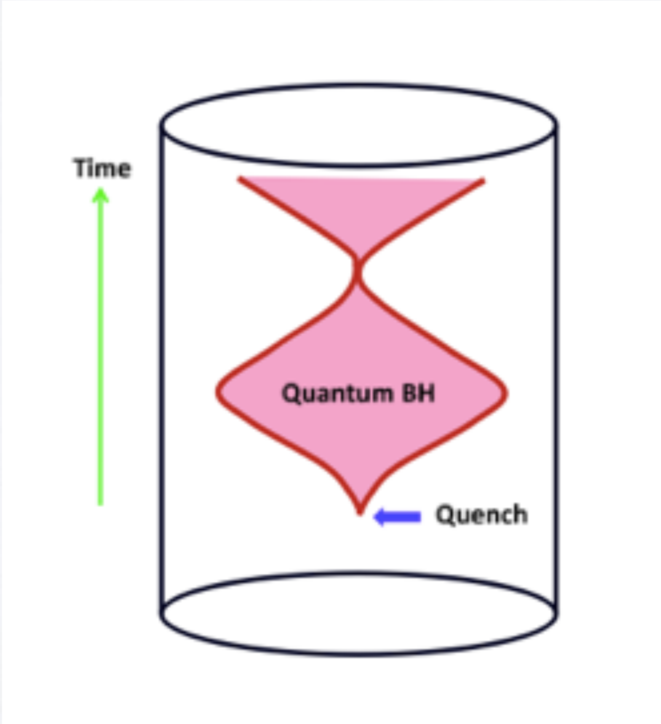
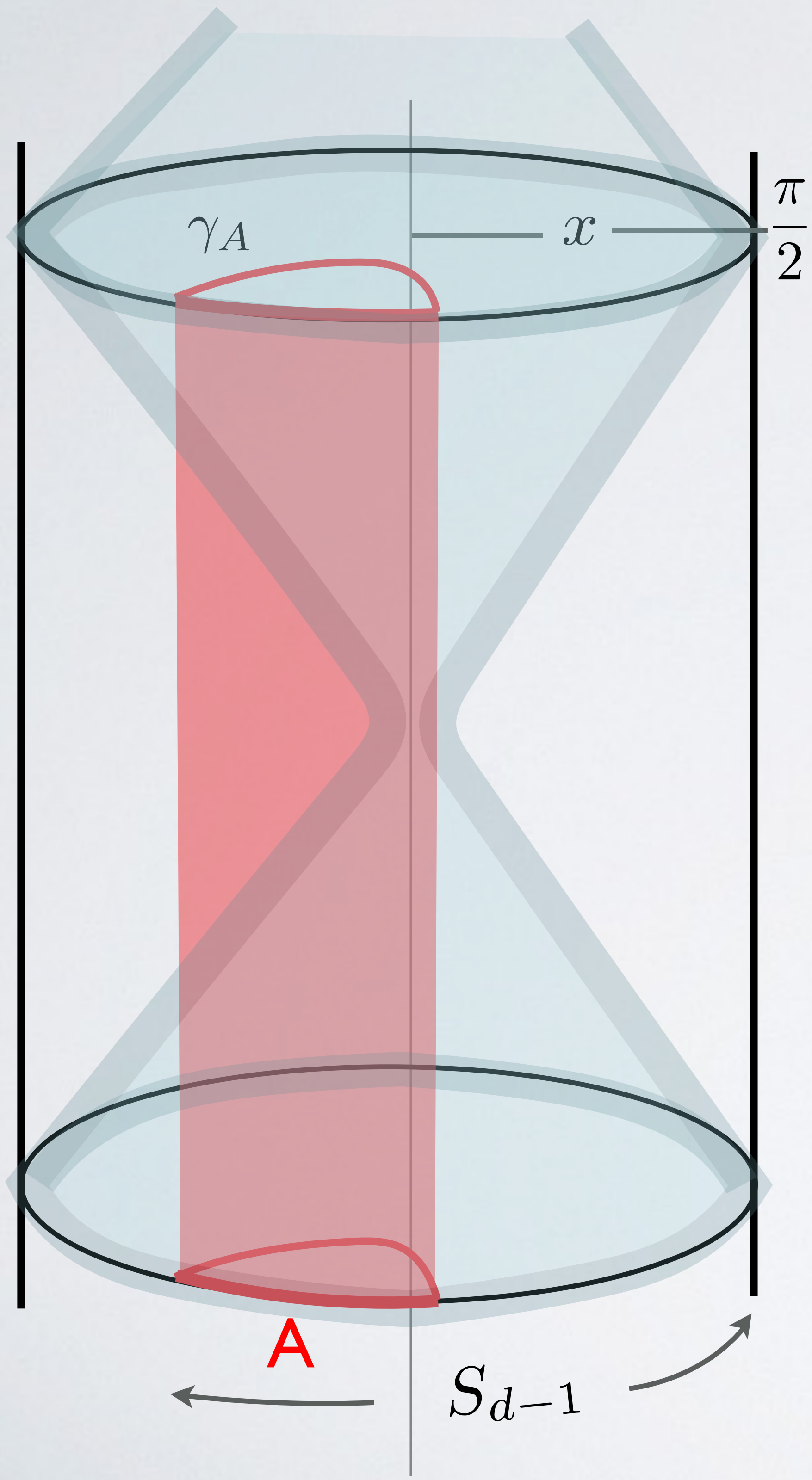


$$ds^2 = \frac{1}{\cos^2 x}\left(-dt^2 + dx^2 + \sin^2 x \, d\Omega_{d-1}^2\right)$$



ENTANGLEMENT EVOLUTION IN GLOBAL AdS



Collapse of a massless scalar field in AdS$_{d+1}$

$$S = \int d^{d+1}x\sqrt{g}\left(\frac{1}{2\kappa^2}R + \frac{d(d-1)}{l^2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi\right)$$

homogeneous ansatz (Bizon & Rostworowski 2011)

$$ds^2 = \frac{1}{\cos^2 x}\left(-A(x,t)e^{-2\delta(x,t)}dt^2 + \frac{dx^2}{A(x,t)} + \sin^2 x\, d\Omega_d^2\right)$$

$$0 \leq x \leq \pi/2$$

Equations of motion + boundary conditions

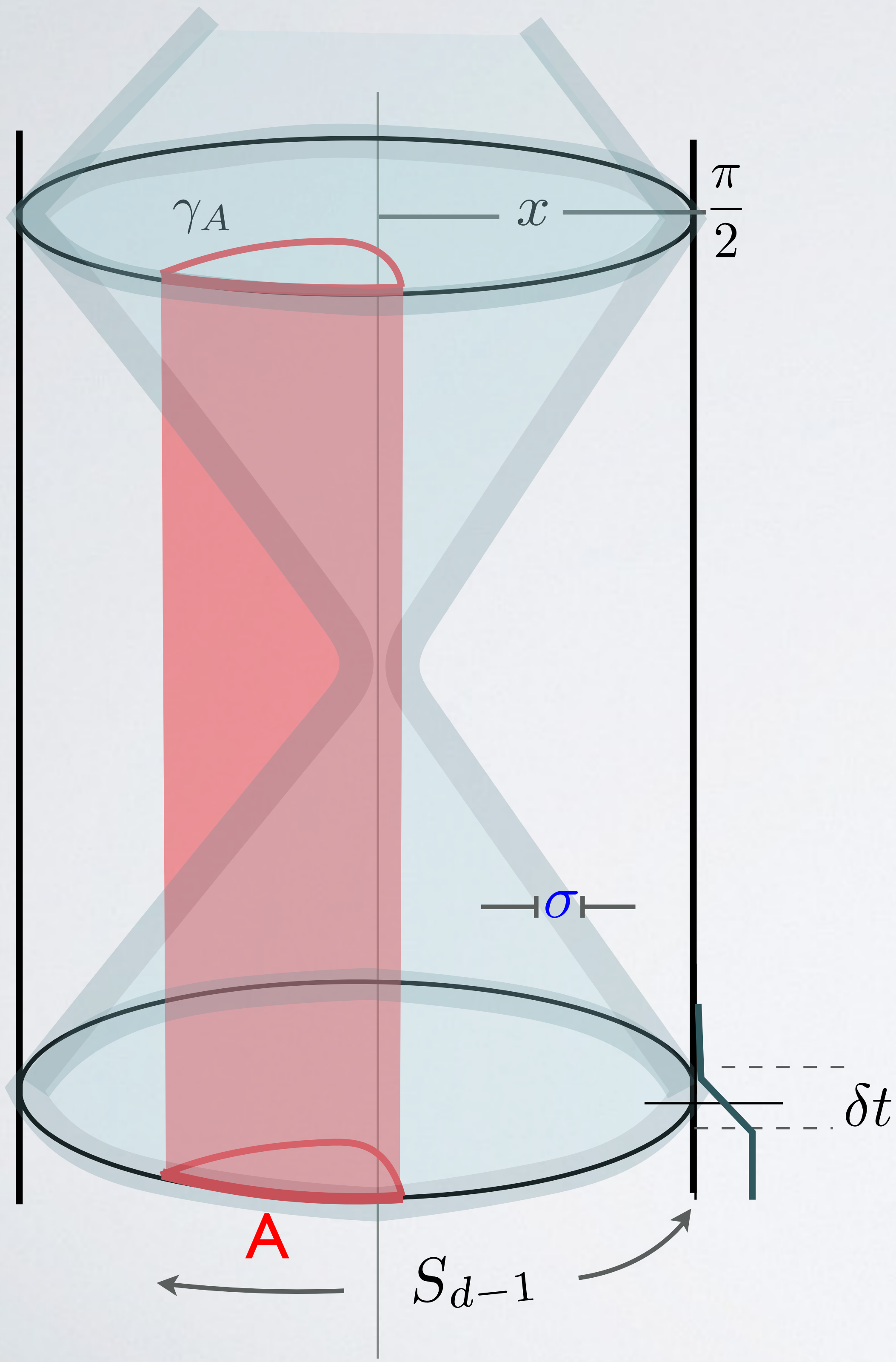
$$\phi(x,t) \sim (\tfrac{\pi}{2} - x)^d \phi_\infty + \ldots$$

$$A(x,t) \sim 1 - (\tfrac{\pi}{2} - x)^d M + \ldots$$

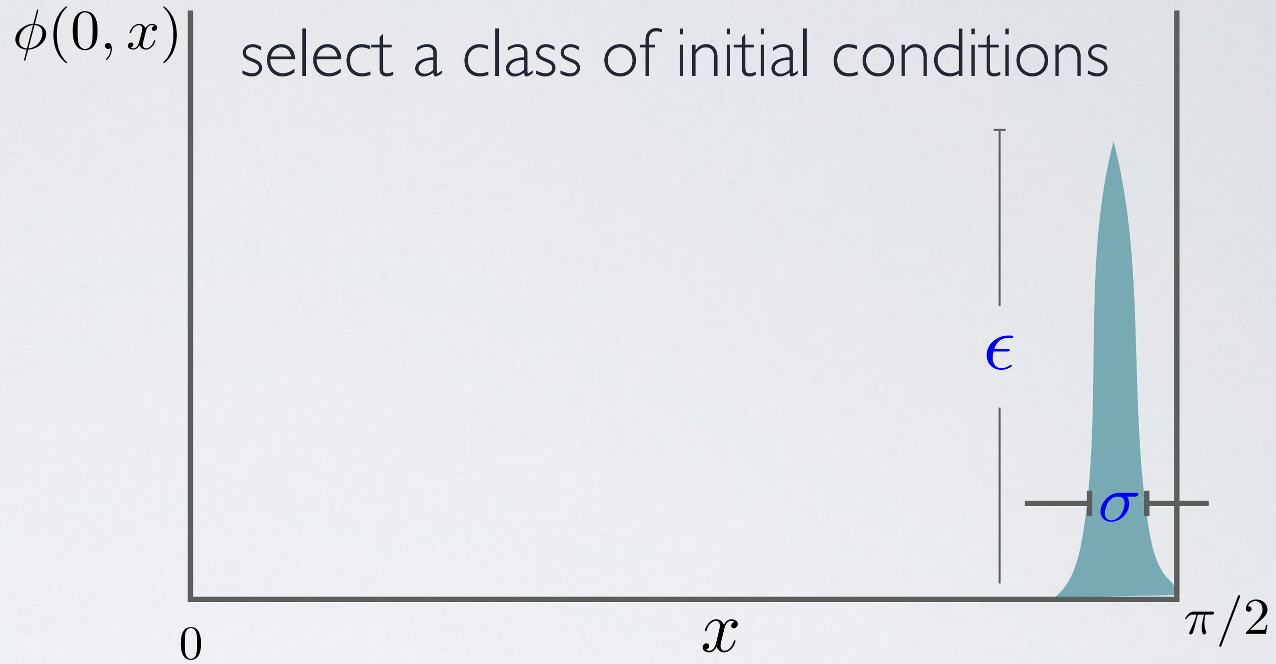
$$\delta(x,t) \sim (\tfrac{\pi}{2} - x)^{2d} \phi_\infty + \ldots$$

apparent horizon forms whenever $A(x_h, t_h) = 0$

ENTANGLEMENT EVOLUTION IN GLOBAL AdS



select a class of initial conditions



$$\phi(0,x) = \epsilon\,\frac{12}{\pi}\exp\left(-\frac{4\tan^2(\frac{\pi}{2}-x)}{\sigma^2}\right)\cos^d x$$

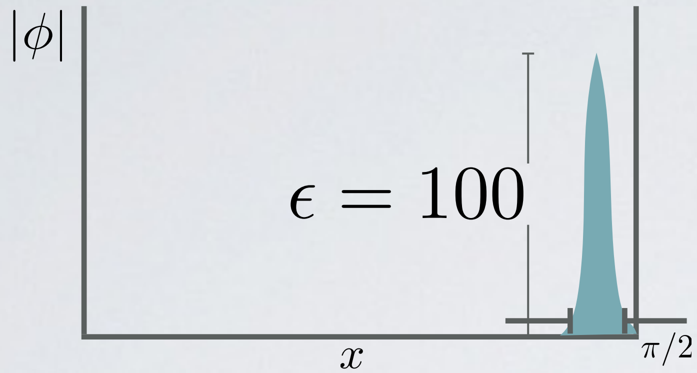
ϵ is related to the quench energy

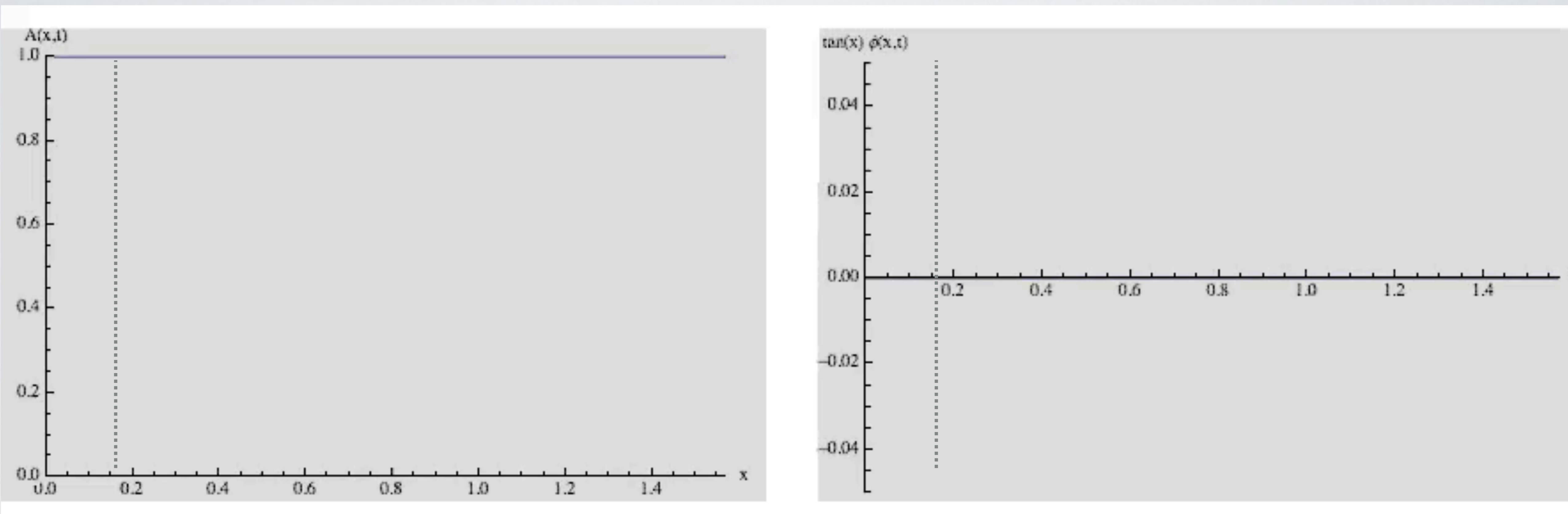
σ is related to the quenching time δt

$$M(\epsilon,\sigma) = \epsilon^2 f(\sigma)$$

AdS$_4$: COLLAPSES

$\sigma = 1/16$



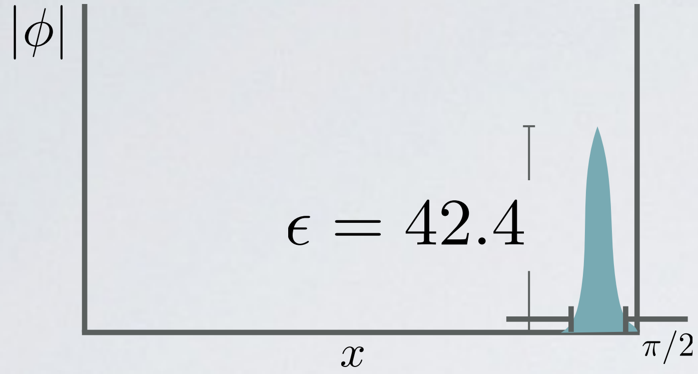


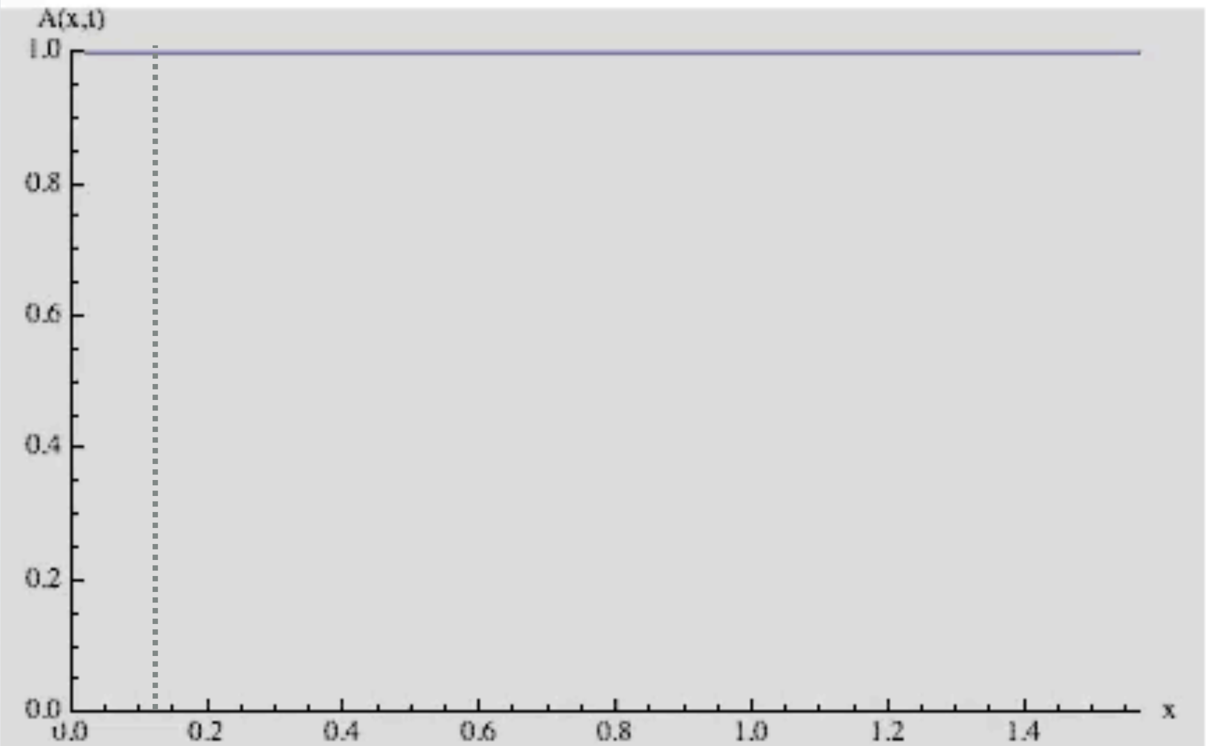
$$ds^2 = \frac{1}{\cos^2 x}\left(-A(x,t)e^{-2\delta(x,t)}dt^2 + \frac{dx^2}{A(x,t)} + \sin^2 x\, d\Omega_d^2\right)$$

