Holographic Anyonic Superfluids

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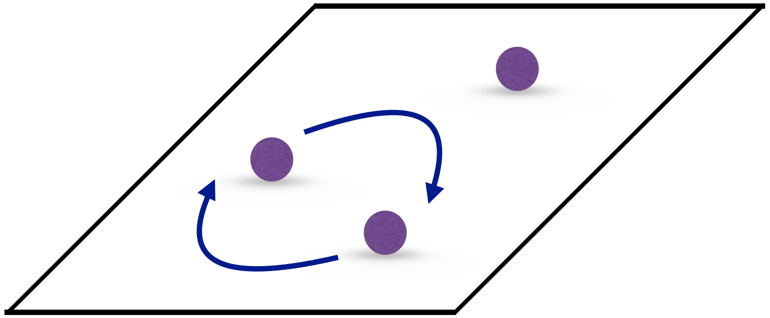
Plan

- Anyons, $SL(2,\mathbb{Z})$, and Quantum Hall Effect
- Superfluids and Anyon Superfliuds
- A Holographic Anyon Superfluid
- A Flowing Holographic Anyon Superfluid

Anyons

particles in 2+1 dim can have arbitrary statistics



$$|\psi_1\psi_2\rangle = e^{i\theta}|\psi_2\psi_1\rangle$$

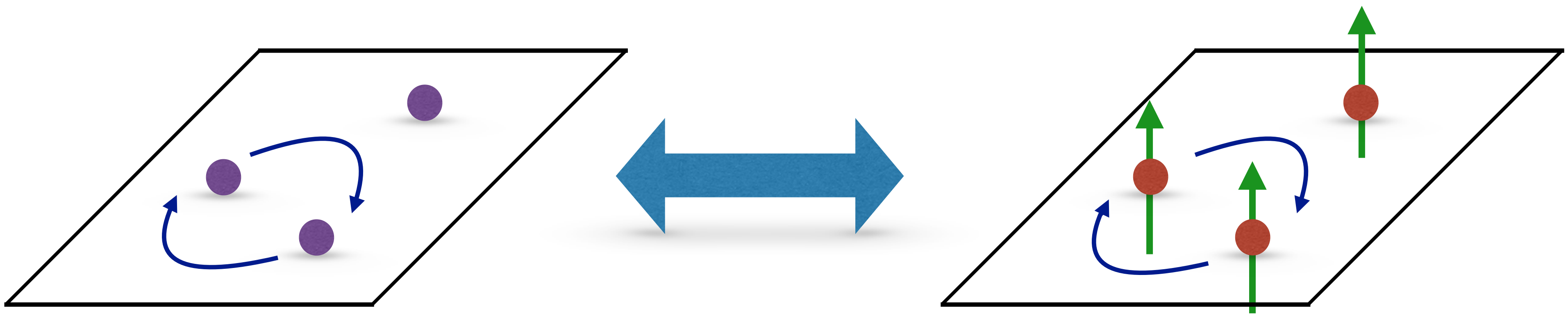
- $\theta = 0$ bosons
- $\theta = \pi$ fermions
- $\theta = \pi n/m$ anyons

Wilczek

Alternate Description

charged particles with n magnetic fluxes attached

statistical phase $\theta \longleftrightarrow$ Aharonov-Bohm phase πn



anyons

fermions with n fluxes

$$\theta_{stat} = \pi\,(1-n)$$

$$\theta_{stat} + \theta_{AB} = \pi - \pi n$$

Flux attachment and *SL*(2,$\mathbb{Z}$)

2+1 dim CFT

Witten
Burgess, Dolan

- *U*(1) current - J
- external vector - $\mathcal{A}$
- define $\mathcal{B} = \frac{1}{2\pi} * d\mathcal{A}$

mapping to CFT'

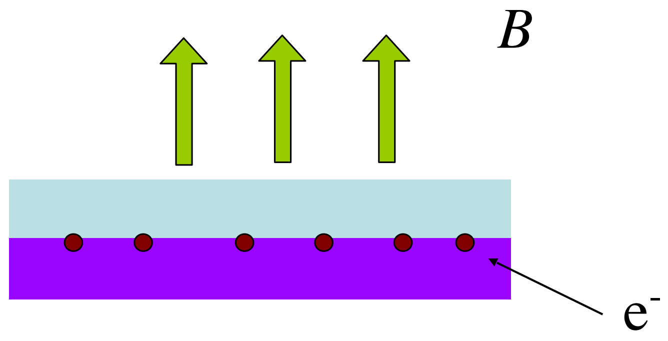
- add Chern-Simons term for $\mathcal{A}$: $J' = J + \mathcal{B}$
- make $\mathcal{A}$ dynamical: $J' = \mathcal{B}$
- generate *SL*(2,$\mathbb{Z}$)

$$\begin{pmatrix} J' \\ \mathcal{B}' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} J \\ \mathcal{B} \end{pmatrix}$$

An example:
Anyons and the Quantum Hall Effect

Set-up:

- e^- in 2+1-dim
- magnetic field B
- low temperature T

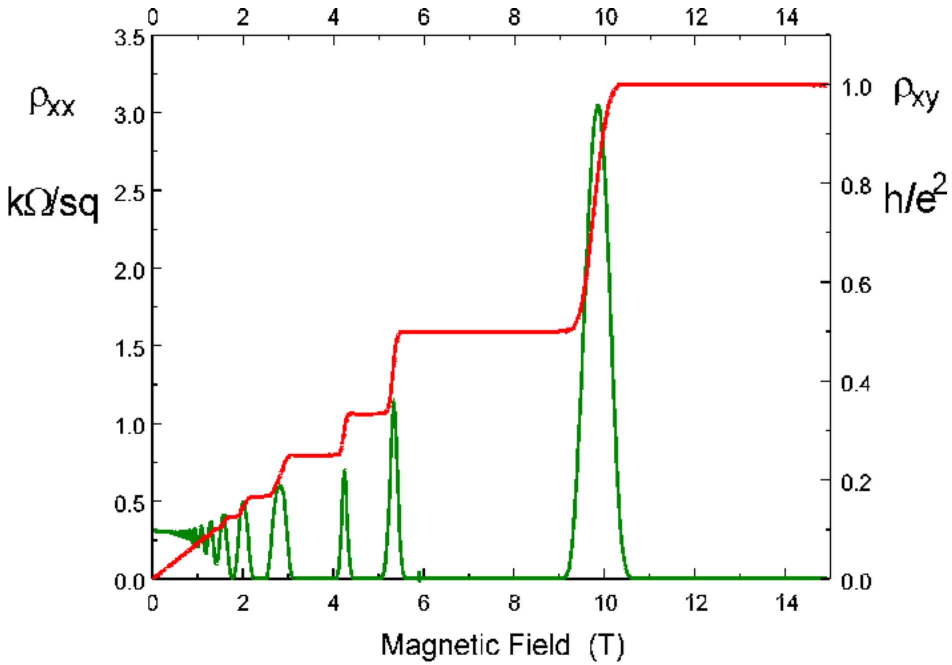


Conductivity

Long. $$\sigma_{xx} = \frac{J_x}{E_x} = 0$$

Hall $$\sigma_{xy} = \frac{J_y}{E_x} = \frac{e^2}{2\pi\hbar}\nu$$

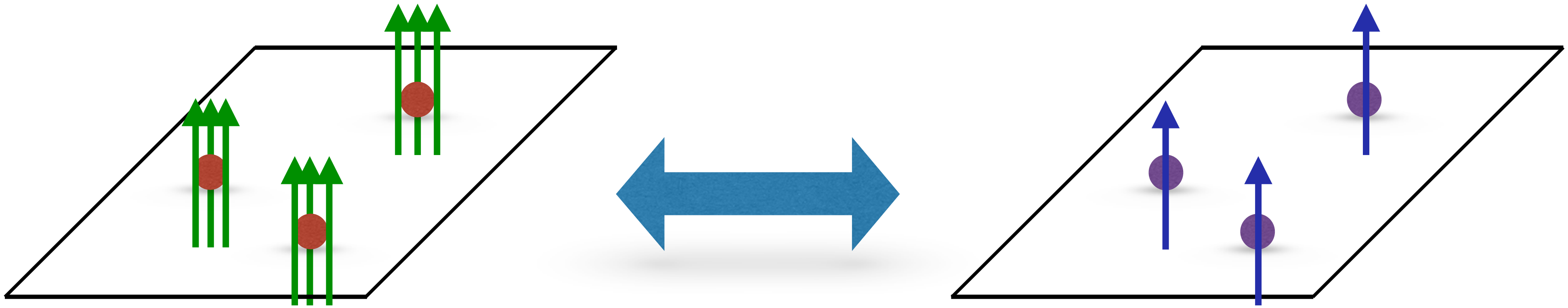
Filling Fraction



An example:
Anyons and the Quantum Hall Effect

QH fluid:

