Non-Equilibrium 2-point functions in AdS/CFT: Formalism and an example

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Motivation

- The purpose of todays talk is to study far from equilibrium processes in AdS/CFT
- Learn more about the duality in extreme environments

  - Learn more about black holes

  - Learn more about the dictionary between bulk and boundary

  - Test whether the duality gives sensible results
- Tool for strongly coupled dynamics in QFT

  - Search for universality at strong coupling

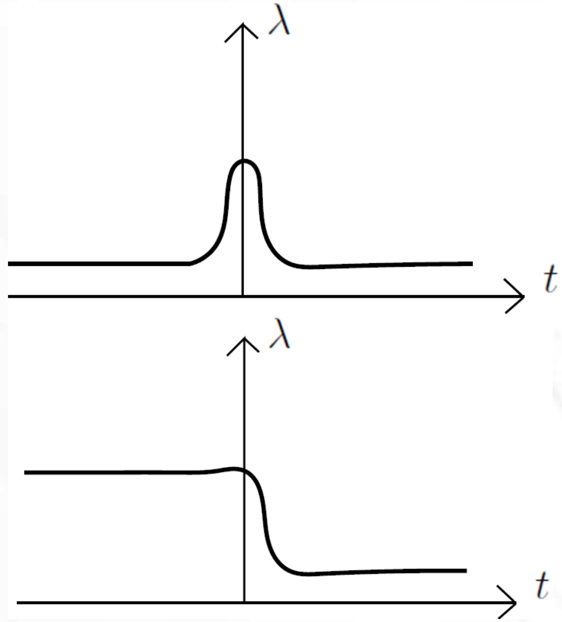
  - Experimental systems to keep in mind: Cold atom systems, condensed matter systems, Heavy ion collisions etc.

Non-Equilibrium example

- Take a QFT and prepare it in the vacuum state
- Excite the system at t=o homogeneously in space → injects a finite energy density into the system
- E.g. a time dependent coupling

$$H = H_0 + \lambda(t) \int d\mathbf{x}\, \mathcal{O}(\mathbf{x}).$$

Sometimes called a "global quench"



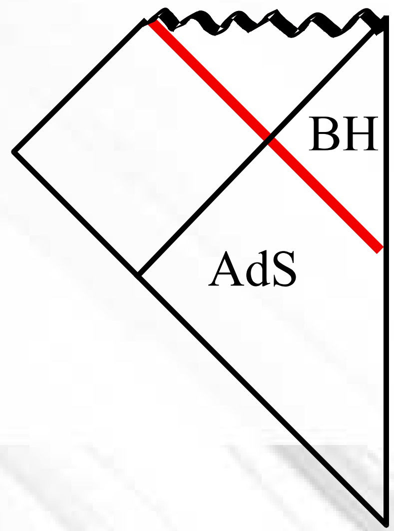
- Want to preserve spatial translational and rotational symmetries for simplicity
- Non-trivial dynamics is mainly in non-local observables

Non-Equilibrium example

- AdS version of the previous setup

$$H = H_0 + \lambda(t) \int d\mathbf{x}\, \mathcal{O}(\mathbf{x}). \qquad S = S_0 - \int d^d x\, J(t) \mathcal{O}(x)$$

- Sources are dual to boundary values of fields
- Start from the vacuum (= AdS) → Suddenly perturb a boundary value of a field → The perturbation starts falling deeper to the bulk and forms a black brane
- A simple analytic model for this process is provided by the Vaidya spacetime, which corresponds to a null shock wave starting from the boundary and forming an AdS-Schwarchild black brane



AdS version of the story

- Dictionary:

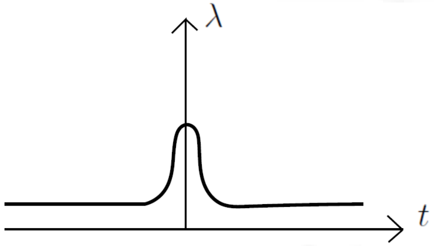
  - Thermalization ⟷ Black hole formation

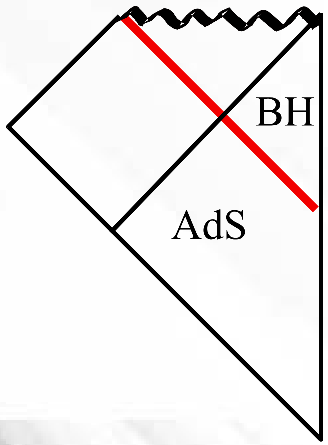
  - Thermalization time scale? ⟷ When does the black hole form?

  (In gravity there is no preferred time coordinate, so there is no one correct answer)

  - More precise question: When/How does a specific observable thermalize? Choose to look at correlation functions of local operators.

$$H = H_0 + \lambda(t) \int d\mathbf{x}\, \mathcal{O}(\mathbf{x}).$$





Outline

1. On correlation functions

- Heuristic picture of correlators
- Formalism in non-equilibrium QFT

2. AdS/CFT dictionary out of equilibrium

- Review of different dictionaries
- Sketch of a proof of equivalence of the two “best” dictionaries

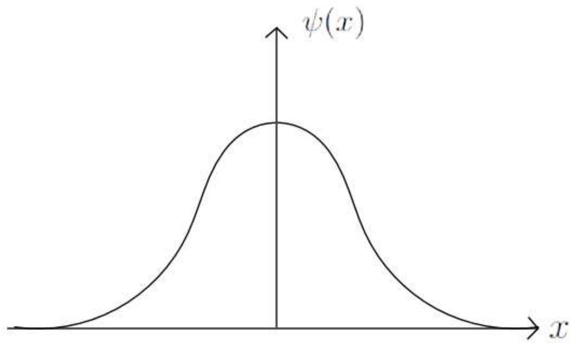
3. Explicit example of 2-point functions in a collapsing spacetime

- Method
- Results

Correlation functions

- Example 1: the Harmonic Oscillator

$$H = \frac{1}{2}p^2 + \frac{1}{2}x^2$$



- Classical ground state x=0
- Quantum ground state

$$\psi(x) = Ce^{-x^2/2}$$

- Prepare the same ground state and measure the position of the particle: On average find $\langle x \rangle = 0$, but due to quantum fluctuations the single measurements give non-zero values and the distribution of measured values has a width

$$\Delta x^2 = \langle x^2 \rangle - \langle x \rangle^2 = \int dx\, x^2 |\psi(x)|^2 = \frac{1}{2}$$