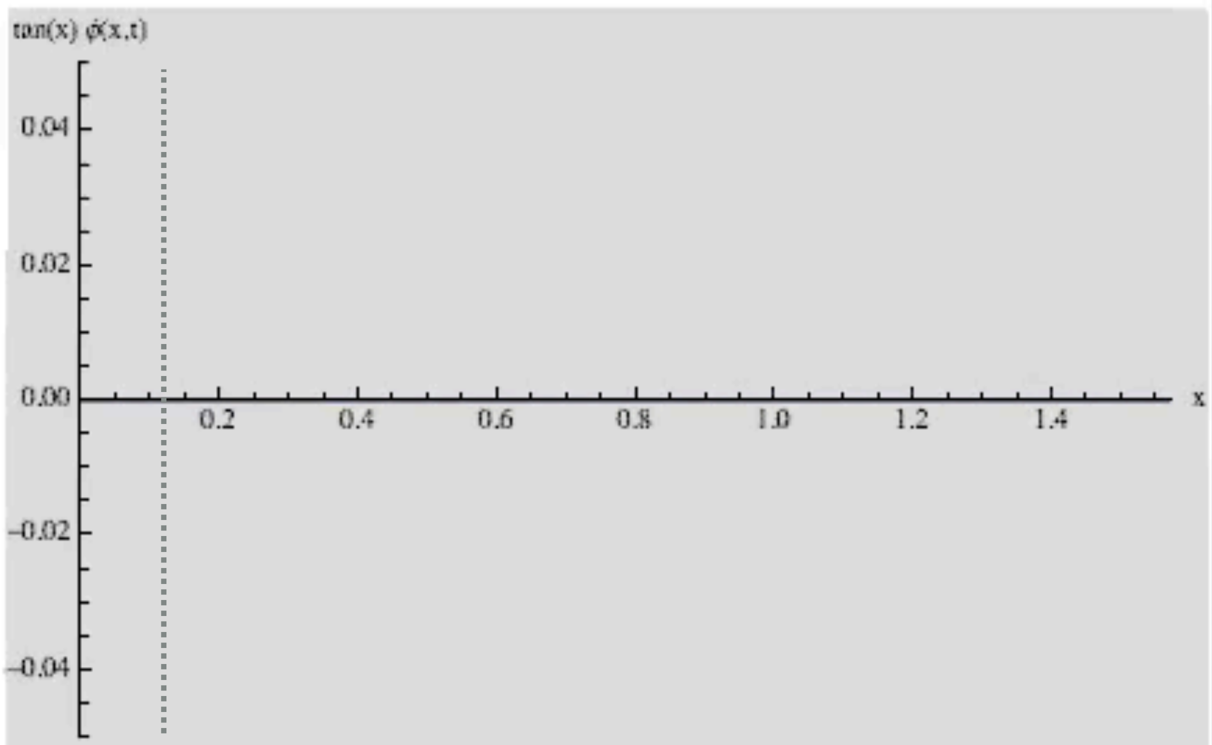
$$0 \leq x \leq \pi/2$$

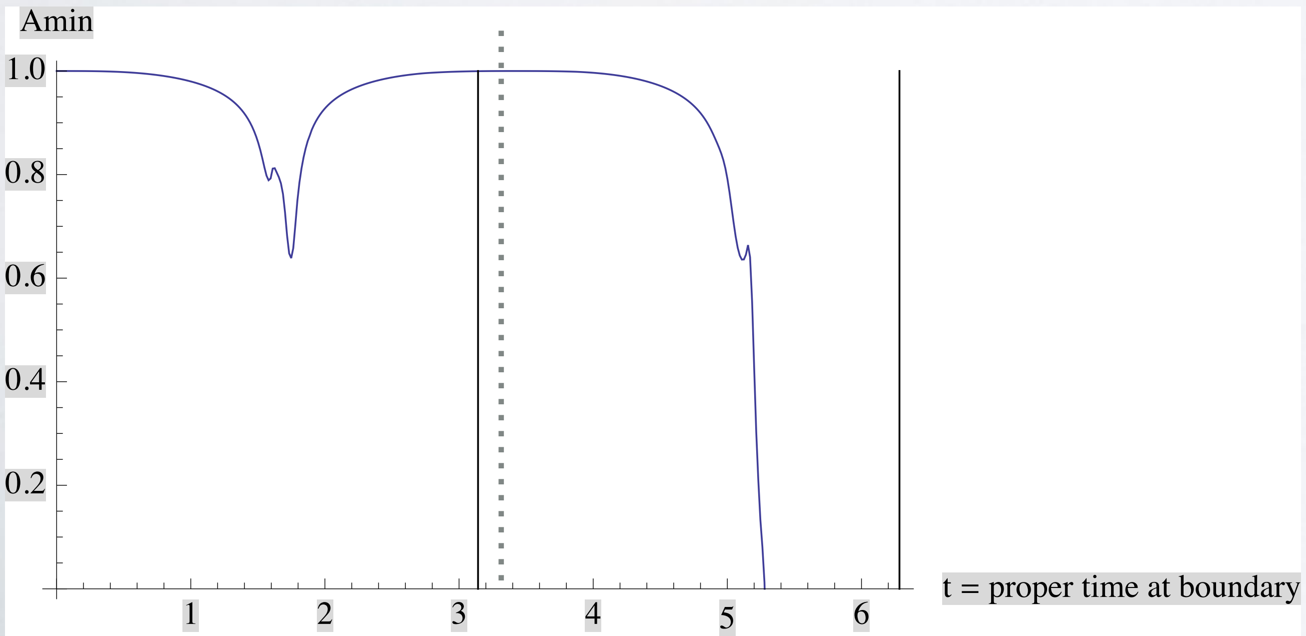
AdS$_4$: COLLAPSES

$\sigma = 1/16$







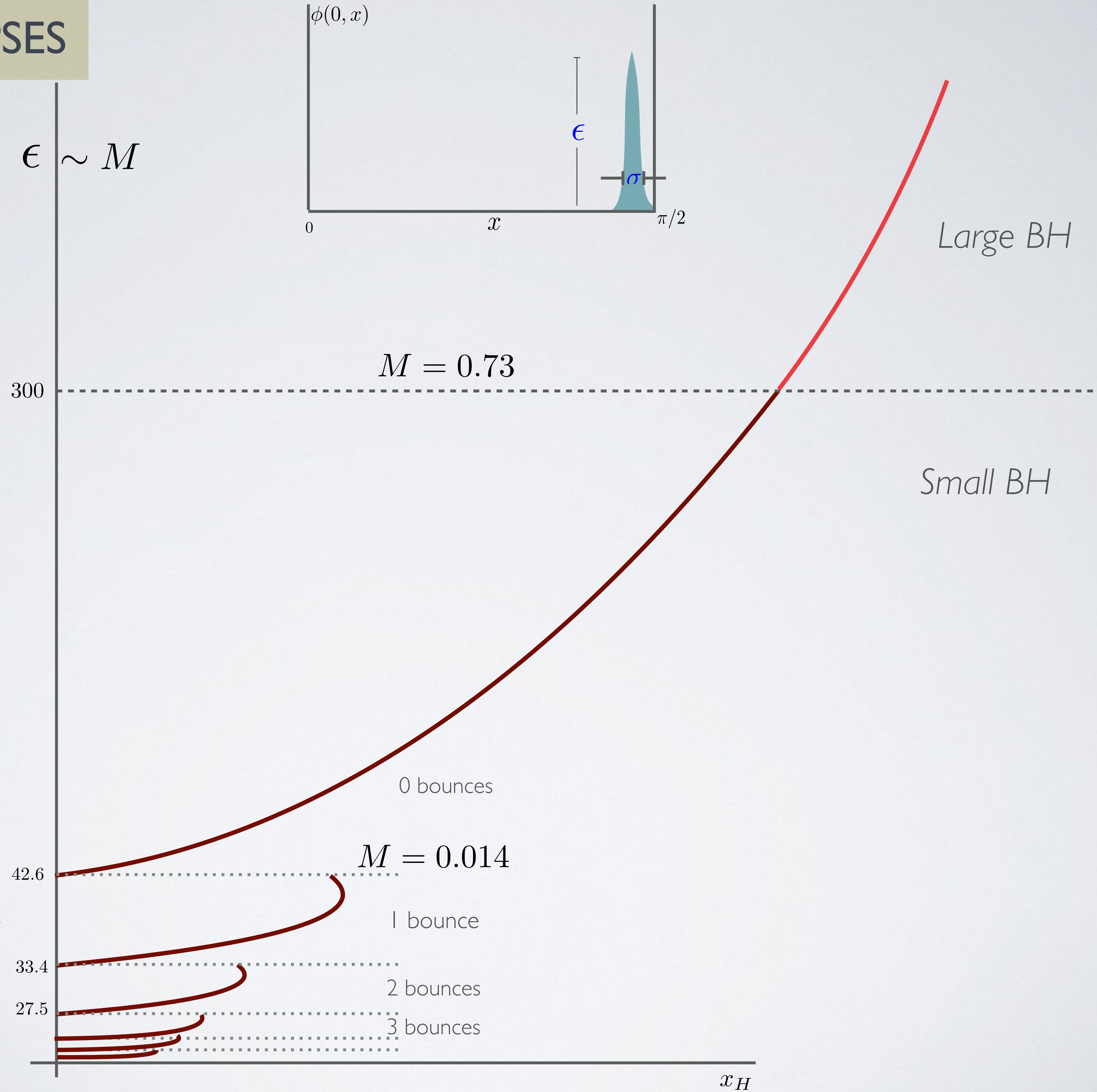


weak turbulence in action

the periodicity is $\geq \pi$!

