Lifshitz as a continuous deformation of AdS

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References

Reference

Based on work with Kostas Skenderis and Marika Taylor 1304.7776 and 1306.3344

Related work includes

[Balasubramanian, McGreevy (2008)], [D.Son (2008)] [Kachru, Liu, Mulligan (2008)], [M.Taylor (2008)], [A.Donos, J.P.Gauntlett (2009, 2010)], [S.Ross (2011)], [Baggio, de Boer, Holsheimer (2011)], [Mann, McNees (2011)], [Cassani, Faedo (2011)],[Amado, Faedo (2011)],[Andrade, Ross (2012, 2013)],[Gath, Hartong, Monteiro, Obers (2013)]...

Outline

- Introduction
- Holographic Dictionary
- Lifshitz symmetric field theories
- Thermodynamics
- Conclusions

Introduction

- Gauge/gravity dualities (or holography) became a standard tool for extracting strong coupling physics.
- Various condensed matter systems with scaling symmetry provide a natural playground for holographic techniques.
- Typically condensed matter systems are not relativistic. Often these are anisotropic (e.g. time and space play different role).
- To study such systems gravity solutions with non-relativistic isometries have been constructed.
- Holography may provide new universality classes of non-relativistic systems, which are hard to access by usual perturbative methods.

Anisotropic scaling symmetry

Phase transitions or critical points in condensed matter systems often exhibit the symmetry under anisotropic rescaling transformation

$$t \to \lambda^{z} t, \qquad x \to \lambda x,$$

where z is called *dynamical exponent*.

$z = 1$ case leads to relativistic invariance.

Anisotropic scaling symmetry

Two interesting symmetry groups realising anisotropic scaling are

- Lifshitz symmetry (contains spacetime translations, space rotations, dilatations)
- Schrödinger symmetry (contains spacetime translations, space rotations, dilatations and boosts)

In this talk we will concentrate on the Lifshitz case.

A spacetime possessing Lifshitz symmetry as the isometry is called Lifshitz space(time):

$$ds^2 = dr^2 - e^{2zr} dt^2 + e^{2r} dx_i dx^i.$$

It is symmetric under

$$t \to \lambda^z t, \qquad x \to \lambda x, \qquad r \to r - \log \lambda.$$

Approach

The predominant approach in applications of holography is to proceed phenomenologically, i.e. the observables are computed holographically and the results are compared to an experiment.

In this work we address the question:

What is the field theory dual to Lifshitz space?

Approach

Some problems with understanding the dual theory are:

- These geometries have been mostly constructed using bottom up approach.
- The geometry is not asymptotically Anti-de Sitter, and therefore the usual AdS/CFT dictionary does not directly apply.

Our Approach

We tune the parameters of gravity Lifshitz solution in such a way that the dual field theory can be interpreted as a deformation of underlying conformal field theory (CFT).

Approach: The Main Idea

We tune the dynamical exponent z to be close to 1

$$z = 1 + \epsilon^2, \quad \text{with} \quad \epsilon \ll 1.$$

This allows us to view the Lifshitz geometry as a small perturbation of Anti-de Sitter.

We can use AdS/CFT dictionary to interpret and analyse Lifshitz solution in this case.

We expect that the intuition gained from this analysis applies more generally.

Theoretical models and experiments featuring $z \approx 1$

A sample of theoretical models with $z \approx 1$ include those describing:

- Quantum spin systems with quenched disorder
- Quantum Hall systems
- Spin liquids in the presence of non-magnetic disorder
- Quantum transitions to and from the superconducting state in high Tc superconductors
- Approach of the IR fixed point in Hořava-Lifshitz gravity.

Experimental evidence for quantum critical behavior with $z \approx 1$:

- The transition from the insulator to superconductor in the underdoped region of certain high Tc superconductors [Zuev et al. PRL(2005)], [Matthey et al. PRL(2007)],...
- The transition from the superconductor to metal in the overdoped region of certain high Tc superconductors [Lemberger et al. PLB (2011)].

