Turbulent strings in AdS/CFT

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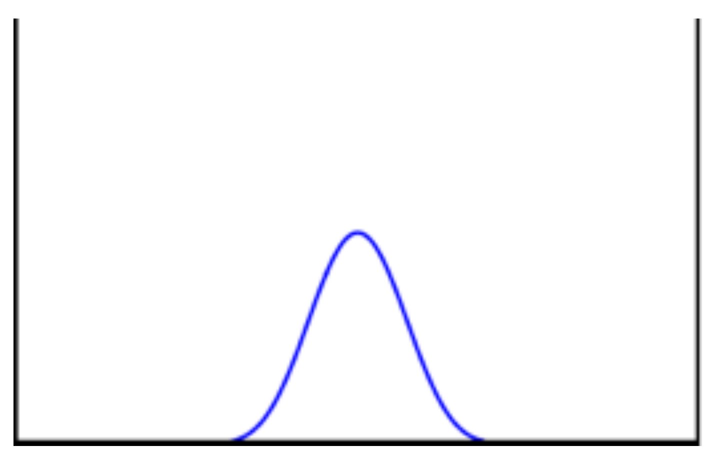
arXiv:1504.02190 with Keiju Murata



12 May 2015@Oxford

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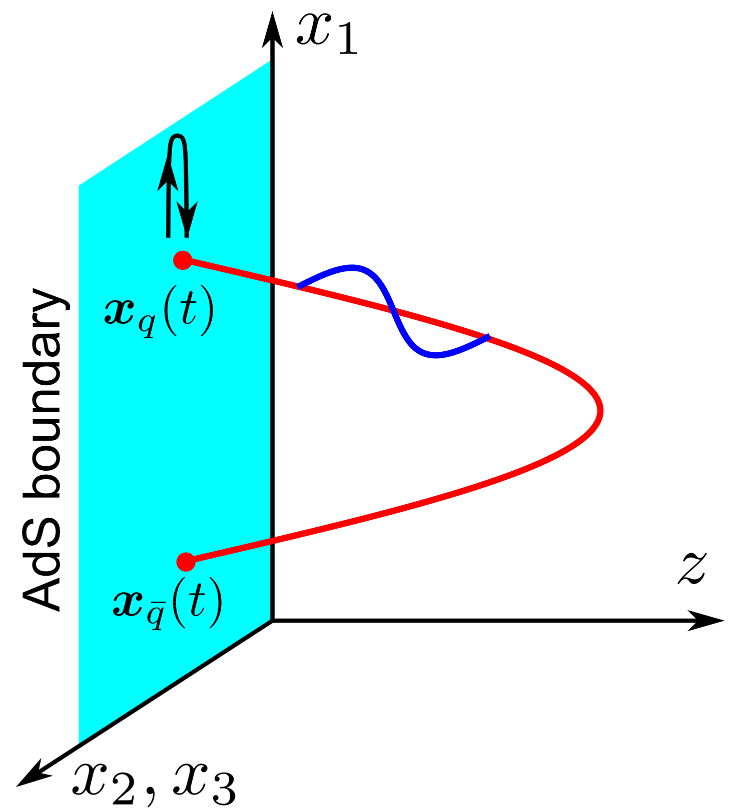
What I will do

Perturb holographic quark-antiquark potential



We solve nonlinear time evolution

Motivation

I think time-dependent dynamics in gauge/gravity duality is interesting

- AdS thermalization

  Relation to real QGP? New BH dynamics?

- AdS turbulent instability

  What is essential? AdS? Einstein? Nonlinearity?

- c.f.) Dynamical meson melting

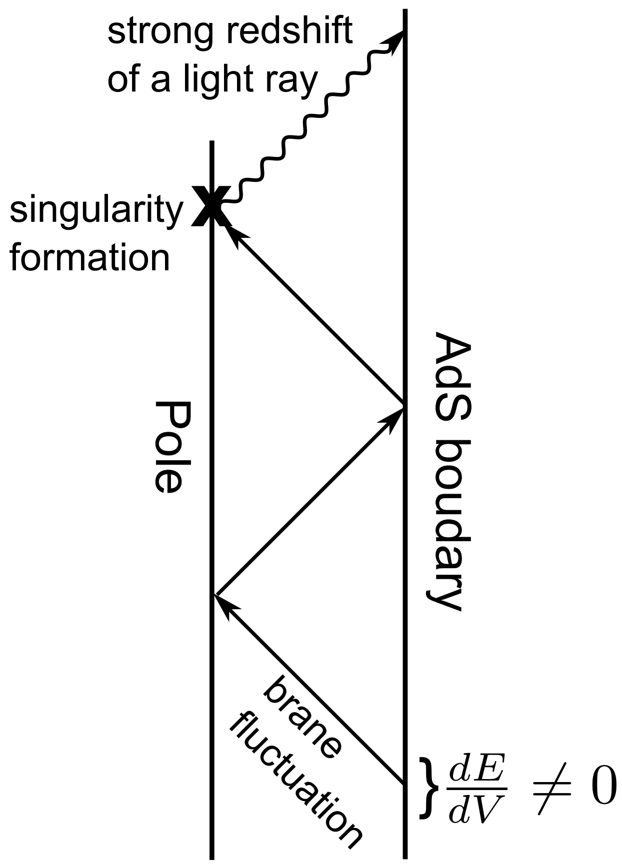
  Time evolution in thermalizing D3/D7

  [TI-Kinoshita-Murata-Tanahashi]

Turbulent instability in D3/D7

Singularity formation after some wave reflections

[Hashimoto-Kinoshita-Oka-Murata]



- Electric field quench: 0→E
- "Meson turbulence"
- Probably due to nonlinearity in DBI

Considering F1 would be simpler

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Time-like holographic Wilson loop

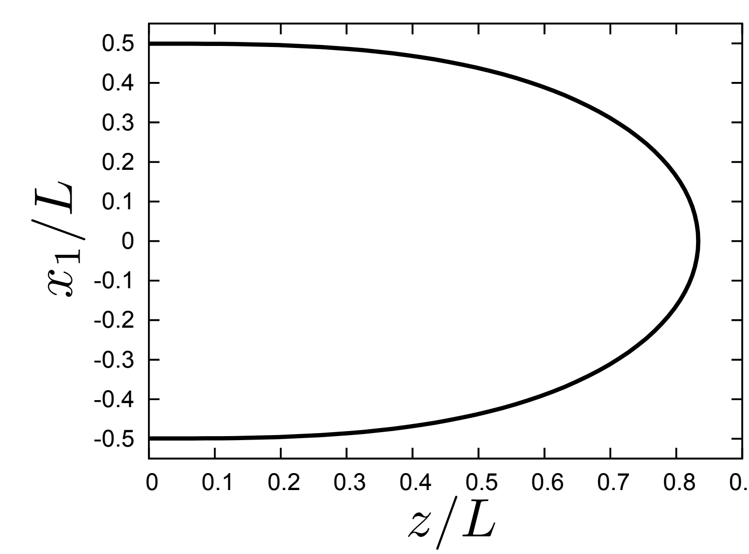
[Maldacena, Rey-Yee]

$AdS_5 x S^5$ $$ds^2 = \frac{\ell^2}{z^2}\left(-dt^2 + dz^2 + d\boldsymbol{x}^2\right) + \ell^2 d\Omega_5^2$$

Static gauge: (τ,σ)=(t,z)
Target space embedding: $x_1 = X_1(z)$

Solution for separation L

$$X_1(z) = \pm z_0 \int_{z/z_0}^{1} dw \frac{w^2}{\sqrt{1-w^4}}$$
$$= \pm z_0 \left[\Gamma_0 + F(z/z_0; i) - E(z/z_0; i)\right]$$



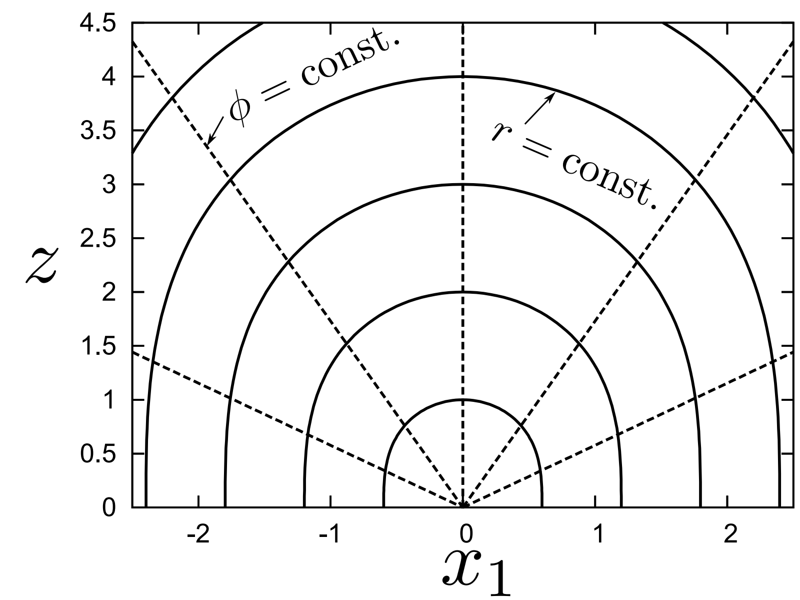
z_0: string tip
Γ_0=0.599

$$\frac{L}{2} = z_0 \Gamma_0$$

A convenient parametrization

Inverse function of F(z;k) is sn(x;k)

$$F(\mathrm{sn}(x;k);k) = x$$



Polar-like coordinates (r,ϕ) where the static solution is r=z_0

$$z = rf(\phi) = r\,\mathrm{sn}(\phi;i)$$

$$x_1 = rg(\phi) = r\begin{cases}\phi - E(\mathrm{sn}(\phi;i);i) + \Gamma_0 & (\phi \le \beta_0/2) \\ \phi + E(\mathrm{sn}(\phi;i);i) - \Gamma_0 - \beta_0 & (\phi > \beta_0/2)\end{cases}$$