AdS_4 : COLLAPSES

$$\sigma = \frac{1}{16}$$



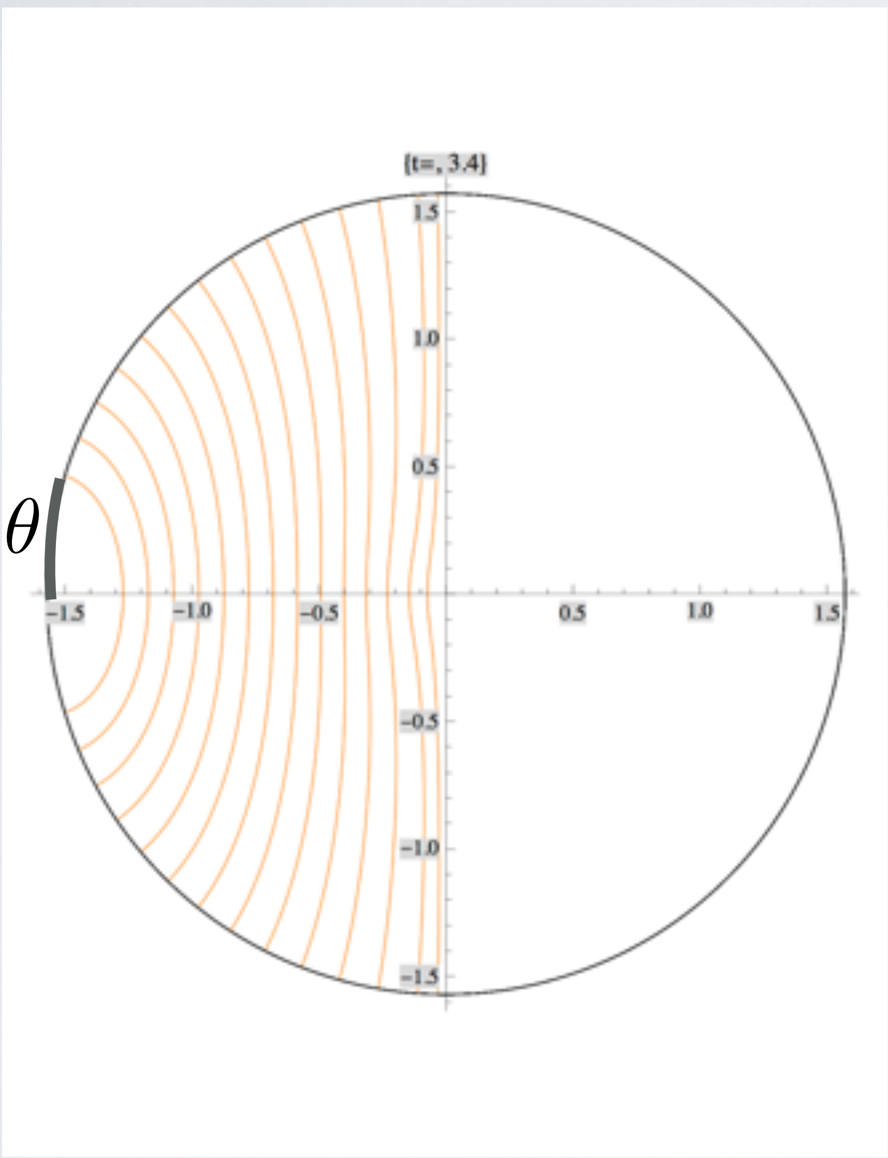
M. Choptuik 1992

P. Bizón & A. Rostworowski 2011

AdS$_4$: ENTANGLEMENT ENTROPY

Pre-collapse

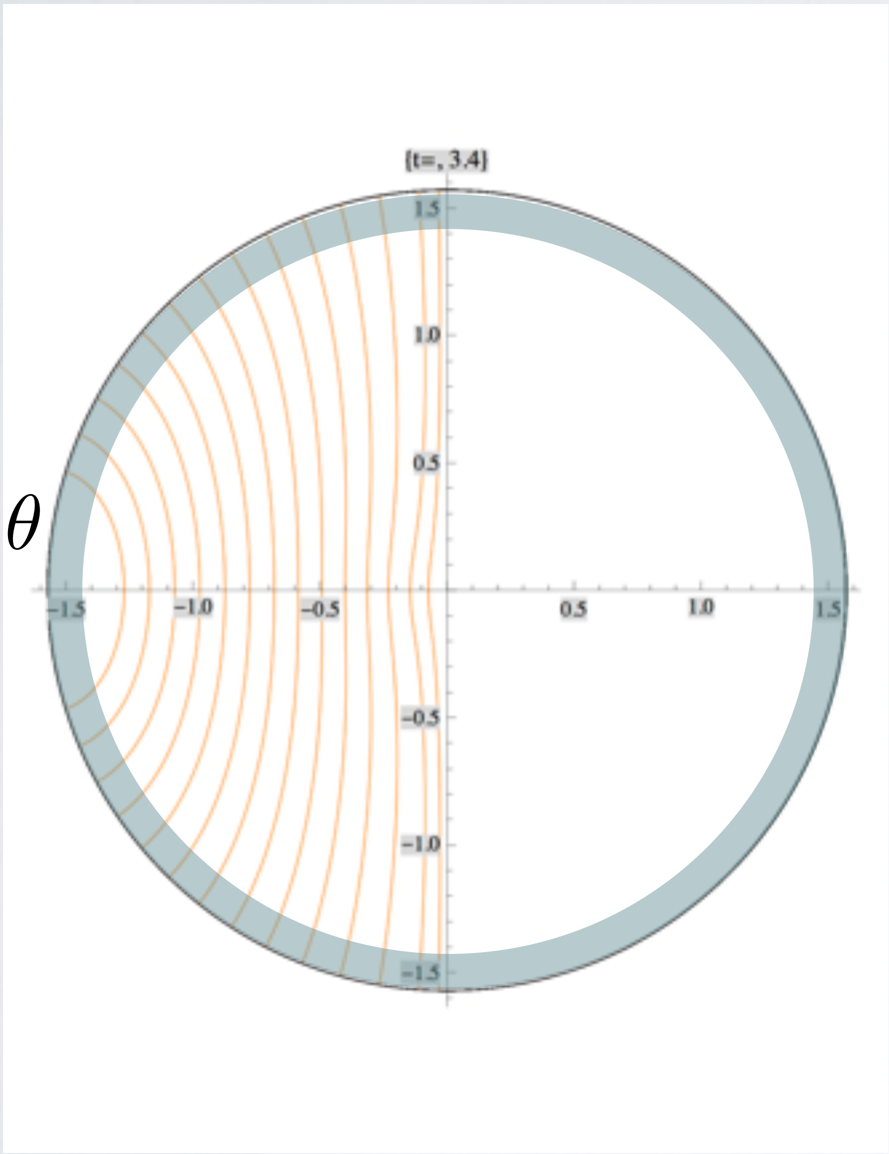
$$\theta = 0.3, 0.4, ..., 1.4$$

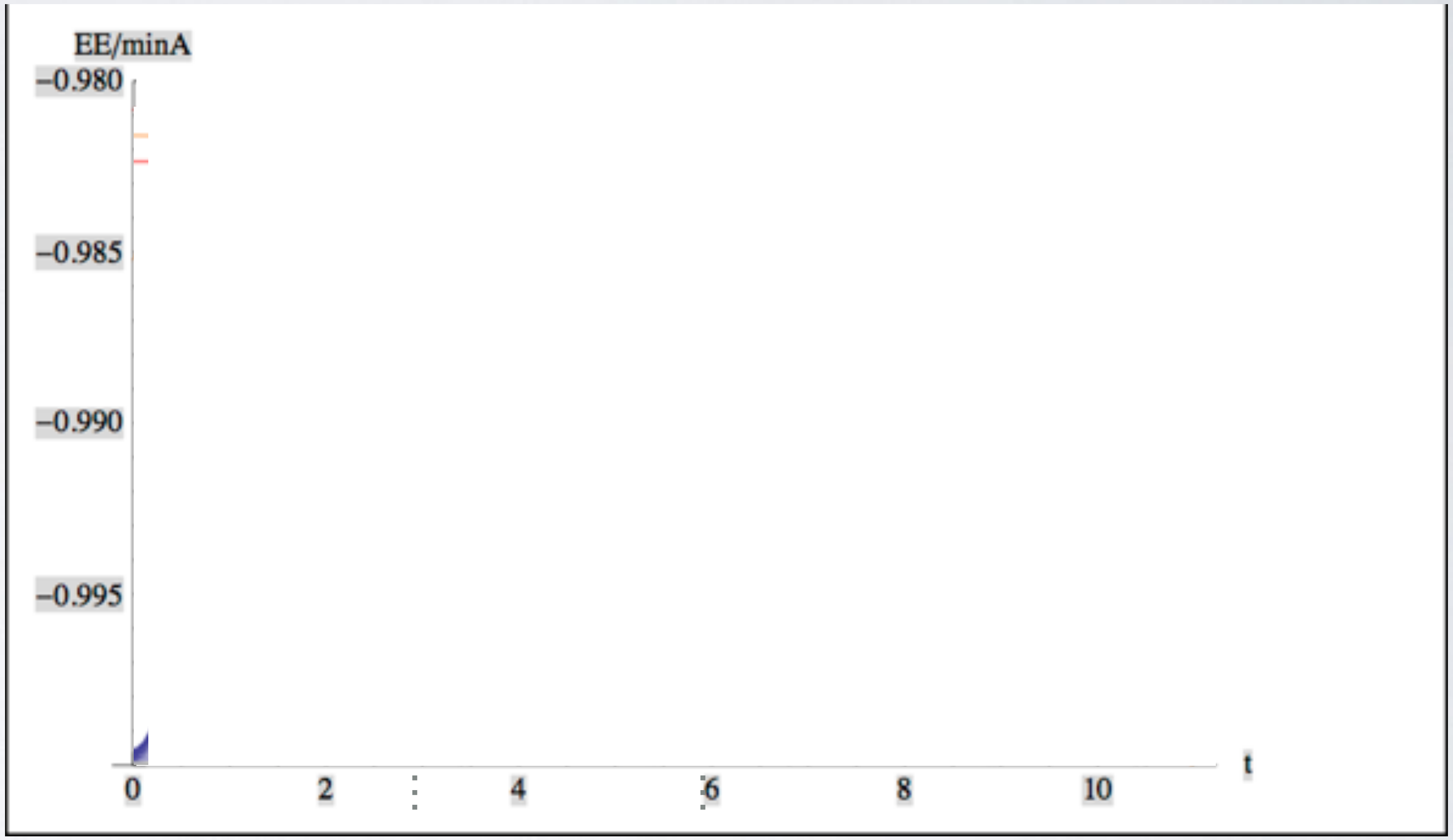


J. Abajo-Arrastia, E da Silva, E. López, J.M. & A. Serantes.

Pre-collapse

$$\theta = 0.3, 0.4, ..., 1.4$$

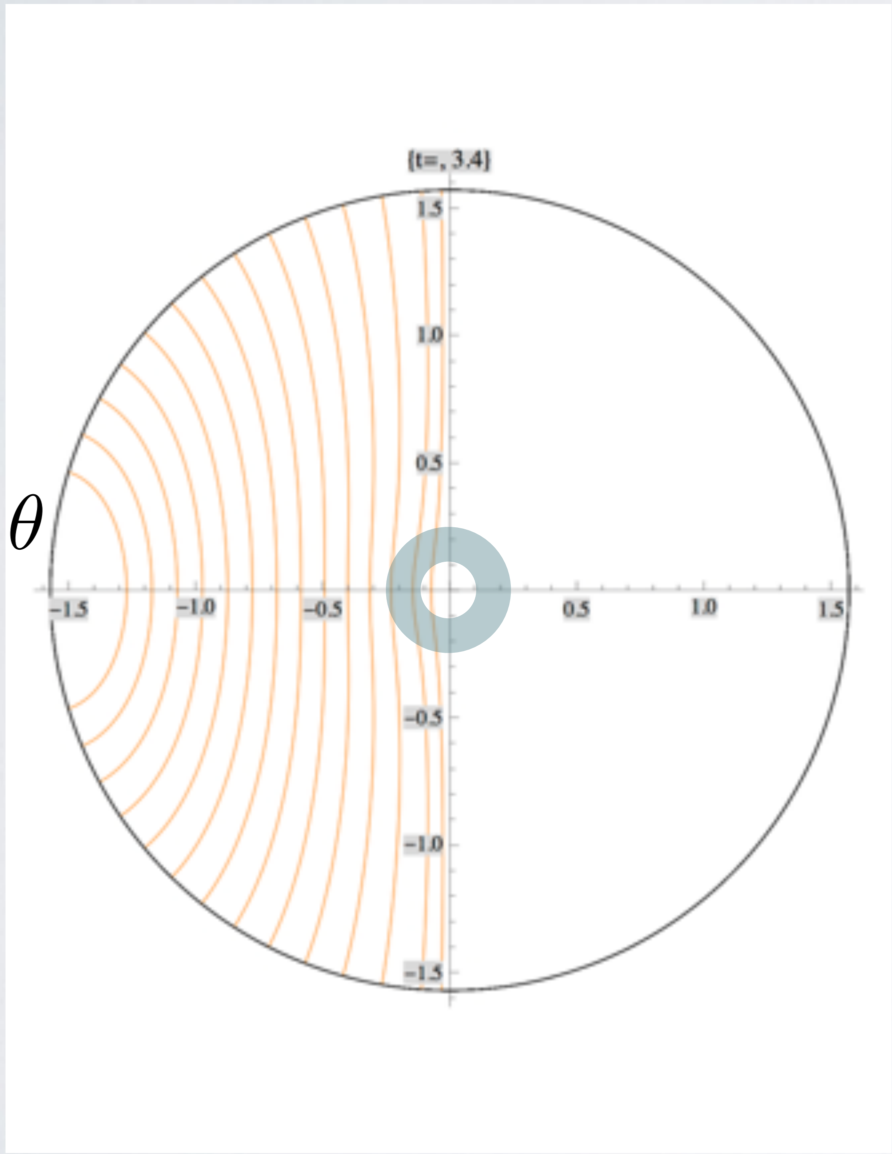


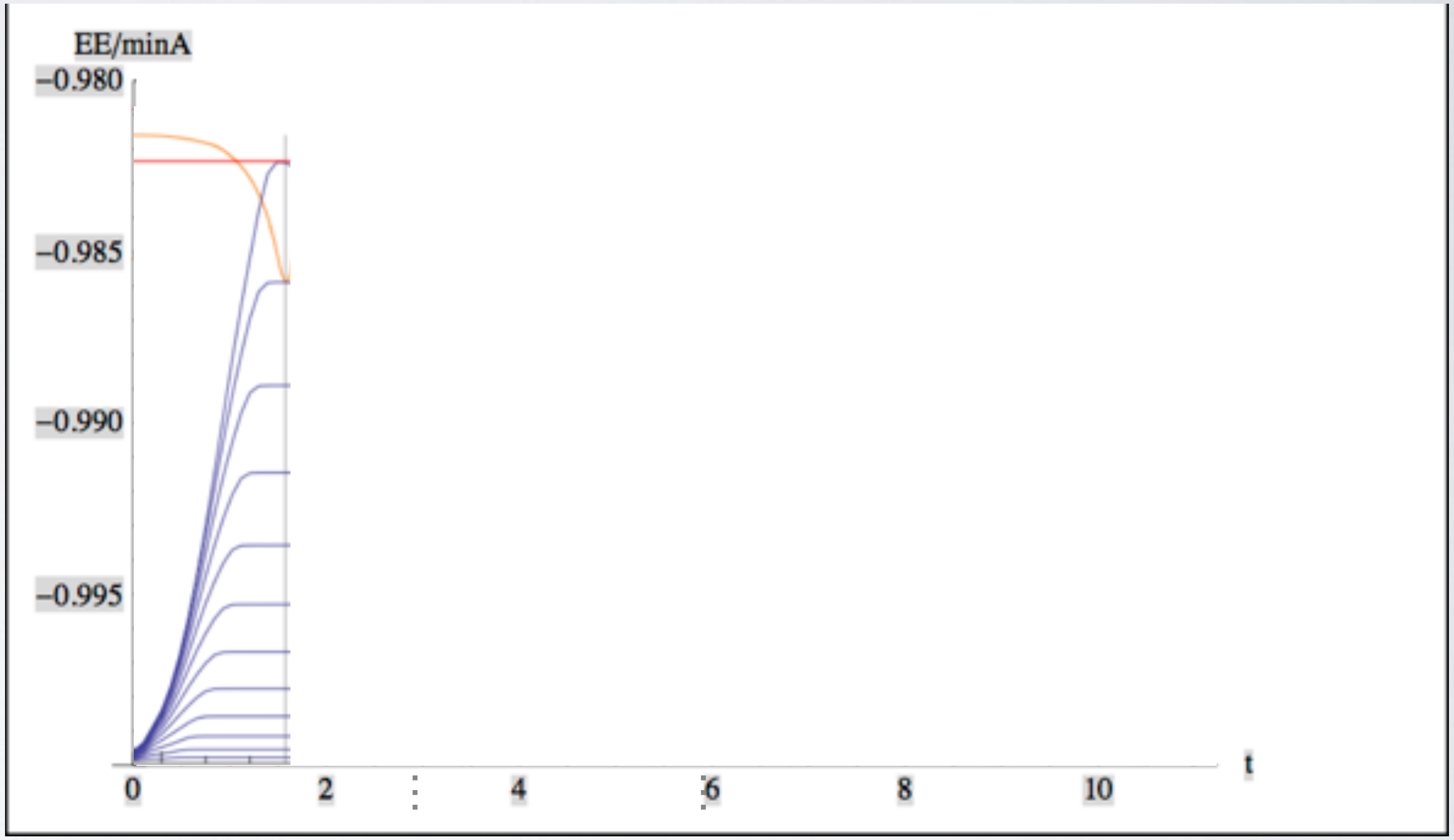


J. Abajo-Arrastia, E da Silva, E. López, J.M. & A. Serantes.

Pre-collapse

$$\theta = 0.3, 0.4, ..., 1.4$$

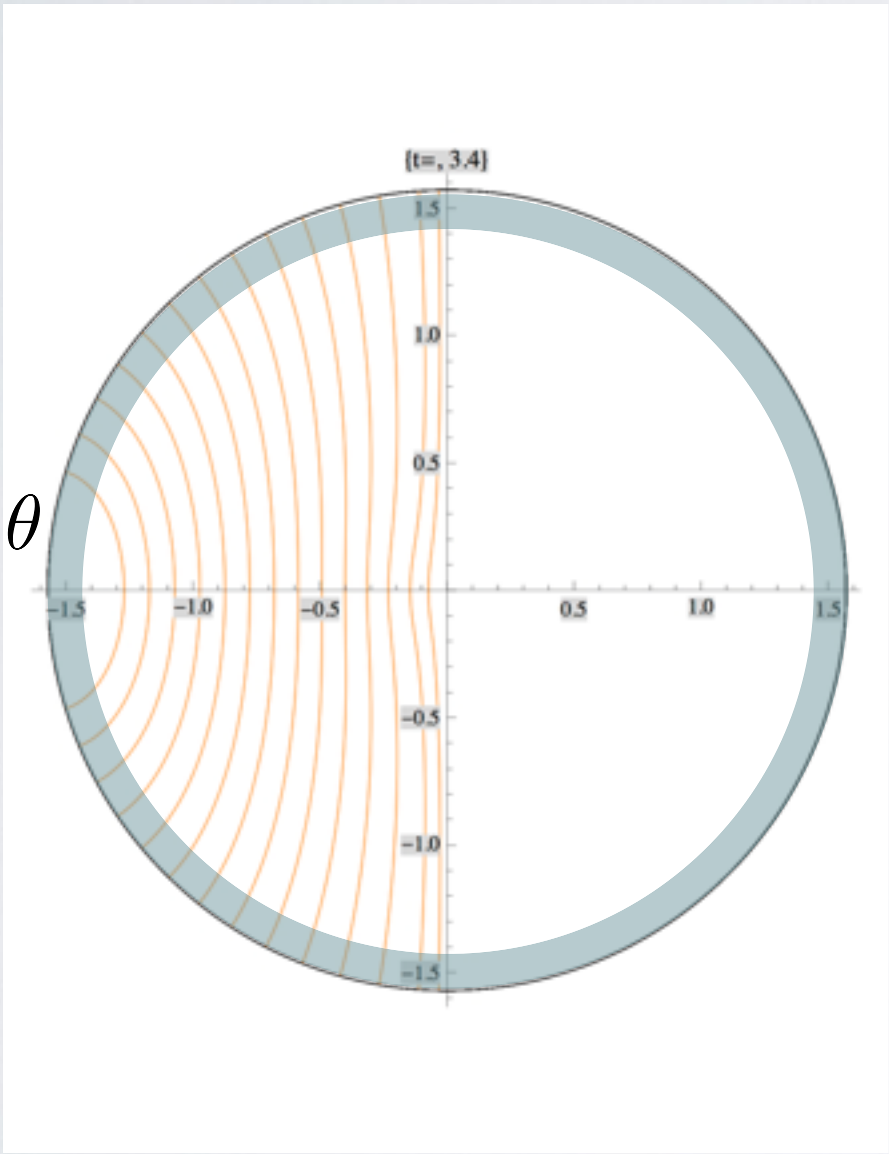