The Model

One of the simplest theories exhibiting Lifshitz solutions is the so-called Einstein-Proca model

$$S = \frac{1}{16\pi G_{d+1}} \int d^{d+1}x\sqrt{-G}\left[R + d(d-1) - \frac{1}{4}F^2 - \frac{1}{2}M^2A^2\right].$$

When

$$M^2 = \frac{zd(d-1)^2}{z^2 + z(d-2) + (d-1)^2}$$

this model allows Lifshitz solution

$$\begin{aligned} ds^2 &= dr^2 - e^{2zr/l}dt^2 + e^{2r/l}dx^i dx_i \\ A &= \mathcal{A}e^{zr/l}dt, \qquad \mathcal{A}^2 = \frac{2(z-1)}{z}. \end{aligned}$$

The Model

Let us focus now on the case $z \approx 1 + \epsilon^2$ with $\epsilon \ll 1$. According to standard AdS/CFT dictionary, this theory expanded around AdS critical point describes relativistic CFT, which has a vector primary operator J_i of dimension

$$\Delta = \frac{1}{2}(d + \sqrt{(d-2)^2 + 4M^2})$$
$$\approx d + \frac{d-2}{d}\epsilon^2 + \frac{(-2d^3 + 6d^2 - 7d + 4)}{d^3(d-1)}\epsilon^4 + \cdots .$$

This same theory also has a Lifshitz critical point.

The Model

Recall that the asymptotic expansion of the bulk vector field is given by

$$A_i = e^{(\Delta-d+1)r} A_{(0)i} + \cdots + e^{-(\Delta-1)r} A_{(d)i} + \cdots .$$

We now interpret the Lifshitz solution with $z \approx 1 + \epsilon^2$ as a small perturbation of AdS. The metric is AdS up to order ϵ^2 while the massive vector becomes

$$A_{(0)t} = \sqrt{2}\epsilon(1 + O(\epsilon^2)).$$

Interpretation

Thus to order ϵ the Lifshitz solution has holographic interpretation as a deformation of the CFT by a vector primary operator J_t of dimension d

$$S_{CFT} \to S_{CFT} + \sqrt{2} \int d^d x \epsilon J^t.$$

Holographic Dictionary

We set up the holographic dictionary working perturbatively in ϵ.

- Holographic renormalization for arbitrary z was studied in [Ross (2011)], [Baggio et al. (2011)], [Griffin et al. (2011)].
- In contrast to previous approaches we have in mind deforming (by an irrelevant operator) AdS space into Lifshitz and do not assume particular fall-off behaviour for the bulk fields, but derive bulk solution for arbitrary Dirichlet data.

Holographic Dictionary

We parametrize the metric and the vector field as

$$ds^2 = dr^2 + e^{2r} g_{ij} dx^i dx^j,$$
$$g_{ij}(x, r; \epsilon) = g_{[0]ij}(x, r) + \epsilon^2 g_{[2]ij}(x, r) + \dots$$
$$A_i(x, r; \epsilon) = \epsilon e^r A_{(0)i}(x) + \dots .$$

For simplicity we will show the results for position x-independent sources. In this case $A_r = 0$.

Holographic Dictionary at Order ϵ^0

At the leading order in ϵ the result is well-known [de Haro, Skenderis, Solodukhin (2000)]

$$g_{[0]ij}(x,r) = g_{[0](0)ij} + e^{-dr} g_{[0](d)ij} + \dots .$$

$g_{[0](0)ij}(x)$ is the source. $g_{[0](d)ij}(x)$ is not determined locally by the source and is related to the renormalized stress-energy 1-point function $< T_{ij}(x) >$.

Holographic Dictionary at Order ϵ

At order ϵ the massive vector becomes

$$A_i(r;\epsilon) = \epsilon e^r \Big(A_{(0)i} + e^{-dr}(r\ a_{(d)i} + A_{(d)i}) + \dots \Big),$$

with $a_{(d)i} = g_{[0](d)ij} A^j_{(0)}$.

We introduced new source $A_{(0)i}$ at this order. Coefficient $A_{(d)i}$ is again undetermined by asymptotic analysis and will be related to the 1-point function $< J_i >$ of the vector operator.