Correlation functions

- Example 2: Free scalar QFT

$$H = \frac{1}{2}\int d\mathbf{x}\left(\Pi^2 + (\nabla\phi)^2 + m^2\phi^2\right)$$

- Quantum ground state is again a Gaussian (as we are dealing with a set of coupled harmonic oscillators)

$$\Psi\left[\phi\right] = \mathcal{N} e^{-\frac{1}{4}\int d\mathbf{x} d\mathbf{y} \phi(\mathbf{x}) K(\mathbf{x},\mathbf{y}) \phi(\mathbf{y})} \qquad K(\mathbf{x},\mathbf{y}) = 2\int \frac{d\mathbf{k}}{(2\pi)^D}\sqrt{k^2+m^2}e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}$$

- By measuring the field at two spatially separated points x and y, and recording the measured values can construct

$$\langle\phi(\mathbf{x})\phi(\mathbf{y})\rangle = \int [d\phi]\,|\Psi\left[\phi\right]|^2\phi(\mathbf{x})\phi(\mathbf{y}) = K^{-1}(\mathbf{x},\mathbf{y}) \approx e^{-m|\mathbf{x}-\mathbf{y}|}$$

- This tells us two things, the measured values at spatially separated points are correlated (due to entanglement) and the wavefunction has a width due to quantum mechanics

Correlation functions

- Example 3: Free scalar QFT with a classical source

$$H = \frac{1}{2}\int d\mathbf{x}\left(\Pi^2 + (\nabla\phi)^2 + m^2\phi^2 + 2J\phi\right)$$

- Treat the current term as an interaction and use the Dirac interaction picture. Then states evolve as

$$|\psi(t)\rangle = T(e^{-i\int_0^t dt' d\mathbf{x}' \, J(x')\phi(x')})|\psi(0)\rangle$$

- What is the average value of the field at some point x, after turning on a small source?

$$\langle\phi(x)\rangle = \langle\psi(0)|T(e^{i\int_0^t dt' d\mathbf{x}' \, J(x')\phi(x')})\phi(x)T(e^{-i\int_0^t dt' d\mathbf{x}' \, J(x')\phi(x')})|\psi(0)\rangle$$

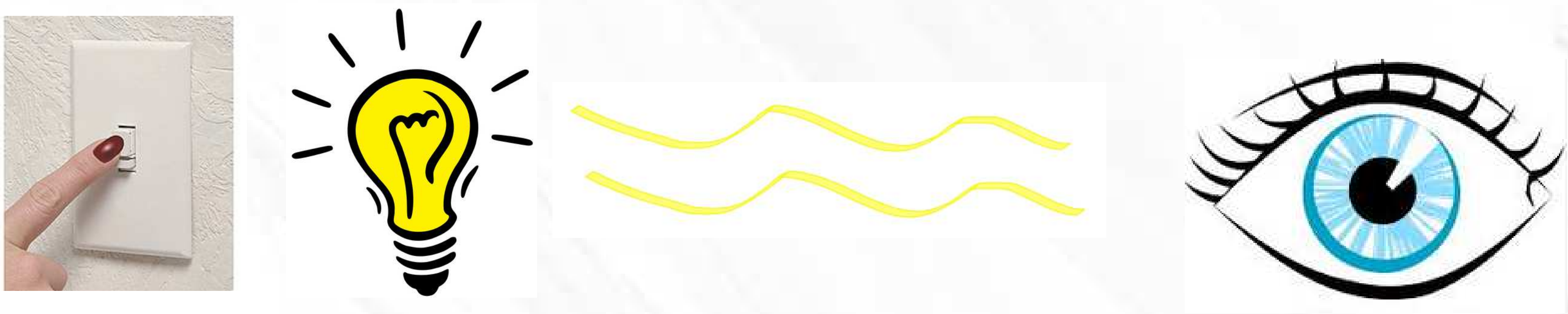
$$\approx \langle\psi(0)|\phi(x)|\psi(0)\rangle - i\int dt' d\mathbf{x}' J(x')\theta(t-t')\langle\psi(0)|\left[\phi(x),\phi(x')\right]|\psi(0)\rangle$$

Correlation functions

- For a localized source $J(x) = J_0 \delta(x)$

$$\delta\langle\phi(x)\rangle = -iJ_0 G_R(x,0)$$

- The analogous question in QED would be: turn on a current in the light bulb at x'=0, what is the amount of light you will see at x?



- The retarded correlator quantifies response

Correlation functions

- Lessons:

  - Different correlation functions answer to different physical questions:

  One point functions = Average results for observables

  Spacelike separated correlator = Quantum fluctuations in the state

  Retarded correlator = Response of the system

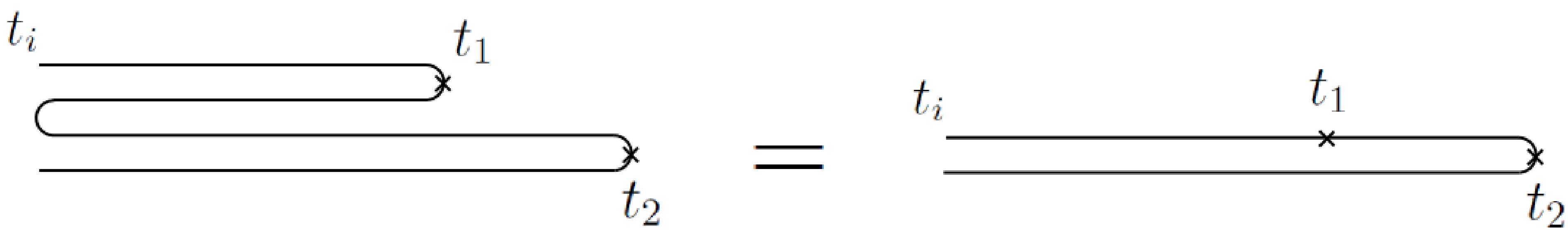
Correlation functions: Formalism

- In the following will work in the Heisenberg picture

$$|\psi(t)\rangle = |\psi(t_i)\rangle \qquad A(t) = U^\dagger(t,t_i)A(t_i)U(t,t_i)$$

- Consider the two point function

$$\begin{aligned}\langle\psi|A(t_2)B(t_1)|\psi\rangle &= \langle\psi|U^\dagger(t_2,t_i)A(t_i)U(t_2,t_i)U^\dagger(t_1,t_i)B(t_i)U(t_1,t_i)|\psi\rangle \\ &= \langle\psi|U^\dagger(t_2,t_i)A(t_i)U(t_2,t_1)B(t_i)U(t_1,t_i)|\psi\rangle\end{aligned}$$