- Filling fraction $\nu = 2\pi \frac{D}{B}$
- $SL(2,\mathbb{Z})$ maps between QH states: $\nu' = \frac{a\nu - b}{-c\nu + d}$



FQH fluid of fermions ν

IQH fluid of anyons $\theta = \pi/\nu$

Special ex:

$\nu = 1/3$

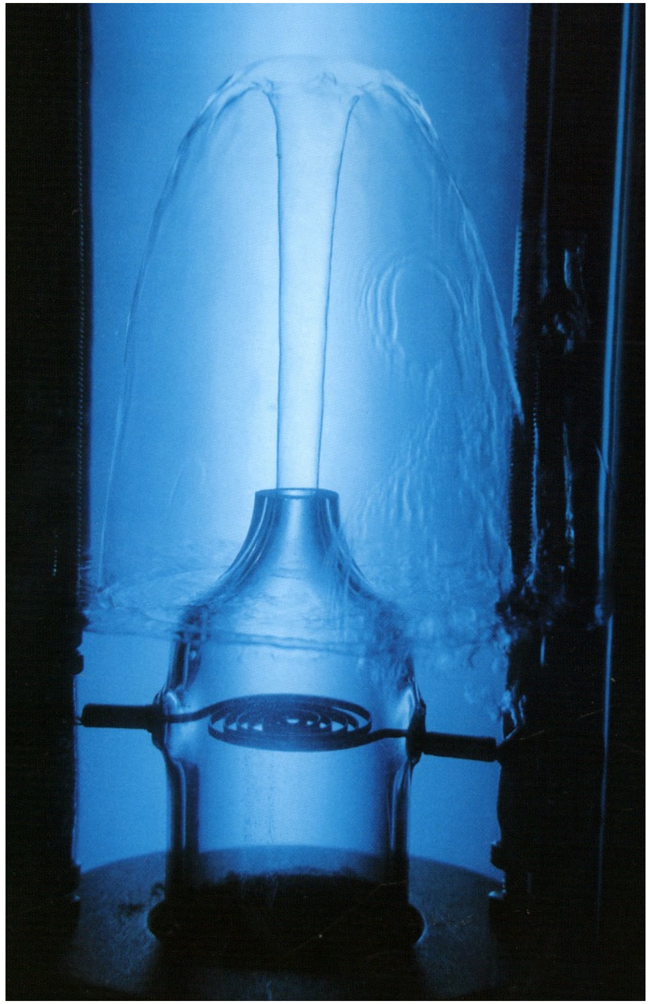
$\theta = 3\pi$ - fermions **Jain**

Superfluids

- Flow without resistance
- For example:
  - Liquid ^{4}He, $T < 2.17$K
  - Holographic dual of hairy BH
- Spontaneously broken global symmetry

$\Downarrow$

massless mode



^{4}He fountain

Anyon Superfluids

Anyons in $B = 0$ $\Rightarrow$ Superfluid **Laughlin**

Start with:

- QH fluid of fermions, filling fraction ν
- Background E_x $\Rightarrow$ Hall current $J_y = \dfrac{\nu}{2\pi} E_x$

SL(2,$\mathbb{Z}$) with $d/c = \nu$

$J'_y = d\ J_y \neq 0$ current

$B' = E'_x = 0$ no sources

$\theta' = \pi \left(1 - \dfrac{1}{\nu}\right)$ anyons

Superfluidity without symmetry breaking

Usually:

massless mode $\longleftrightarrow$ spontaneous symmetry breaking

For anyons:

massless mode $\longleftrightarrow$ Spontaneous **Fact** Violation

$$[T_x, T_y] \neq 0$$

Chen, Wilczek, Witten, Halperin, Giddings

Holographic Models of the QHE

Brane intersections with $\#ND=6$

- fundamental fermions
- probe Dq in Dp background
- ~~SUSY~~
- Chern-Simons terms

Familiar example: Sakai-Sugimoto D4-D8-$\overline{\text{D8}}$

QHE Models:

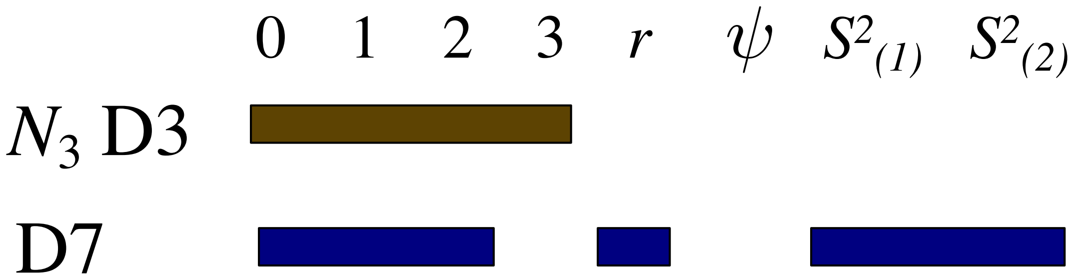
- D3-D7': **Bergman, NJ, GL, ML**

  2+1-dim defect, ν irrational

- D2-D8': **Järvinen, NJ, ML**

  fully 2+1-dim, $\nu = 1$

D3-D7' Model



$$d\Omega_5^2 = d\psi^2 + \cos^2\psi \; d\Omega^2_{2(1)} + \sin^2\psi \; d\Omega^2_{2(2)}$$

Probe D7:

- wraps $S^2 \times S^2 \subset S^5$
- fermions on 2+1-dim defect
- embedding $\psi(r)$
  - tachyonic
  - stabilize with wrapped flux on S^2

Add charges and magnetic field

Magnetic Field $F_{12} = B$

Charge Density $F_{0r} = A_0'(r)$

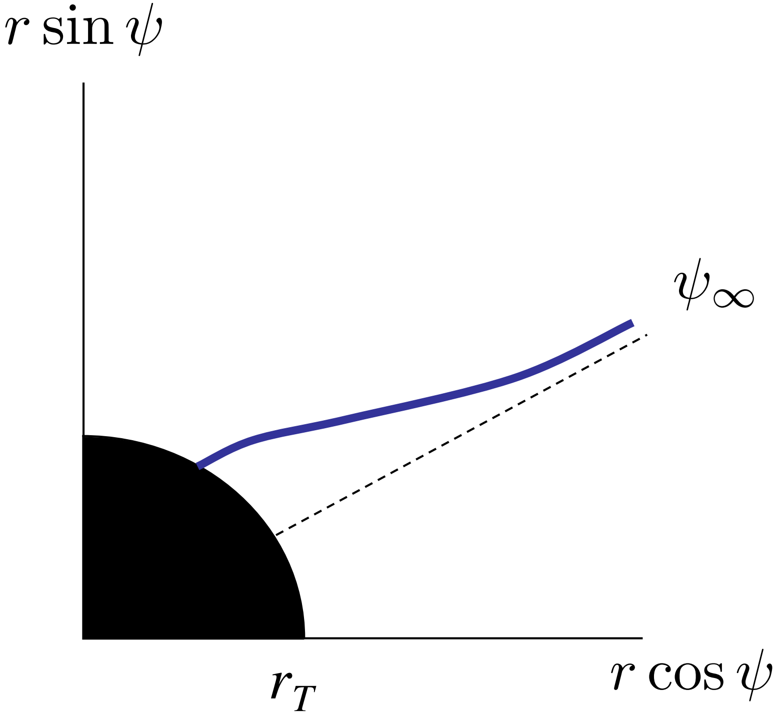
Sources for A_0:

- “Fractionalized”: charged BH
- “Mesonic”: Induced by Chern-Simons

$$S_{CS} \sim \int C_5 \wedge F \wedge F$$

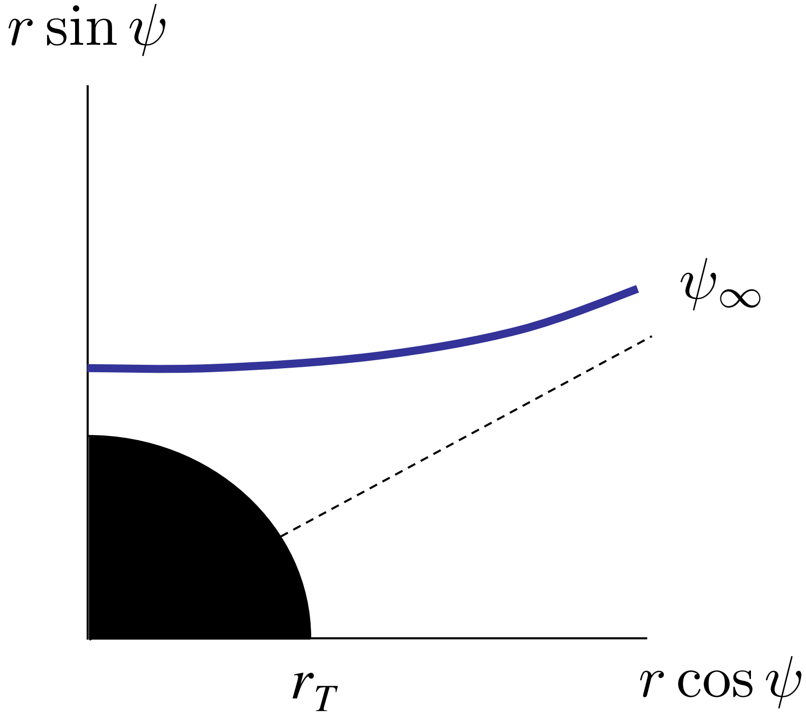
Embeddings

Black Hole



D7 enters horizon
metallic

Minkowski

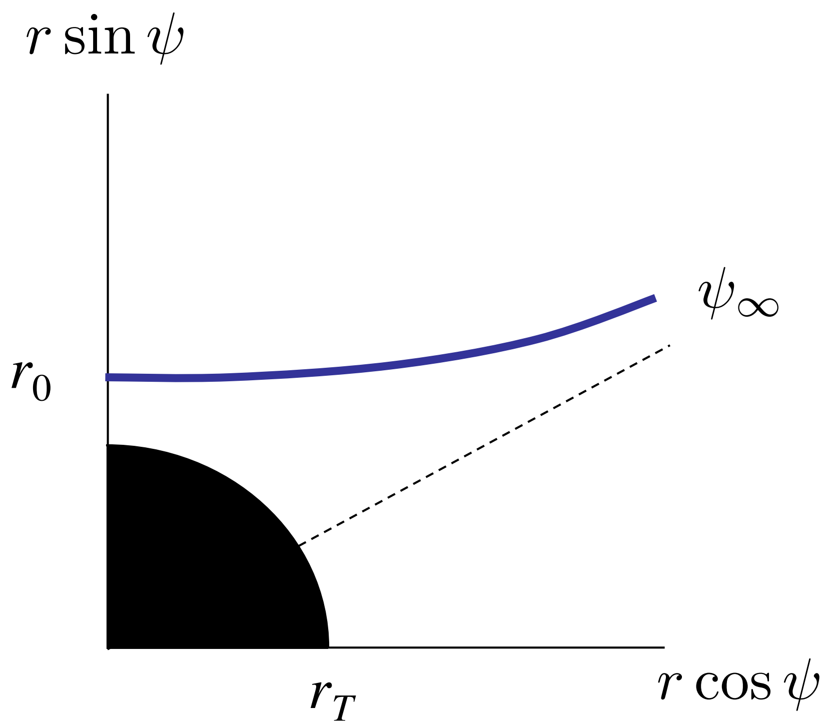


D7 ends where S^2 shrinks
QH fluid

Minkowski Embedding - QH state

- no sources at tip $\Longrightarrow$ $$\nu = 2\pi \frac{D}{B} \approx 1 - \frac{2\psi_\infty}{\pi}$$
- gap for charged excitations $m_{gap} = r_0$

$$\sigma_{xy} = \frac{\nu}{2\pi}$$



Alternative Quantization

Standard Dirichlet conditions:

bulk gauge field A fixed at boundary

$$\delta S_D = \int_\partial J_\mu \delta A^\mu$$

Change to Neumann condition:

$$S_N = S_D - \int_\partial J_\mu A^\mu \quad \Rightarrow \quad \delta S_N = -\int_\partial A^\mu \delta J_\mu$$

define $\quad J = \frac{1}{2\pi} * dv \quad$ and recall $\quad \mathcal{B} = \frac{1}{2\pi} * d\mathcal{A}$

$$\Rightarrow \quad \delta S_N = -\int_\partial \mathcal{B}_\mu \delta v^\mu$$

General alternative quantization

Dirichlet to Neumann = Bulk electric-magnetic duality

General boundary variation implements $SL(2,\mathbb{Z})$

$$\delta S = \int_{\partial} \left(aJ_\mu + b\mathcal{B}_\mu\right)\left(c\delta v^\mu + d\delta A^\mu\right)$$

For anyons with $\boldsymbol{B}$=0, choose $d/c = \nu$

Fluctuations

Bulk remains unchanged under $SL(2,\mathbb{Z})$

Lowest modes:

$$\left.\begin{array}{l} 1.\ \ \delta J' \\ 2.\ \ \delta\psi \end{array}\right\} \text{ with fixed } \mathcal{B}'$$

In original $SL(2,\mathbb{Z})$ variables:

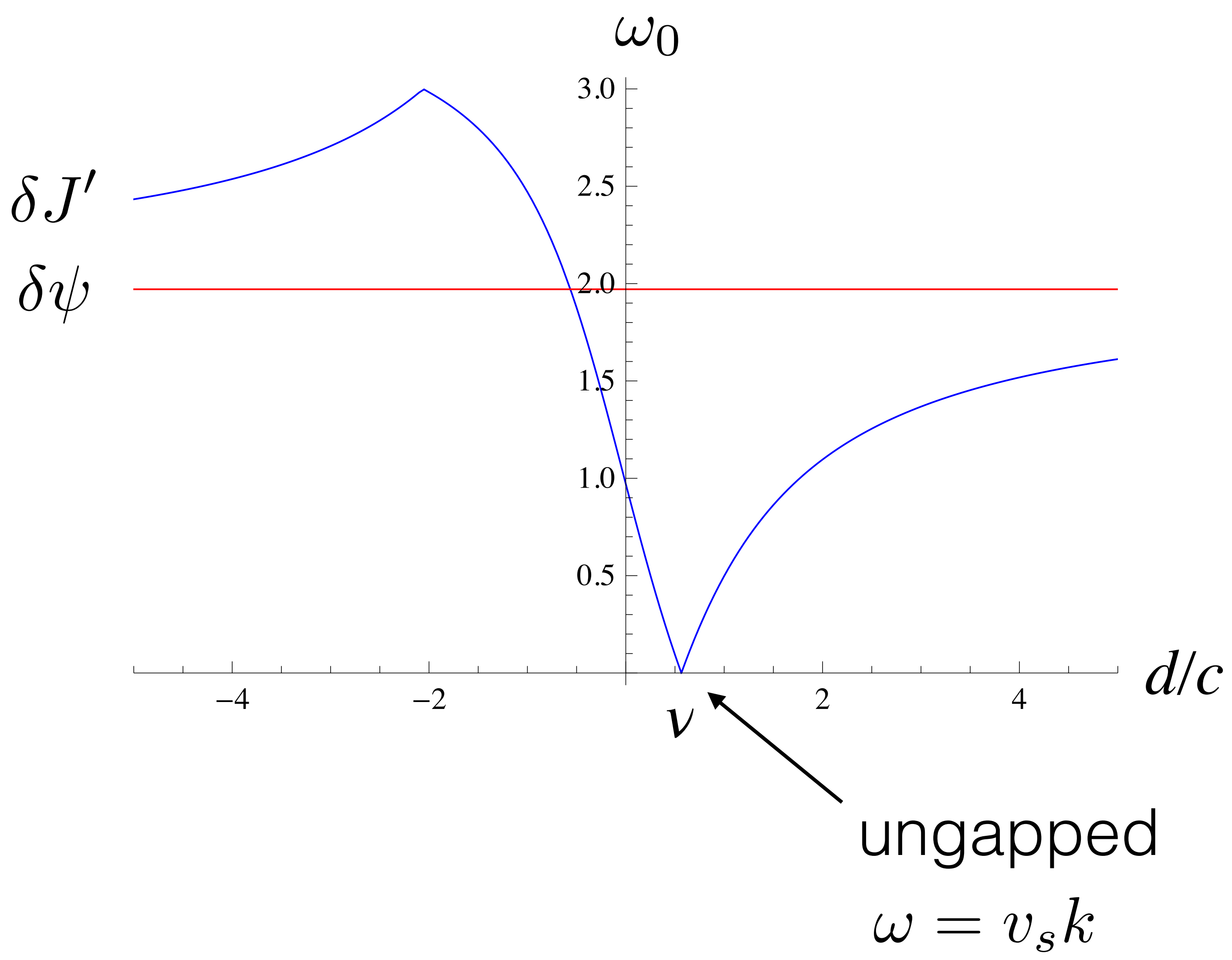
$$0 = \delta B' = c\,\delta D + d\,\delta B$$

$$\Downarrow$$

$$\frac{\delta D}{\delta B} = \frac{d}{c}$$

if $\dfrac{d}{c} = \nu$ $\Rightarrow$ extra $\boldsymbol{D}$ is compensated by extra $\boldsymbol{B}$