Holographic Dictionary at Order ϵ^2

At order ϵ^2 the vector field backreacts on the metric

$$g_{ij} = g_{[0](0)ij} + e^{-dr} g_{[0](d)ij} + \epsilon^2 \Big(r\, h_{[2](0)ij} + e^{-dr} (r\, h_{[2](d)ij} + g_{[2](d)ij}) \Big).$$

The coefficients $h_{[2](0)ij}$ and $h_{[2](d)ij}$ are determined locally in terms of the source $g_{[0](0)ij}$. $h_{[2](0)ij}$ renormalises the background metric

$$g_{[0](0)ij} \to g_{[0](0)ij} + \epsilon^2 r\, h_{[2](0)ij},$$

$$h_{[2](0)ij} = -A_{(0)i} A_{(0)j} + \frac{1}{2(d-1)} A_{(0)k} A^k_{(0)} g_{[0](0)ij}.$$

$g_{[2](d)ij}$ is determined only partially.

Holographic Dictionary: Renormalization

The counterterm

$$S_{\text{ct}} = -\frac{1}{32\pi G_{d+1}} \int d^d x \sqrt{-\gamma} \Big(4(d-1) - \gamma^{ij} A_i A_j \Big)$$

suffices to render the action finite to order ϵ^2.

The renormalized 1-point functions are obtained by differentiating the renormalized action $S_{\text{ren}} = S_{\text{bare}} + S_{\text{ct}}$ with respect to the sources. The vector 1-point function is

$$\langle J^i \rangle = -\frac{1}{\sqrt{-g_{[0](0)}}} \frac{\delta S_{\text{ren}}}{\delta A_{(0)i}} = -\frac{1}{16\pi G_{d+1}} \left(d A^i_{(d)} - g^{ij}_{[0](d)} A_{(0)j} \right).$$

Holographic Dictionary: Renormalization

The 1-point function of the stress-energy tensor:

$$\langle T_{ij}\rangle = \langle T_{ij}\rangle_{[0]} + \epsilon^2 \langle T_{ij}\rangle_{[2]} + \cdots$$

with

$$\langle T_{ij}\rangle_{[0]} = -\frac{2}{\sqrt{-g_{[0](0)}}}\frac{\delta S_{[0]\mathrm{ren}}}{\delta g^{ij}_{[0](0)}} = \frac{d}{16\pi G_{d+1}} g_{[0](d)ij}$$

and

$$\begin{aligned}\langle T_{ij}\rangle_{[2]} = \frac{1}{16\pi G_{d+1}}\Big[& d\, g_{[2](d)ij} - (A_{(0)i}A_{(d)j} + A_{(0)j}A_{(d)i}) - A_{(0)k}A^k_{(d)}\eta_{ij} \\ & + \frac{d-1}{d}(A_{(0)i}g_{[0](d)jk} + A_{(0)j}g_{[0](d)ik})A^k_{(0)} \\ & + \frac{d^2-d+2}{2d(d-1)}A^k_{(0)}g_{[0](d)kl}A^l_{(0)}\eta_{ij} - \frac{d-2}{4(d-1)}A_{(0)k}A^k_{(0)}g_{[0](d)ij}\Big].\end{aligned}$$

Holographic Dictionary: Ward Identities

The holographic stress-energy tensor satisfies

$$\nabla^j \langle T_{ij} \rangle = A_{(0)i} \nabla_j \left\langle J^j \right\rangle - \left\langle J^j \right\rangle F_{(0)ij}.$$

Computing the trace of the second order stress energy tensor gives the complete anomaly through order ϵ^2

$$\langle T^i_i \rangle - \frac{1}{2} A^i_{(0)} \langle T_{ij} \rangle A^j_{(0)} = A_{(0)i} \langle J^i \rangle.$$

Recall that on very general grounds

$$\langle T^i_i \rangle = \sum_k \beta_k \langle O_k \rangle.$$

The terms quadratic in $A_{(0)i}$ can be thought of as a beta function contribution to the trace Ward identity.

Recovering Lifshitz Invariance

Let us now fix the source terms to be those corresponding to the Lifshitz solution with $z = 1 + \epsilon^2$:

$$A_{(0)t} = \sqrt{2}, \qquad g_{[0](0)ij} = \eta_{ij}.$$

The trace Ward identity becomes

$$z\langle T^t_t \rangle + \langle T^a_a \rangle = 0,$$

which is precisely the condition for Lifshitz invariance!