- Important to notice the time-evolution backwards in time (in particle physics often consider amplitudes from initial to final states so there is only forwards time-evolution)

Correlation functions: Formalism

- Apply this to QFT and go to the path integral formalism

$$\langle\psi|A(t_2)B(t_1)|\psi\rangle = \langle\psi|U^\dagger(t_2,t_i)A(t_i)U(t_2,t_1)B(t_i)U(t_1,t_i)|\psi\rangle$$

$$= \int [d\varphi_+, d\varphi_-]\, e^{iS[\varphi_+]-iS[\varphi_-]}\langle\psi|\varphi_-(t_i)\rangle\langle\varphi_+(t_i)|\psi\rangle A(\varphi(t_2))B(\varphi(t_1))$$

- For the moment, the initial state wavefunction

$$\langle\varphi_+|\psi\rangle = \Psi\left[\varphi_+\right]$$

is arbitrary. It is our initial data.

- Can also define a generating functional, from which correlation functions are obtained by differentiation

$$Z\left[J_+, J_-\right] = \int [d\varphi_+, d\varphi_-]\, e^{iS[\varphi_+]-iS[\varphi_-]+i\int(J_+\varphi_+ + J_-\varphi_-)}\langle\psi|\varphi_-(t_i)\rangle\langle\varphi_+(t_i)|\psi\rangle$$

Correlation functions: Formalism

- Example of an initial state wavefunction: the ground state
- Consider the following quantity

$$\langle \varphi | e^{-\tau H} | \varphi_0 \rangle = \sum_n e^{-\tau E_n} \langle \varphi | n \rangle \langle n | \varphi_0 \rangle$$

- In the large tau limit this is dominated by the ground state, and thus

$$\langle \varphi | 0 \rangle \propto \lim_{\tau \to \infty} \langle \varphi | e^{-\tau H} | \varphi_0 \rangle = \lim_{\tau \to \infty} \int [d\varphi_E] \,|_{\varphi_E(\tau=0)=\varphi}\, e^{-\int_{-\tau}^{0} d\tau d\mathbf{x} \mathcal{L}_E(\varphi_E)}$$

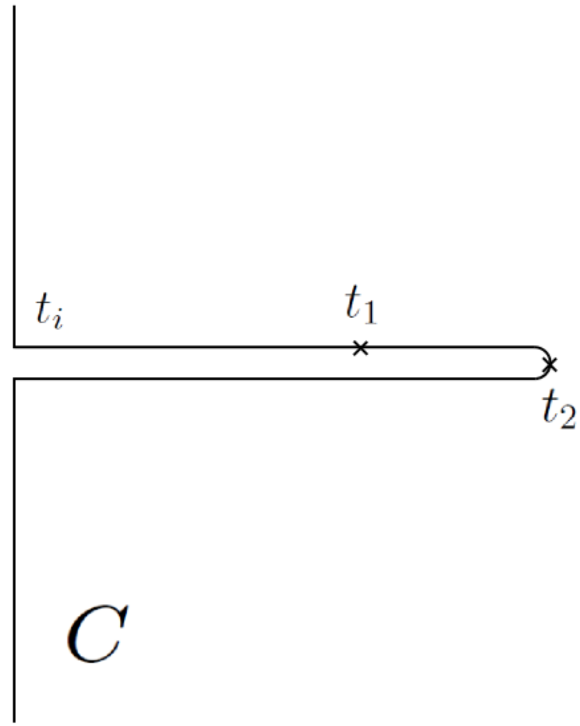
- By adding non-trivial sources to the Euclidean action, one can prepare more general states. In the following specialize to states that can be prepared in this way

Correlation functions: Formalism

- Collecting all the pieces we obtain the generating functional

$$Z[J_+, J_-] = \int [d\varphi_C] \, e^{iS_C[\varphi_C] + i\int_C \phi_C J_C}$$

- Non-equilibrium correlators can be calculated from a generating functional that is obtained by gluing together Euclidean and Lorentzian spacetimes and performing a path integral over the fields in all of the parts.



The AdS/CFT dictionary

- Recall the standard AdS/CFT dictionary

$$Z_{CFT}[J] = \int [d\Phi]\, e^{-S_E[\Phi]}\Big|_{\Phi = z^{\Delta_-} J + \dots}$$

- Where all the bulk fields are denoted as

$$\Phi = (g, A_\mu, \phi, \dots)$$

- At weak coupling (large-N in CFT), can perform a saddle point approximation

$$Z_{CFT}[J] \approx e^{-S_E[\Phi_{cl}]} \qquad \frac{\delta S_E}{\delta \Phi}\Big|_{\Phi = \Phi_{cl}} = 0$$

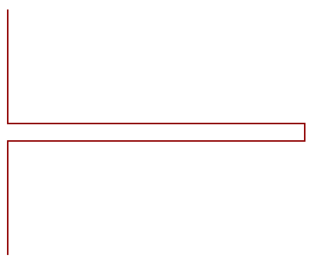
- Leads to a well posed problem as the boundary sources are enough to determine the unique classical solution, since the equations of motion are elliptic

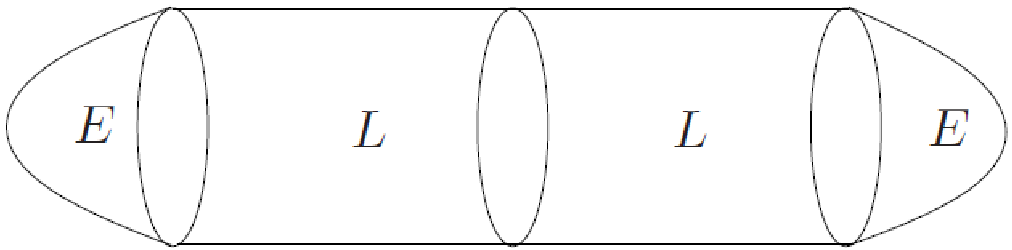
The AdS/CFT dictionary

- For some Lorentzian situations (ground state or thermal state) one can take the Euclidean correlator and analytically continue it to Lorentzian time