Spectrum vs. d/c

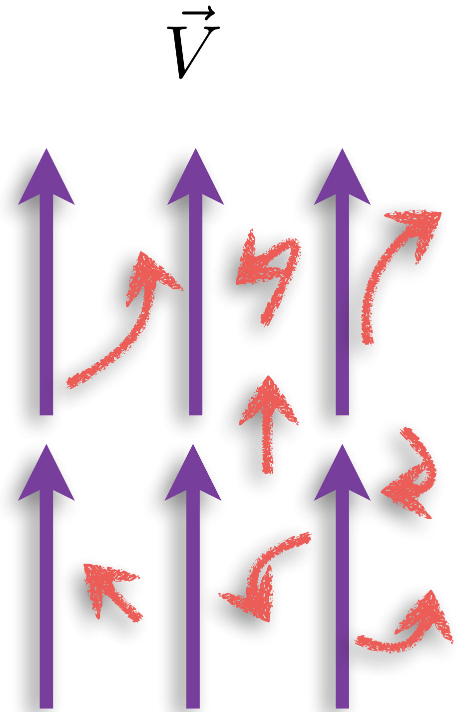


Superfluids can flow

Two component description:

- superfluid with velocity $\vec{V}$
- normal fluid

at low T, gas of phonons



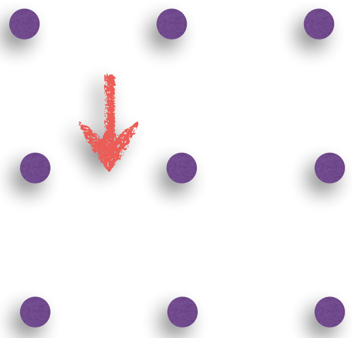
In holographic model:

Superfluid velocity V_y $\longleftrightarrow$ Electric field E_x

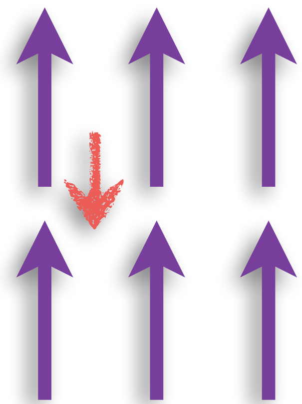
But not too fast!

Superfluid stable only if all excitations have $\omega(k) > 0$

At T=0: $\vec{V} = 0$



Boost by -V →



Phonon Energy $\omega(\vec{k})$ $\qquad$ $\omega' = \gamma\left(\omega - \vec{k}\cdot\vec{V}\right)$

Landau criterion for superfluidity:

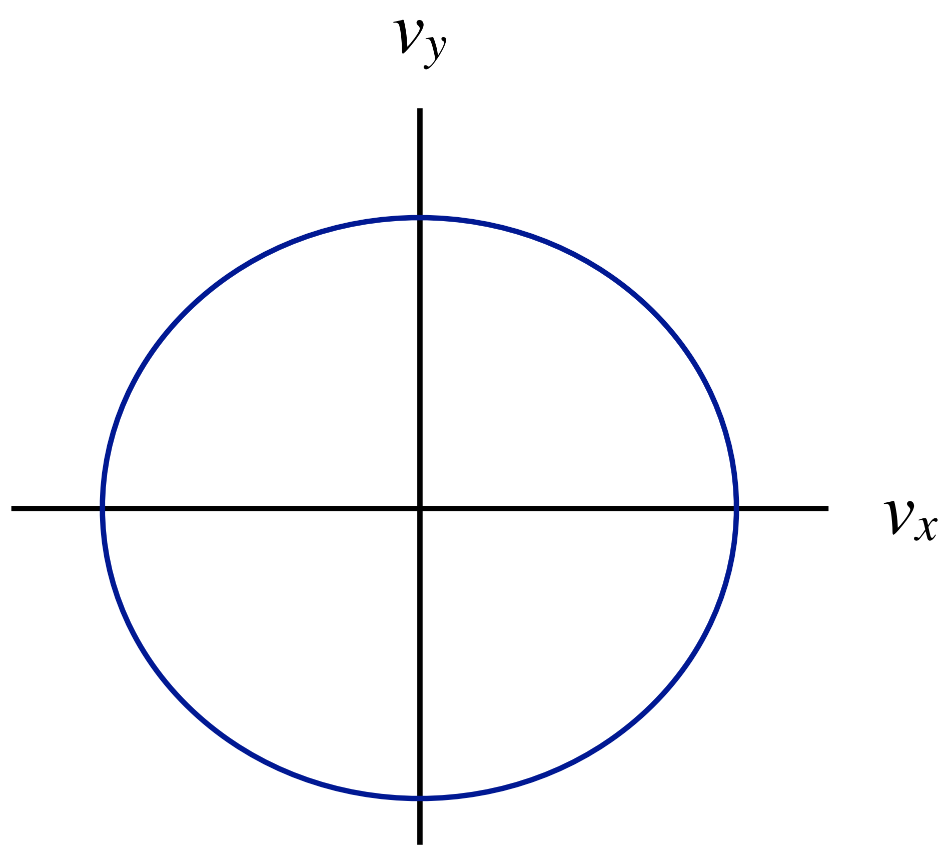
$$\omega'(\vec{k}) > 0 \quad \Longrightarrow \quad V < V^{crit} = v_s$$

Anisotropic sound speed

Now $T \geq 0$, holographic anyonic superfluid:

phonon velocity v_s
vs.
direction

$$V_y = 0$$



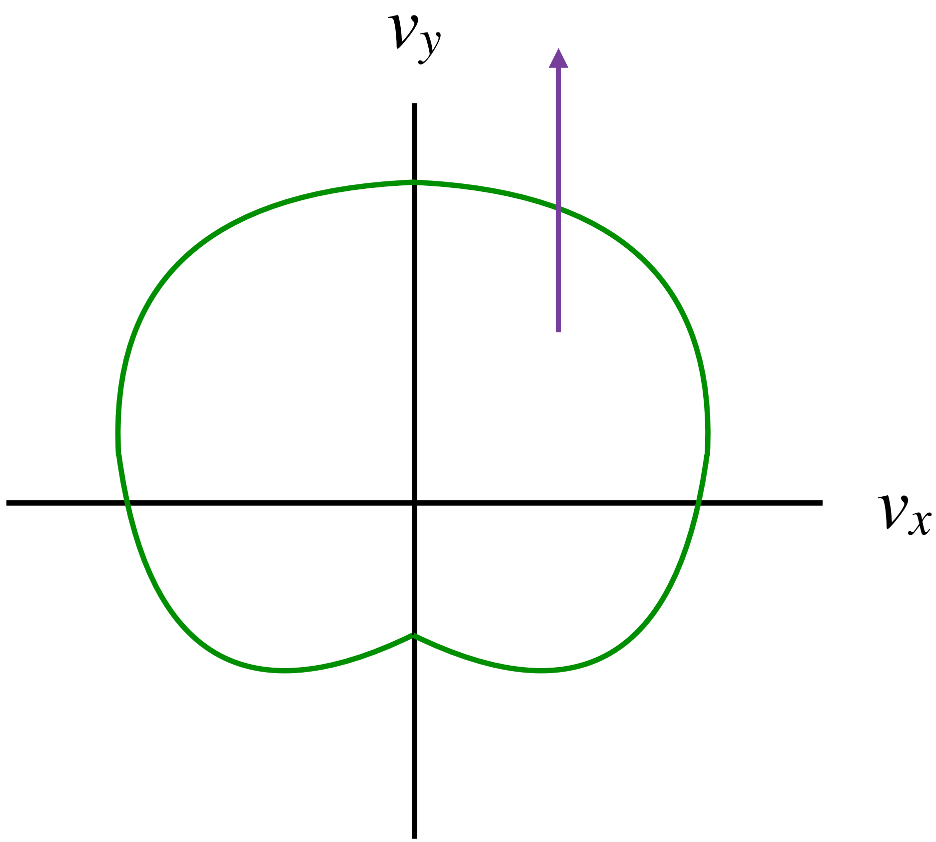
Anisotropic sound speed

Now $T \geq 0$, holographic anyonic superfluid:

phonon velocity v_s
vs.
direction

$$V_y > 0$$

direction of flow



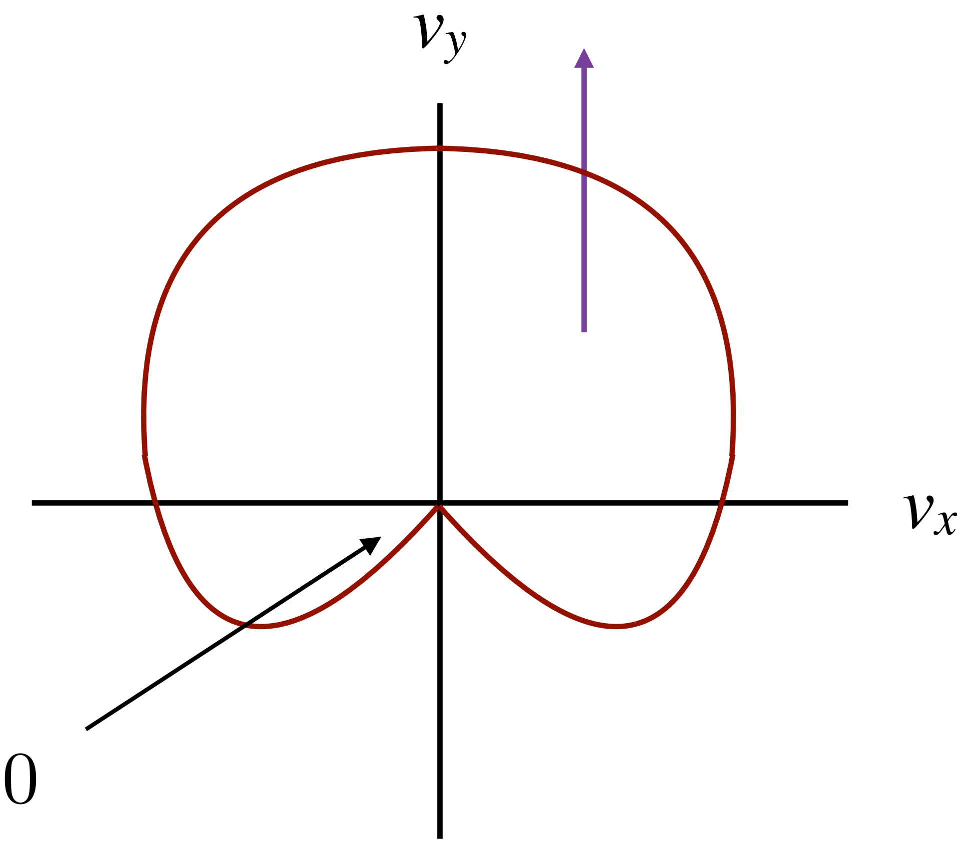
Anisotropic sound speed

Now $T \geq 0$, holographic anyonic superfluid:

phonon velocity v_s
vs.
direction

$$V_y = V^{crit}$$

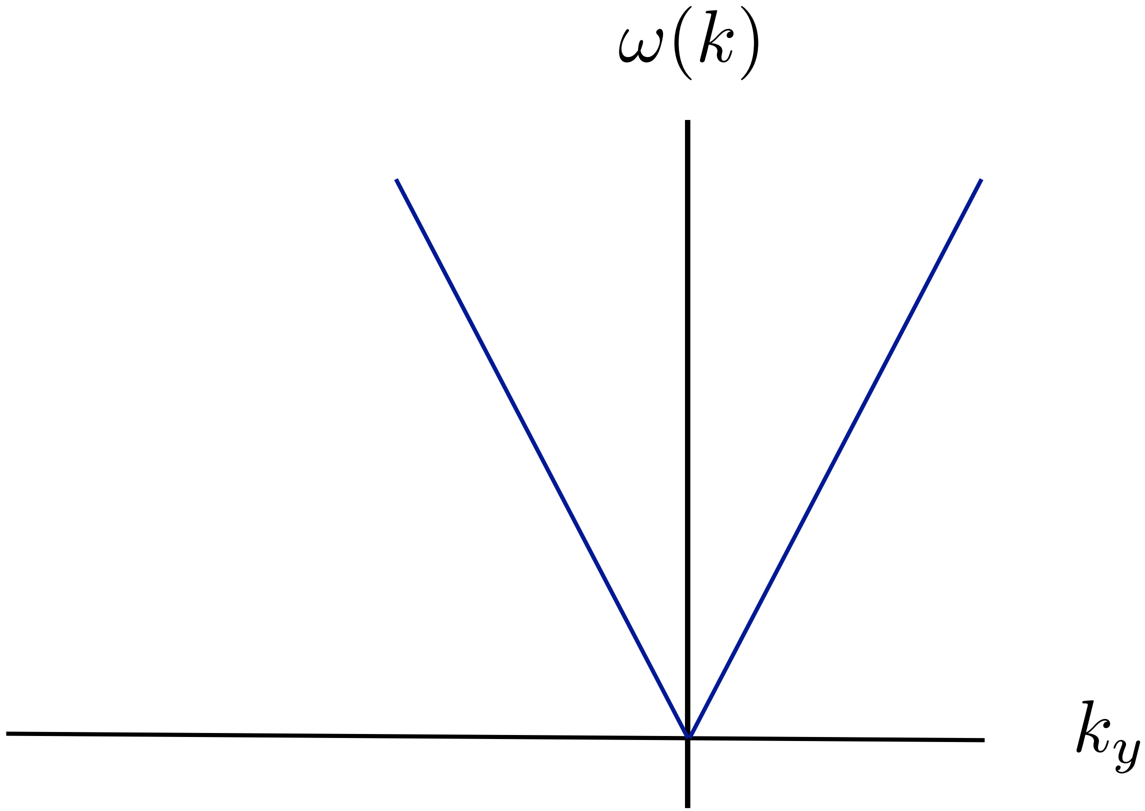
direction of flow



$v_s = 0$
in -y direction

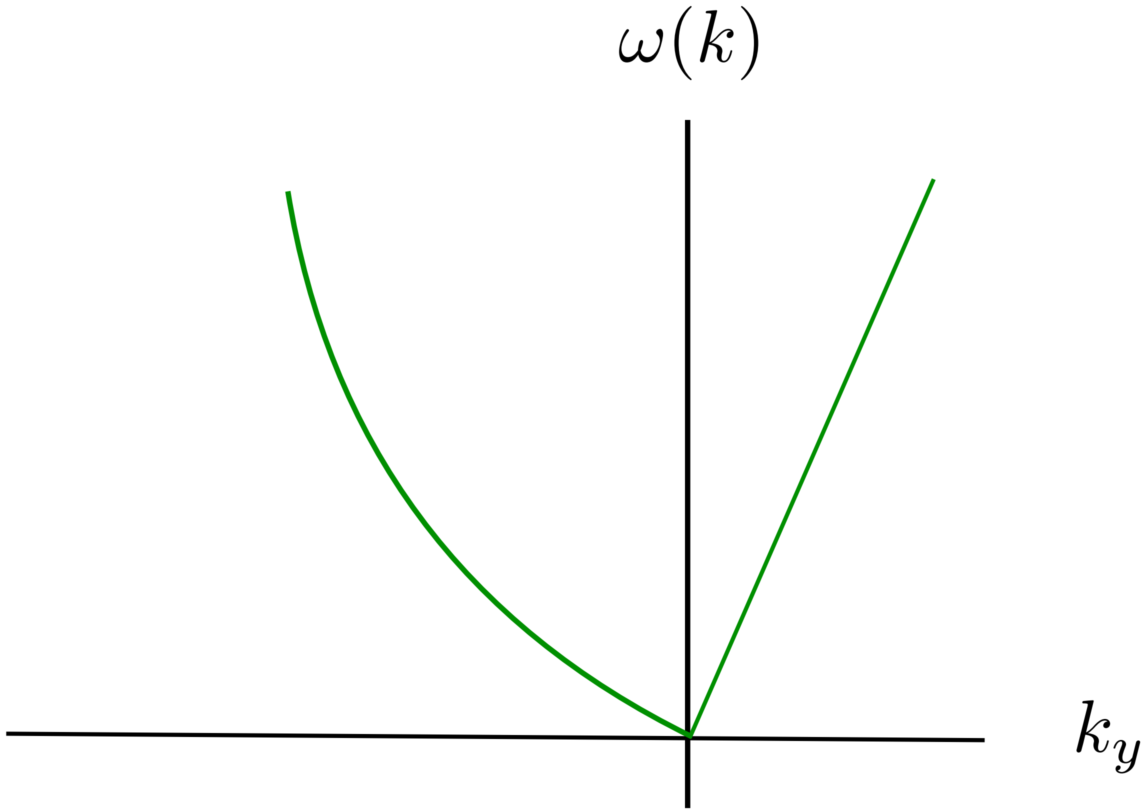
Phonon dispersion

$$V_y = 0$$



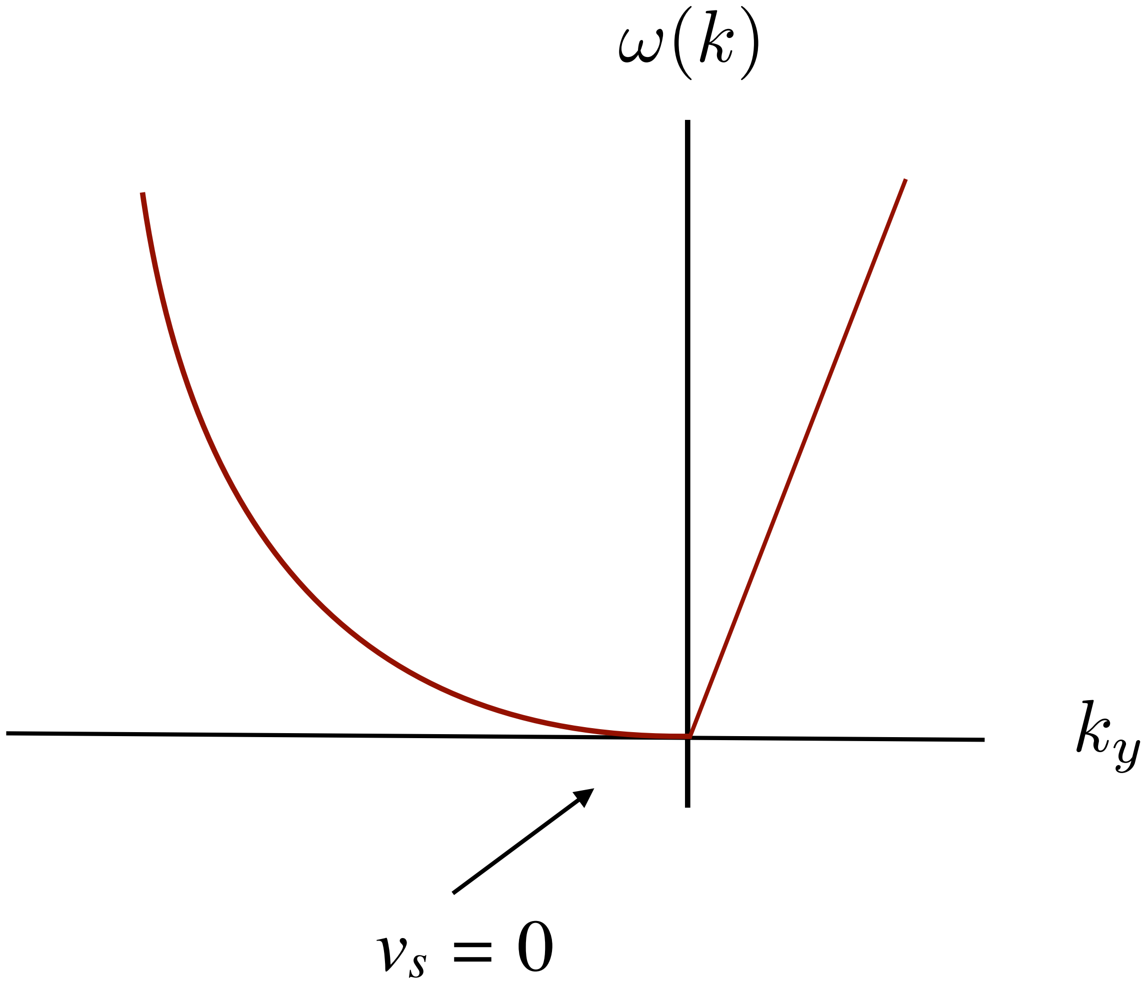
Phonon dispersion

$V_y > 0$



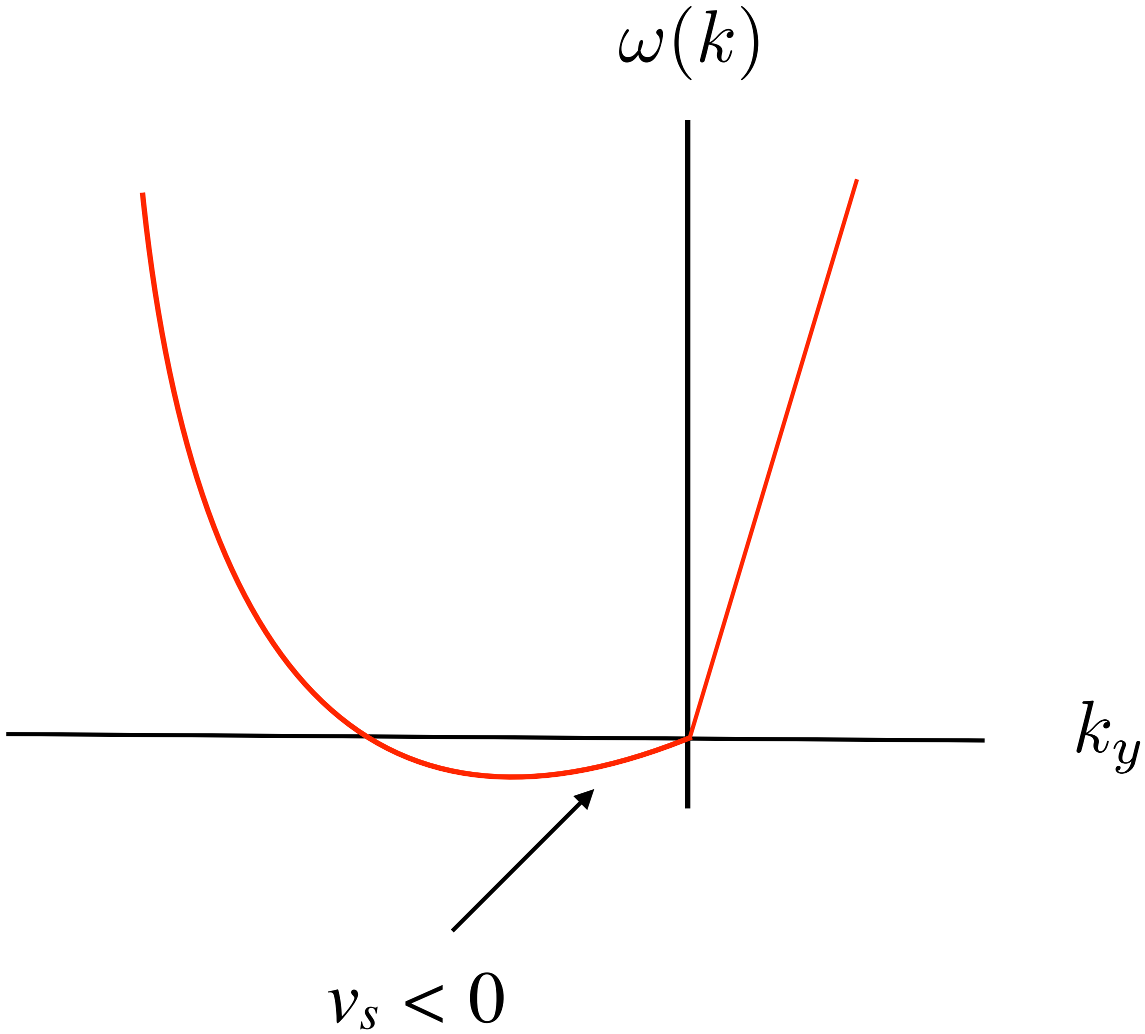
Phonon dispersion

$$V_y = V^{crit}$$