A nice identity $f'(\phi)^2 + g'(\phi)^2 = 1$

Linearized perturbations

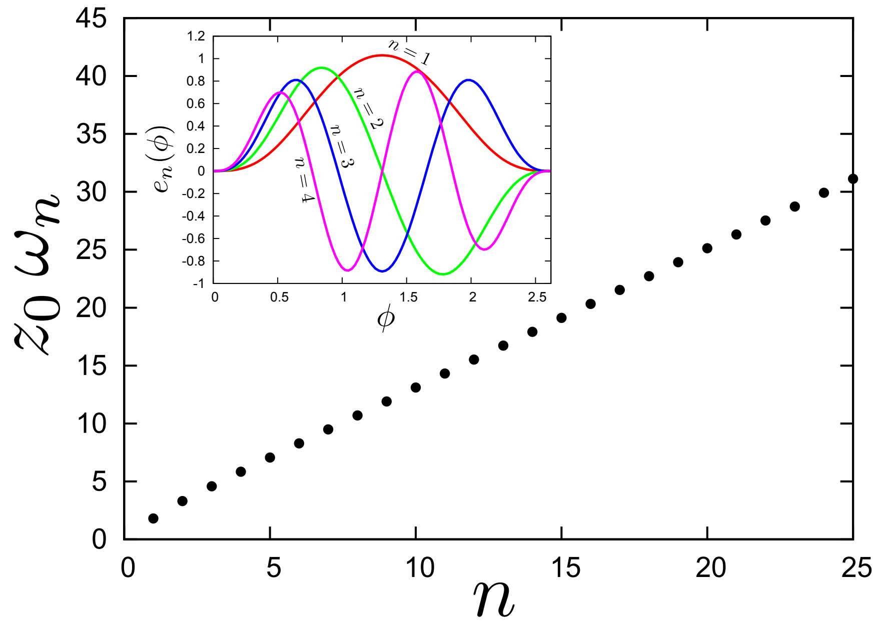
Longitudinal fluctuations around $r=z_0$

$$r = z_0[1 + \chi_1(t,\phi)]$$

[Callan-Guijosa, Klebanov-Maldacena-Thorn]

Linearized EoM for eigenvalues/functions

$$(\partial_t^2 + \mathcal{H})\chi_1 = 0 \qquad \mathcal{H} \equiv -\frac{1}{z_0^2 h}\partial_\phi h \partial_\phi \qquad h \equiv ((g/f)'f)^2$$



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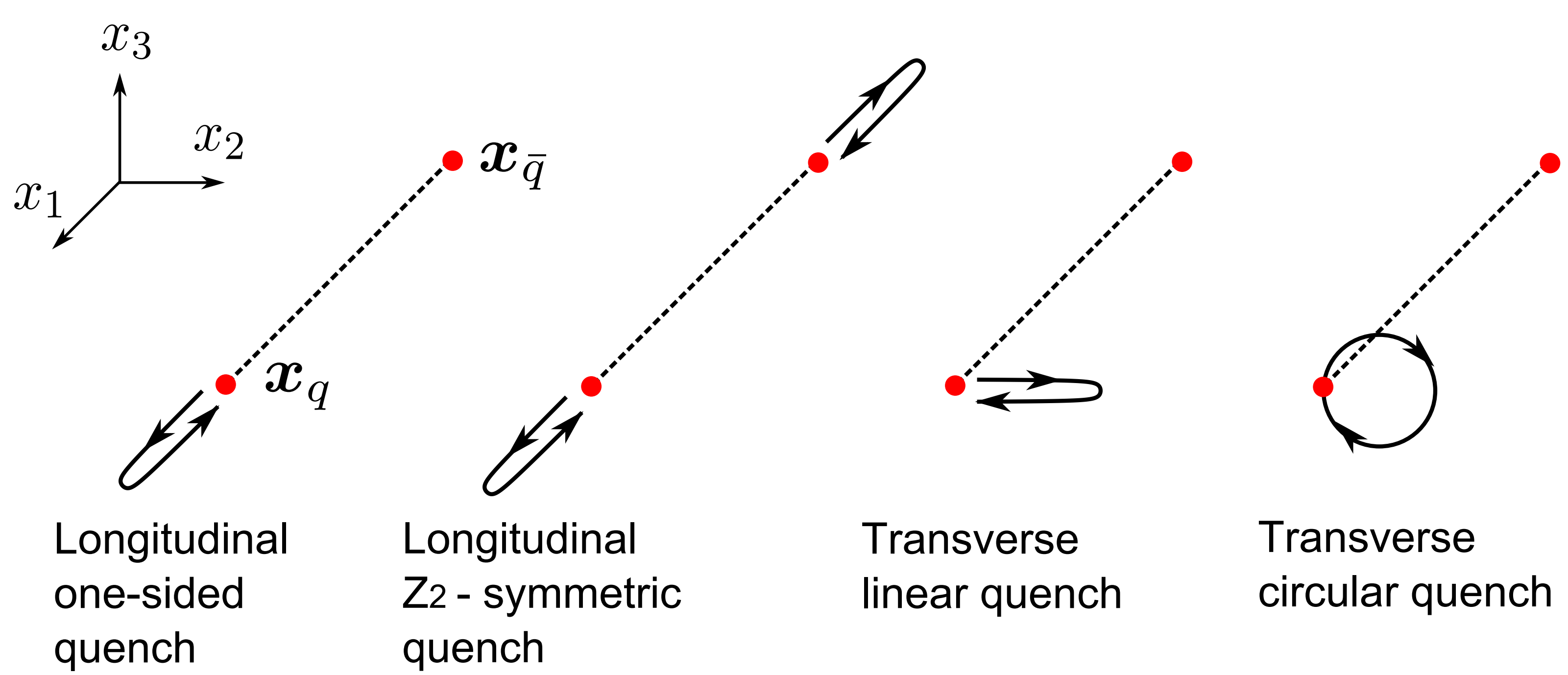
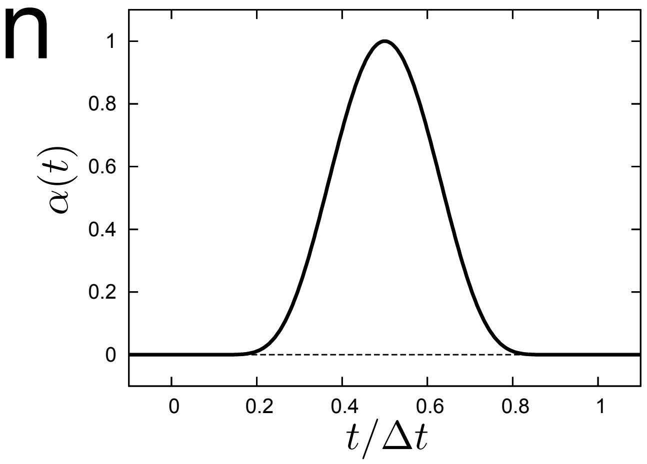
Perturb the string endpoints



Quench profile: a compact C[∞] function

$$\alpha(t) = \exp\left[2\left(\frac{\Delta t}{t-\Delta t} - \frac{\Delta t}{t} + 4\right)\right] \quad (0 < t < \Delta t)$$



Worldsheet double null coordinates

Induced metric $ds^2_{F1} = -2\gamma_{uv} du dv$

Worldsheet: u,v

Target space: T(u,v), Z(u,v), X1,2,3(u,v)

$$\gamma_{uv} = \frac{\ell^2}{Z^2}(-T_{,u}T_{,v} + Z_{,u}Z_{,v} + \boldsymbol{X}_{,u}\cdot\boldsymbol{X}_{,v})$$

Equations of motion

$$T_{,uv} = \frac{1}{Z}(T_{,u}Z_{,v} + Z_{,u}T_{,v})$$

$$Z_{,uv} = \frac{1}{Z}(T_{,u}T_{,v} + Z_{,u}Z_{,v} - \boldsymbol{X}_{,u}\cdot\boldsymbol{X}_{,v})$$

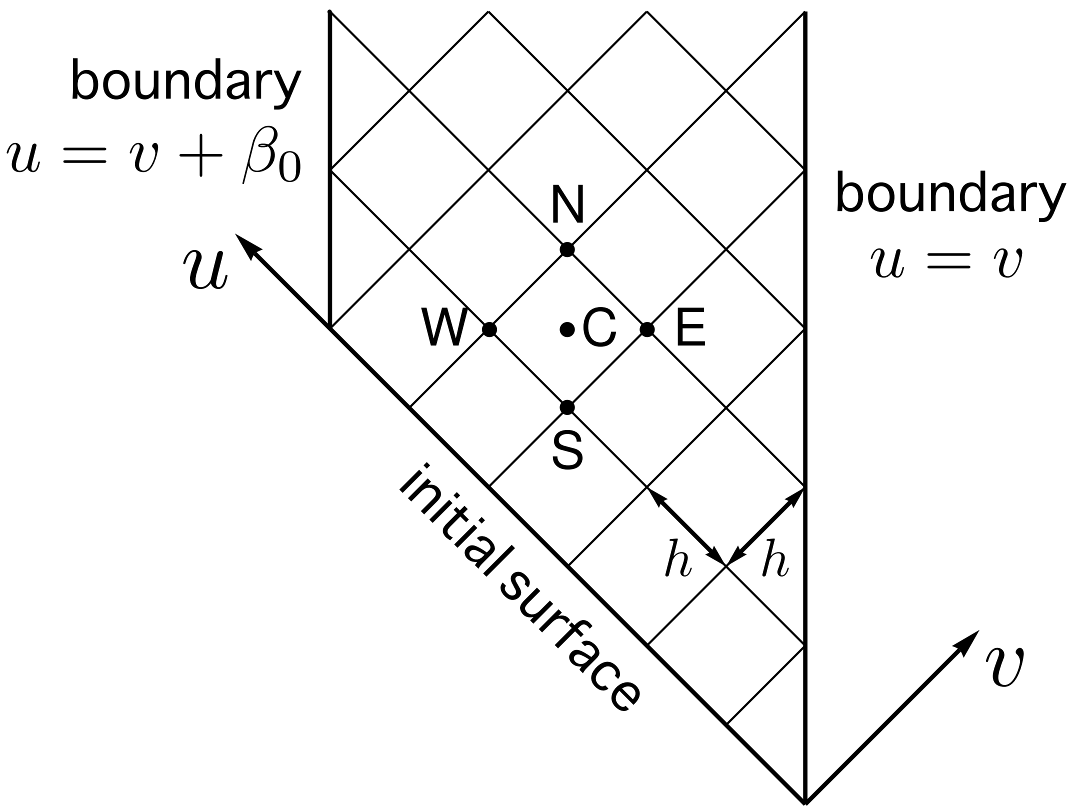
$$\boldsymbol{X}_{,uv} = \frac{1}{Z}(\boldsymbol{X}_{,u}Z_{,v} + Z_{,u}\boldsymbol{X}_{,v})$$

Constraints

$$\gamma_{uu} = \frac{\ell^2}{Z^2}(-T^2_{,u} + Z^2_{,u} + \boldsymbol{X}^2_{,u}) = 0$$

$$\gamma_{vv} = \frac{\ell^2}{Z^2}(-T^2_{,v} + Z^2_{,v} + \boldsymbol{X}^2_{,v}) = 0$$