- One way to generalize the dictionary to non-equilibrium situations is to build a holographic version of the complex time contour path integral. An obvious candidate dictionary is

$$Z_{CFT}\left[J_+, J_-\right] = \int \left[d\Phi_C\right] e^{iS_C\left[\Phi_C\right]}\Big|_{\Phi_\pm = z^{\Delta_-} J_\pm + \dots}$$





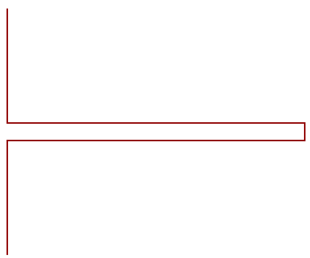
The AdS/CFT dictionary

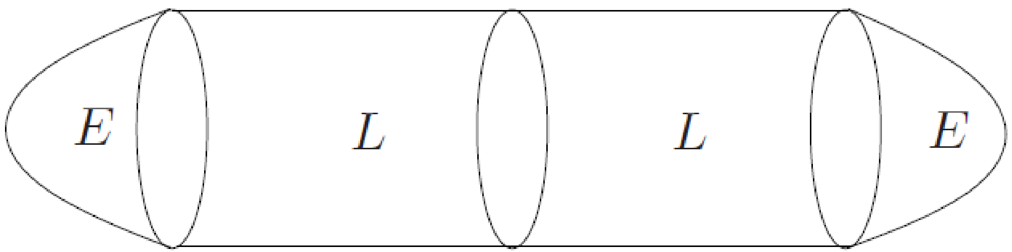
- Again at weak coupling in the bulk, we can perform a saddle point approximation

$$Z_{CFT}\left[J_+, J_-\right] \approx e^{iS[\Phi_+]-iS[\Phi_-]-S_E\left[\Phi_{E,1}\right]-S_E\left[\Phi_{E,2}\right]}$$

$$\frac{\delta}{\delta\Phi}\Big(iS\left[\Phi_+\right]-iS\left[\Phi_-\right]-S_E\left[\Phi_{E,1}\right]-S_E\left[\Phi_{E,2}\right]\Big)=0$$

- Variations on the Lorentzian parts leads to Lorentzian eoms. Variations at the Euclidean parts leads to Euclidean eoms. Variations at the joining surfaces lead to “matching conditions”.





The AdS/CFT dictionary

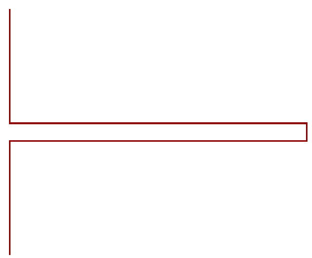
- In the following we will assume that the metric has been appropriately matched and consider a free scalar field in this metric background
- The Euclidean on-shell action of a scalar field can be written in the following form, where K is the inverse of the equal time two point function

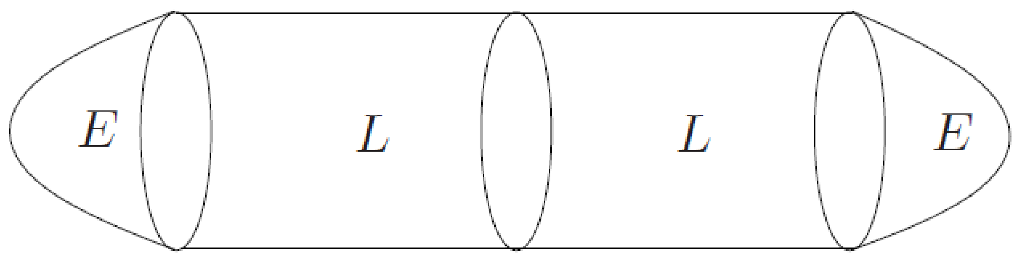
$$S_E = \frac{1}{4}\int_{E\cap L} d\mathbf{x}_1 d\mathbf{x}_2\, \phi(\mathbf{x}_1)K(\mathbf{x}_1,\mathbf{x}_2)\phi(\mathbf{x}_2)$$

- Using this the equations of motion become

$$(\Box - m^2)\phi_\pm = 0 \qquad D^t\phi_+(\boldsymbol{x},t)|_{t=t_i} = -\frac{i}{2}\int d\boldsymbol{x}_1\phi_+(\boldsymbol{x}_1,t_i)K(\boldsymbol{x}_1,\boldsymbol{x})$$

$$D^t\phi_+(\boldsymbol{x},t)|_{t=t_m} = D^t\phi_-(\boldsymbol{x},t)|_{t=t_m} \qquad D^t\phi_-(\boldsymbol{x},t)|_{t=t_i} = \frac{i}{2}\int d\boldsymbol{x}_1\phi_-(\boldsymbol{x}_1,t_i)K(\boldsymbol{x}_1,\boldsymbol{x})$$





The AdS/CFT dictionary

- There is also another independent version of the AdS/CFT dictionary where one identifies bulk and boundary operators

$$\hat{\mathcal{O}}(x) = \lim_{z \to 0} z^{-\Delta} \hat{\Phi}(x, z)$$

- In addition, to calculate correlation functions, one has to make a map between bulk and boundary theory state
- For the states that can be prepared with a Euclidean path integral, this map is the same as before, the bulk wavefunction of the quantum field is

$$\Psi\left[\Phi\right] = \mathcal{N} e^{-S_E[\Phi_E]}\Big|_{\Phi_E(\tau=0)=\Phi}$$

- This is the “extrapolate” dictionary, and is simpler to use in practice

The AdS/CFT dictionary

- We have two versions of the AdS/CFT dictionary, that are supposed to make sense in non-equilibrium situations in a class of initial states
- There are three options:

  - Both of them are wrong

  - One of them is correct and the other one wrong

  - Both of them are correct and lead to the same result
- We will argue that in the case of a free scalar, they lead to the same results. We take this as evidence for the third option.