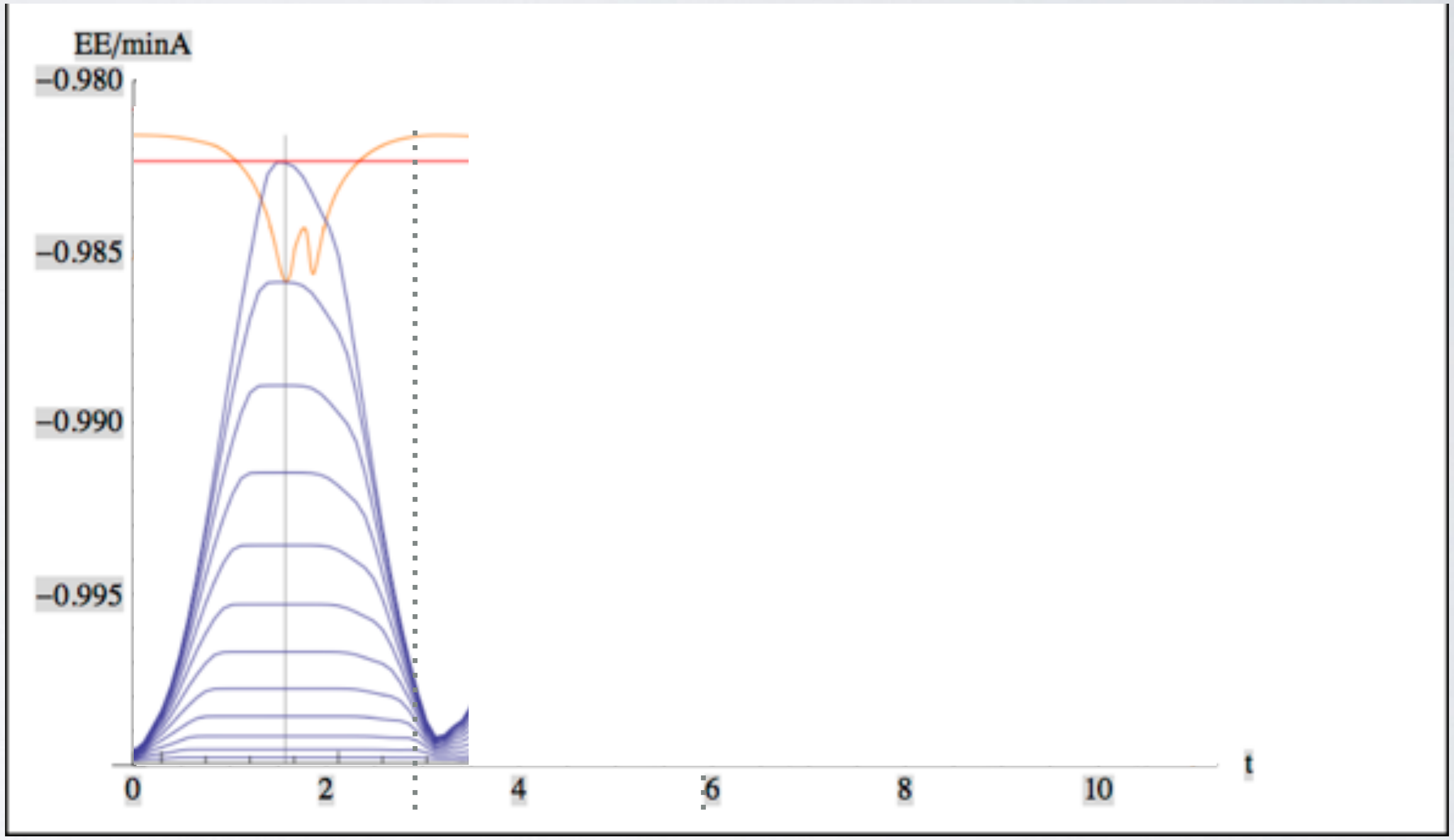


J. Abajo-Arrastia, E da Silva, E. López, J.M. & A. Serantes.

Pre-collapse

$$\theta = 0.3, 0.4, ..., 1.4$$

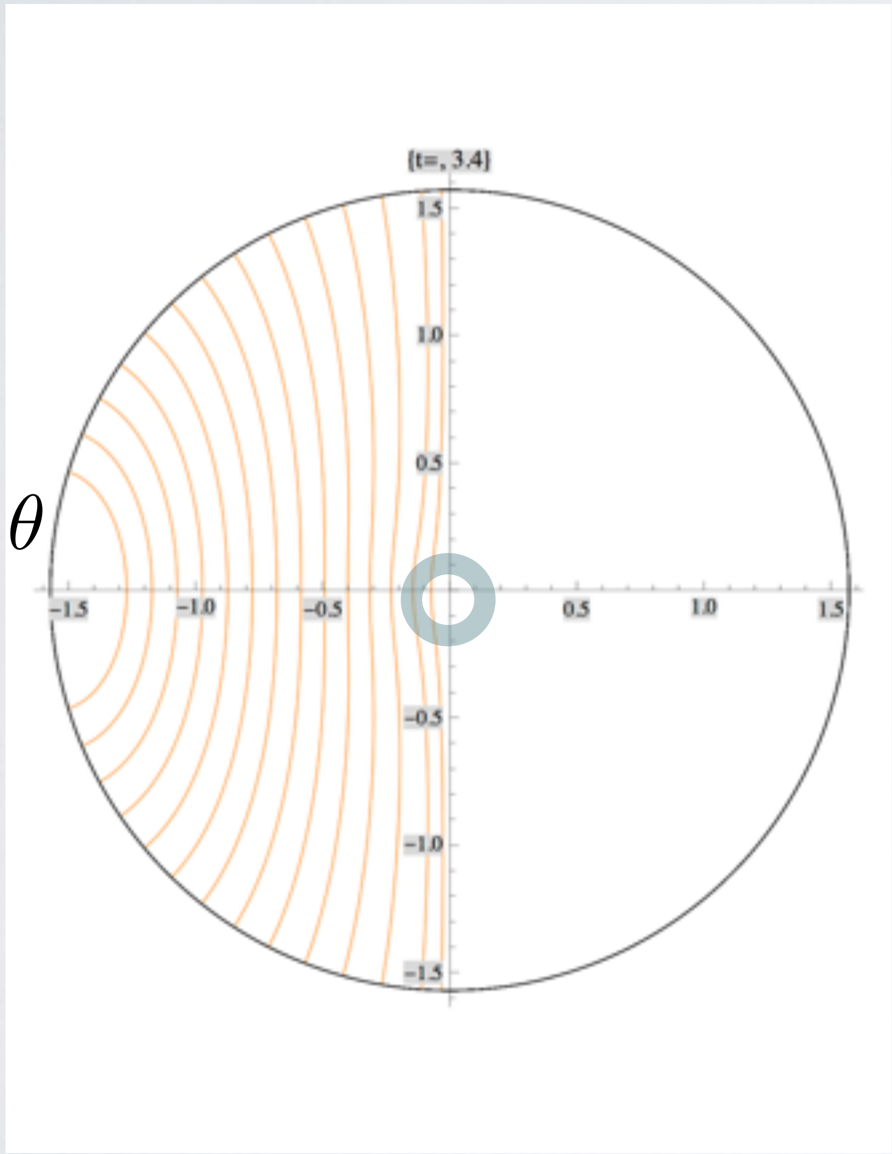


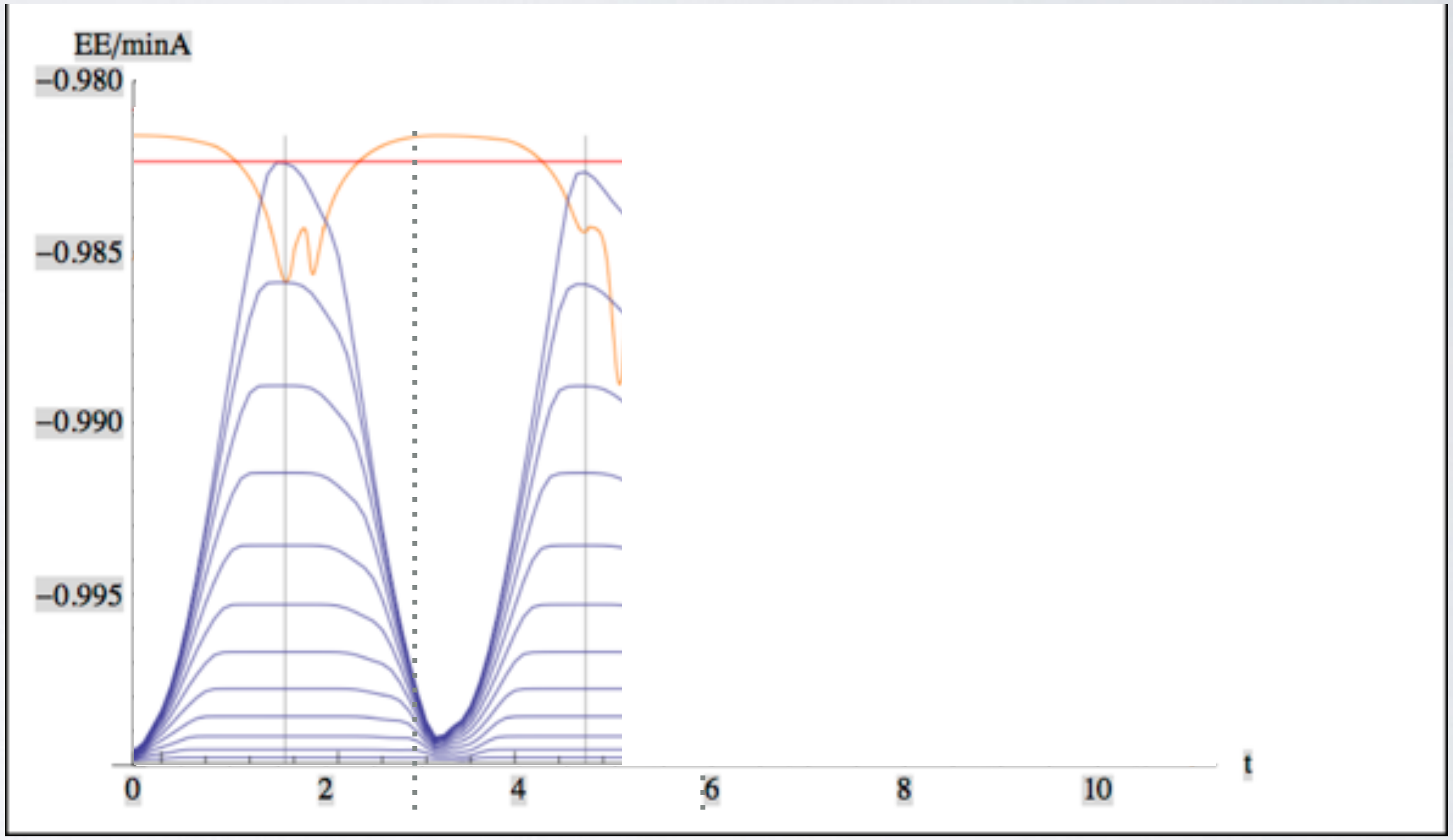


J. Abajo-Arrastia, E da Silva, E. López, J.M. & A. Serantes.

Pre-collapse

$$\theta = 0.3, 0.4, \ldots, 1.4$$

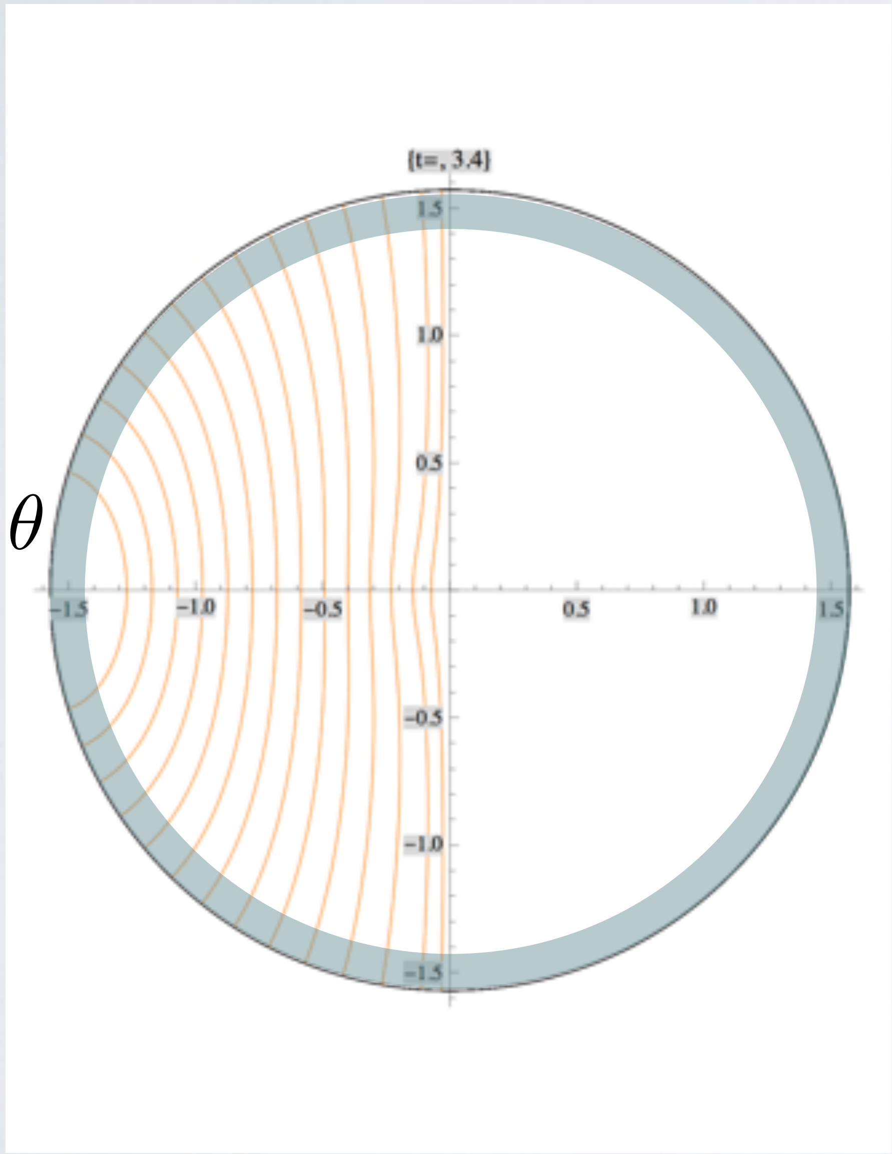


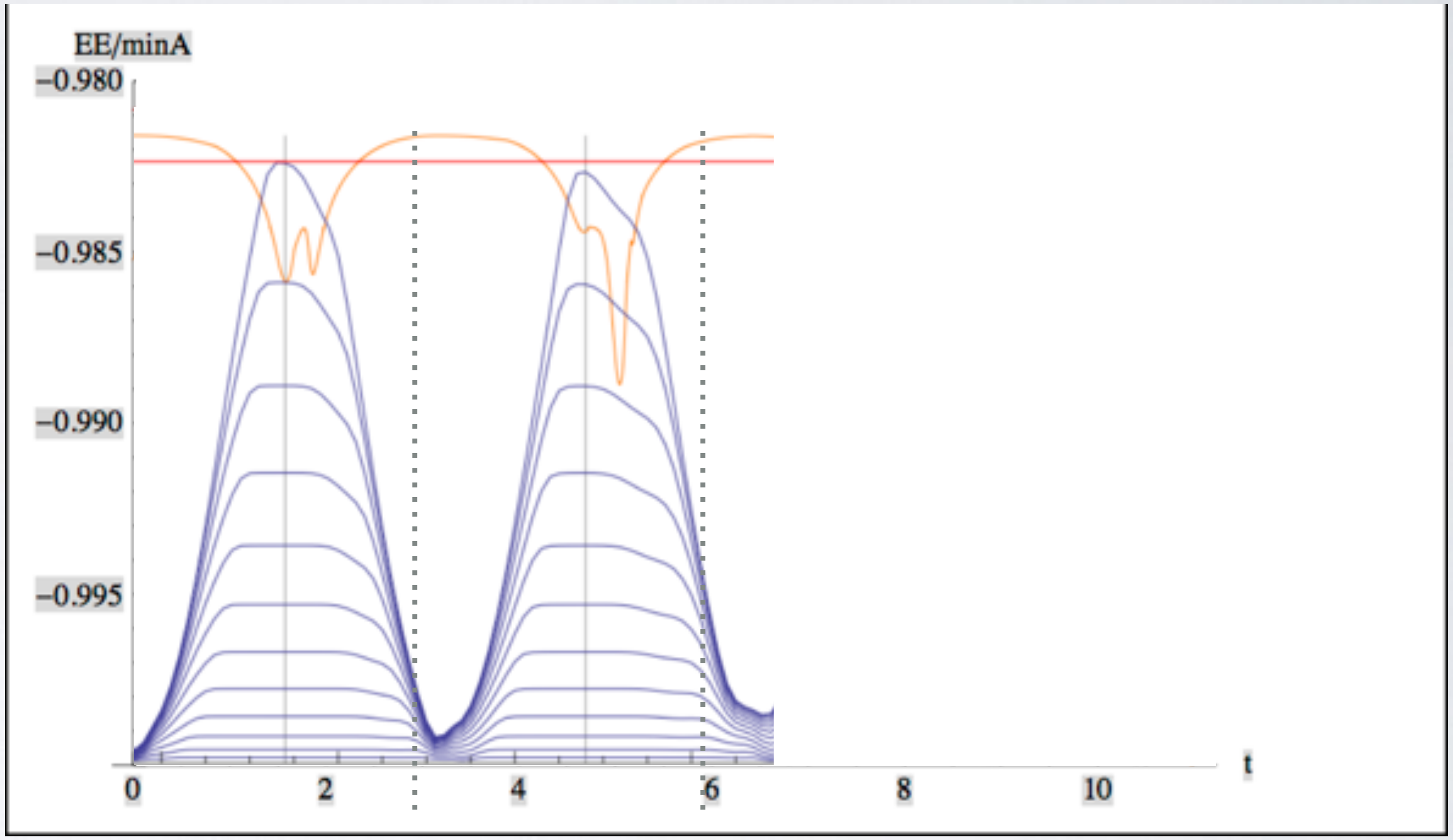


J. Abajo-Arrastia, E da Silva, E. López, J.M. & A. Serantes.

Pre-collapse

$$\theta = 0.3, 0.4, ..., 1.4$$

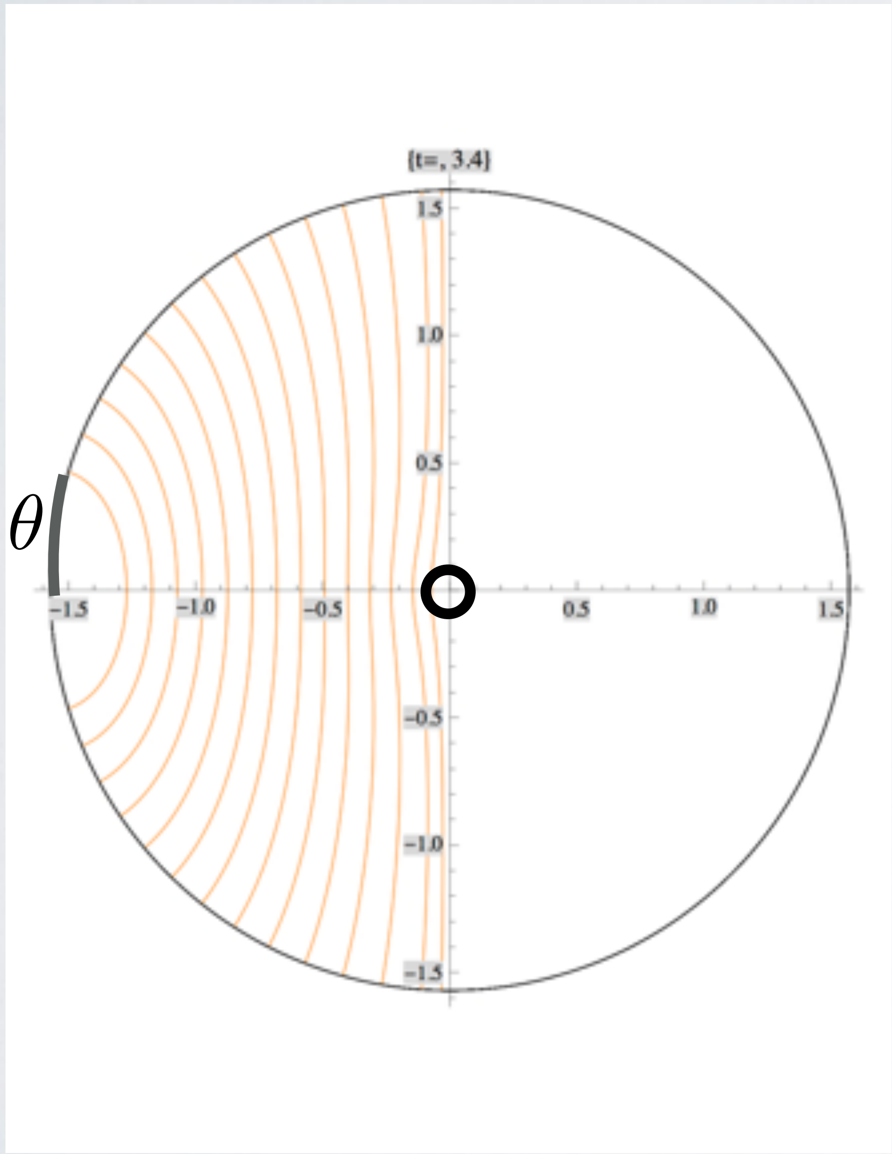