Discretization



To solve EoMs, we use O(h^2) central finite differential

$$\Psi_{,uv}|_C = \frac{\Psi_N - \Psi_E - \Psi_W + \Psi_S}{h^2}$$

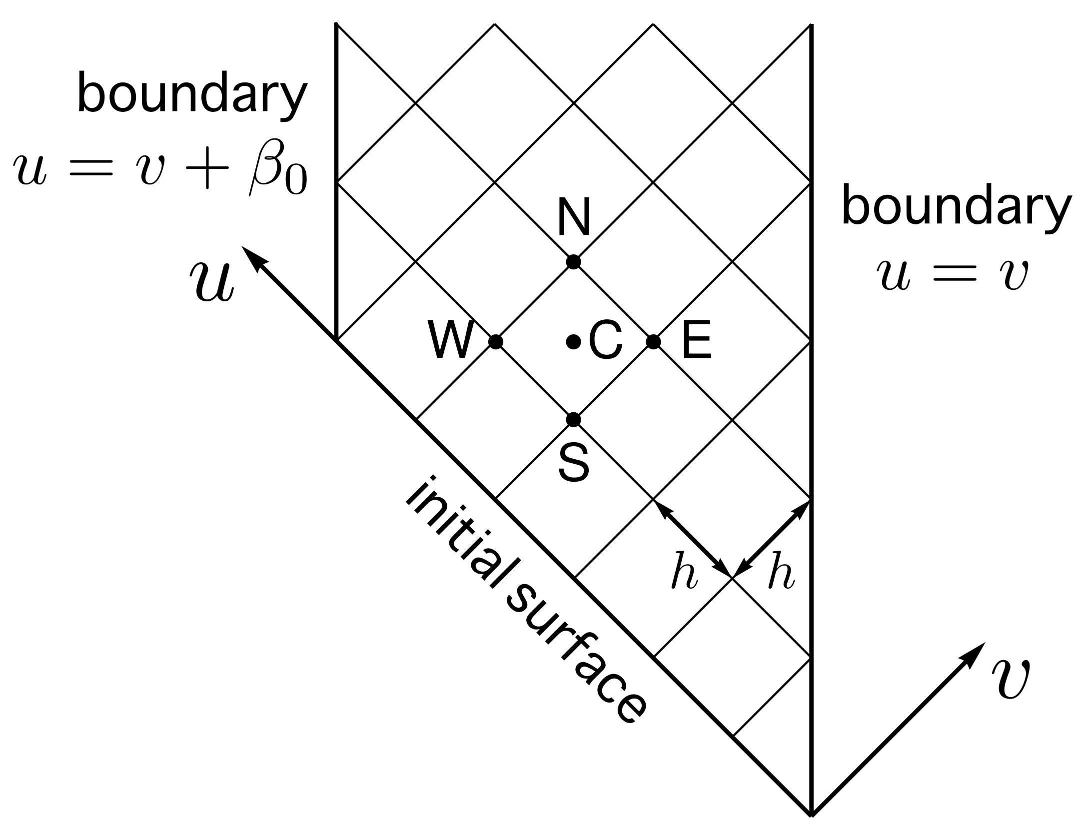
$$\Psi_{,u}|_C = \frac{\Psi_N - \Psi_E + \Psi_W - \Psi_S}{2h}$$

$$\Psi_{,v}|_C = \frac{\Psi_N + \Psi_E - \Psi_W - \Psi_S}{2h}$$

$$\Psi|_C = \frac{\Psi_E + \Psi_W}{2}$$

Compute N by using EWS data

Initial data



Initial data satisfies the constraint

$$\gamma_{uu} = \frac{\ell^2}{Z^2}(-T_{,u}^2 + Z_{,u}^2 + \boldsymbol{X}_{,u}^2) = 0$$

Solution (gauge: ϕ=u when v=0)

$$T(u,0) = z_0\, u$$
$$Z(u,0) = z_0\, f(u)$$
$$X_1(u,0) = z_0\, g(u)$$

where we used $f'(\phi)^2 + g'(\phi)^2 = 1$

Boundary quench is then added at $0 < T_{bdry} < \Delta t$

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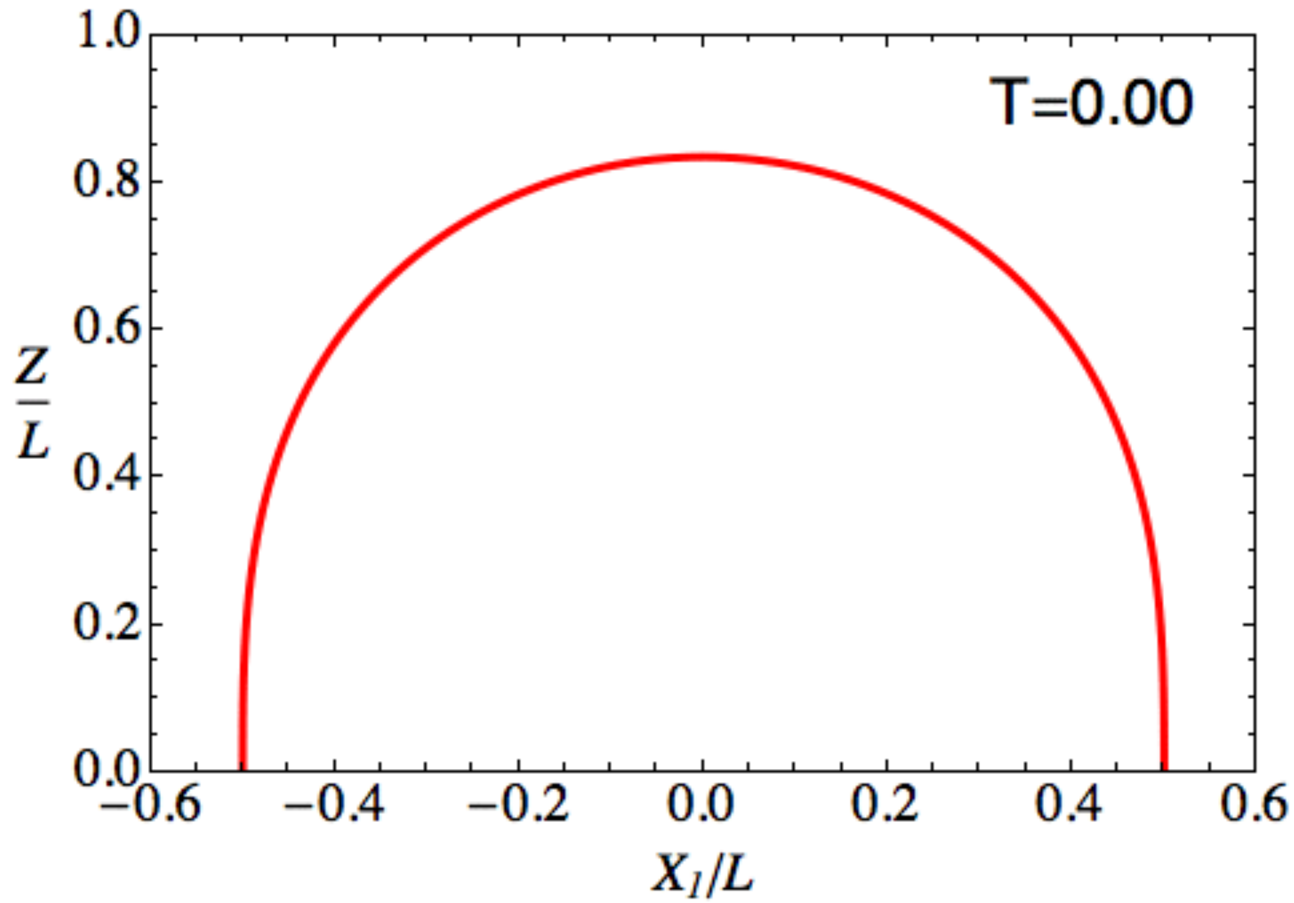
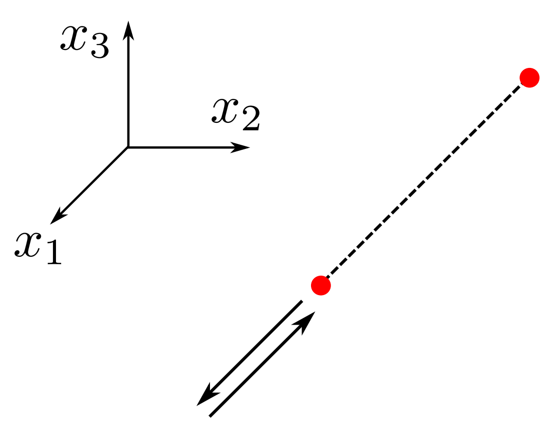
Longitudinal one-sided quench