The AdS/CFT dictionary

- We will prove the equivalence by constructing a solution to the equations of motion following from the gluing approach
- The ansatz for the solution is motivated by the "extrapolate" dictionary
- First we need to slightly reformulate the problem

$$(\Box - m^2)\phi_\pm = 0 \qquad D^t\phi_+(\boldsymbol{x},t)|_{t=t_i} = -\frac{i}{2}\int d\boldsymbol{x}_1\phi_+(\boldsymbol{x}_1,t_i)K(\boldsymbol{x}_1,\boldsymbol{x})$$

$$D^t\phi_+(\boldsymbol{x},t)|_{t=t_m} = D^t\phi_-(\boldsymbol{x},t)|_{t=t_m} \qquad D^t\phi_-(\boldsymbol{x},t)|_{t=t_i} = \frac{i}{2}\int d\boldsymbol{x}_1\phi_-(\boldsymbol{x}_1,t_i)K(\boldsymbol{x}_1,\boldsymbol{x})$$

$$\phi_\pm(x_B,z) = z^{\Delta_-}J_\pm(x_B) + \dots$$

- A standard approach to solving equations like this is to define bulk to boundary propagators (in the following take $J_- = 0$)

$$\phi_+(x) = \int d\boldsymbol{x}_B K_{++}(x,x_B)J_+(x_B) \qquad \phi_-(x) = \int d\boldsymbol{x}_B K_{-+}(x,x_B)J_+(x_B)$$

The AdS/CFT dictionary

- The bulk to boundary propagators have to satisfy all the same equations of motion as the scalar field itself, except that the boundary condition near the AdS boundary is different

$$K_{\alpha\beta}(x_1, z_1; x_2) = z_1^{\Delta_-} \delta_{\alpha\beta} \delta^{d-1}(x_1 - x_2) + z_1^{\Delta} K^{(1)}_{\alpha\beta} + \dots$$

- The gluing dictionary leads to the correlators

$$\langle T\mathcal{O}(x_1)\mathcal{O}(x_2)\rangle = \lim_{z_1 \to 0} z_1^{-\Delta} K_{++}(x_1, z_1; x_2)$$

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle = \lim_{z_1 \to 0} z_1^{-\Delta} K_{-+}(x_1, z_1; x_2)$$

- On the other hand the corresponding correlators according to the "extrapolate" dictionary are

$$\langle T\mathcal{O}(x_1)\mathcal{O}(x_2)\rangle = \lim_{z_1, z_2 \to 0} z_1^{-\Delta} z_2^{-\Delta} \langle T\phi(x_1, z_1)\phi(x_2, z_2)\rangle$$

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle = \lim_{z_1, z_2 \to 0} z_1^{-\Delta} z_2^{-\Delta} \langle \phi(x_1, z_1)\phi(x_2, z_2)\rangle$$

The AdS/CFT dictionary

- Assuming that the dictionaries are equivalent leads to the identities

$$K_{++}(x_1, z_1; x_2) = \lim_{z_2 \to 0} z_2^{-\Delta} \langle T\phi(x_1, z_1)\phi(x_2, z_2)\rangle$$

$$K_{-+}(x_1, z_1; x_2) = \lim_{z_2 \to 0} z_2^{-\Delta} \langle \phi(x_1, z_1)\phi(x_2, z_2)\rangle$$

- So proving the equivalence of the dictionaries is equivalent to showing that the above K's satisfy all the equations of motion arising from the gluing construction

The AdS/CFT dictionary

- It is clear that they satisfy the correct bulk equation of motion as

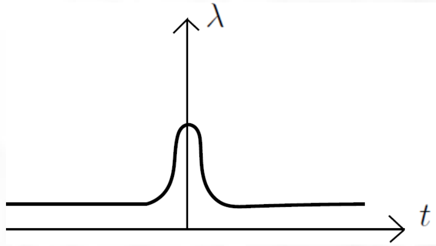
$$(\Box - m^2)\langle T\phi(x_1)\phi(x_2)\rangle = i\delta(x_1 - x_2)$$

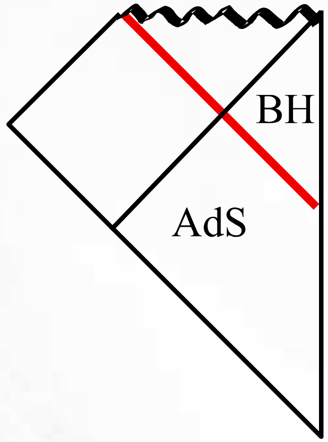
$$(\Box - m^2)\langle \phi(x_1)\phi(x_2)\rangle = 0$$

- The initial and final conditions are the trickiest to show. There need to use the fact that the kernel K in the wavefunction is the inverse of the bulk to bulk correlator.
- The delta function boundary condition at AdS boundary follows from the delta function on the right hand side of the Klein-Gordon equation for the bulk correlator.
- This is the proof.

AdS-Vaidya correlator

- Consider the example in the beginning





- The Vaidya spacetime provides a simple analytic example of the above process

$$ds^2 = \frac{1}{z^2}\left[-(1-\theta(v)z^2)dv^2 - 2dv\,dz + dx^2\right]$$

- By itself this does not solve the vacuum Einstein's equations, but needs a source. In a realistic case, this would be a scalar field that is collapsing, and the theta function would be a smooth function.

AdS-Vaidya correlator

- We will want to work out the correlation functions in this spacetime.
- Energy-momentum tensor one point functions become time independent immediately
- Consider a scalar field

$$S = -\frac{1}{2}\int d^dx\sqrt{-g}\left(\partial_\mu\phi\partial^\mu\phi + m^2\phi^2\right)$$

- The simplest case is when $m^2 = -3/4$ (there is a hidden Weyl symmetry in this case)
- We want the two point function of the scalar. Use the extrapolate dictionary, and work in the Heisenberg picture. State is the initial AdS vacuum.

AdS-Vaidya correlator

- We choose to calculate the time ordered 2-point correlator (all others can be obtained from this one)

$$G_F(x_2, x_1) = \langle \psi | T \phi(x_2) \phi(x_1) | \psi \rangle$$

- From the Heisenberg equation of motion, it follows that

$$(\Box_1 - m^2) G_F(x_2, x_1) = i \frac{\delta(x_2 - x_1)}{\sqrt{-g}} = (\Box_2 - m^2) G_F(x_2, x_1)$$

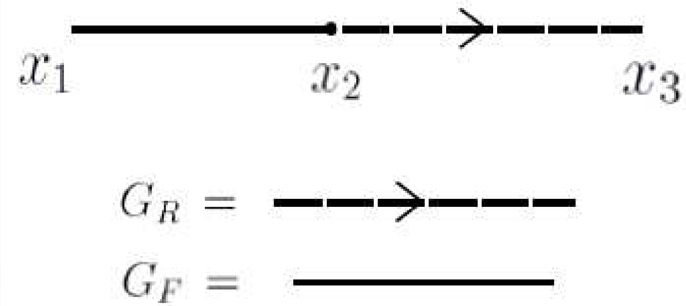
- Thus, we are lead to solve a 6 dimensional PDE.
- The initial data is given by the initial state (the AdS vacuum)

$$G_F^{\mathrm{AdS}}(v_2, x_2, z_2; v_1, x_1, z_1) = \frac{\sqrt{z_1 z_2}}{4\pi} \left(\frac{1}{\sqrt{-(v_2 - v_1)^2 - 2(v_2 - v_1)(z_2 - z_1) + (x_2 - x_1)^2 + i\epsilon}} - \frac{1}{\sqrt{-(v_2 - v_1)^2 - 2(v_2 - v_1)(z_2 - z_1) + 4 z_1 z_2 + (x_2 - x_1)^2 + i\epsilon}} \right)$$