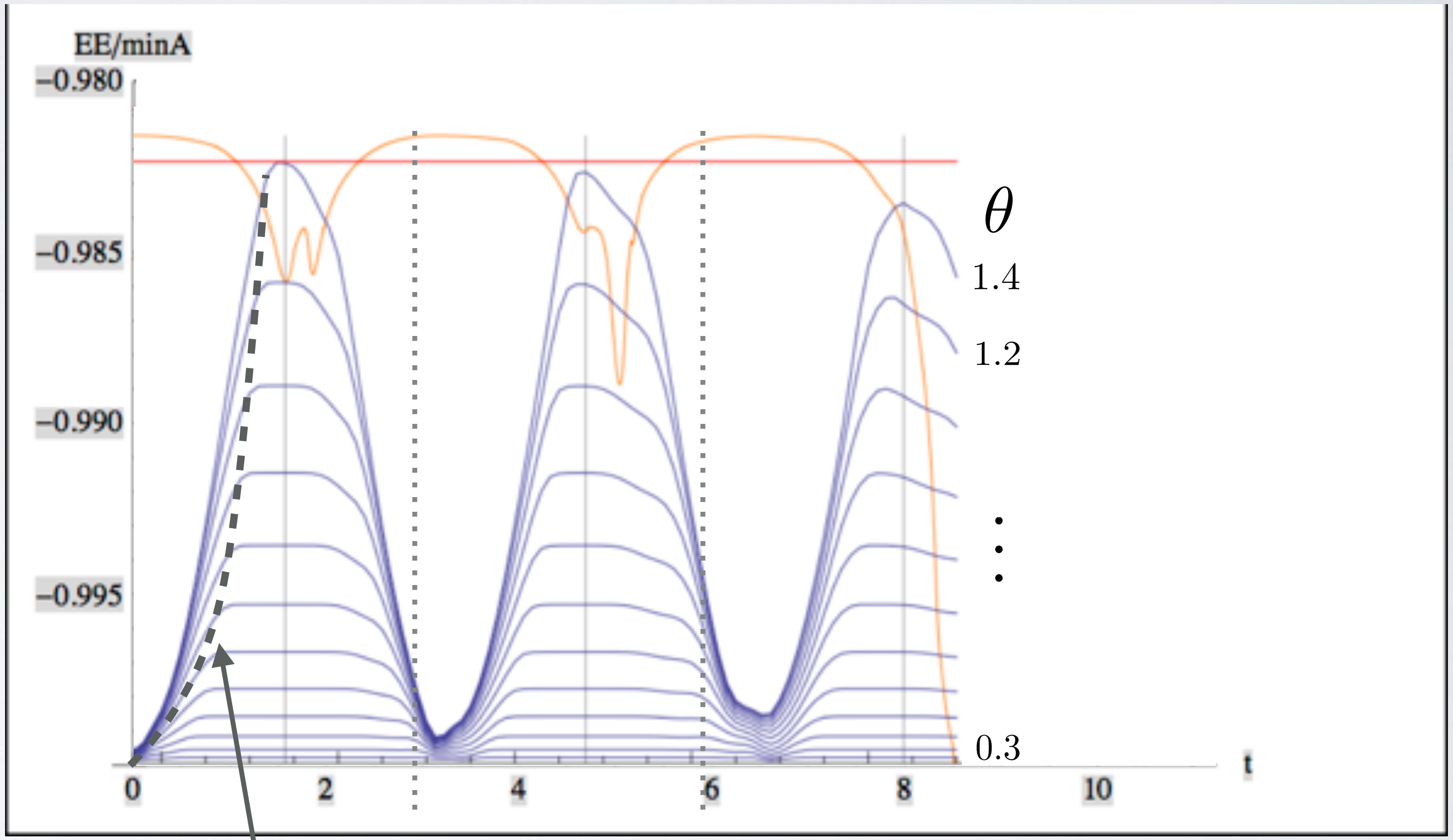


J. Abajo-Arrastia, E da Silva, E. López, J.M. & A. Serantes.

Pre-collapse

$$\theta = 0.3, 0.4, ..., 1.4$$

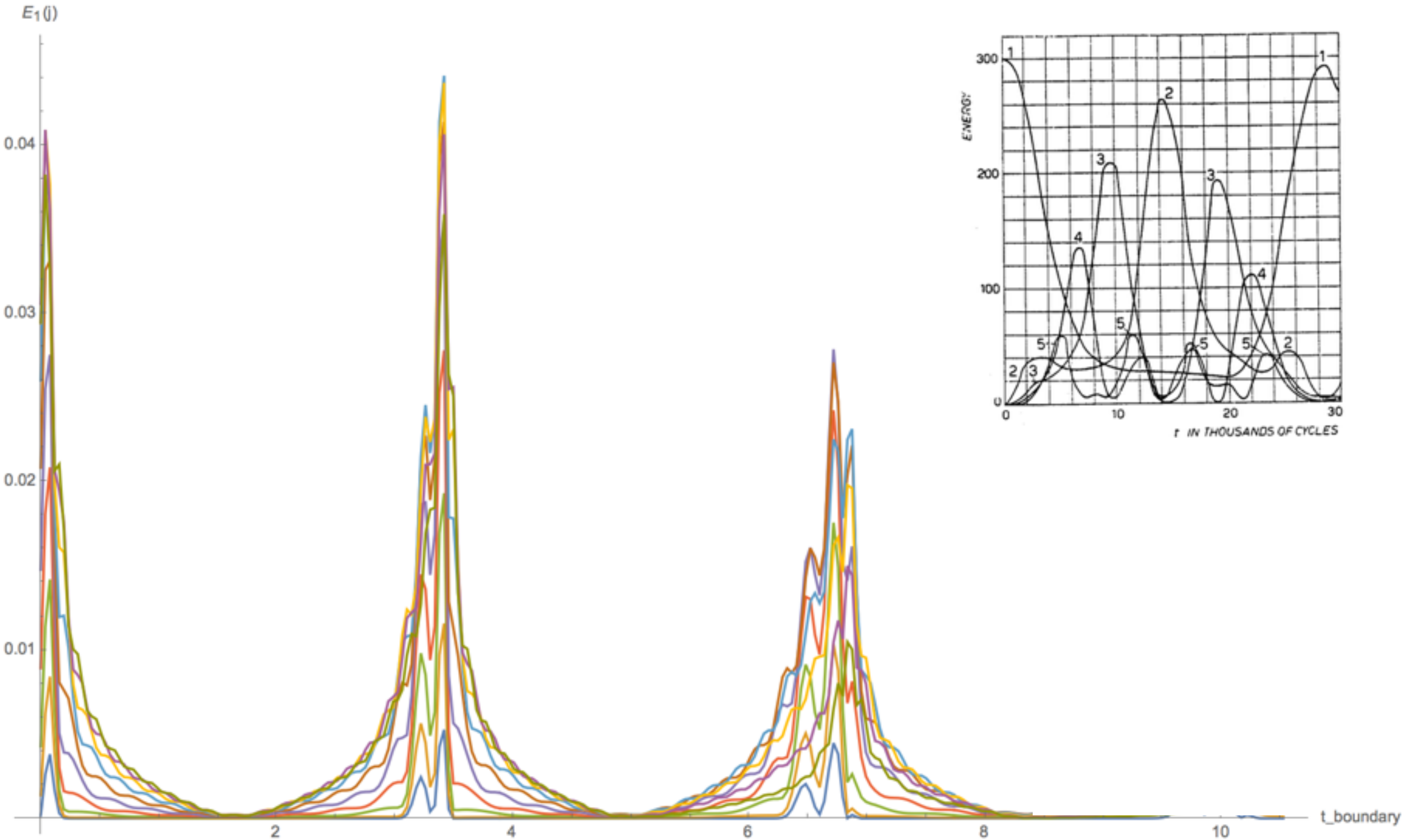




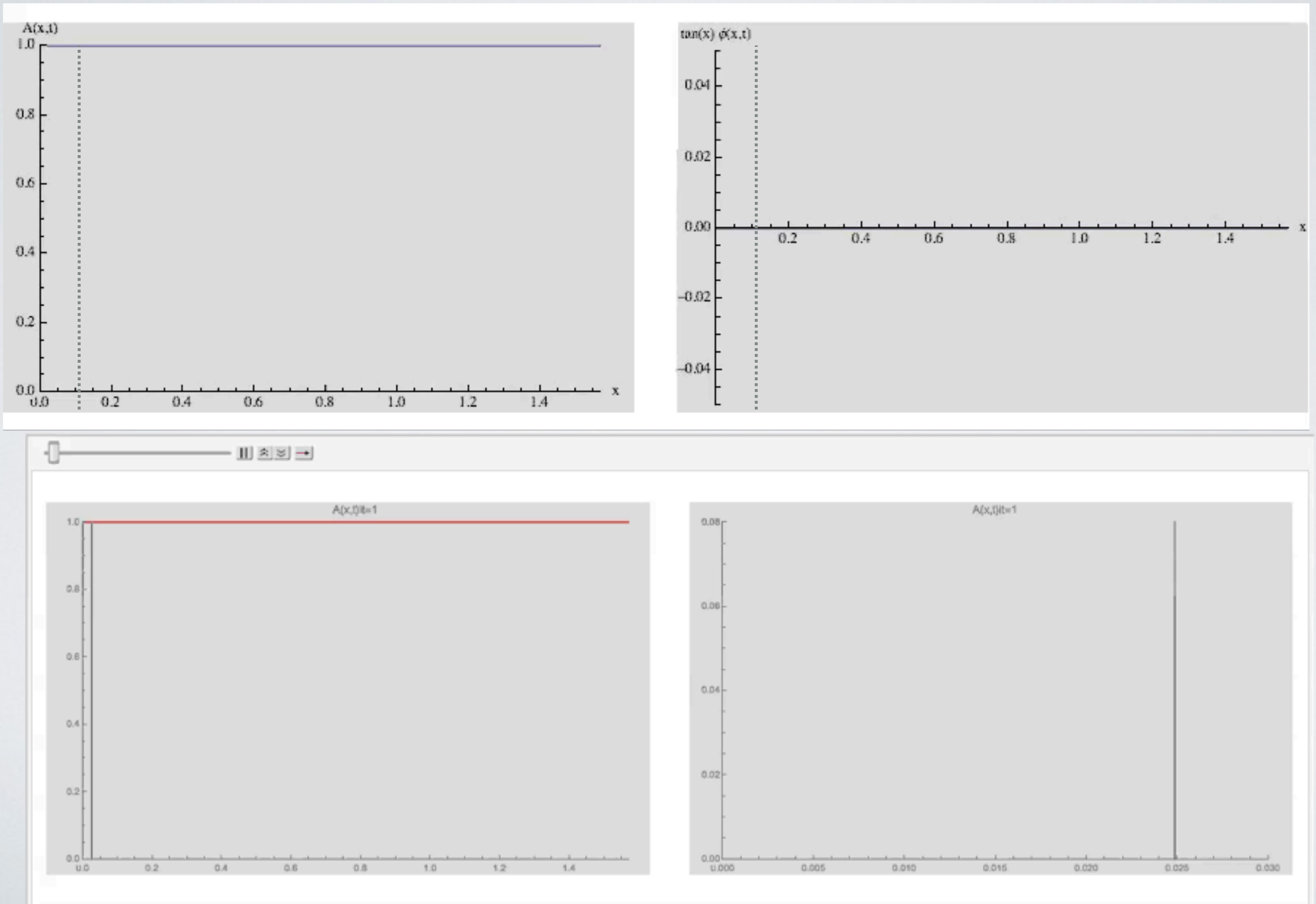
$$t = \theta = l/2$$

Javier Abajo-Arrastia, Emilia da Silva, Esperanza López, J.M. & Alexandre Serantes. arXiv:1403.2632

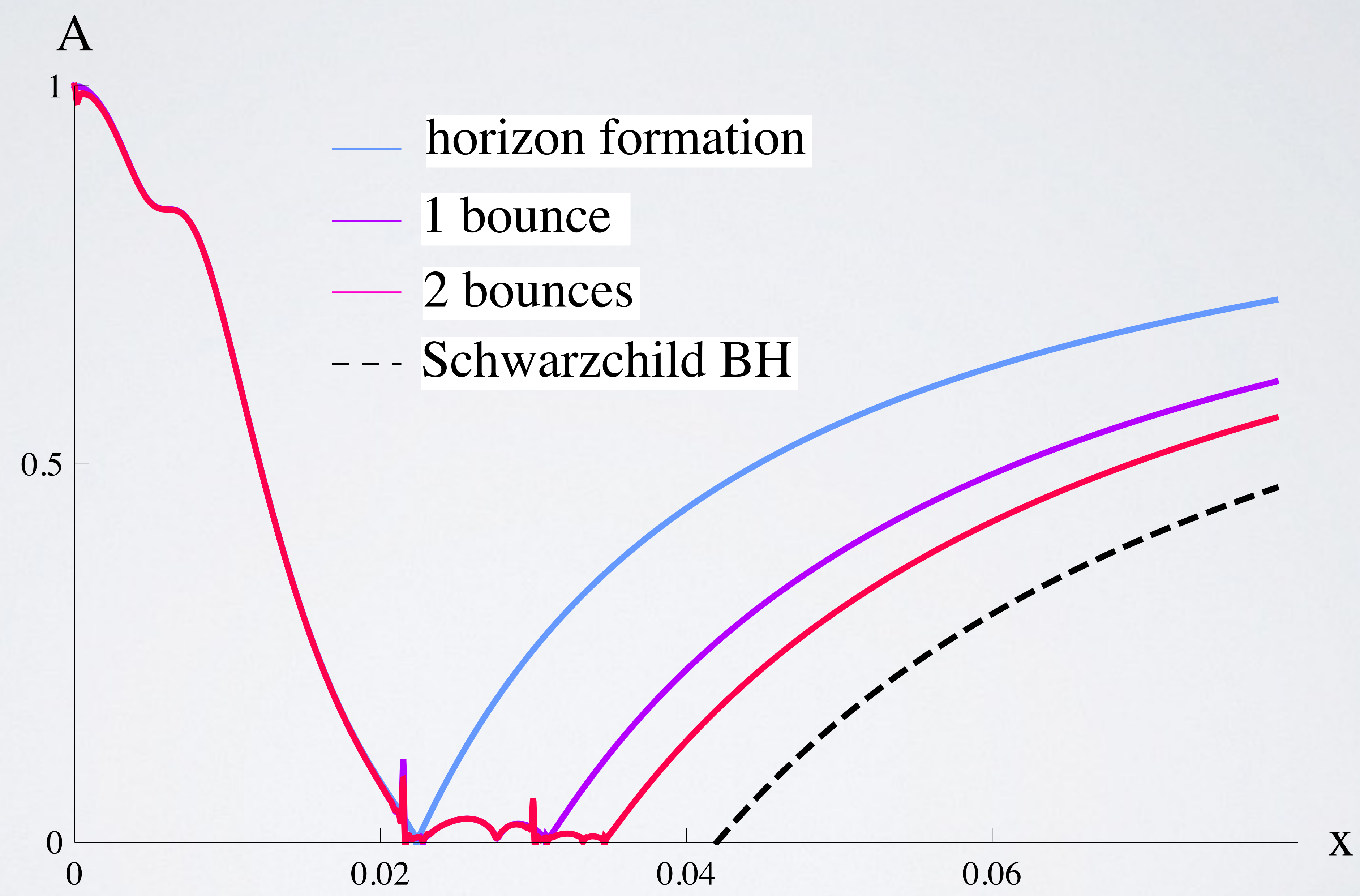
AdS_4 : SPECTRAL ANALYSIS



AdS_4 : ABSORTIVE PHASE

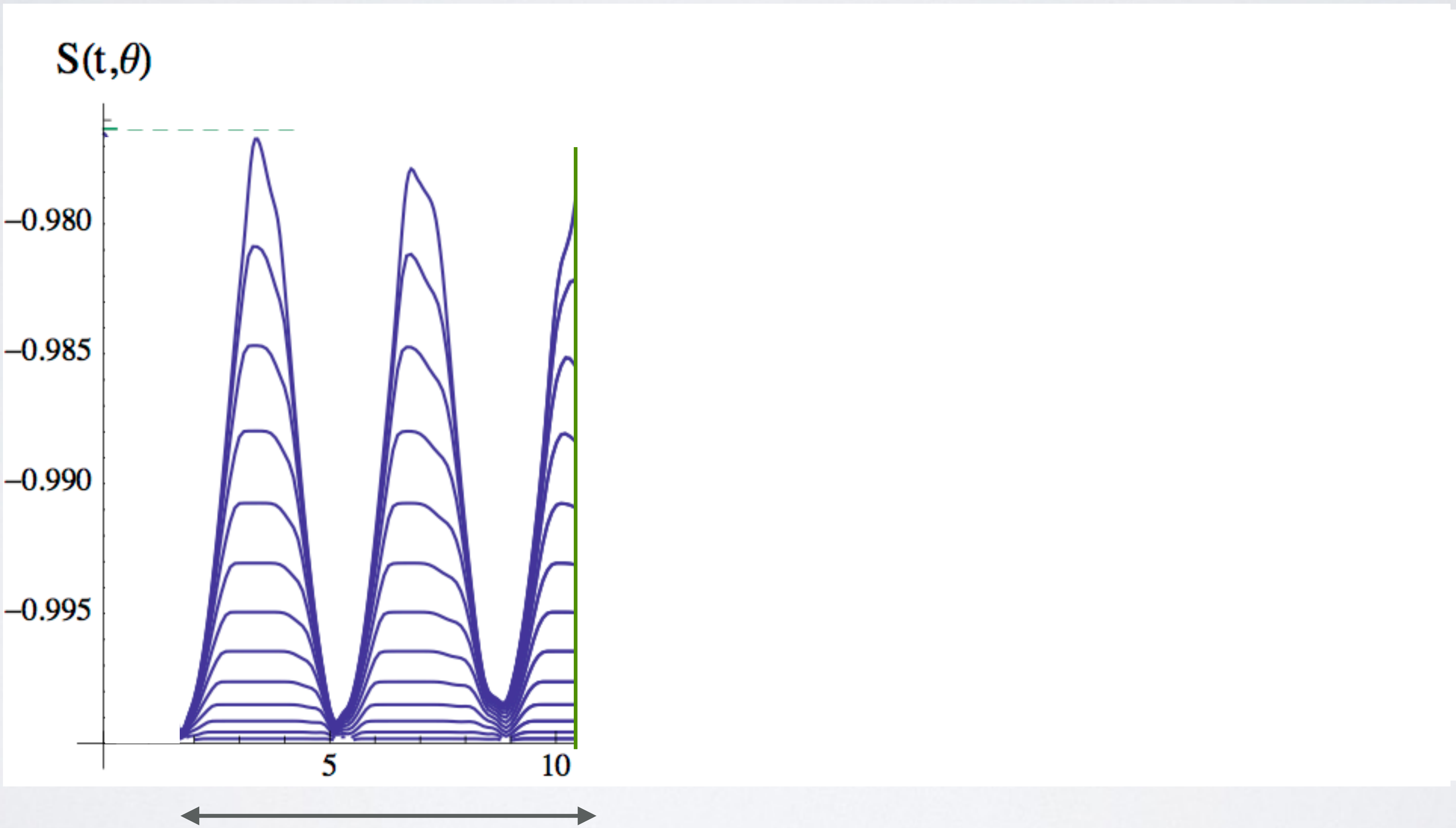


AdS_4 : ABSORTIVE PHASE



AdS$_4$: ABSORTIVE PHASE

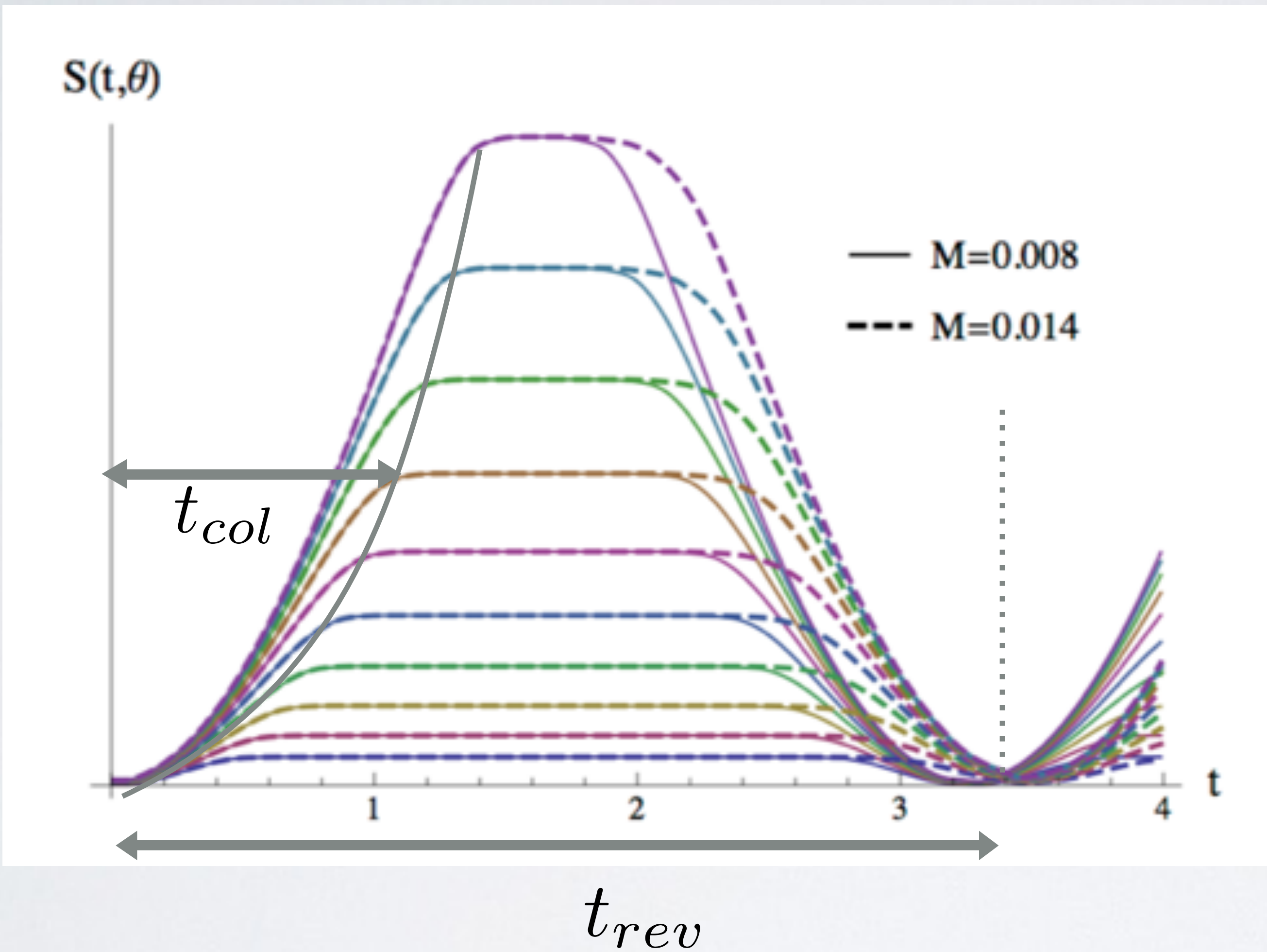
Post-collapse: two regimes, elastic and absorptive