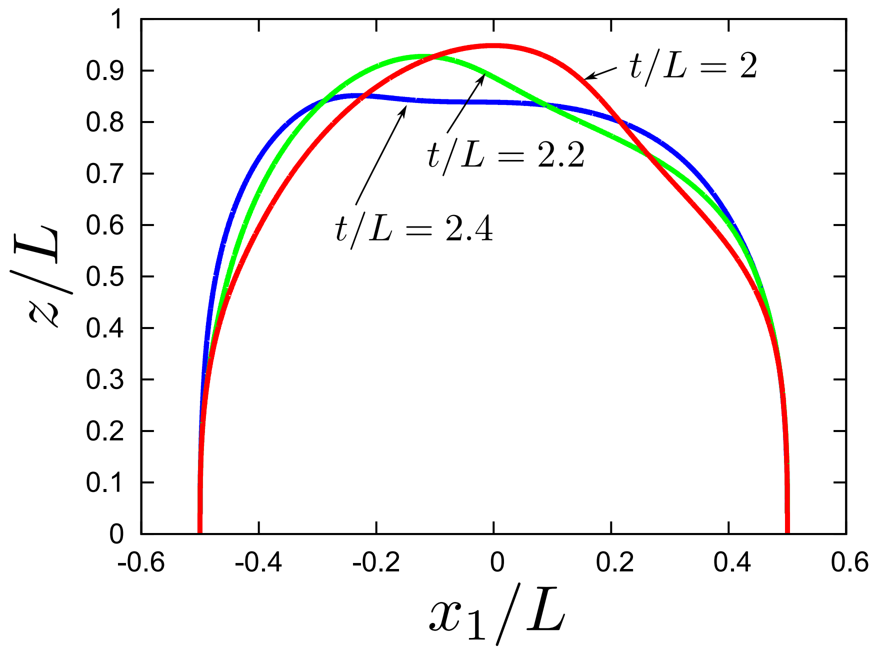
ε=0.03, Δt/L=2

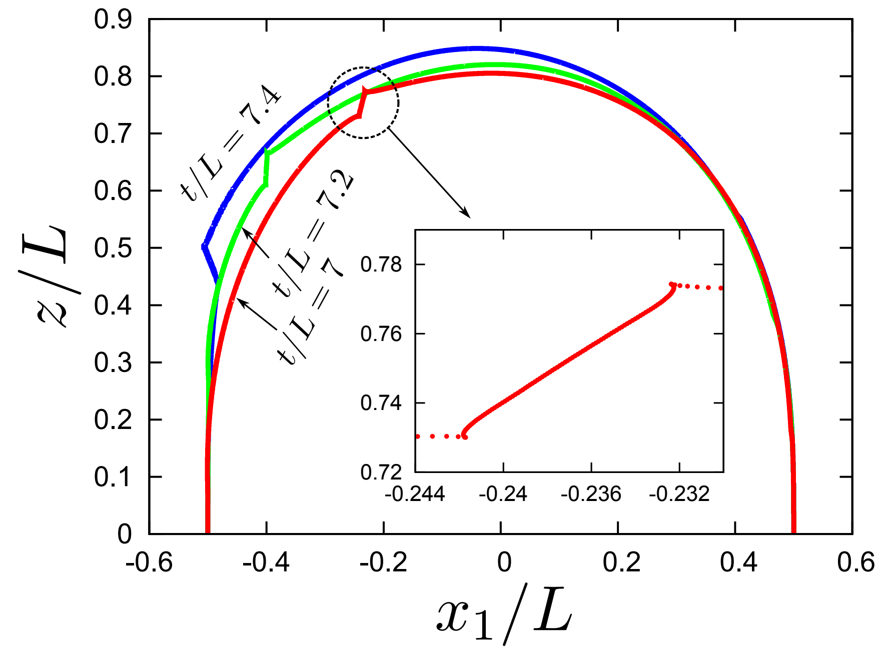


Amplitude: ε=Δx/L

Duration: Δt/L

Cusp formation





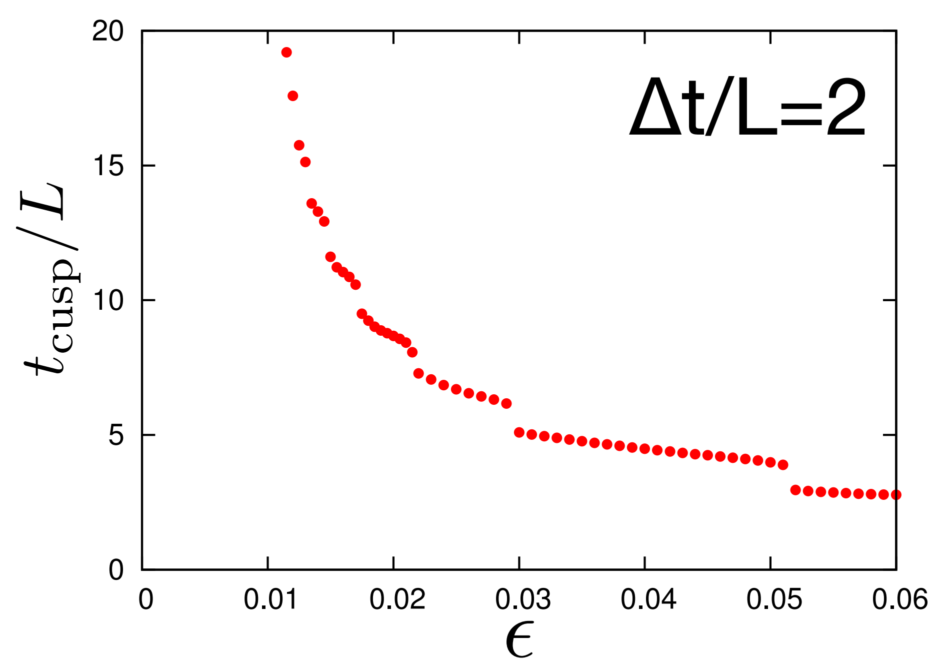
- Cusps are seen in target space (x,z)-coordinates
- Fields on worldsheet (u,v)-coordinates are regular
- Cusps are created in a pair (around t/L~5)

Analysis 1: Cusp detection

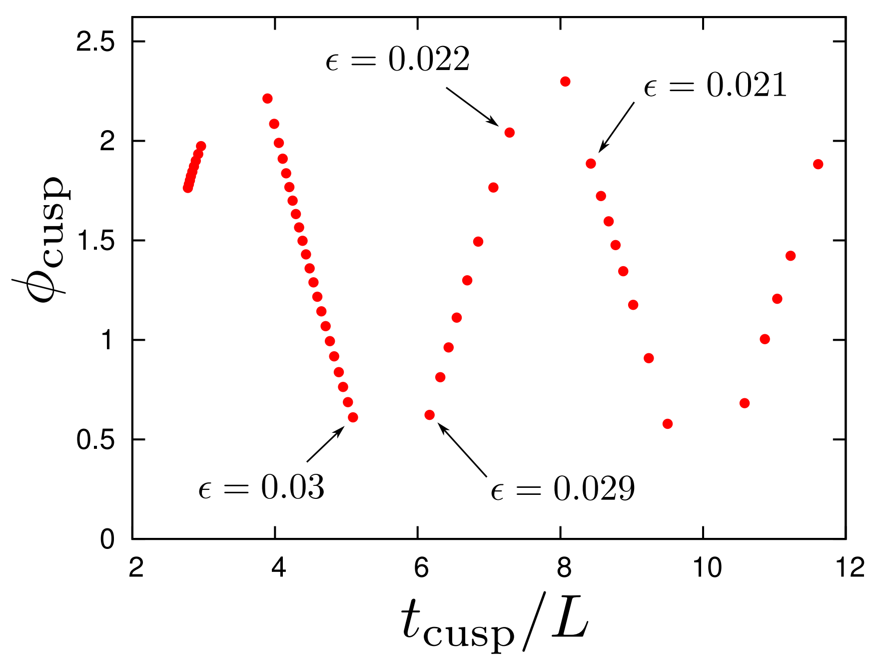
The conditions satisfied at a cusp:

$$J_z \equiv T_{,u} Z_{,v} - T_{,v} Z_{,u} = 0$$

$$J_i \equiv T_{,u} X_{i,v} - T_{,v} X_{i,u} = 0$$



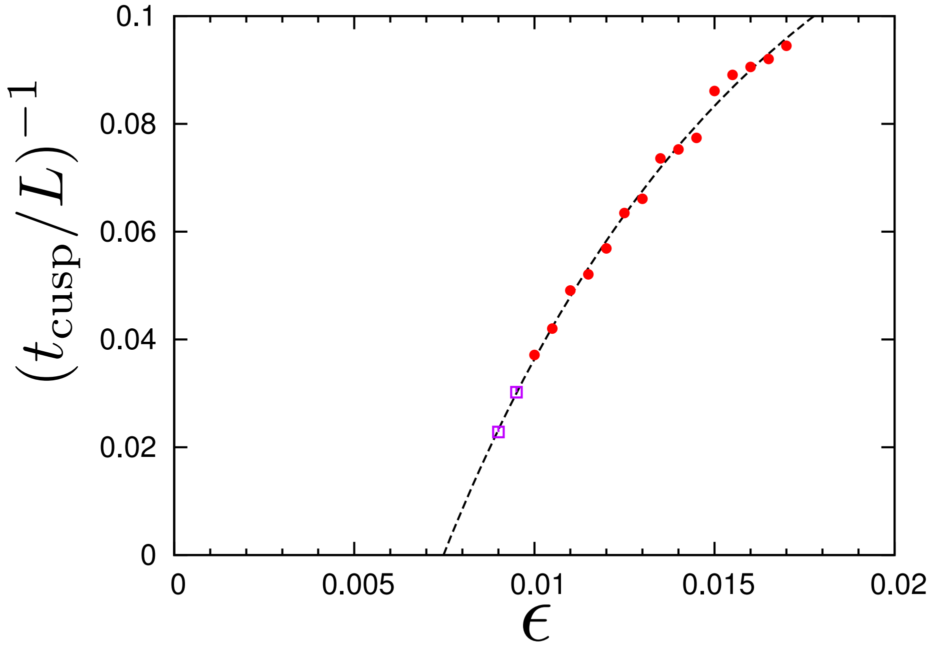
Cusp formation time when ε is changed

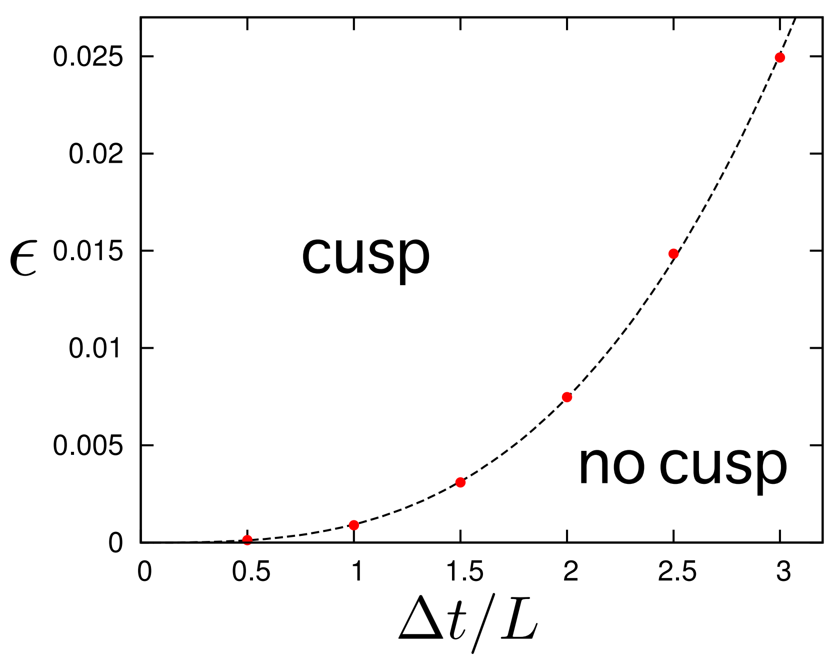


Corresponding formation points

Critical amplitude

There is a minimal amplitude for cusp formation





An extrapolation to t_{cusp}~∞:
ε_{crit}~0.075 for Δt/L=2

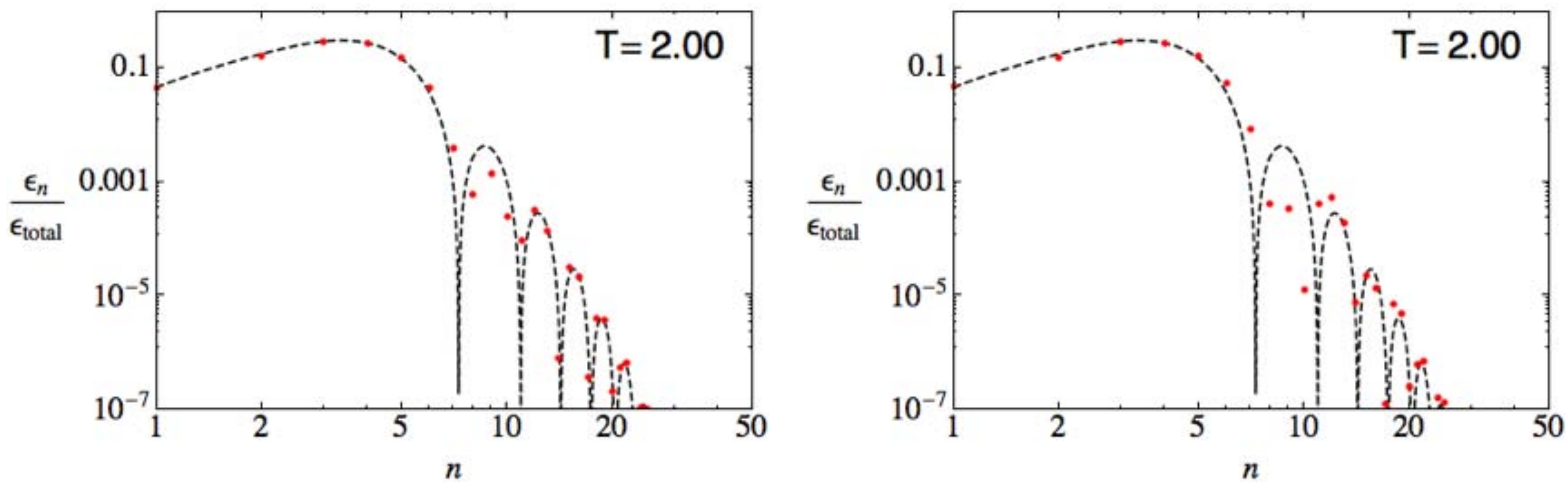
Scaling (in small Δt/L)
ε_{crit}~$(\Delta t/L)^3$