Lifshitz Invariant Field Theories

We can construct a current

$$l_i = T_{ij}\,\xi^j,$$

where $\delta x^i = \xi^i$ is a Lifshitz rescaling

$$\xi^t = z\,x^0, \qquad \xi^i = x^i.$$

Its divergence is

$$\partial^i l_i = zT^t_t + T^a_a.$$

Lifshitz Invariance

In Lifsthitz invariant theory the conserved stress-energy tensor satisfies Lifshitz trace identity

$$z\langle T^t_t\rangle + \langle T^a_a\rangle = 0.$$

Lifshitz Invariant Field Theories

Recall that on very general grounds

$$\langle T_i^i \rangle = \sum_k \beta_k \langle O_k \rangle.$$

Lifshitz Invariance

- The deformed theory is still scale invariant!
- The deformation is irrelevant from relativistic point of view but it is marginal with respect to non-relativistic Lifshitz symmetry!
- The beta function does not induce an RG flow, but changes the nature of the fixed point.
- The deformed theory is invariant with respect to anisotropic rescaling.

Lifshitz Invariant Field Theories

The Lifshitz theory we obtained here is different from those discussed in the literature. In particular the action

$$S = \int dtd^d x(\dot{\phi}^2 + \phi(-\partial^2)^z \phi)$$

for $z = 1 + \epsilon^2$ is not of the form we found.

Question

Is the deformation picture specific for holographic construction or does it provide a whole new universality class of Lifshitz theories?

Lifshitz Invariant Field Theories

Answer

Any CFT deformed by any dimension d vector primary operator generically flows to a Lifshitz invariant fixed point!

Lifshitz Invariant Field Theories

The theories we consider have the form

$$S = S_{\mathrm{CFT}} + \sqrt{2}\epsilon \int d^d x J^t.$$

Some Properties

- Translational invariance is preserved $\rightarrow$ stress-energy tensor is conserved.
- Lorentz invariance is broken $\rightarrow$ stress-energy tensor is not symmetric.
- Classically stress-energy tensor is traceless $\rightarrow$ the classical theory is $z = 1$ non-relativistic CFT.
- Quantum theory is $z = 1 + \epsilon^2$ Lifshitz theory.

Quantum Theory

To analyse the quantum theory we use conformal perturbation theory

$$
\begin{aligned}
Z[\epsilon] &= Z_{CFT} - \epsilon \int d^d x A_{(0)i} \langle J^i(x) \rangle_{CFT} \\
&+ \frac{\epsilon^2}{2} \int_{|x-y|>\Lambda} d^d x d^d y A_{(0)i}(x) A_{(0)j}(y) \langle J^i(x) J^j(y) \rangle_{CFT} + \ldots
\end{aligned}
$$

The first non-trivial effect is at order ϵ^2. To compute it we use the OPE of $J^i(x) J^j(y)$.

Quantum Theory

The general form of the OPE is

$$J_i(x)J_j(0) \sim \sum C_{ij}^k \frac{\mathcal{O}_k}{x^{2d-\Delta_k}},$$

The OPE contains the following universal terms

$$J_i(x)J_j(0) \sim C_J \frac{I_{ij}}{x^{2d}} + \cdots + \mathcal{A}_{ij}{}^{kl} \frac{T_{kl}}{x^d} + \ldots,$$

where

$$I_{ij} = \delta_{ij} - 2\frac{x_i x_j}{x^2}.$$

The OPE coefficient $\mathcal{A}_{ij}{}^{kl}$ is completely fixed by conformal invariance in $d = 2$ while there is a 2-parameter family of coefficients in $d > 2$.

Quantum Theory

From the OPE we can immediately derive the leading divergence in the partition function

$$\frac{C_J \epsilon^2}{2} \int d^d x d^d y (A^\mu(x) + \ldots) A^\nu(x) \frac{I_{\mu\nu}(y-x)}{(y-x)^{2d}} + \ldots$$
$$\sim \epsilon^2 \Lambda^d \int d^d x A_\mu(x) A^\mu(x) + \ldots .$$

If we identify field theory UV cutoff Λ with the holographic regulator e^{r_0} we recognise the volume divergence from our holographic computation.