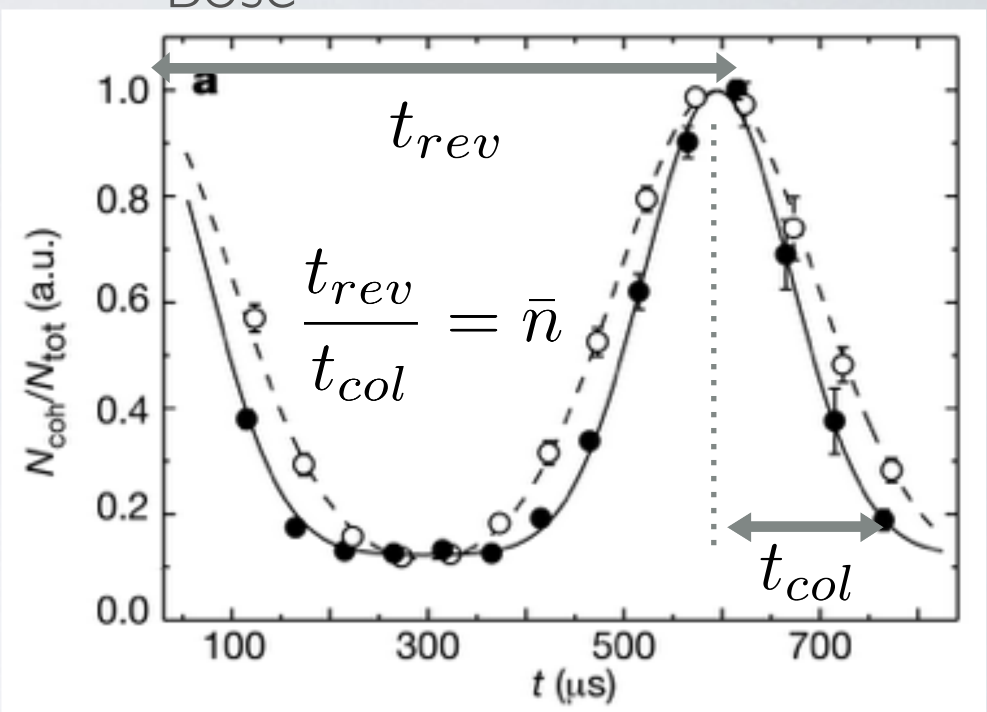
$$T \sim 1/\epsilon^2$$

AdS4 : CHANGE INITIAL CONDITIONS ϵ

Bouncing phase: period changes with mass: two time scales



Bose



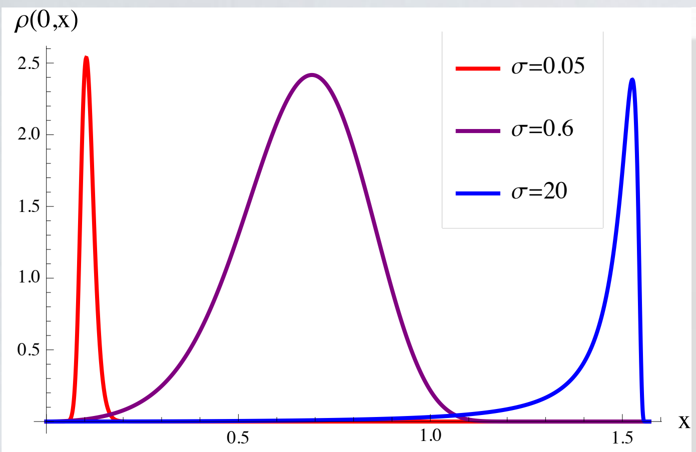
collapse time: $t_{col} = \theta$

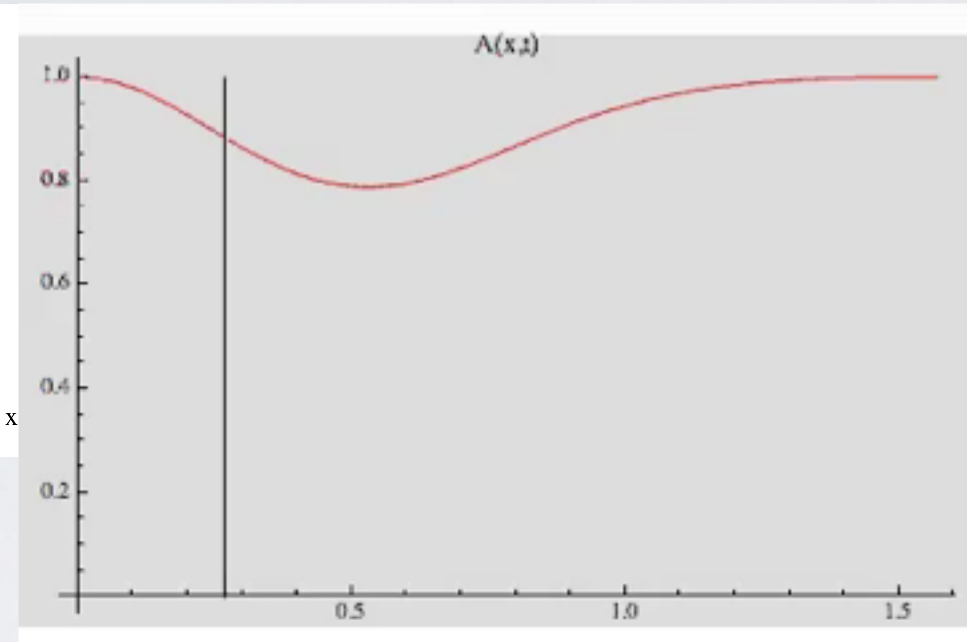
revival time: $t_{rev} = f(M)$

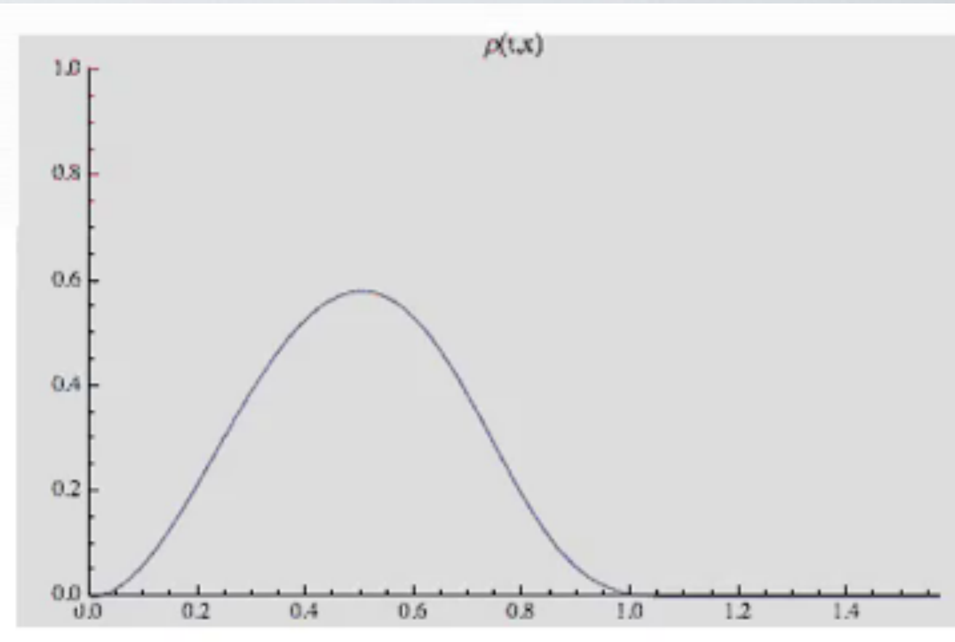
AdS$_4$: CHANGE INITIAL CONDITIONS σ

- **no collapse** occurs for $0.3 \leq \sigma \leq 16$

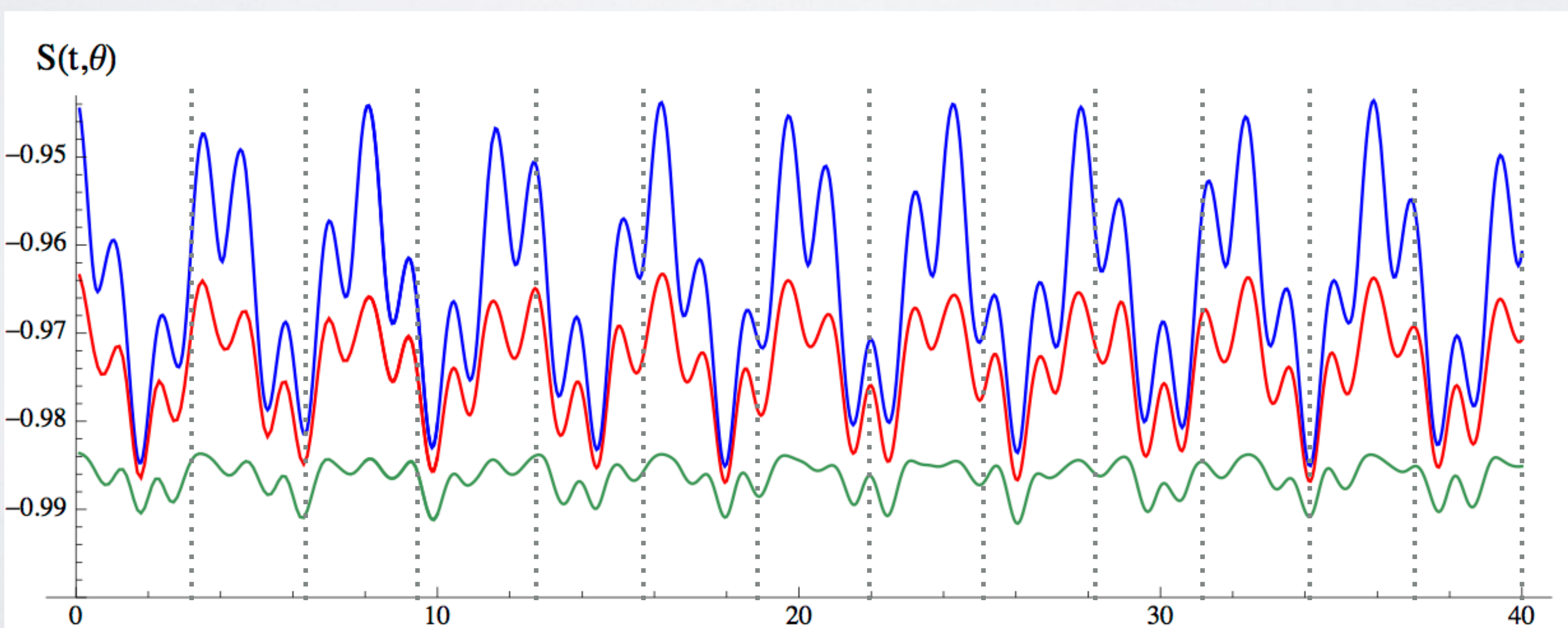
Buchel, Liebling & Lehner 1304.4166



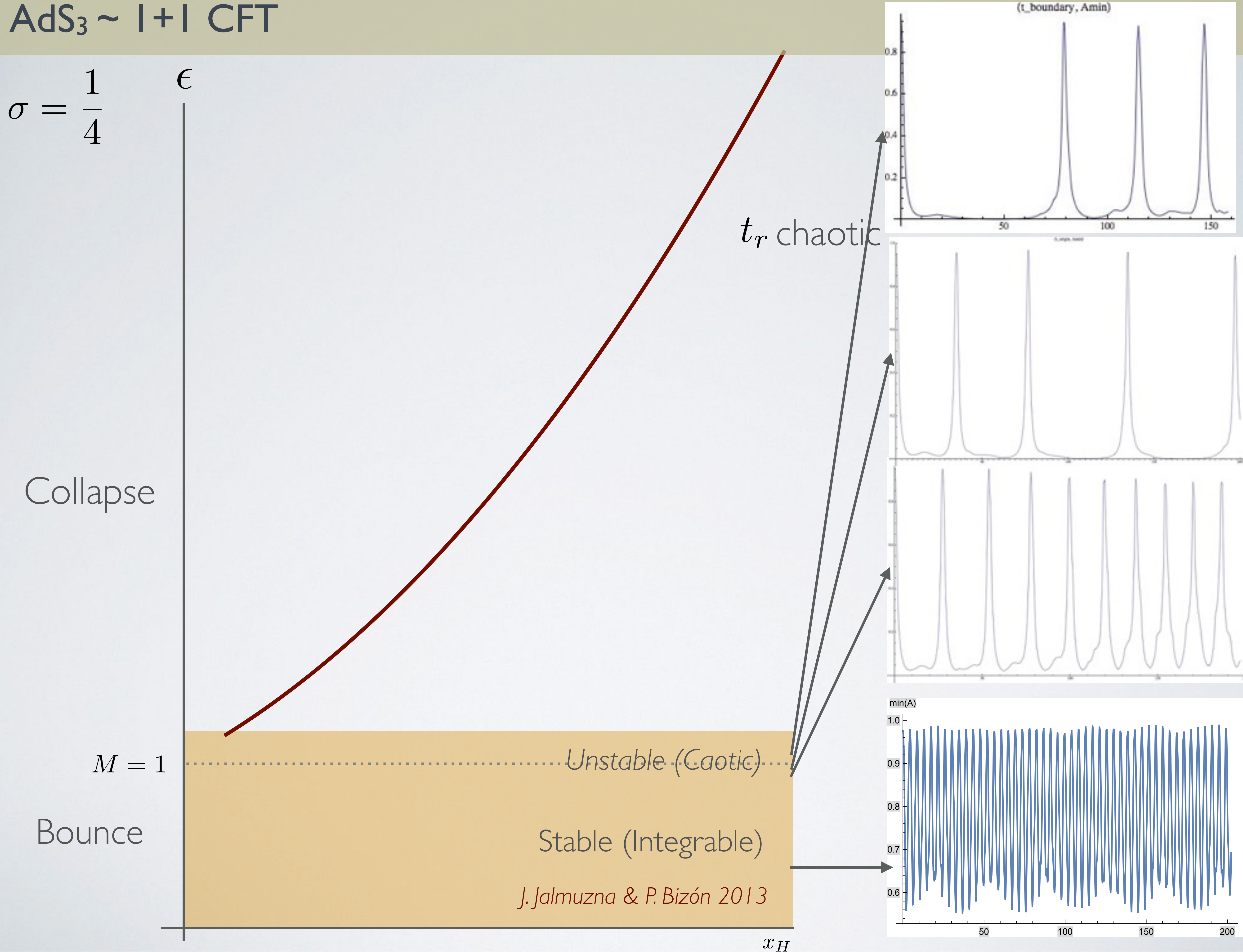




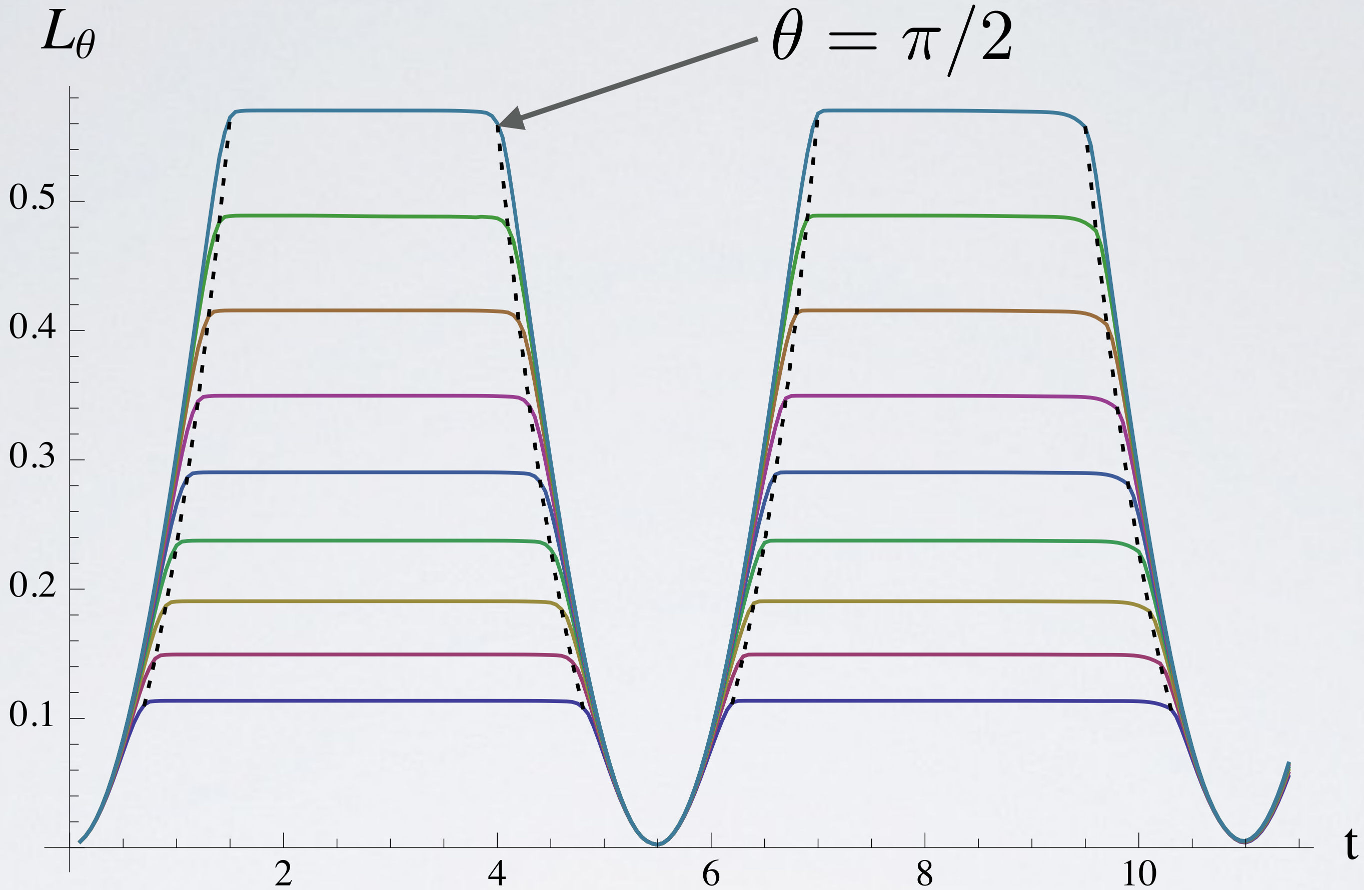
- new $\pi/3$ period appears

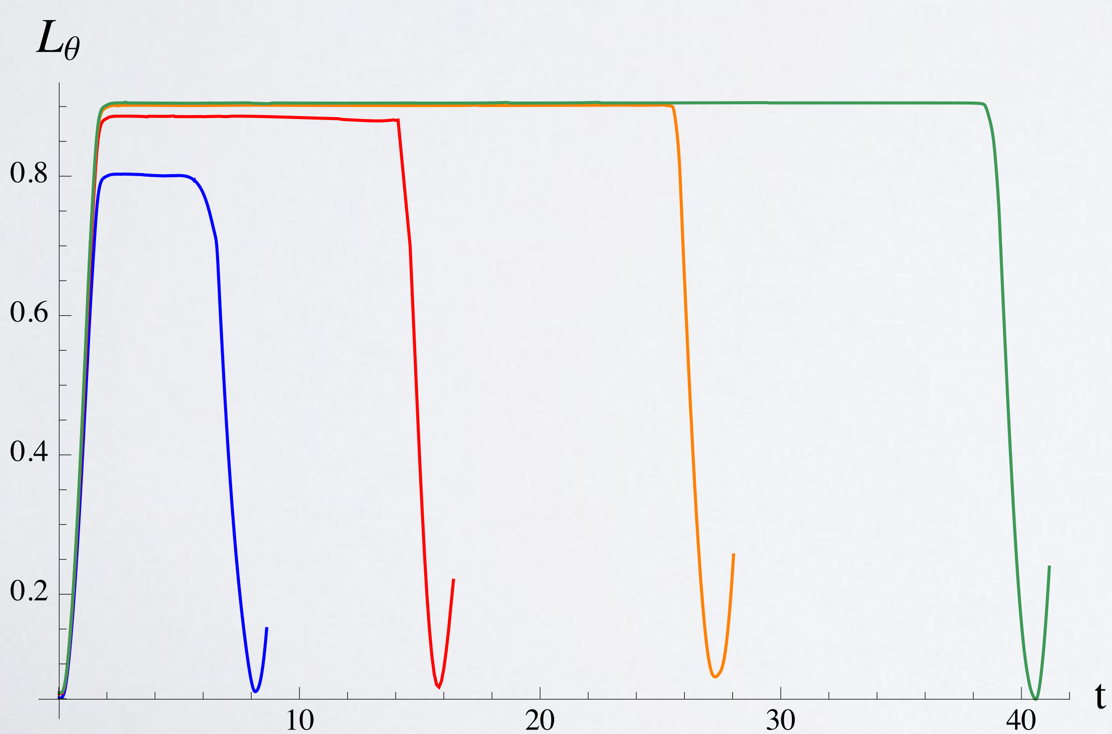