Analysis 2: Energy spectrum

Decompose nonlinear solutions in linear eigenmodes e_n

$$\chi_1 = \sum_{n=1}^{\infty} c_n(t) e_n(\phi) \qquad \varepsilon_n(t) = \frac{\sqrt{\lambda} z_0}{4\pi} \left(\dot{c}_n{}^2 + \omega_n^2 c_n{}^2 \right)$$

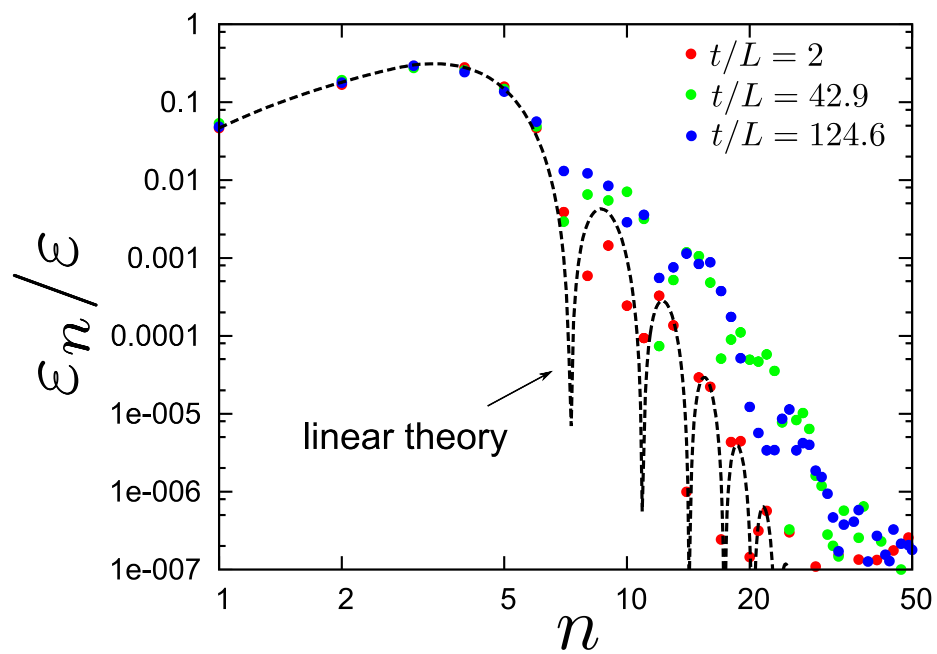
Log-log plot



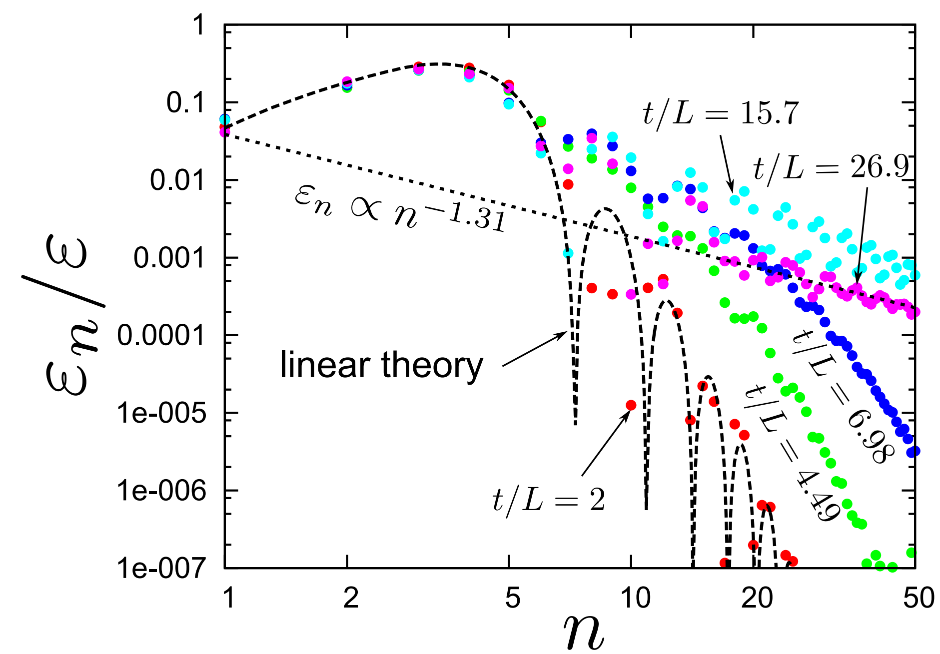
ε=0.005, Δt/L=2 (no cusp)   ε=0.01 (cusps T~27)

***Dashed lines: linearized computations

Energy cascade



ε=0.005, Δt/L=2 (no cusp)



ε=0.01 (cusps T~27)

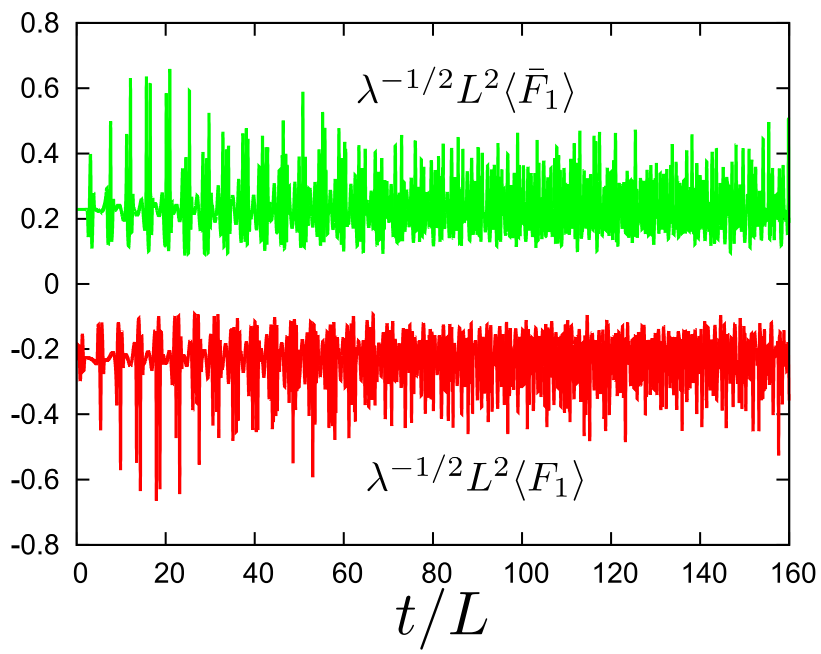
Cusp formation: direct energy cascade → power law

No cusp: no clear power law

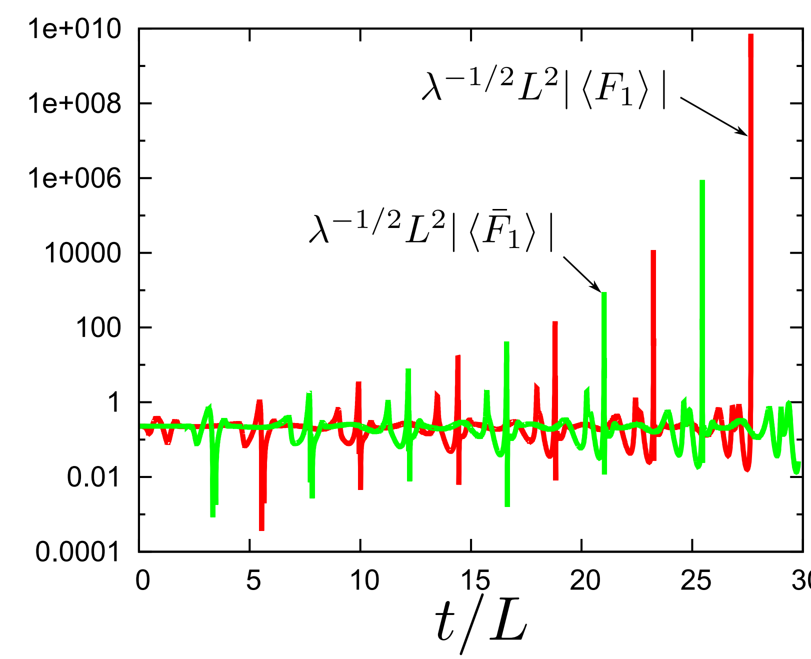
Analysis 3: Forces on the endpoints

$$\langle \boldsymbol{F}(t) \rangle = \frac{\delta S_{\text{on-shell}}}{\delta \boldsymbol{x}_q}$$

Force diverges when a cusp reaches the boundary



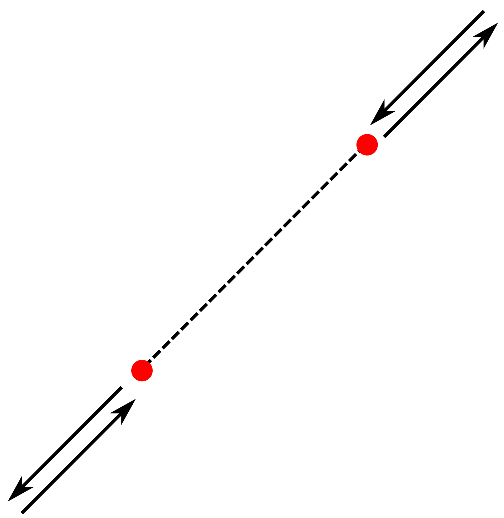
ε=0.005, Δt/L=2 (no cusp)