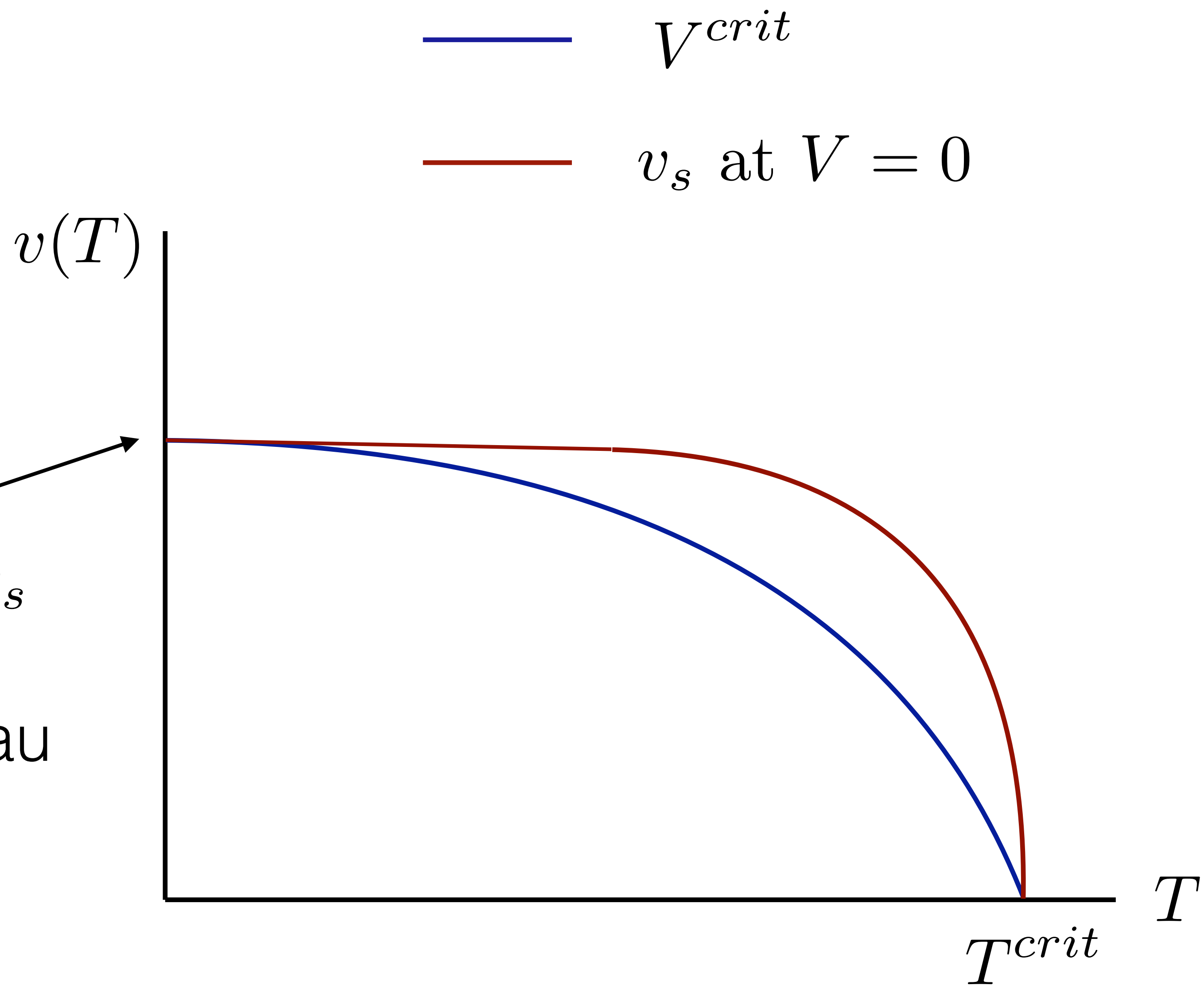


Phonon dispersion

$$V_y > V^{crit}$$



Critical Velocity vs. Temperature

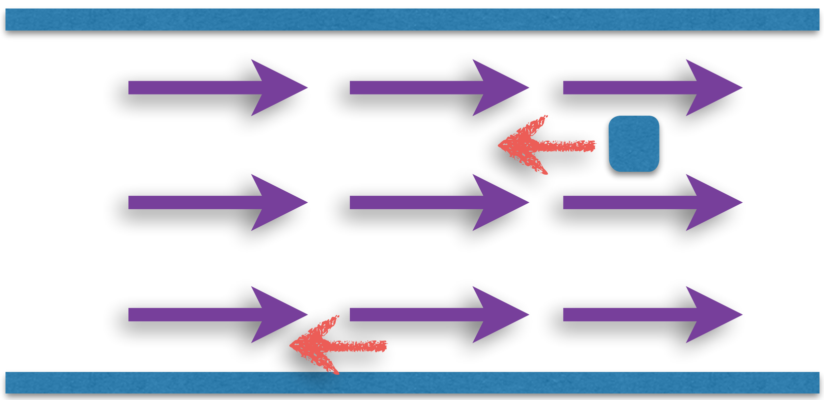


$V^{crit} = v_s$
at $T = 0$
by Landau
criterion

Instability ?

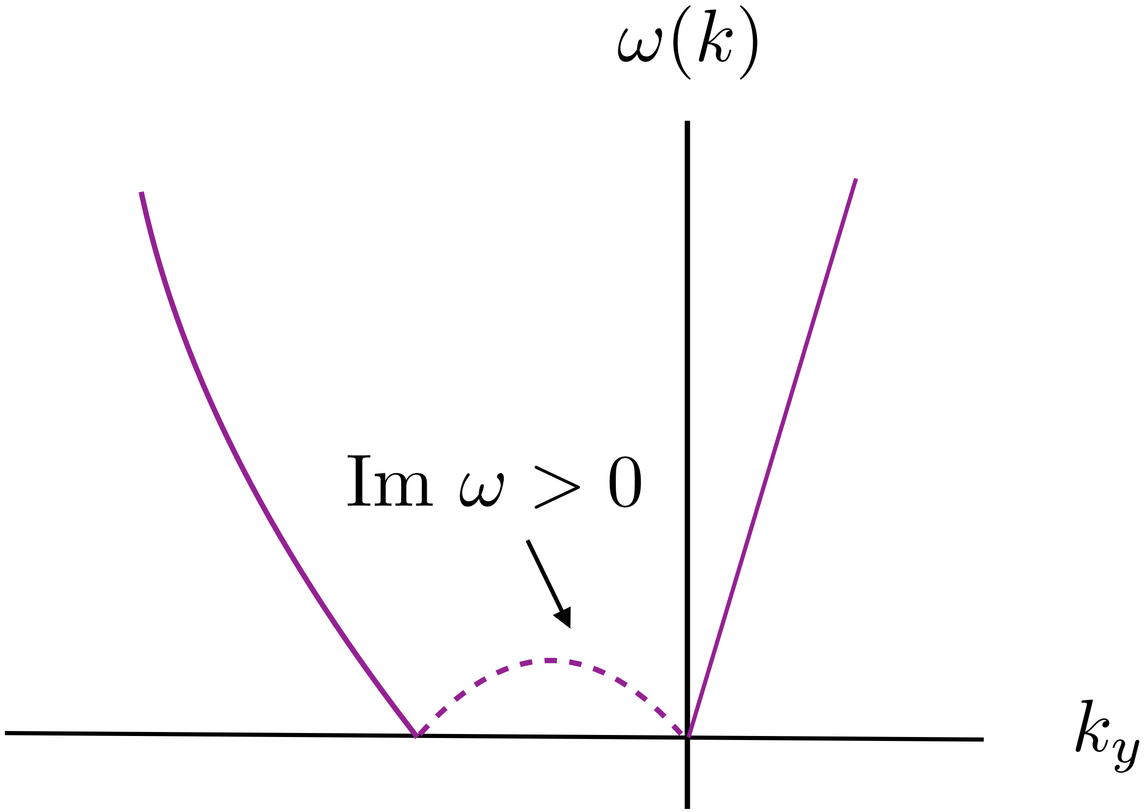
At $T = 0$ and $V_y > V^{crit}$

- Lorentz invar. $\Rightarrow$ Infinite superfluid stable
- Barrier/boundary can excite modes with $\omega < 0$