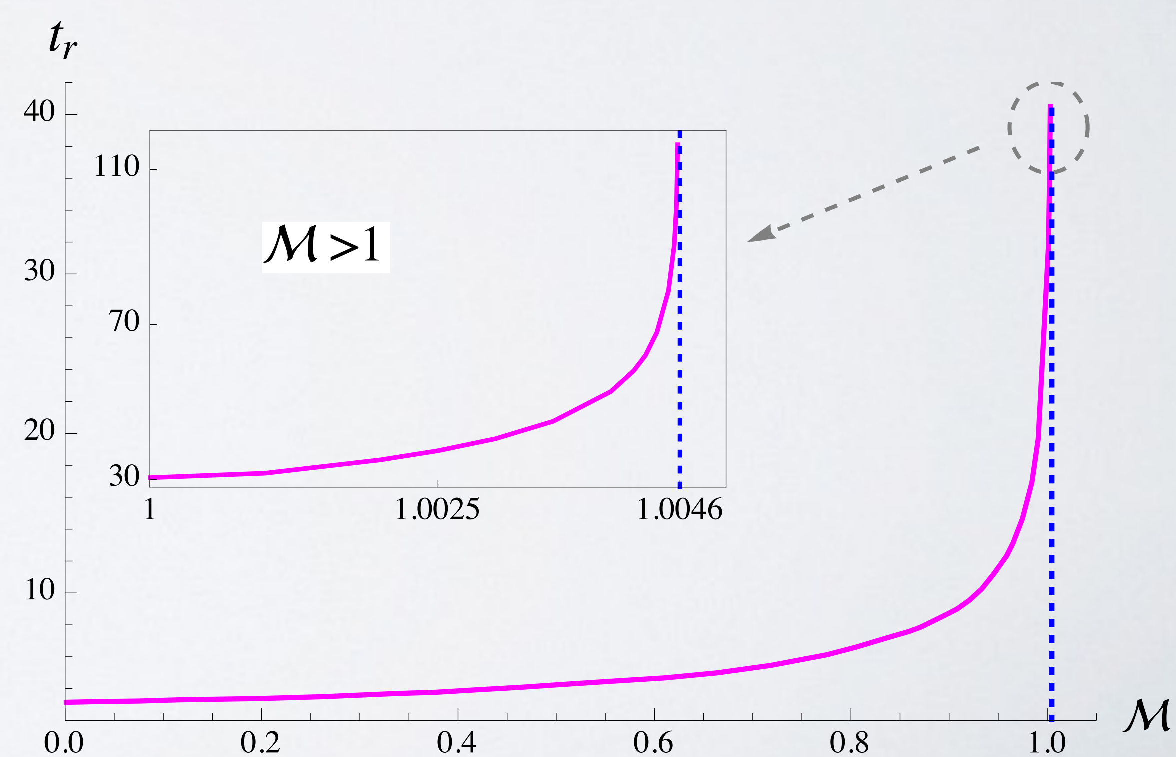
$AdS_3 \sim 1+1$ CFT



AdS$_3$ ~ 1+1 CFT

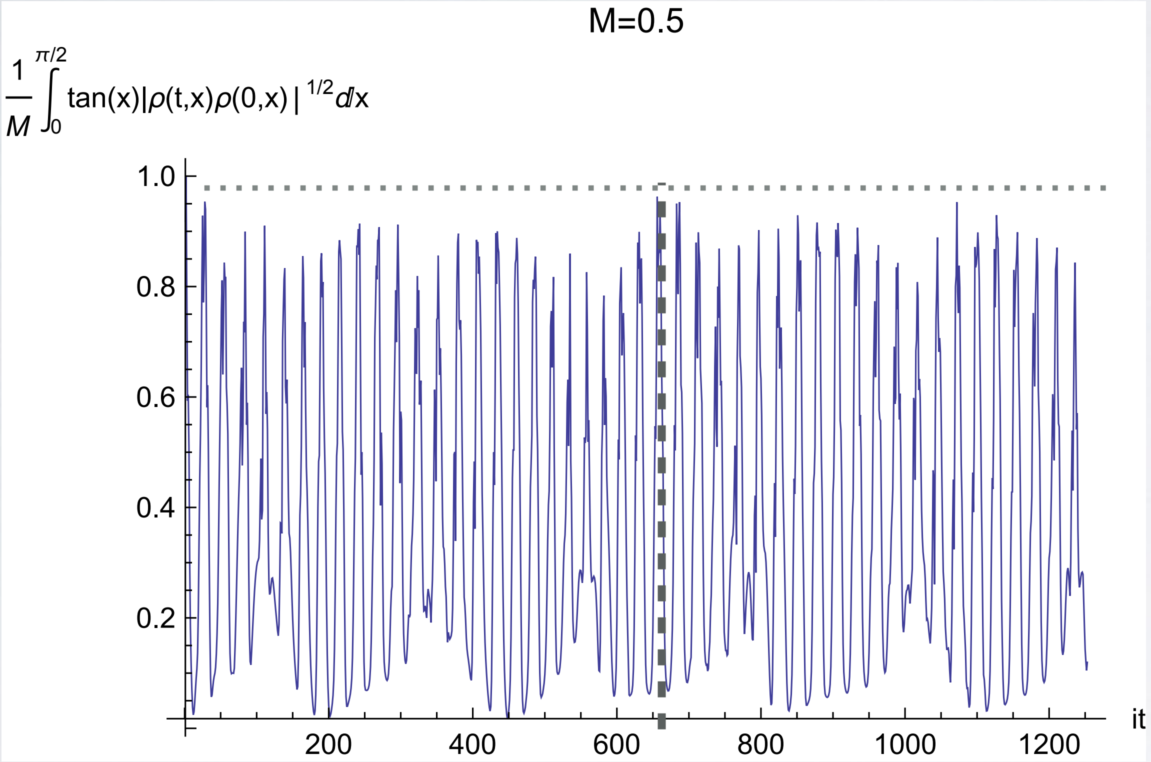

L_θ
$\theta = \pi/2$
0.5
0.4
0.3
0.2
0.1
2
4
6
8
10
t

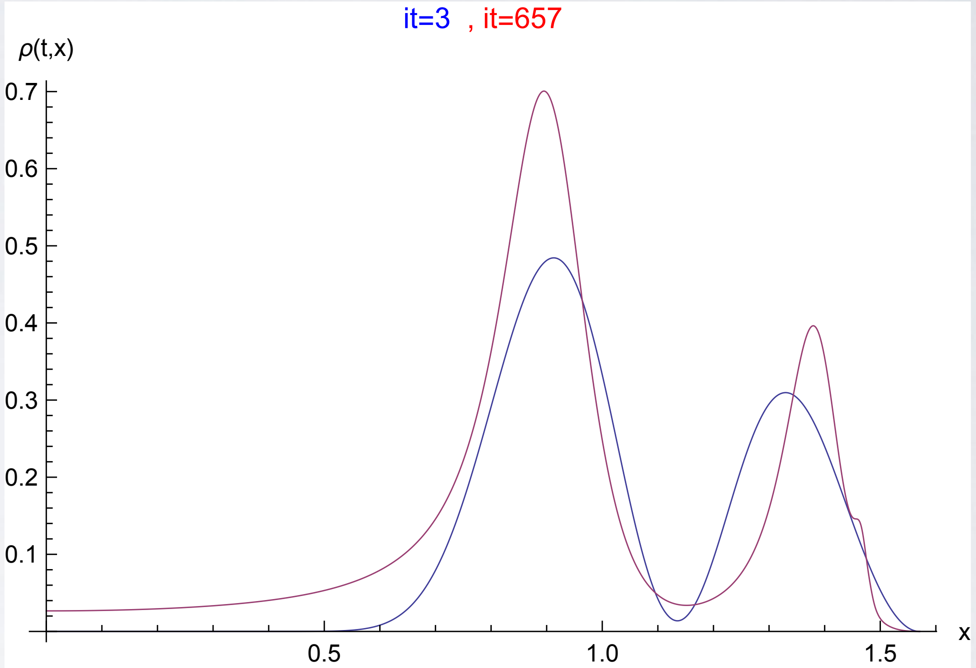

L_θ
0.8
0.6
0.4
0.2
10
20
30
40
t


t_r
40
30
20
10
110
70
30
$\mathcal{M} > 1$
1
1.0025
1.0046
0.0
0.2
0.4
0.6
0.8
1.0
$\mathcal{M}$

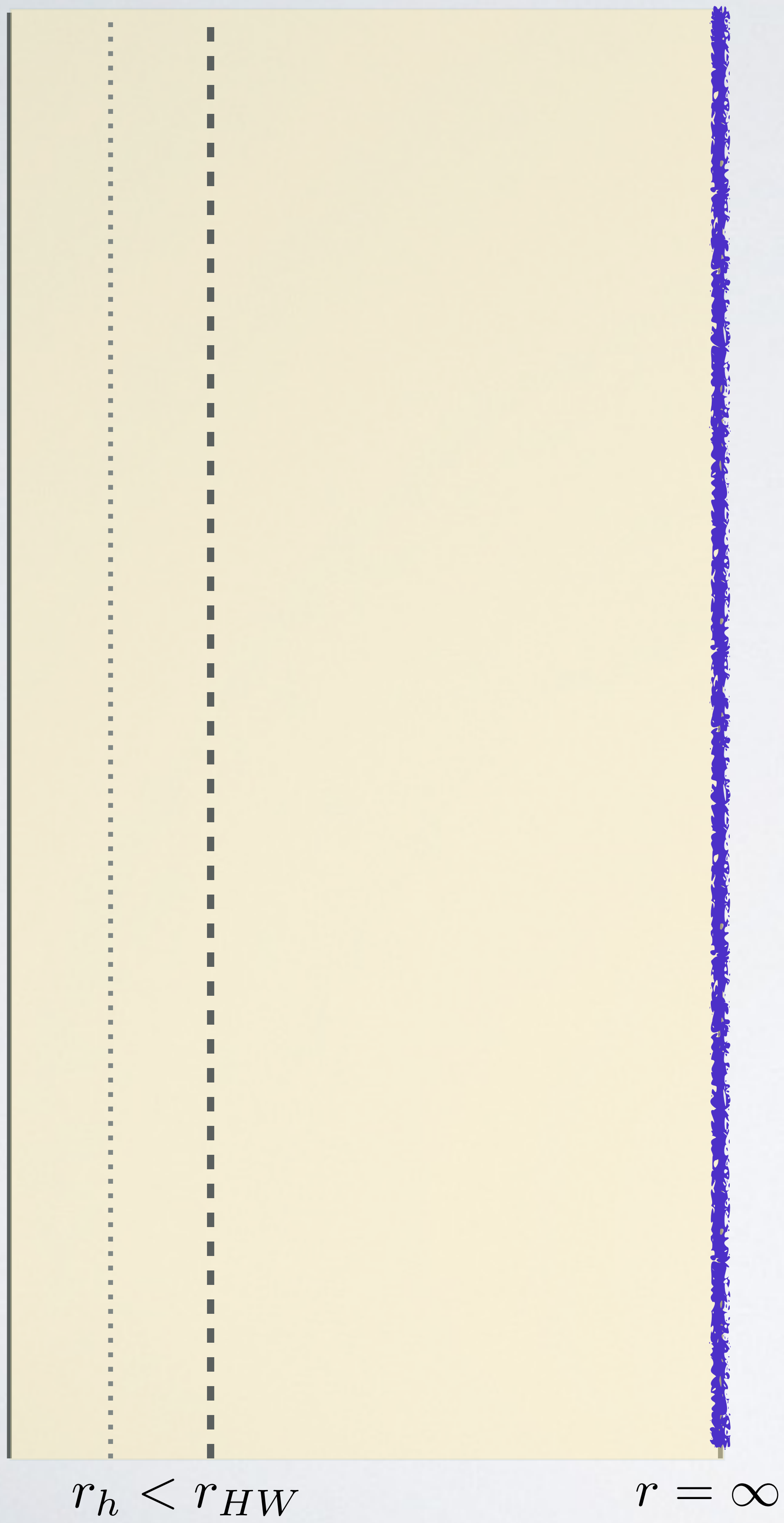
Autocorrelation superperiod

$$C(t) = \frac{1}{M}\int_0^{\pi/2} \tan(x)|\rho(t,x)\rho(0,x)|^{1/2}dx$$



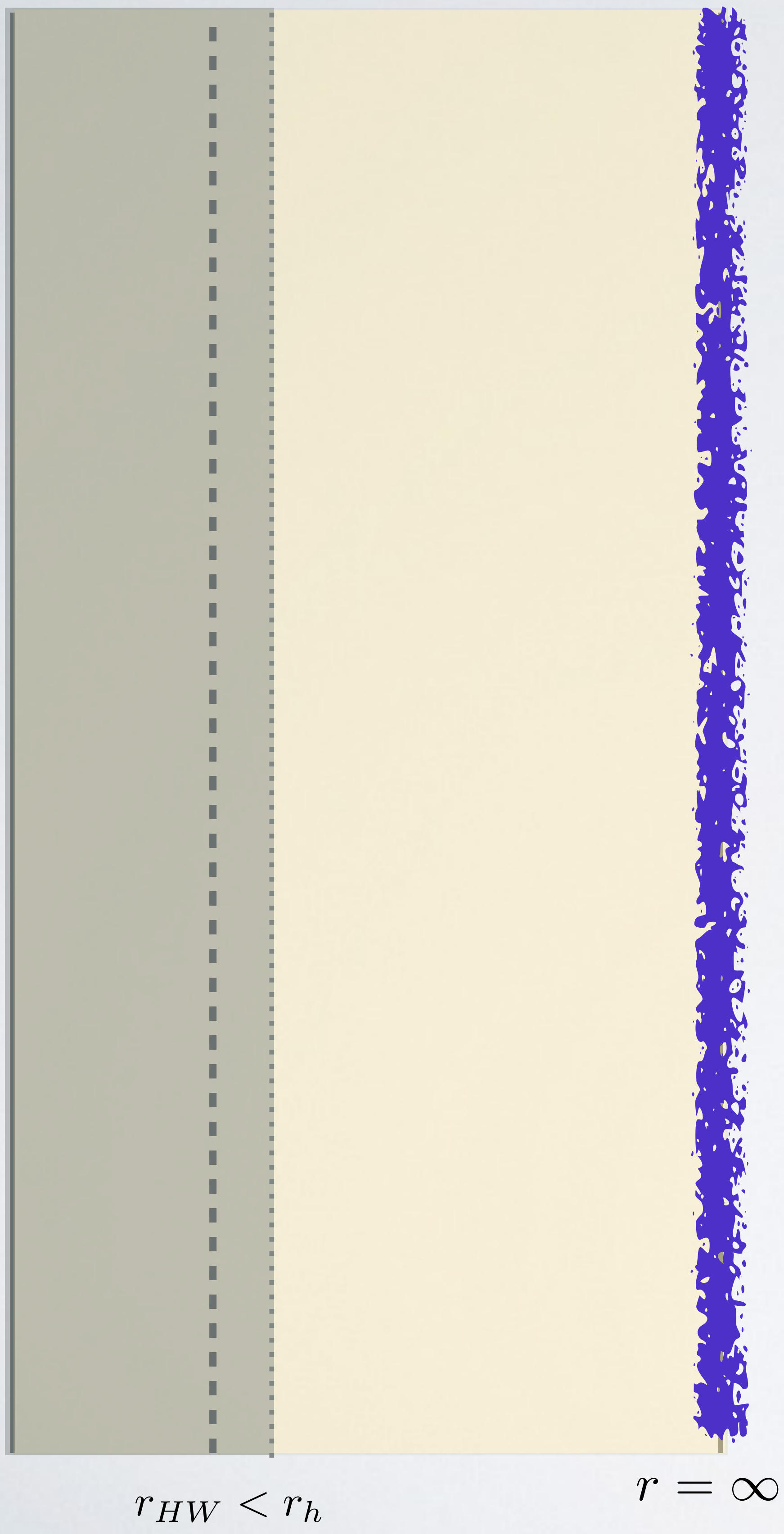


PLANAR AdS_5 with a Hard Wall



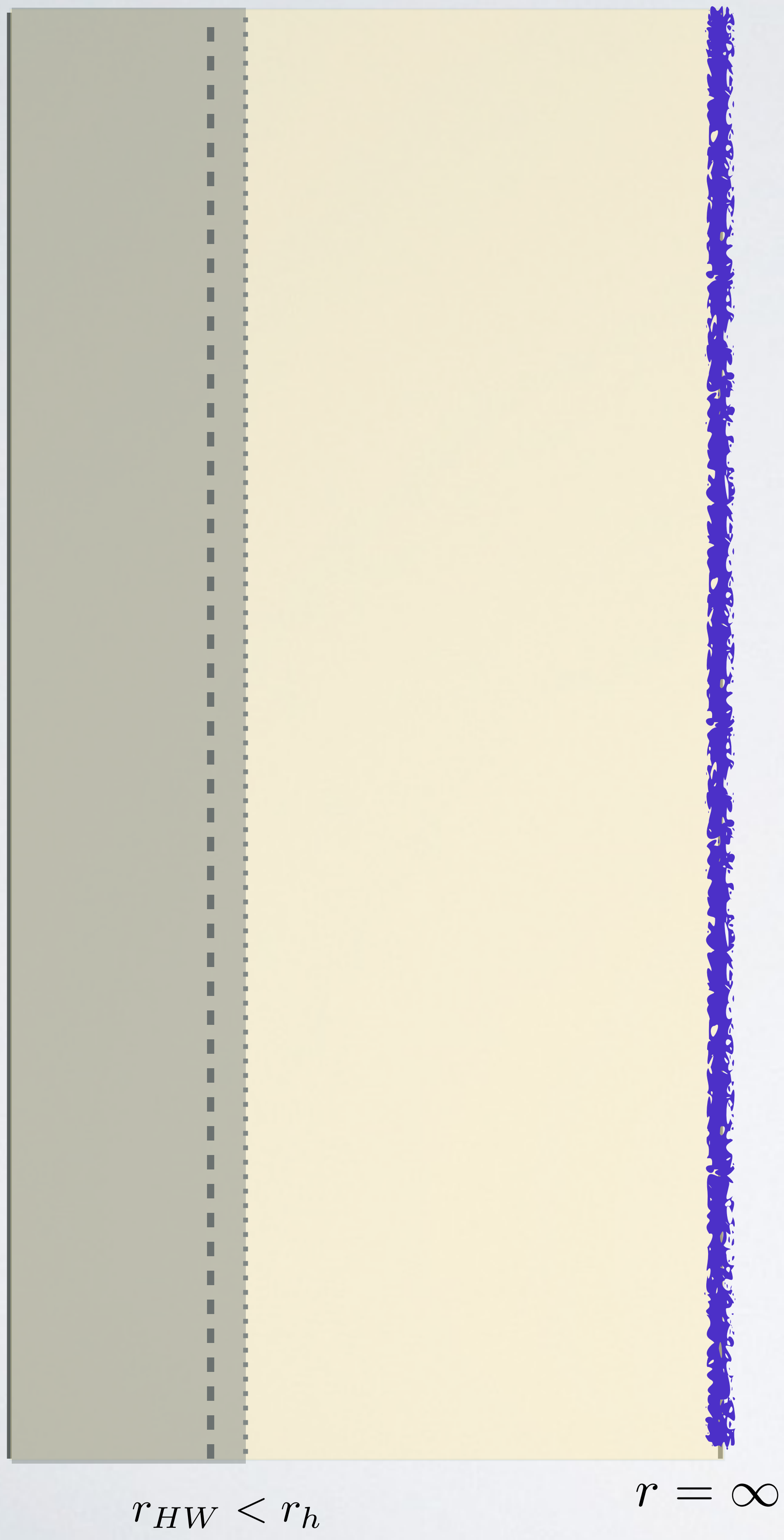
Craps, Lindgren, Taliotis, Vanhoof & Zhang 1406.1454