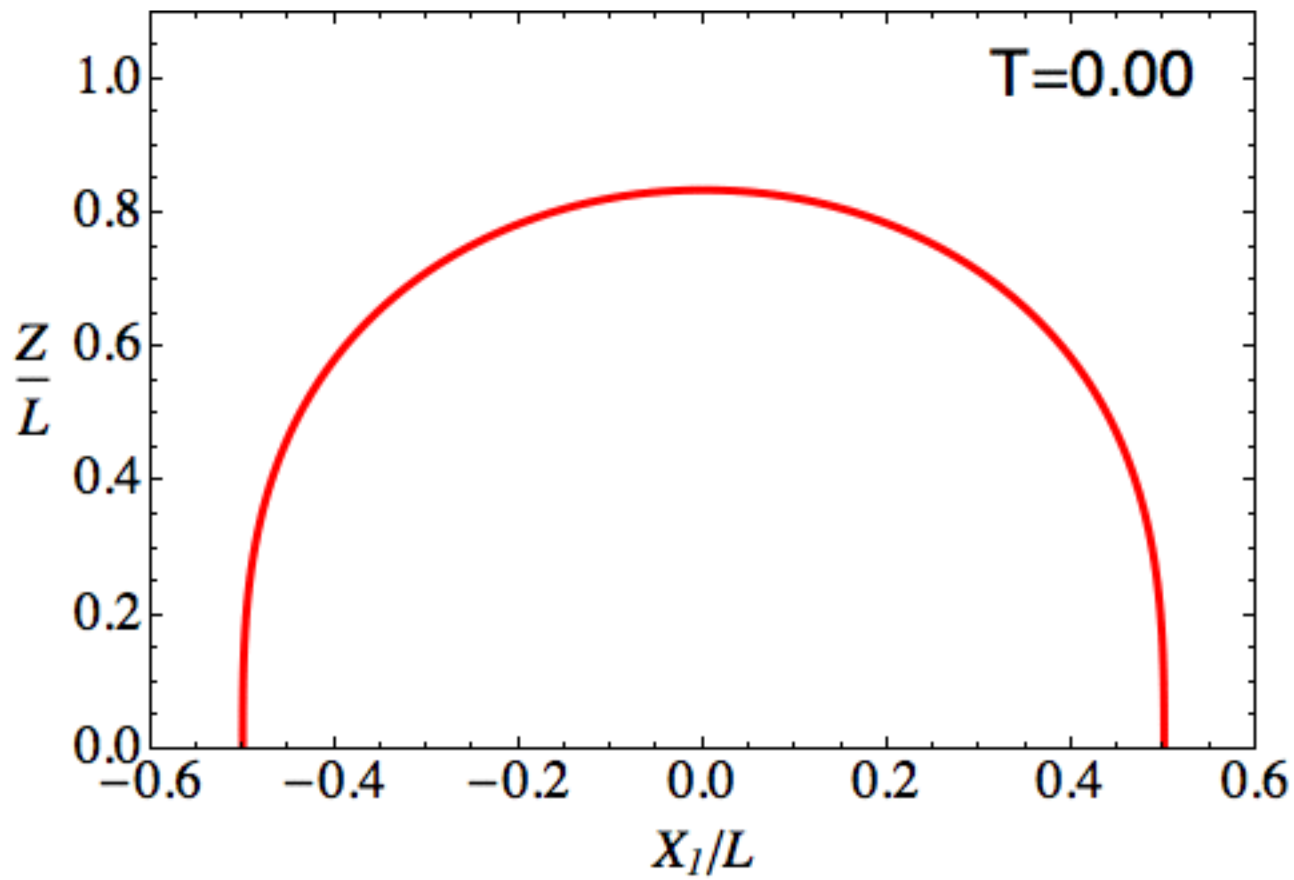


ε=0.01 (cusps T~27)