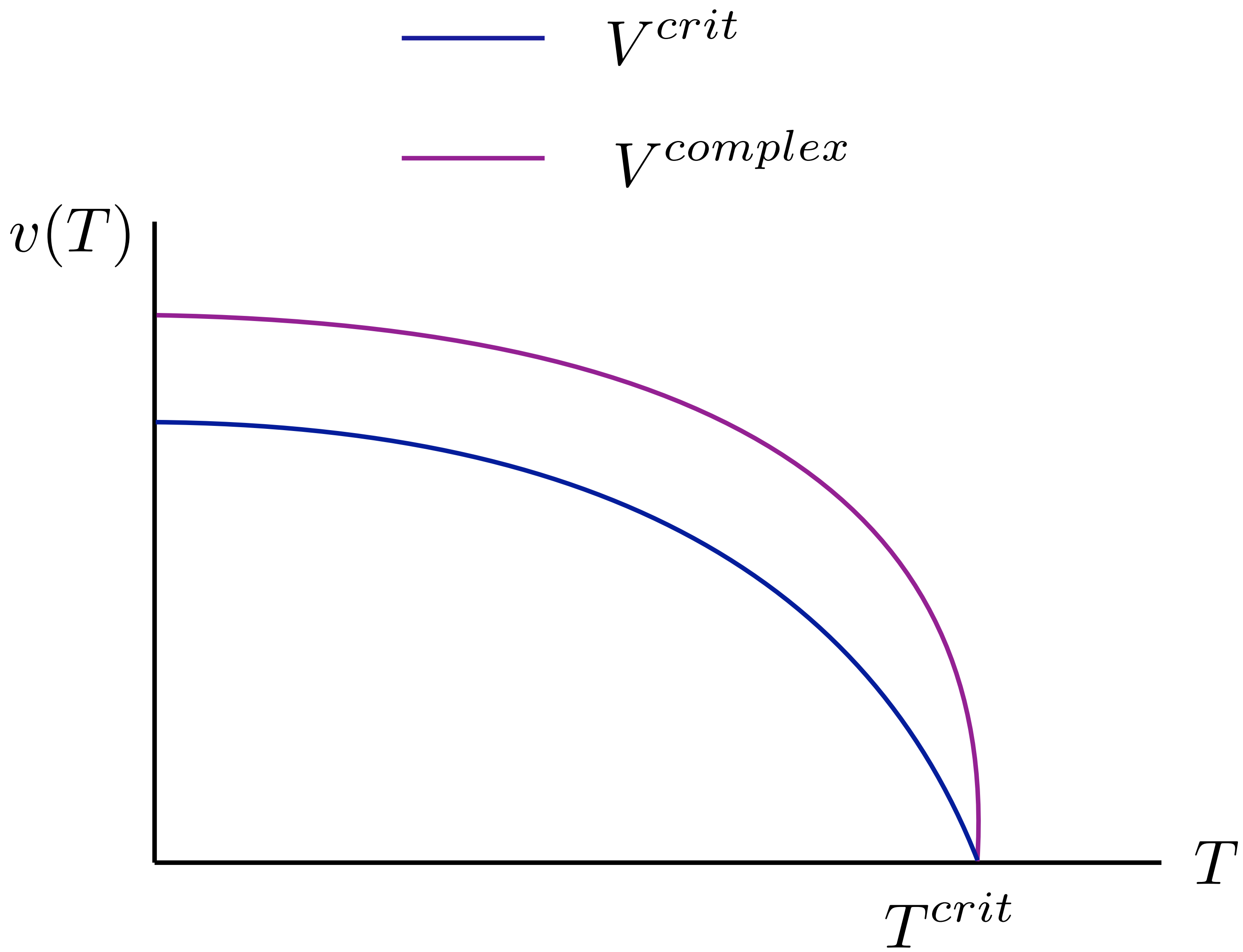


Instability

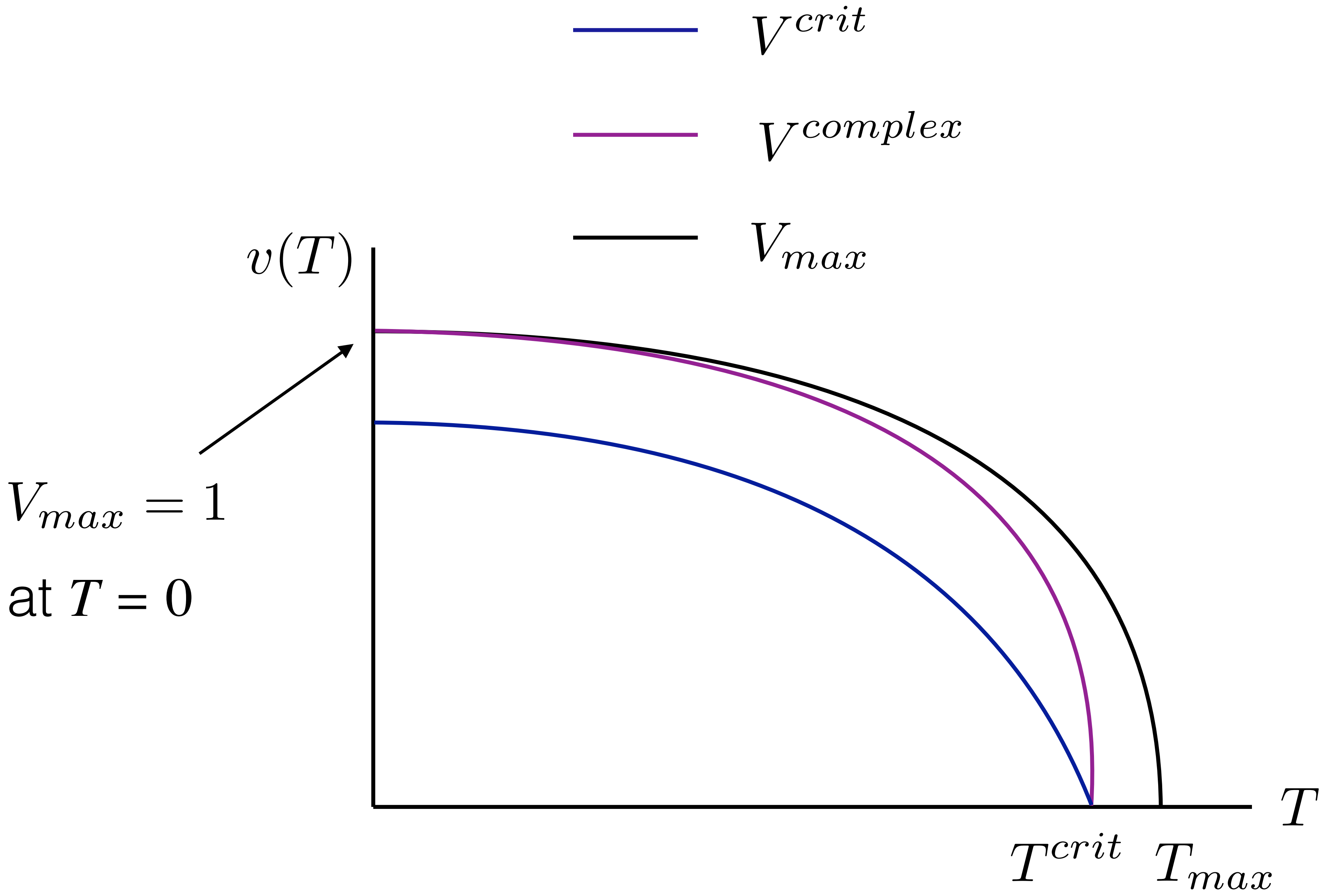
For $V_y > V^{complex} > V^{crit}$



Velocity vs. Temperature



Velocity vs. Temperature



Summary

- Anyonic superfluid: non-standard superfluid
- Related to QH fluid by $SL(2,\mathbb{Z})$
- Holographic model of strongly-coupled anyon superflulid
  - $T \geq 0$
  - $V^{complex} > V^{crit}$
  - ground state at large V?