Quantum Theory

The term in the OPE containing stress-energy tensor leads to a logarithmic divergence

$$\epsilon^2 \log \Lambda \int d^d x \beta_{ij} T^{ij}(x)$$

and thus renormalizes the background metric

$$g_{ij}(x, \Lambda) = \eta_{ij} + \epsilon^2 \log \Lambda \beta_{ij}.$$

Recall, from holographic computation

$$g_{[0](0)ij} \to g_{[0](0)ij} + \epsilon^2 r_0\, h_{[2](0)ij}.$$

The beta-function contributions in field-theoretic and holographic computation agree!

Recall that on very general grounds

$$\langle T_i^i \rangle = \sum_k \beta_k \langle O_k \rangle .$$

The dilatation Ward identity now becomes

$$\langle T_i^i \rangle = \beta_{ij} \langle T^{ij} \rangle \qquad => \qquad z\langle T_t^t \rangle + \langle T_a^a \rangle = 0.$$

This establishes the Lifshitz invariance of this particular class of theories.

Moreover, in $d = 2$ conformal perturbation theory precisely reproduces the trace anomaly found holographically.

Lifshitz Invariant Theories: Some simple examples

- Consider a CFT of a free boson X and two free fermions ψ and $\bar{\psi}$ in $d = 2$ dimensions. There is dimension 2 vector

$$J_\mu = i\partial_\mu X \psi \bar{\psi},$$

which deforms free CFT into a Lifshitz theory (cf. [Balasubramanian, Berkooz, Ross, Simón (2013)]).

- Another simple example in $d = 3$ dimensions is given by a free theory of two scalars with stress-energy tensor

$$T_{\mu\nu} = \partial_\mu \phi_1 \partial_\nu \phi_1 - \frac{1}{8}\Big(\partial_\mu \partial_\nu + \delta_{\mu\nu}\partial^2\Big)\phi_1^2 + (1 \leftrightarrow 2)$$

deformed by dimension 3 vector

$$J_\mu = (\phi_1^2 - \phi_2^2)(\phi_2 \partial_\mu \phi_1 - \phi_1 \partial_\mu \phi_2).$$

Correlation functions in $d = 2$

Similarly as conformal symmetry in $d = 2$ dimensions, Lifshitz symmetry in $d = 2$ is powerful enough and fixes the correlation functions of conserved stress-energy tensor

$$J_{\mu\nu} = T^{CFT}_{\mu\nu} - A_\mu J_\nu.$$

On general grounds

$$\langle \mathcal{O}_{\Delta_{L1}}(x) \mathcal{O}_{\Delta_{L2}}(0) \rangle = \frac{f(\chi)}{x^{\Delta_{L1}+\Delta_{L2}}},$$

where the function f depends on the dimensionless ratio $\chi = t/x^z$. In particular the scaling dimensions of the stress-energy tensor

$$\Delta_L(J_{tt}) = \Delta_L(J_{xx}) = 1 + z, \quad \Delta_L(J_{tx}) = 2z, \quad \Delta_L(J_{xt}) = 2.$$

Correlation functions in $d = 2$: The Strategy

- Diffeomorphism Ward identity relates n-point correlation functions of stress energy tensor in the presence of sources, e.g. in CFT

$$\bar{\partial} \langle T_{ww}(w) T_{ww}(0) \rangle = -\partial \langle T_{w\bar{w}}(w) T_{ww}(0) \rangle \,.$$

- Trace Ward identity ($\langle T_{w\bar{w}} \rangle \sim R[h]$) allows one to compute one of the local 2-point functions.
- Integratiing diffeomorphism Ward identity one obtains all (local and non-local) 2-point correlation functions of stress-energy tensor with itself!

Correlation functions in $d = 2$

Ward identities alone fix the 2-point functions $\langle J_{\mu\nu}(y) J_{\rho\sigma}(x) \rangle$, e.g.

$$\langle J_{tt}(t,x) J_{tt}(0) \rangle = \frac{c}{(2\pi)^2} x^{-2(1+z)} \Big[(1-\epsilon^2) \frac{\chi^4 - 6\chi^2 + 1}{(\chi^2+1)^4} - \frac{1}{12}\epsilon^2 \frac{\chi^2-1}{(\chi^2+1)^3} + \epsilon^2 \frac{(\chi^2-1)(\chi^4-14\chi^2+1)}{(\chi^2+1)^5} (\log(1+\chi^2) - \frac{9}{4}) + \dots \Big].$$

Higher n-point functions can also be derived using Ward identities.