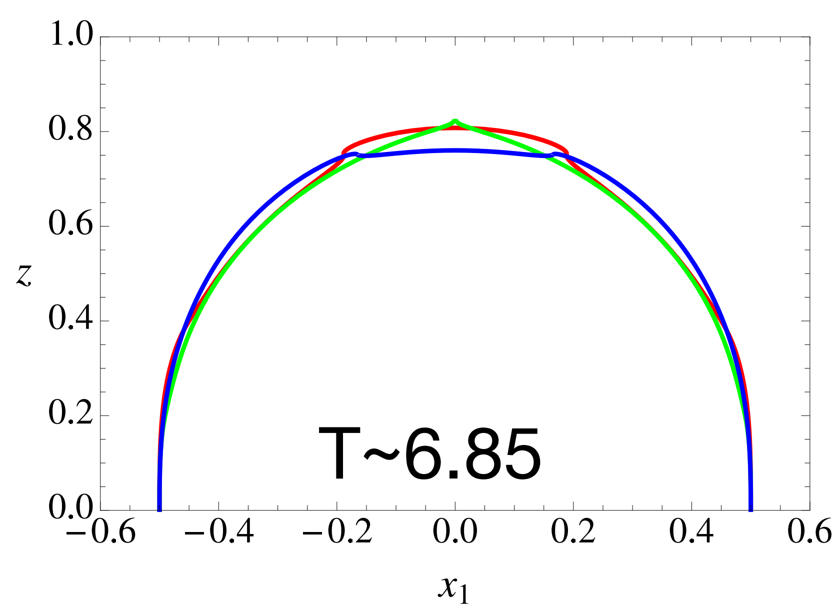
***Red: x=L/2, green: x=-L/2

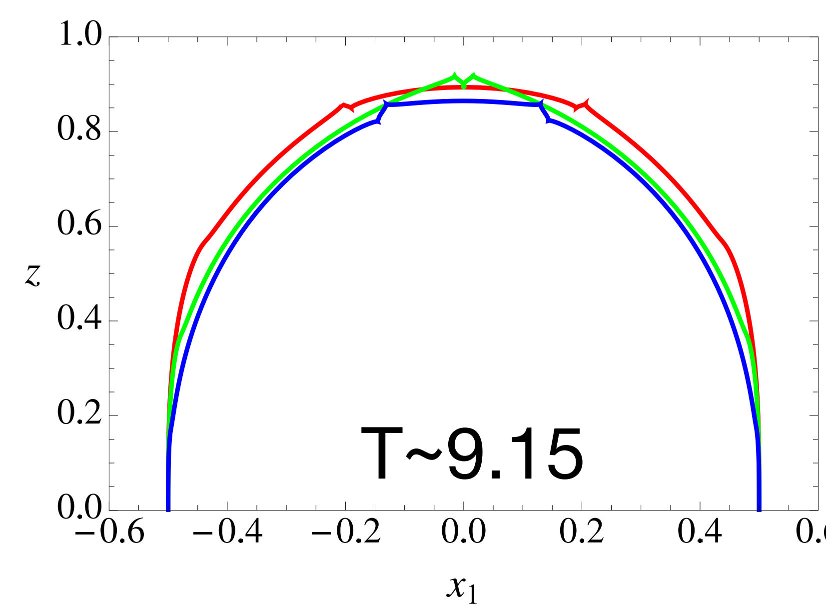
Z_2-symmetric quench



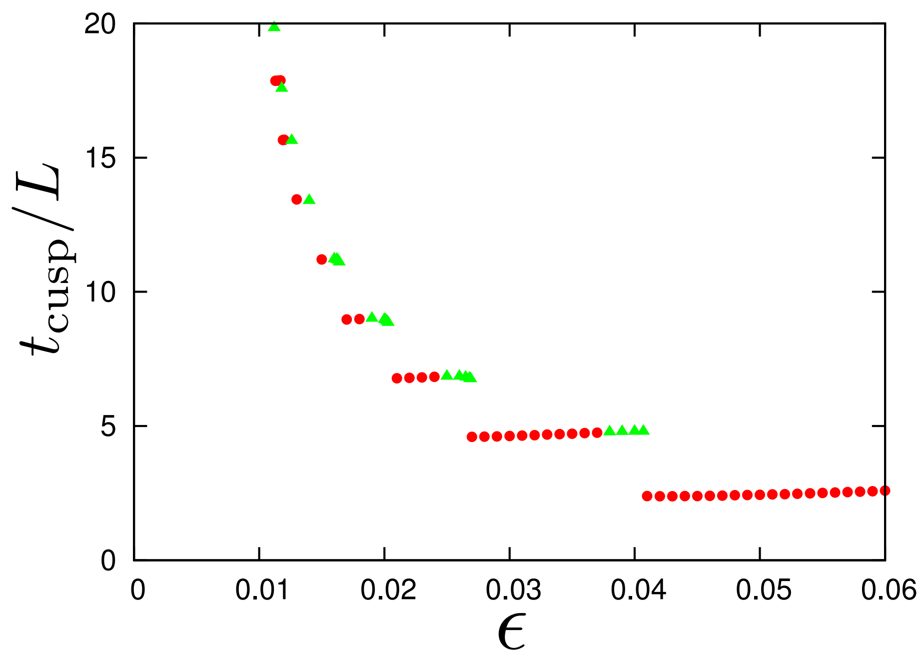


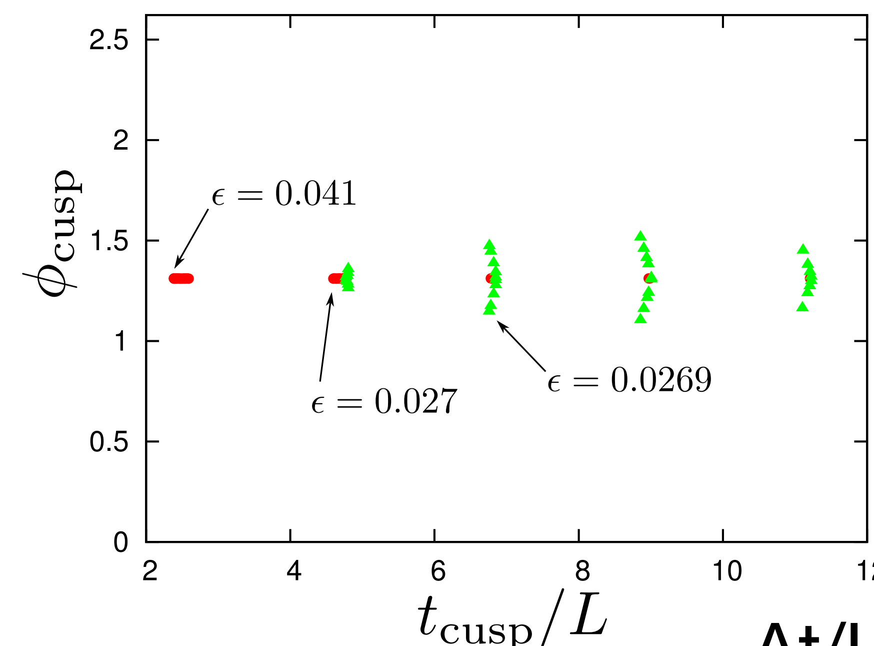
ε=0.025
Δt/L=2





Z_2-symmetric quench

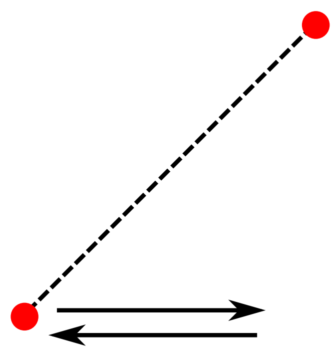


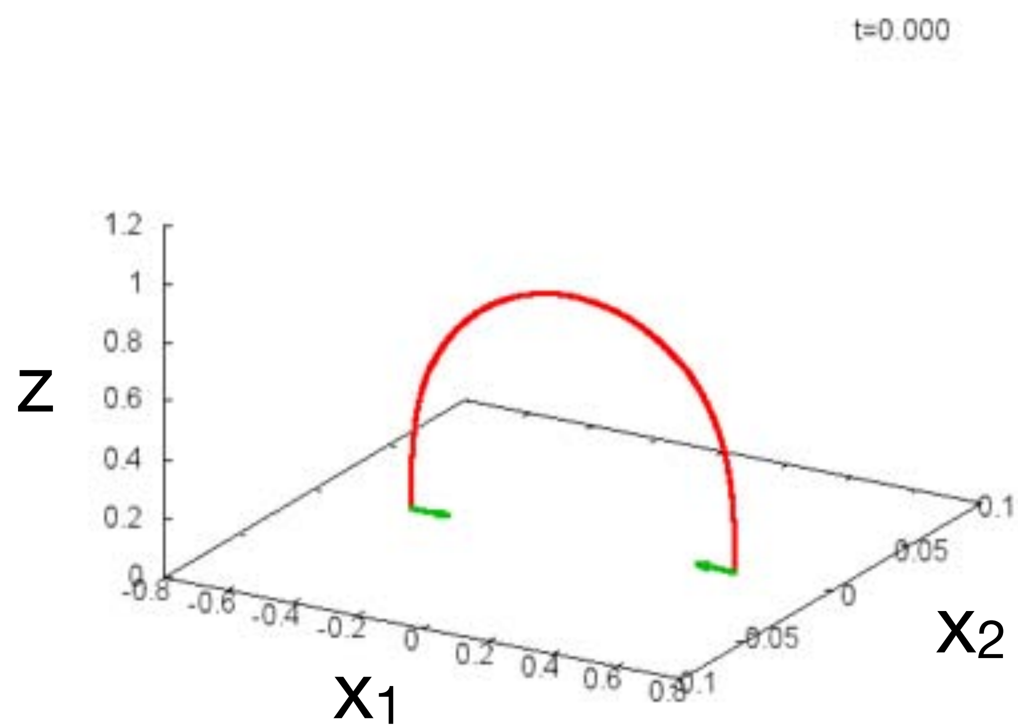


$\Delta t/L=2$

- Formation times are discretized by wave collisions
- First cusp formations by such collisions (red ●). The cusps are pair-created and annihilated.
- Traveling cusps can be formed first (green ▲)

Transverse linear quench



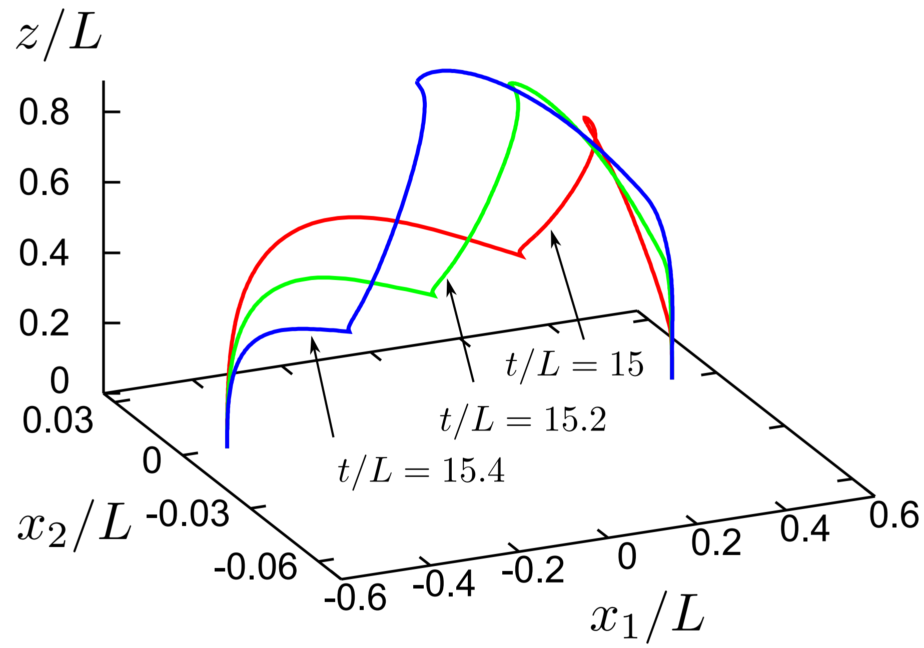


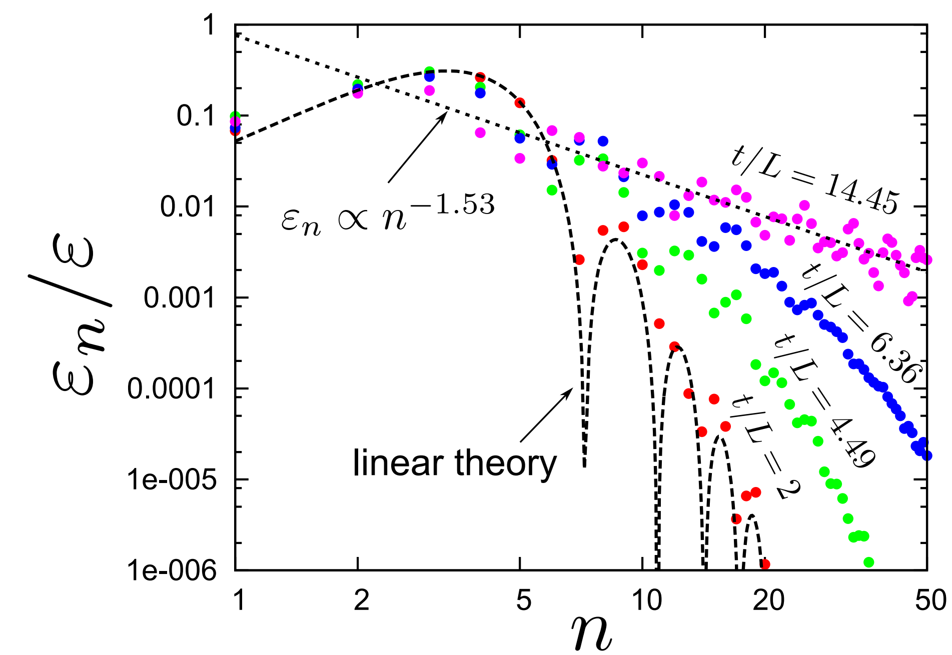
ε=0.03, $\Delta t/L$=2

***Green arrows: forces

String oscillates in 1+3 dim (t,z,x_1,x_2)

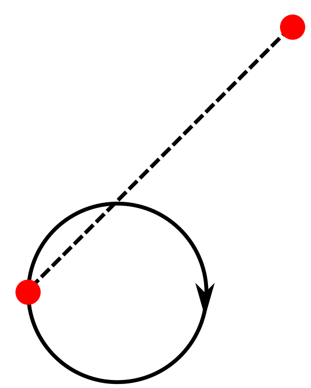
Transverse linear quench

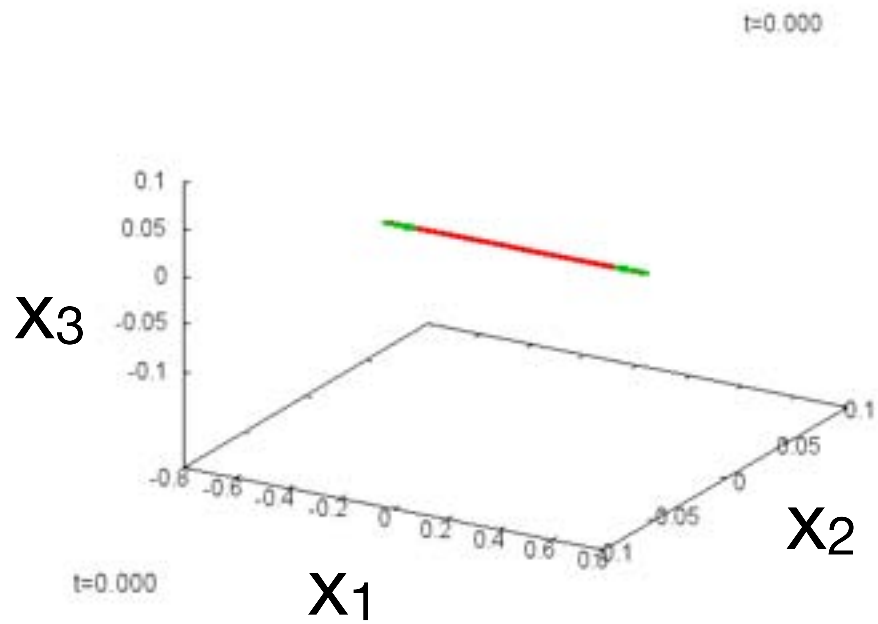




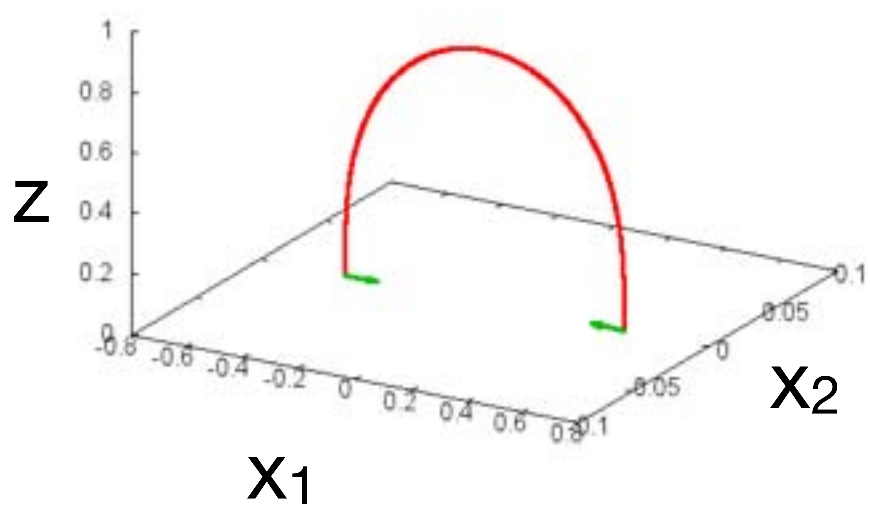
- Cusps are formed at T~14.45
- The energy spectrum keeps a power law