AdS-Vaidya correlator

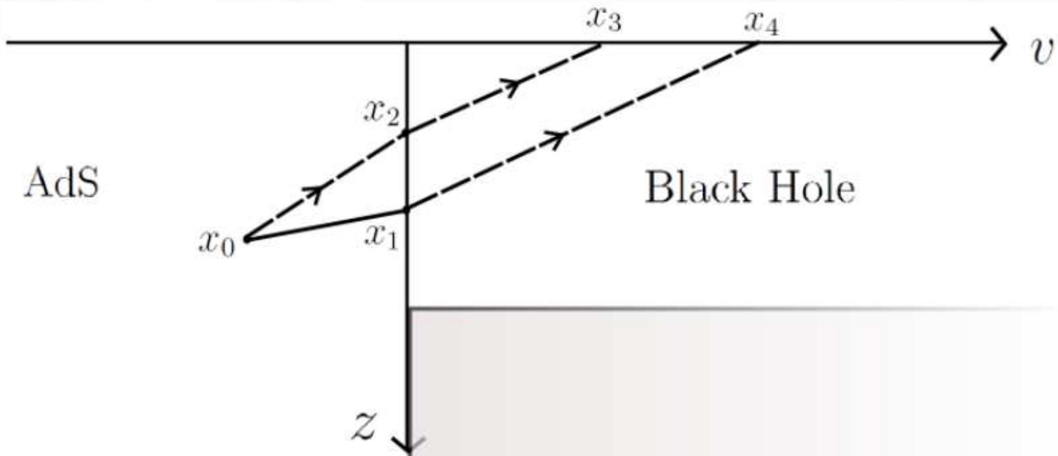
- Since the eom is linear we can use the method of Green's functions

$$G_F(x_3,x_1) = i\int_{t_2=\text{const}} dx_2\, G_F(x_2,x_1)\overleftrightarrow{D}^t G_R(x_3,x_2)$$



- We will use the above formula 3 times as follows

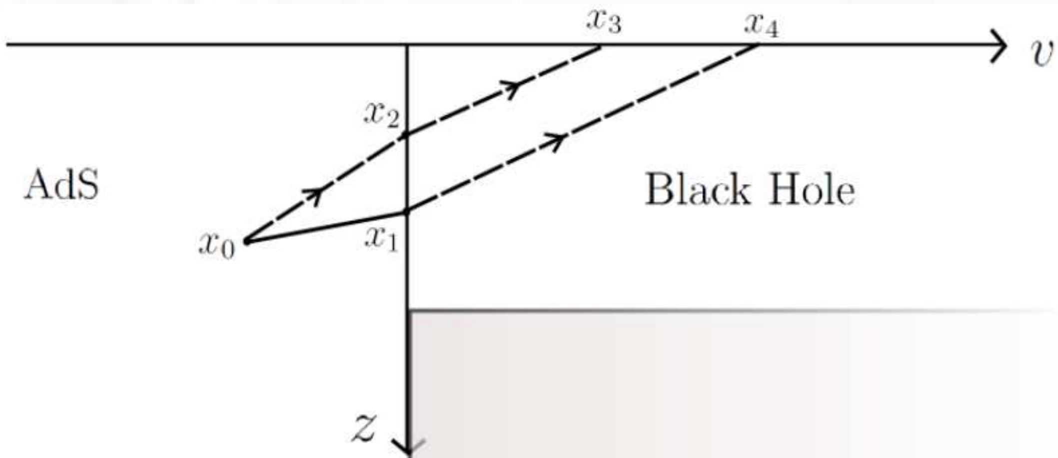
$$G_F(x_4,x_3) = -i\int dx_2 dx_1 dx_0 \left(G_F(x_1,x_0)\overleftrightarrow{D}^{t_1} G_R(x_4,x_1)\right)\overleftrightarrow{D}^{t_0}\left(G_R(x_2,x_0)\overleftrightarrow{D}^{t_2}G_R(x_3,x_2)\right)$$



AdS-Vaidya correlator

- This is useful because the retarded correlator happens to be independent of the initial state (a proof in the next slide)
- Thus, we can use the thermal retarded correlator in the black hole part, which is analytically known

$$
\begin{aligned}
&G_F^{\mathrm{BTZ}}(v_2, x_2, z_2; v_1, x_1, z_1) \\
&= \sqrt{\frac{z_1 z_2}{32\pi^2}} \left(\frac{1}{\sqrt{\cosh(x_2 - x_1) - z_1 z_2 - (1 - z_1 z_2)\cosh(v_2 - v_1) - (z_2 - z_1)\sinh(v_2 - v_1) + i\epsilon}} \right. \\
&\qquad \left. - \frac{1}{\sqrt{\cosh(x_2 - x_1) + z_1 z_2 - (1 - z_1 z_2)\cosh(v_2 - v_1) - (z_2 - z_1)\sinh(v_2 - v_1) + i\epsilon}} \right)
\end{aligned}
$$



AdS-Vaidya correlator

- The retarded correlator is independent of the state because

  1) It satisfies a second order differential equation

  2) The initial data is all determined by the equal time commutation relations

$$G_R(x_2, x_1) = \theta(t_2 - t_1)\langle[\phi(x_2), \phi(x_1)]\rangle$$

- At equal times we thus have

$$G_R = 0$$

$$\partial_{t_2} G_R \propto \langle[\phi(x_1), \Pi(x_2)]\rangle|_{t_1=t_2} = i\delta(\boldsymbol{x}_1 - \boldsymbol{x}_2)$$

AdS-Vaidya correlator

- The task is to compute the 6 dimensional integral

$$G_F(x_4,x_3) = -i \int dx_2 dx_1 dx_0 \left(G_F(x_1,x_0) \overleftrightarrow{D}^{t_1} G_R(x_4,x_1) \right) \overleftrightarrow{D}^{t_0} \left(G_R(x_2,x_0) \overleftrightarrow{D}^{t_2} G_R(x_3,x_2) \right)$$