Summary of field theory results

- CFTs deformed by dimension d vector operator generically are Lifshitz invariant.
- We found a whole new universality class of Lifshitz invariant theories.
- Results of conformal perturbation theory match those from holography.
- Lifshitz invariance fixes correlation functions of stress-energy tensor.

Lifshitz black brane

- In condensed matter theory one is primely interested in the physics at finite temperature.
- Holographically one needs to construct a black hole/brane with Lifshitz asymptotics.
- There are no analytic Lifshitz black brane solutions in Einstein-Proca model for generic z. There are Lifshitz black holes in Einstein-Maxwell-Dilaton model [M.Taylor (2008)], but these have running dilaton.
- Here we present an *analytic* solution with $z \approx 1$.

Lifshitz black brane

We construct the black brane solution perturbatively in ϵ.
At order ϵ^0 it is given by neutral Schwarzschild-AdS black brane

$$ds^2 = \frac{dy^2}{y^2(1-\frac{y_h^d}{y^d})} - y^2(1-\frac{y_h^d}{y^d})dt^2 + y^2 dx\cdot dx.$$

At order ϵ^1 the vector gets non-trivial profile

$$A_t(y) = \epsilon A_{(0)t}\frac{\pi}{\sin\frac{\pi}{d}}\frac{d-1}{d^2}y(1-\frac{y_h^d}{y^d})\ {}_2F_1(\frac{1}{d},\frac{d-1}{d};2;1-\frac{y_h^d}{y^d}).$$

Lifshitz black brane

At order ϵ^2 the vector backreacts on the geometry

$$ds^2 = \frac{dy^2}{c(y)} - dt^2 c(y) b(y)^2 + y^2 dx \cdot dx,$$

with

$$c(y) = y^2(1 - \frac{y_h^d}{y^d}) + \epsilon^2 A_{(0)t}^2 \Delta c(y); \qquad b(y) = 1 + \epsilon^2 A_{(0)}^2 \Delta b(y).$$

$\Delta c(y)$ and $\Delta b(y)$ satisfy decoupled first order differential equations. For example, $\Delta c(y)$ can be expressed analytically as a product of two hypergeometric functions.

The renormalized 1-point functions are given by

$$\left\langle J^t \right\rangle = y_h^d \frac{\epsilon A_{(0)t}}{16\pi G_{d+1}}\Big(- \frac{2d-1}{2} + \frac{d-1}{d} k(d) + (d-1)\log y_h \Big).$$

$$\langle T_{tt}\rangle_{[2]} = y_h^d \frac{A_{(0)t}A_{(0)t}}{16\pi G_{d+1}}\Big(\frac{2d-1}{4} - \frac{d-1}{2}\Big(\log y_h + \frac{k(d)}{d} + \frac{2c_h}{y_h^2}\Big)\Big),$$

$$\langle T_{ij}\rangle_{[2]} = y_h^d \frac{A_{(0)t}A_{(0)t}}{16\pi G_{d+1}}\Big(- \frac{1}{4(d-1)} + \frac{\log y_h}{2} + \frac{k(d)}{2d} - \frac{c_h}{y_h^2}\Big)\delta_{ij},$$

where $k(d)$ is known function of d.

These correlation functions satisfy correct Ward identities.

Lifshitz black brane: Thermodynamics

The mass M is defined using the conserved current

$$Q_j = (\langle T_{ij} \rangle - A_i \langle J_j \rangle)\xi^i,$$

as

$$M = \int_{t=const} \sqrt{g}(\langle T_{tt} \rangle - A_t \langle J_t \rangle).$$

Entropy S is defined as the area of the horizon and we deduce temperature T by requiring that there is no conical singularity in analytically continued geometry.

Lifshitz black brane: Thermodynamics

Thermodynamic quantities satisfy (also [Bertoldi et al. (2009)],...)