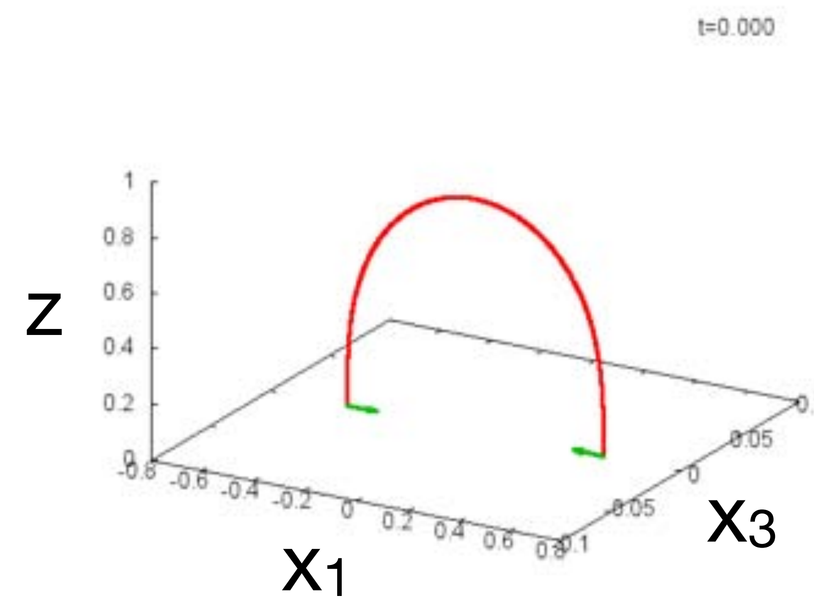
Transverse circular quench





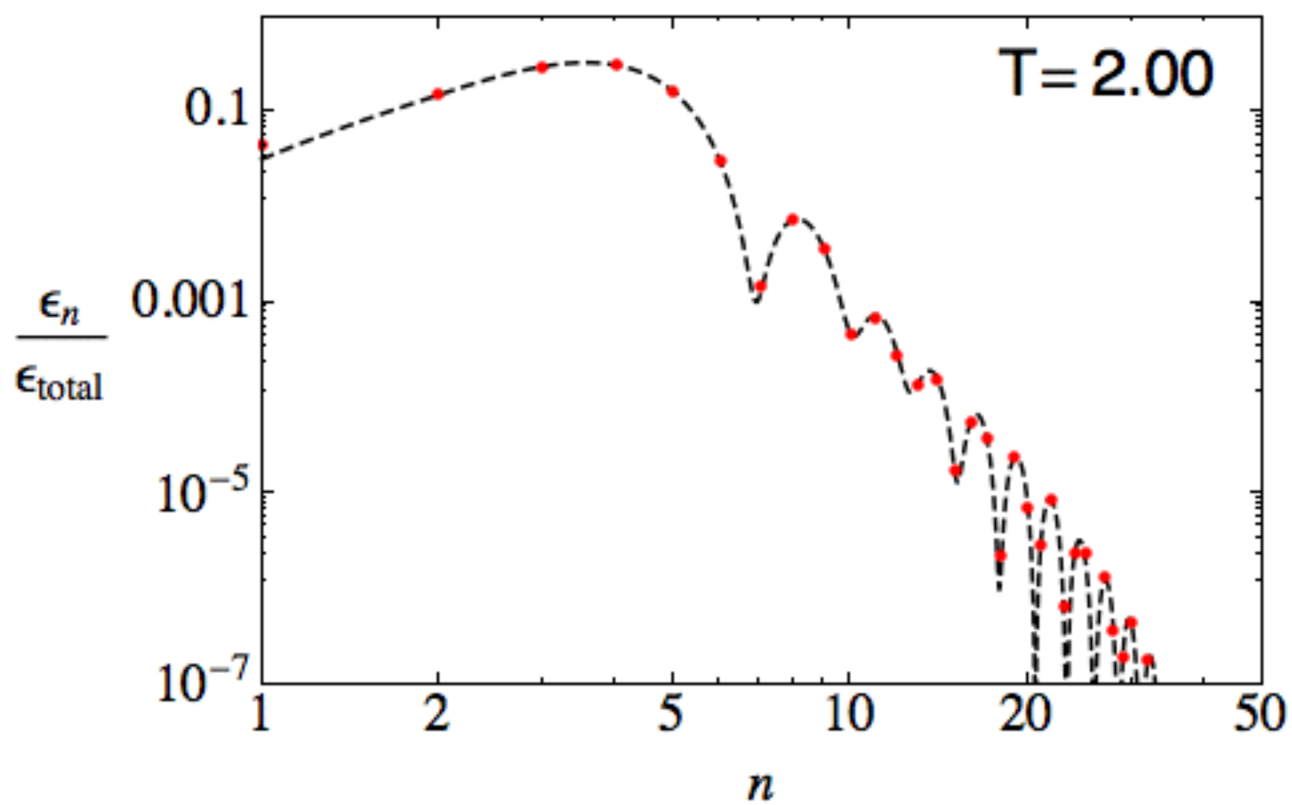
ε=0.02, $\Delta t/L$=2





String oscillates in all 1+4 dim (t,z,x_1,x_2,x_3)

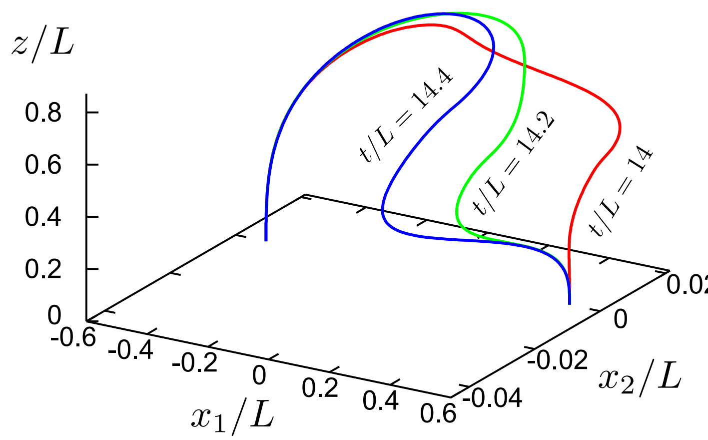
Energy spectrum (Log-log plot)

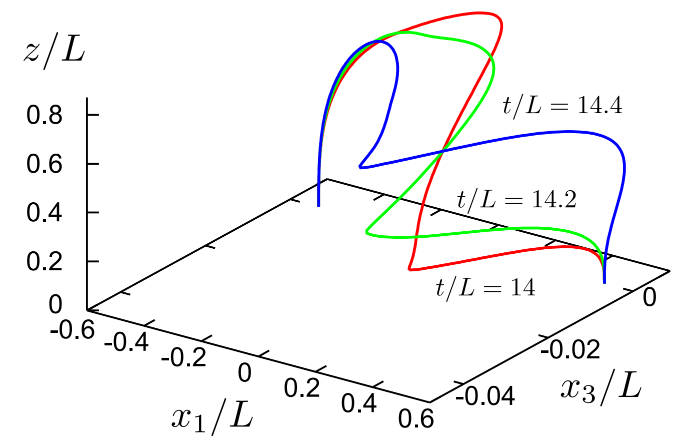


No cusp: no sustaining power law

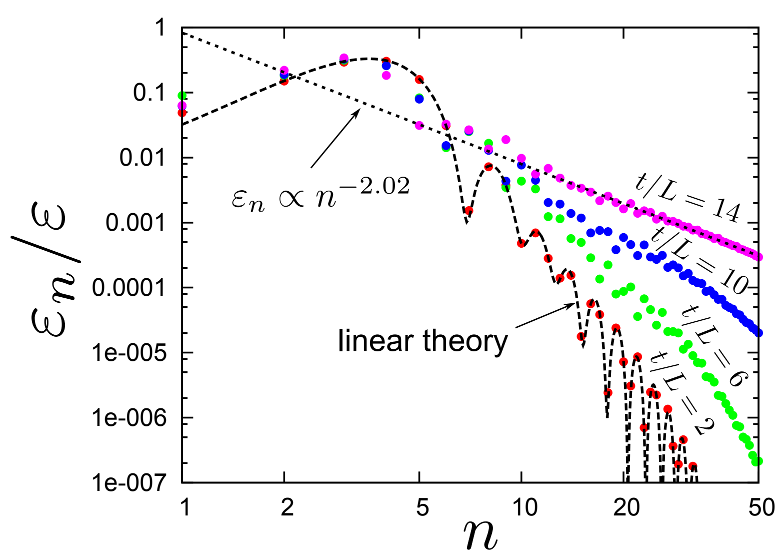
c.f.) Probability of cusp formation is zero if dim>4

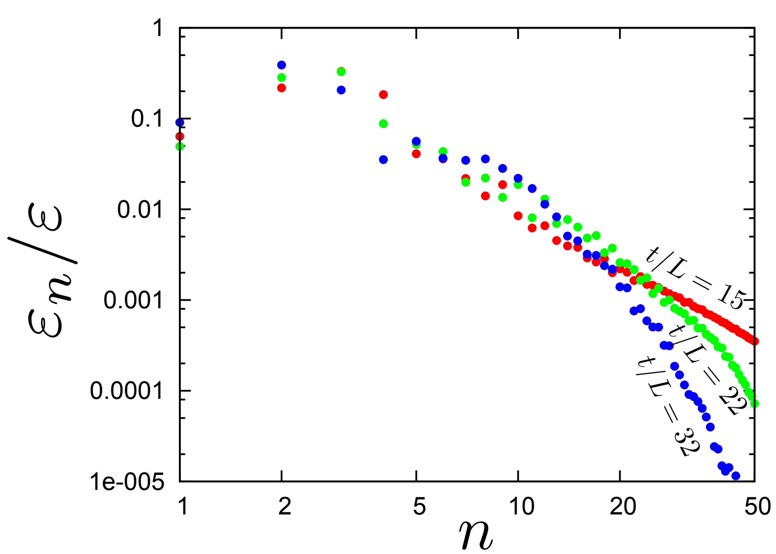
Transverse circular quench





Cuspy, but not real cusps





Direct cascade → inverse cascade

Summary

We computed nonlinear dynamics of the quark-antiquark fundamental string in AdS

- Cusps and turbulent behavior in ≦ 1+3 dim
- No cusp and direct/inverse cascades in 1+4 dim

c.f.) Cosmic strings in flat space

Discussion

Gravitational backreaction may be necessary

- Curvature diverges at the cusps
- AdS gravitational wave bursts?
- Boundary interpretation: gluon bursts?

Future works

- Large amplitude/finite temperature
- Non-conformal backgrounds
- Application to drag force

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