PLANAR AdS_5 with a Hard Wall

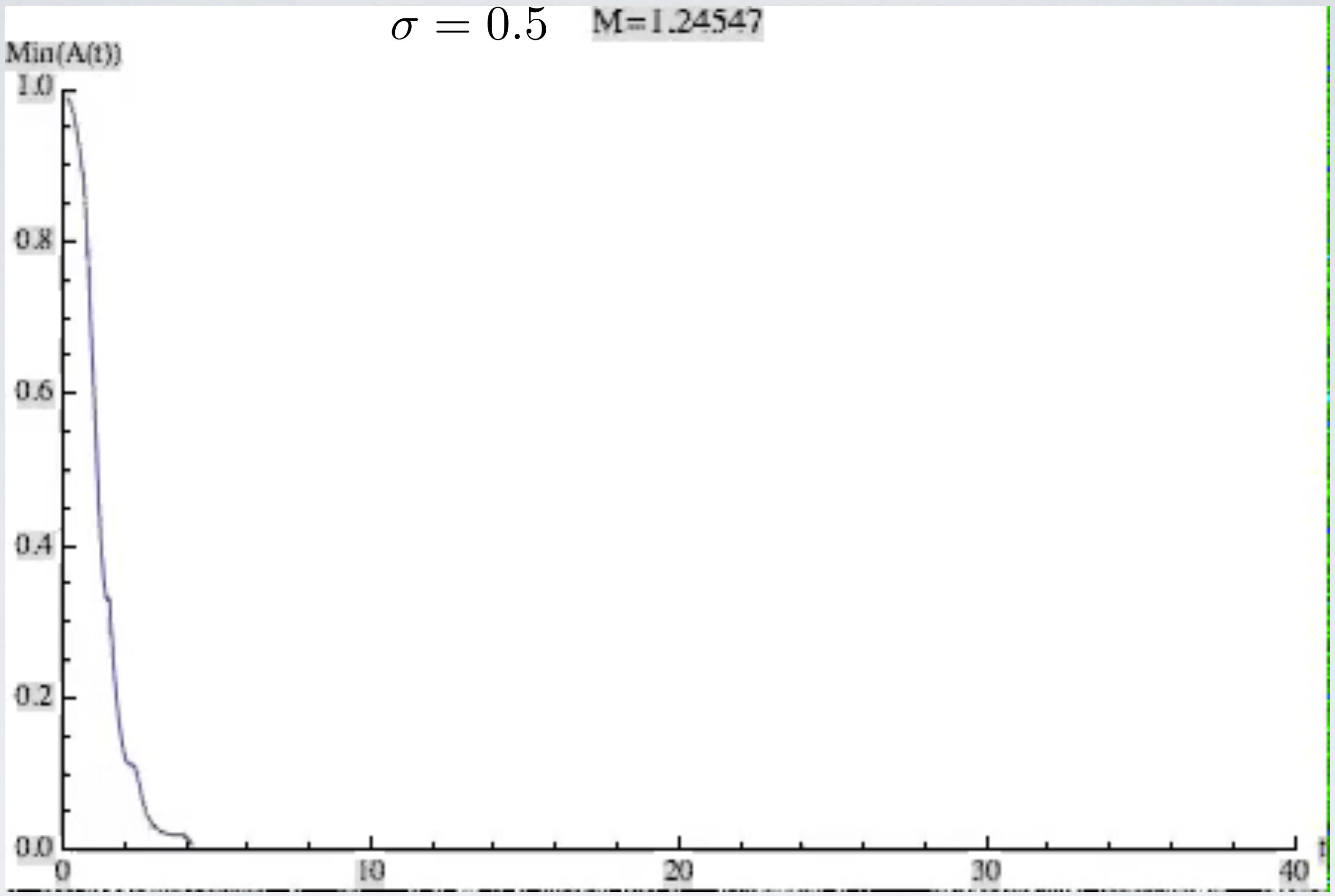


Craps, Lindgren, Taliotis, Vanhoof & Zhang 1406.1454

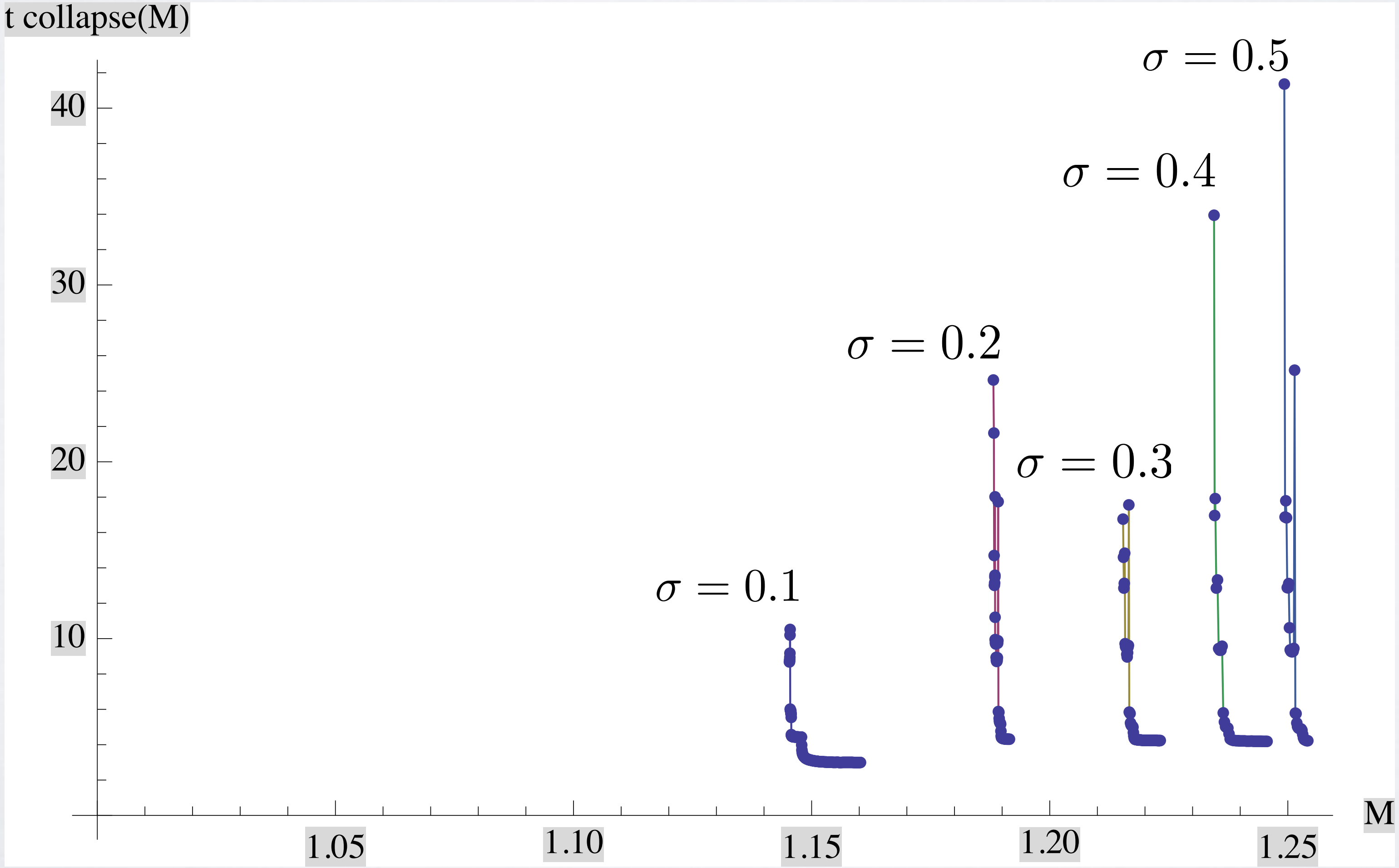
PLANAR AdS_5 with a Hard Wall



da SIlva, J.M., A. Serantes, E. López 1508.xxxx



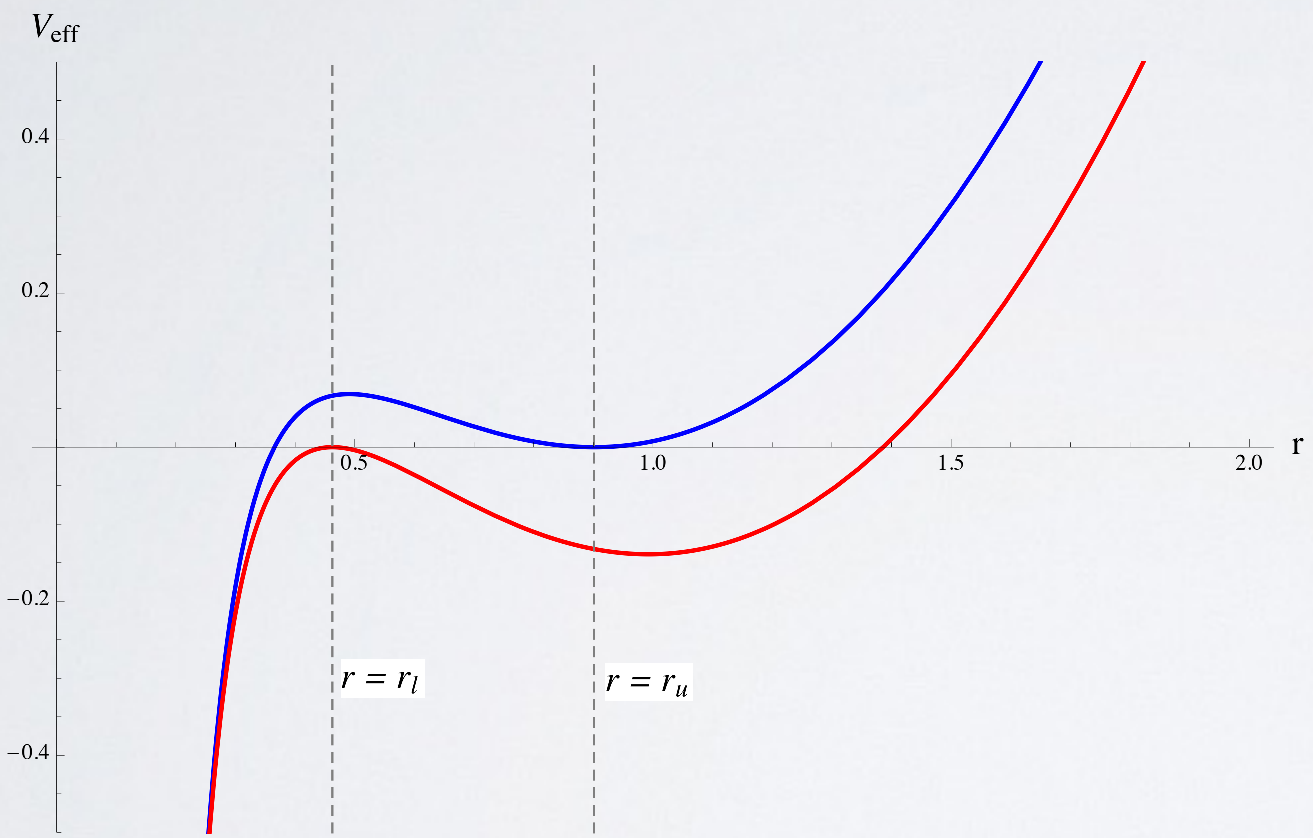
da SIlva, J.M., A. Serantes, E. López 1508.xxxx

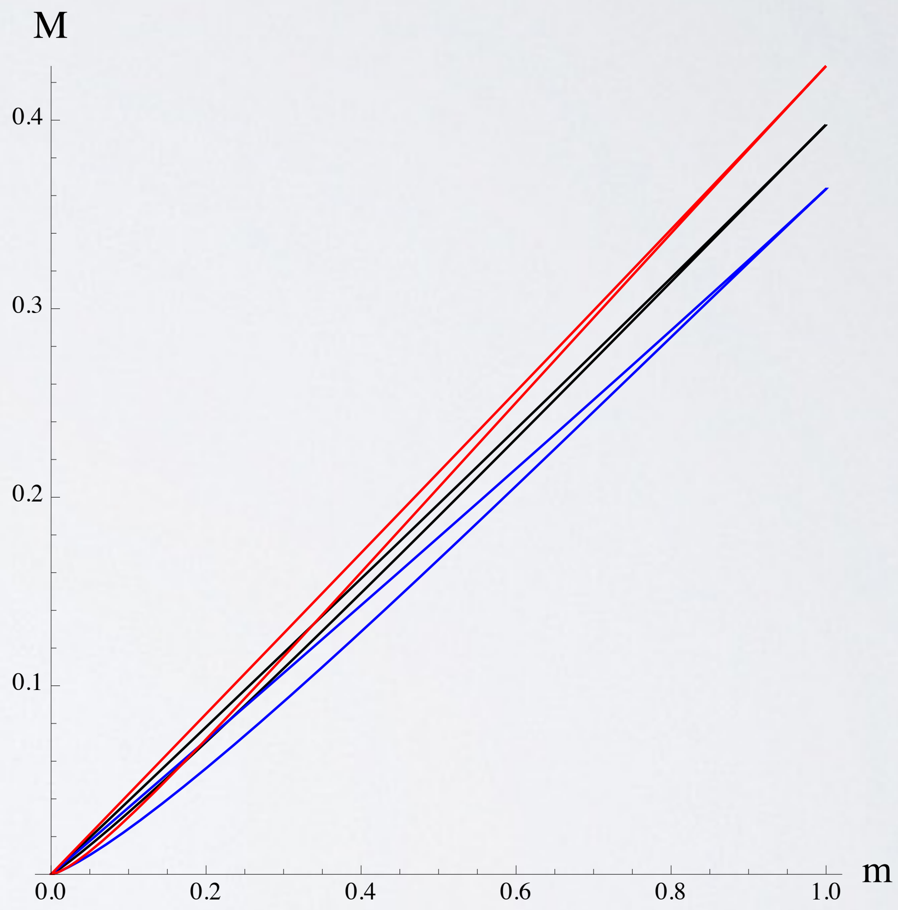


Thin Shell Oscillations in AdS

Equation of state $$p = \left(\frac{\alpha}{d}\right)\sigma$$

Effective potential $$V_{eff} = 1 + r_s^2 - \frac{1}{2} m r_s^{1-d} - \frac{m^2}{4M^2} r_s^{2\alpha} - \frac{1}{4} M^2 r_s^{-2(d-1+\alpha)}$$





Keranen, Nishimura, Stricker, Taanila, & Vuorinen, 1405.7015

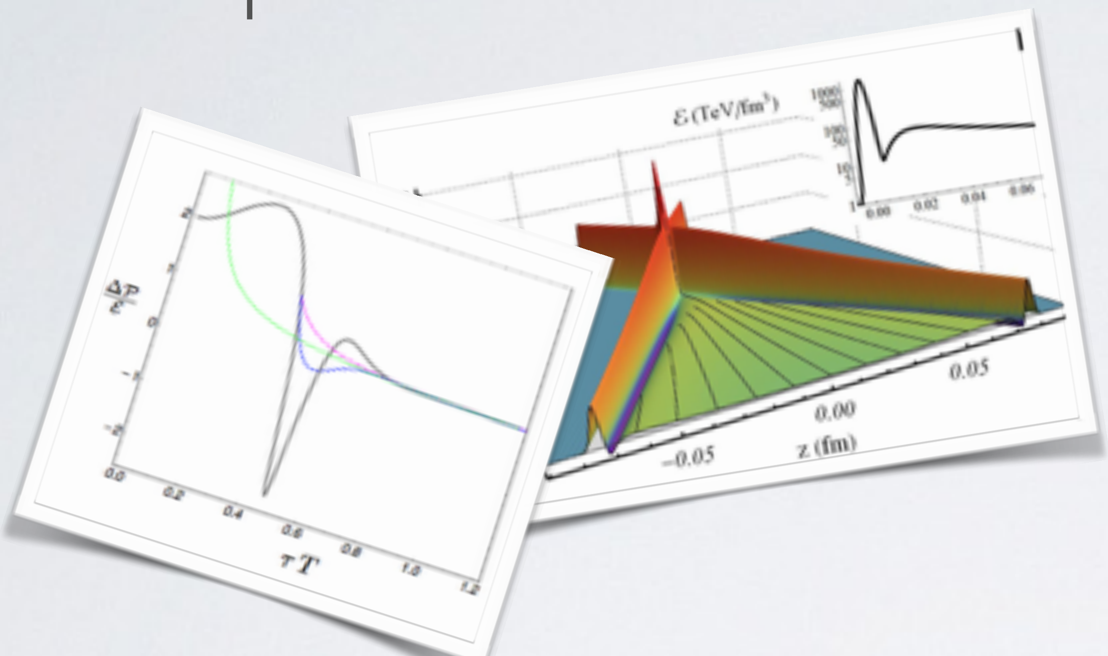
Serantes & J.M. 1507.xxxx

Conclusions & outlook

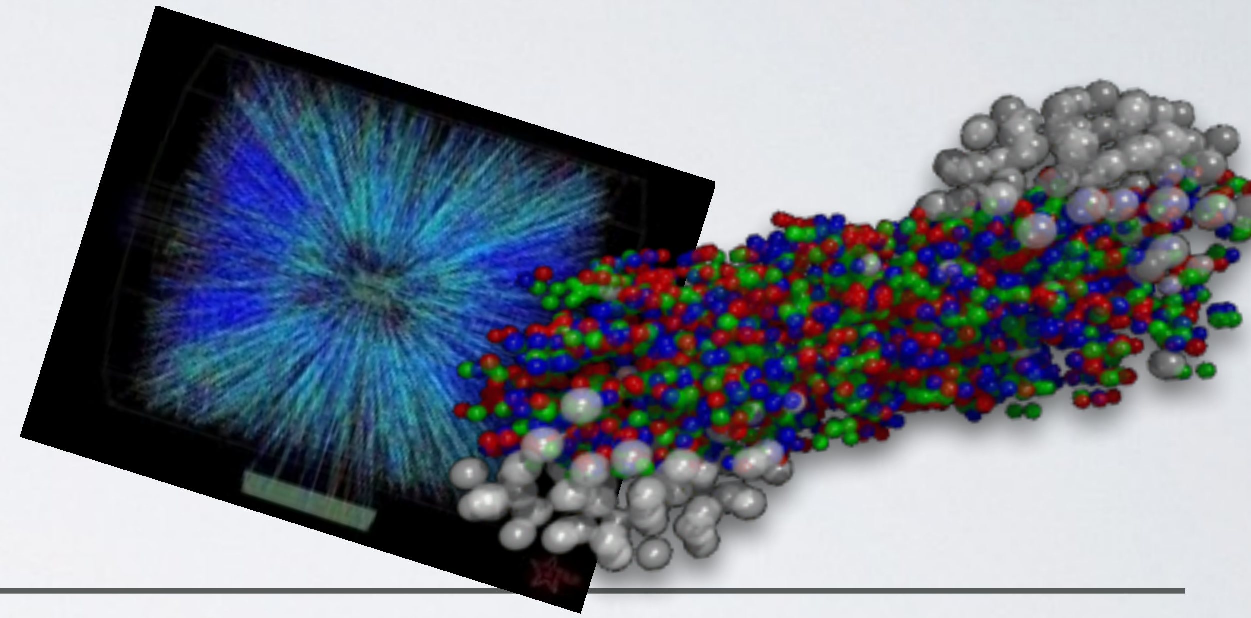
- landscape of evolutions in global AdS for different initial conditions and dimensions ¿$\longleftrightarrow$? routes to thermalization in closed quantum systems at strong coupling
- revivals allow for precission measurements
- search for generic features: two regimes, two timescales, collapse blocking, additional π/d frequency, etc.
- need some exact results from the QFT side at strong coupling and large c in different dimensions and geometries
- collapse in AdS_3 may shed light on the role of integrability breaking
- the massive case needs better understanding/formulation

AdS/CFT

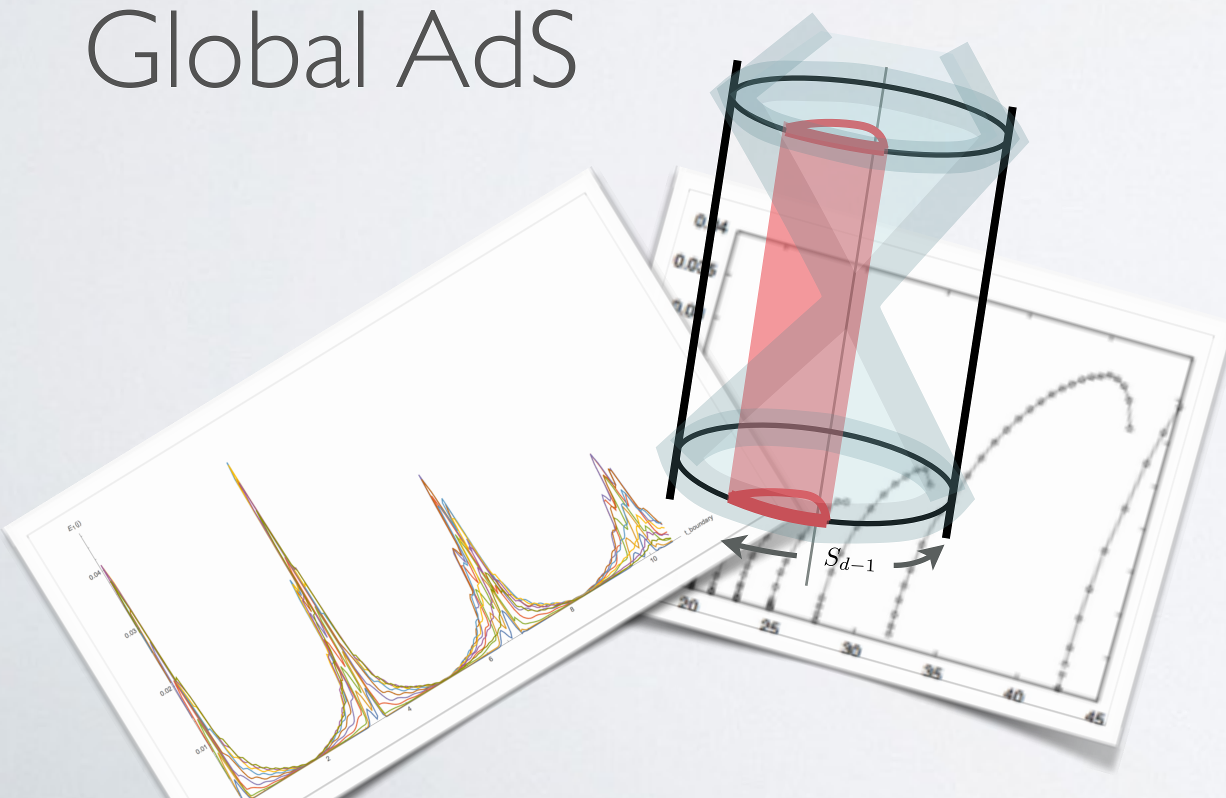
Poincaré patch



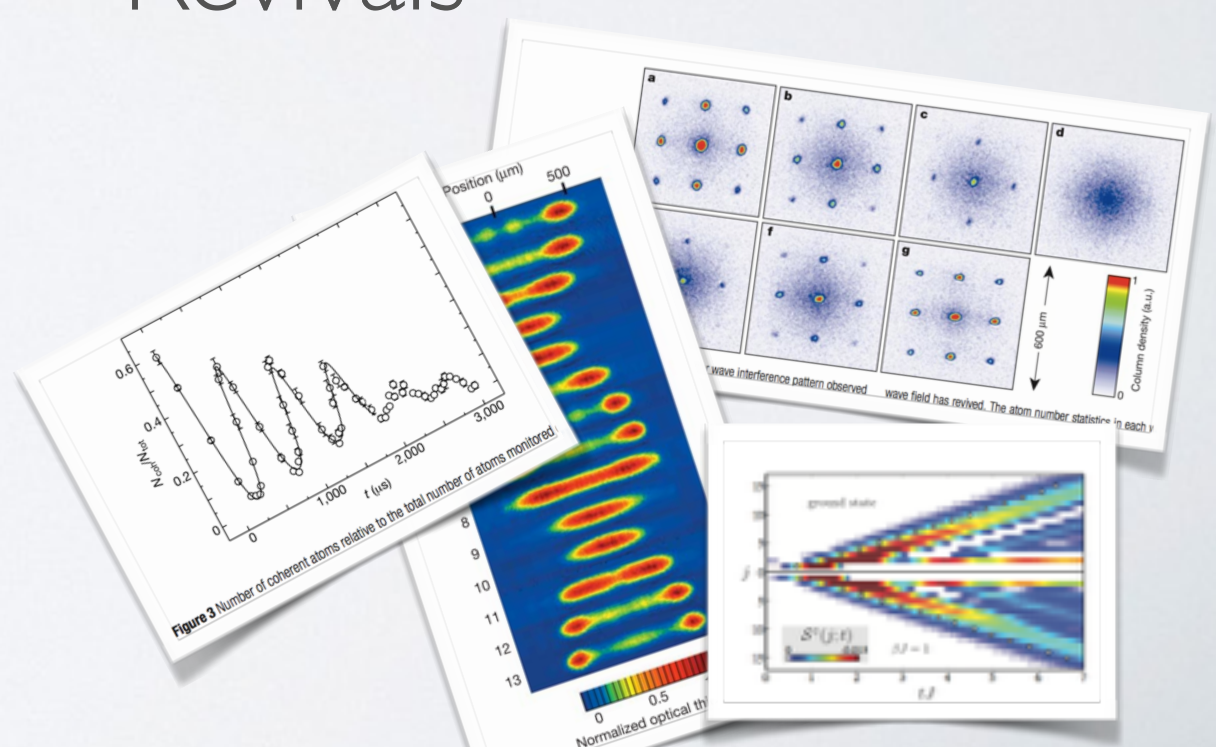
Heavy Ion Collisions



Global AdS



Revivals



thank you