Thermodynamic relation

$$M = \frac{d-1}{d+z-1} TS.$$

From explicit expressions one can also read off

Scaling relation

$$T \sim S^{\frac{z}{d-1}}.$$

These together imply the 1st law of thermodynamics. Evaluating the renormalized on-shell action we confirmed that the free energy F is given by

First law of thermodynamics

$$F = M - TS.$$

Top Down Models

- First class [Donos, Gauntlett (2010)]: $10d$ and $11d$ embeddings of Lifshitz geometry with $z = 2$.
- Second class [Gregory, Parameswaran, Tasinato, Zavala (2010)]: Lifshitz solutions for generic z in Romans gauged supergravity. The $z \sim 1$ solutions in this models belong to the same universality class, i.e. they can be viewed as deformations of relativistic fixed points by a dimension d vector operator J_t.
- Unfortunately, the second class models suffer from Breitenlohner-Freedman type instabilities...

Conclusions

Main Points

- We have interpreted the field theory dual to Lifshitz space as the deformation of CFT by a time-component of dimension d vector primary operator.
- We have performed holographic renormalization and developed holographic dictionary for this class of theories.
- Pure field theory arguments show that such deformations provide a whole new universality class of Lifshitz invariant theories.
- We constructed an analytic Lifshitz black brane and developed thermodynamics for it.

Additional slides follow

Holographic Dictionary: Ward Identities in $d = 2$

In $d = 2$ boundary dimensions for position-dependent sources there is a trace anomaly

$$\langle T^i_i \rangle_{[2]} = A_{(0)i} \langle J^i \rangle + \mathcal{A},$$

with

$$\mathcal{A} \propto \frac{12\pi}{c} A^i_{(0)} \langle T_{ij} \rangle_{[0]} A^j_{(0)} - \frac{1}{4} F_{(0)ij} F^{ij}_{(0)} + \frac{1}{2} (\nabla_i A^i_{(0)})^2 - \frac{R}{4} A^i_{(0)} A_{(0)i}.$$

Holographic Dictionary: Ward Identities in $d = 2$

$$\mathcal{A} \propto \frac{12\pi}{c} A^i_{(0)} \langle T_{ij} \rangle_{[0]} A^j_{(0)} - \frac{1}{4} F_{(0)ij} F^{ij}_{(0)} + \frac{1}{2} (\nabla_i A^i_{(0)})^2 - \frac{R}{4} A^i_{(0)} A_{(0)i}.$$

Remarkably, a related Weyl invariant action for $d > 2$ has appeared in [Deser, Nepomechie (1984)]

$$\mathcal{L} = \frac{d-4}{2} S_{ij} A^i A^j - \frac{1}{4} F_{ij} F^{ij} - \frac{d-4}{2d} (\nabla_i A^i)^2 - \frac{d-4}{8(d-1)} R A_i A^i,$$

where the so-called Schouten tensor is

$$S_{ij} = \frac{1}{d-2} \Big(R_{ij} - \frac{R}{2(d-1)} g_{ij} \Big).$$

Interestingly, $g_{[0](2)ij} \sim \langle T_{ij} \rangle_{[0]}$ has the same Weyl transformation property as S_{ij}! $\mathcal{A}$ is the 2-dimensional generalisation of [Deser, Nepomechie (1984)] action!

Correlation functions in $d = 2$: CFT example

In CFT the diffeomorphism and trace Ward identities are

$$\begin{aligned} \bar{\partial}\langle T_{ww}\rangle + \partial\langle T_{w\bar{w}}\rangle &= 0 \rightarrow, \\ \rightarrow \bar{\partial}\langle T_{ww}(w)\, T_{ww}(0)\rangle &= -\partial\langle T_{w\bar{w}}(w)\, T_{ww}(0)\rangle\,, \\ \langle T_{w\bar{w}}\rangle &= \frac{1}{4}\cdot\frac{c}{24\pi}R[h]. \end{aligned} \tag{1}$$

Taking the functional derivative of (1) wrt h^{ww} we find

$$\langle T_{w\bar{w}}(w)\, T_{ww}(0)\rangle = \frac{1}{4}\cdot\frac{c}{24\pi}4\partial^2\delta^2(w,\bar{w}) = \frac{c}{24\pi}\partial^2\frac{1}{2\pi}\bar{\partial}\partial\log|w|^2,$$

and

$$\langle T_{ww}(w)\, T_{ww}(0)\rangle = -\frac{\partial}{\bar{\partial}}\langle T_{w\bar{w}}(w)\, T_{ww}(0)\rangle = \frac{1}{(2\pi)^2}\frac{c/2}{w^4}.$$