- Technical details:

  - The integrand has singularities at lightlike separated points

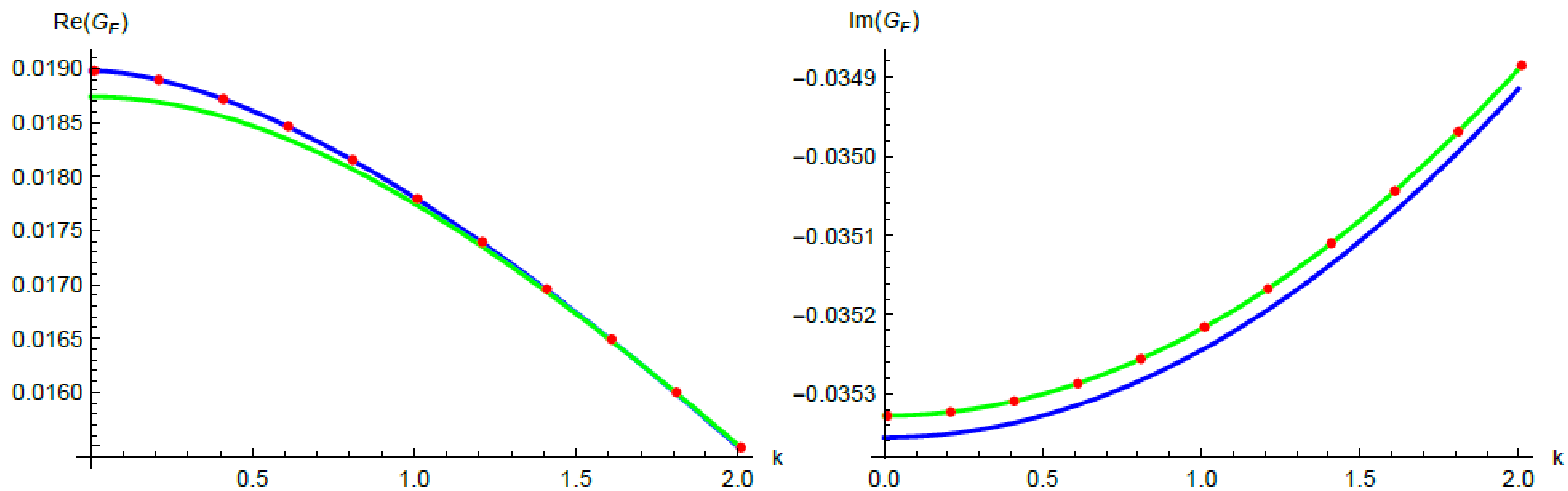
  - It is better to Fourier transform to k-space, which gets rid of 3 integrals and softens the lightcone divergences to logarithmic

AdS-Vaidya correlator

- Then to the results:

  - Blue curve = AdS vacuum correlator

  - Green curve = BTZ thermal correlator



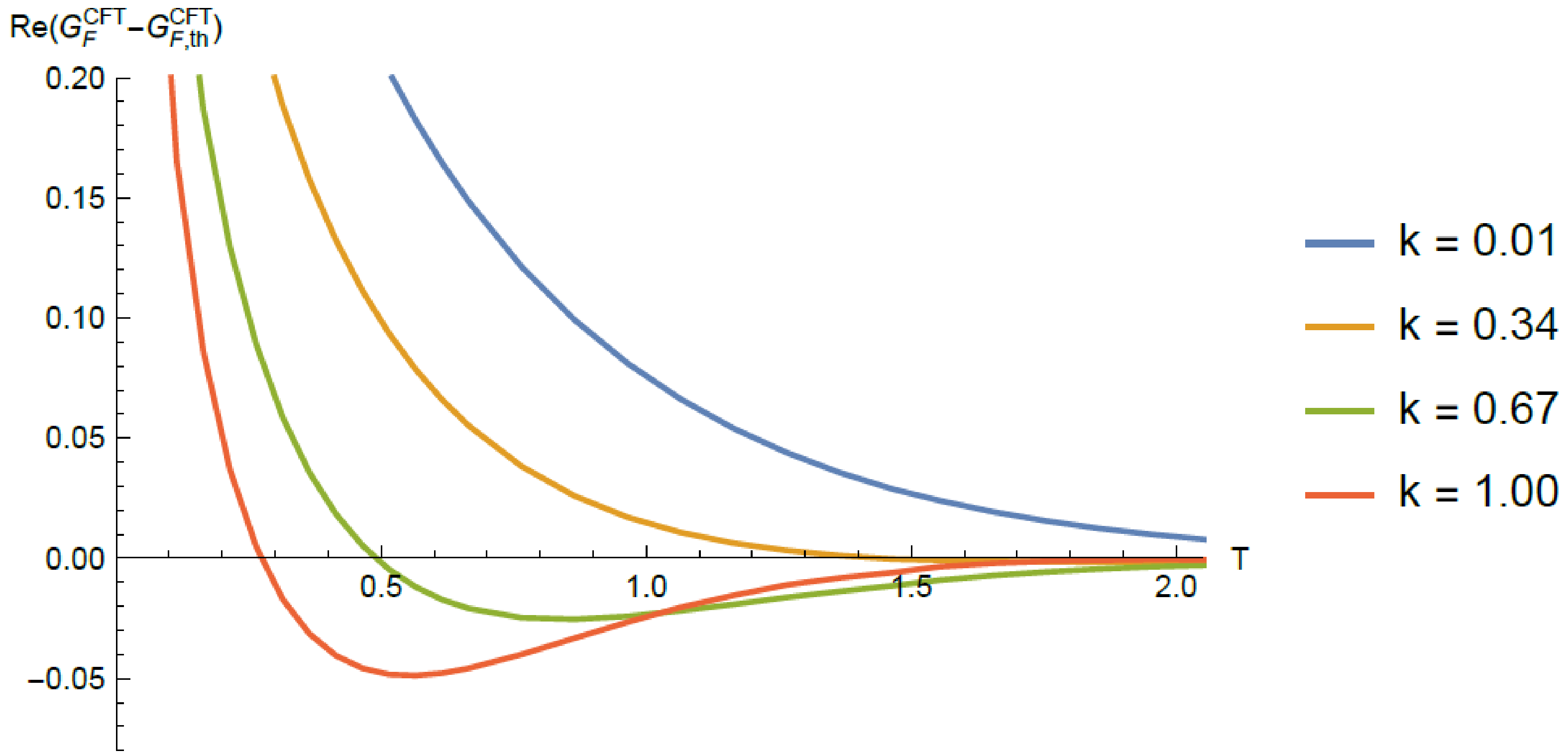
$v_2 = 0.051,\ v_1 = 0.001$ and for $z_1 = 0.10$ and $z_2 = 0.20,$

- The real part is close to the vacuum, while the imaginary part is thermal right away

$$G_R(x_2, x_1) = 2i\theta(t_2 - t_1)\,\mathrm{Im}\,G_F(x_2, x_1)$$

AdS-Vaidya correlator

- Fast (exponential?) approach to thermality. Smallest momentum has the slowest approach

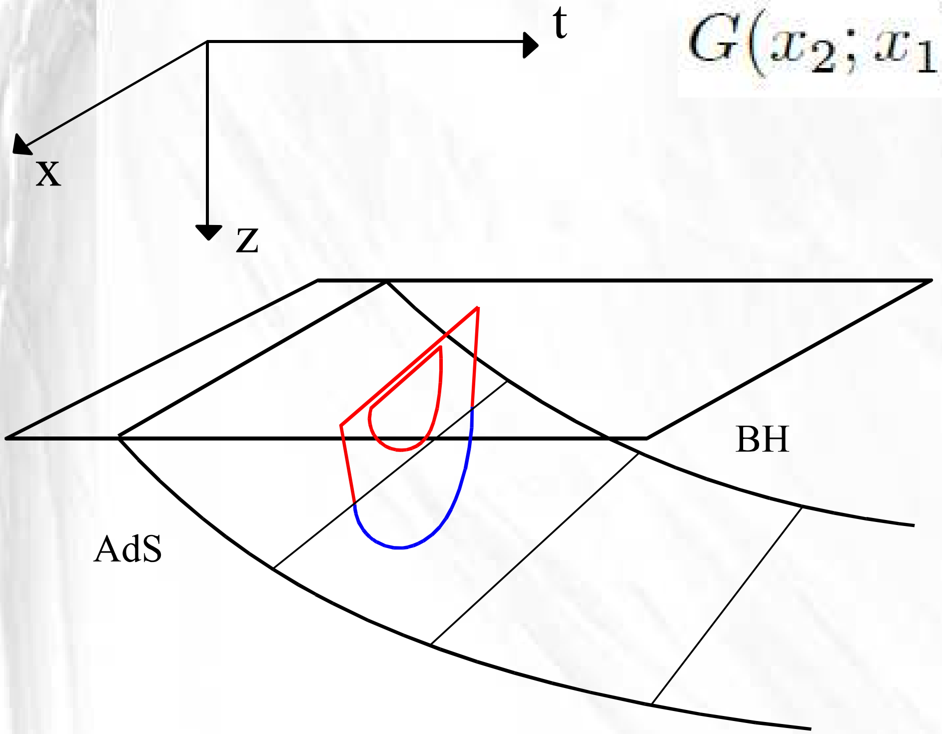


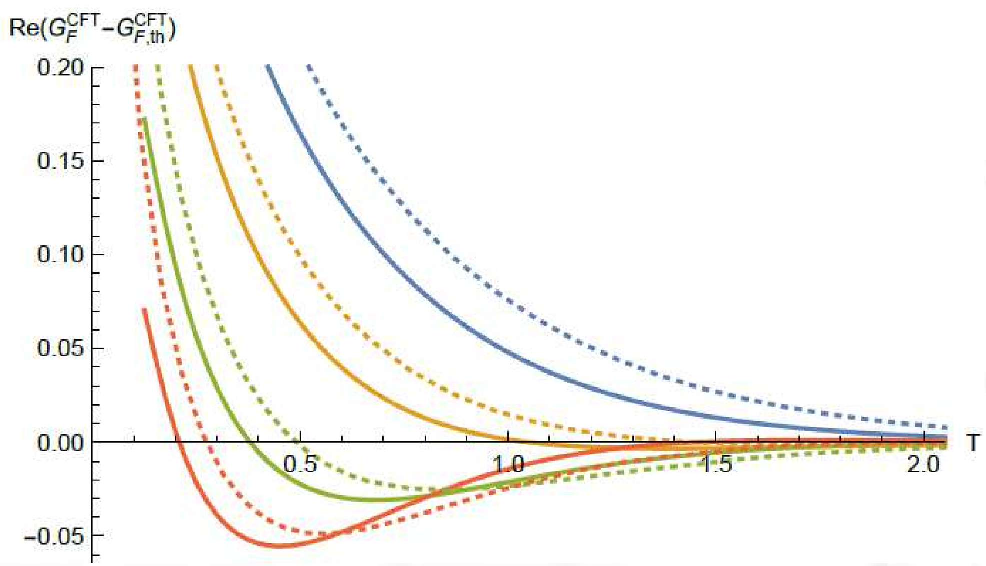
$$\langle T\mathcal{O}(x_1)\mathcal{O}(x_2)\rangle = \lim_{z_1,z_2\to 0} z_1^{-\Delta} z_2^{-\Delta} \langle T\phi(x_1,z_1)\phi(x_2,z_2)\rangle$$

AdS-Vaidya correlator

- At the qualitative level the results are explained by a simple geodesic estimate

$$G(x_2; x_1) \propto e^{-mL[x_{cl}]}$$





$$G(x, t_2; 0, t_1) = \begin{cases} G_{th}(x, t_2; 0, t_1), & |x| < t_1 + t_2 \\ \frac{1}{|x|^{2\Delta}(\cosh(t_1/2)\cosh(t_2/2))^{2\Delta}}, & |x| > t_1 + t_2 \end{cases}$$

Conclusions and open questions

- There are two versions of the AdS/CFT dictionary that are suitable for non-equilibrium settings (for a class of initial states that can be prepared with a Euclidean path integral).
- For a free scalar, the dictionaries agree.
- For 2-point functions of bulk gauge fields and gravity the previous proof propably goes through. One has to work out the appropriate gauge fixings etc.
- For higher point functions could possibly do a perturbative proof of equivalence.
- Also a non-perturbative proof using path integrals is possible, and has been done in Euclidean time for interacting bulk quantum scalar fields

Conclusions and open questions

- What about states that cannot be prepared with Euclidean path integrals?
- What is the CFT dual of the bulk wavefunction?