A universal correction to higher spin entanglement entropy

Michael Ferlaino

Swansea University

November 18th, 2014

University of Oxford

1402.0007, 1405.0015

with Shouvik Datta, Justin R. David, S. Prem Kumar

- AdS/CFT as a strong/weak duality
- Study a different regime where the CFT side is not strongly coupled
  - Vasiliev's higher spin gravity on AdS_4 and large N limit of $\mathrm{O}(N)$ vector models in 3D (Klebanov, Polyakov 2002)
  - Vasiliev's higher spin gravity on AdS_3 and large N limit of W_N minimal models (Gaberdiel, Gopakumar 2010)

- Truncate massless higher spin modes to $s \leq N$ and $\mathrm{SL}_N(\mathbb{R}) \times \mathrm{SL}_N(\mathbb{R})$ CS theory (Blencowe 1989)

- Universal results in 2D CFTs
  - Cardy's entropy formula reproduces BTZ thermal entropy

$$S = 2\pi\sqrt{\frac{c}{6}h} + 2\pi\sqrt{\frac{c}{6}\bar{h}}$$

  - Entanglement entropy for single interval at finite temperature $T = \beta^{-1}$ reproduced by holographic prescription Ryu + Takayanagi

$$S_{\mathrm{EE}}(\Delta) = \frac{c}{3}\ln\left[\frac{\beta}{\pi\epsilon}\sinh\left(\frac{\pi\Delta}{\beta}\right)\right]$$

- No results for CFTs deformed by higher spin operators
  - Generalise Cardy's formula in the presence of higher spin charges
  - Compute RE/EE for finite temperature CFTs with extended W symmetries deformed by higher spin operators
  - How do we holographically compute EE in higher spin theories?

Thermodynamics from CFT

- CFTs deformed by $(3,0)+(0,3)$ higher spin operators

$$\mathcal{L}_{\text{CFT}} \to \mathcal{L}_{\text{CFT}} - \mu W(z) - \mu \bar{W}(\bar{z})$$

- Compute partition function in conformal perturbation theory

$$\frac{Z}{Z_{\text{CFT}}} = 1 + \frac{\mu^2}{2} \int \mathrm{d}^2 z_1 \, \mathrm{d}^2 z_2 \, \langle W(z_1) W(z_2) \rangle_{\text{CFT}} + \cdots$$

$$\langle W(z_1) W(z_2) \rangle_{\text{CFT}} = -\frac{5\pi^3 c}{6\beta^6} \sinh^{-6}\left[\frac{\pi}{\beta}(z_1 - z_2)\right]$$

- Explicitly evaluating the integral

$$\frac{\ln(Z)}{L} = \frac{\pi c}{6\beta} + \mu^2 \frac{8\pi^3 c}{9\beta^3} + \cdots$$

- Match with expansion for higher spin black holes originally computed using CFT modular invariance (Gaberdiel, Hartman, Jin 2012)

- Generalised Cardy's formula in presence of higher spin chemical potential μ

Rényi and entanglement entropies

- 2D system divided in $A \oplus A^c$ where A is a spatial segment on the plane $(z = x + it)$. EE for the subsystem A

$$S_{\mathrm{EE}}(A) = -\mathrm{tr}[\,\rho_A \ln(\rho_A)]$$

- EE computed indirectly through Rényi entropies

$$S_n(A) = \frac{1}{1-n} \ln[\,\mathrm{tr}(\rho_A^n)]$$

$$S_{\mathrm{EE}}(A) = \lim_{n \to 1} S_n(A)$$

Replica trick and twist fields

- Use the replica trick: take n copies of the original system and glue the copies together along the cuts which define subsystem A (Calabrese, Cardy 2009)

Such construction creates the Riemann surface R_n.

- REs by means of twist fields (Calabrese, Cardy 2004)

$$\mathrm{tr}(\rho_A^n) = \frac{Z^{(n)}}{Z^n} = \frac{\langle \bar{\sigma}_n(y_1, \bar{y}_1)\sigma_n(y_2, \bar{y}_2)\rangle_{\mathrm{pla}}}{Z^n}$$

$$h(\sigma_n) = \bar{h}(\sigma_n) = \frac{c}{24}(n - n^{-1})$$

- Correlators on R_n

$$\langle U_s(z)\rangle_{R_n} = \frac{\langle \bar{\sigma}_n(y_1, \bar{y}_1)U_s(z)\sigma_n(y_2, \bar{y}_2)\rangle_{\mathrm{pla}}}{\langle \bar{\sigma}_n(y_1, \bar{y}_1)\sigma_n(y_2, \bar{y}_2)\rangle_{\mathrm{pla}}}$$

- Zero temperature EE: $\langle \bar{\sigma}_n(y_1, \bar{y}_1)\sigma_n(y_2, \bar{y}_2)\rangle_{\mathrm{pla}} = \Delta^{-4h(\sigma_n)}$ (Holzhey, Larsen, Wilczek 1994. Calabrese, Cardy 2004)

$$S_{\mathrm{EE}}(\Delta) = \frac{c}{3}\ln\left(\frac{\Delta}{\epsilon}\right) \qquad \Delta = |y_1 - y_2|$$

- Compute RE/EE for deformed CFTs with extended symmetries

$$Z^{(n)} = \int \mathrm{e}^{-I^{(n)} - \delta I^{(n)}} \qquad \delta I^{(n)} = -\mu \int (W + \bar{W})$$

- Series expand

$$\begin{aligned} Z^{(n)} = &+ \langle \bar{\sigma}_n(y_1, \bar{y}_1) \sigma_n(y_2, \bar{y}_2) \rangle_{\text{pla}} \\ &+ \mu \int \mathrm{d}^2 z \, \langle \bar{\sigma}_n(y_1, \bar{y}_1) \sigma_n(y_2, \bar{y}_2) W(z) \rangle_{\text{pla}} \\ &+ \frac{1}{2} \mu^2 \int \mathrm{d}^2 z_1 \, \mathrm{d}^2 z_2 \, \langle \bar{\sigma}_n(y_1, \bar{y}_1) W(z_1) W(z_2) \sigma_n(y_2, \bar{y}_2) \rangle_{\text{pla}} \\ &+ \cdots \end{aligned}$$

- N free complex fermions/bosons have extended W symmetries (Bergshoeff, Pope, Romans, Sezgin, Shen 1990. Bakas, Kiritsis 1990)
- Explicitly compute correlators for free field theories (Datta, David, MF, Kumar 2014)
- Consider $W_\infty(\lambda)$ CFTs generated by infinite tower of conserved currents $s \geq 2$ and recover free theories at $\lambda = 0, 1$ (Kraus, Perlmutter 2011. Gaberdiel, Gopakumar 2012)

- Leading μ^2 correction

$$G^{(4)} = \langle \bar{\sigma}_n(y_1, \bar{y}_1) W(z_1) W(z_2) \sigma_n(y_2, \bar{y}_2) \rangle_{\text{pla}}$$

- By conformal symmetry

$$G^{(4)} = -\frac{5c}{6\pi^2 (z_1 - z_2)^6 |y_1 - y_2|^{4h(\sigma_n)}} F(x) \qquad x = \frac{(z_1 - y_2)(z_2 - y_1)}{(z_1 - y_1)(z_2 - y_2)}$$

- Can express F as

$$F = 1 + F_1 \eta + F_2 \eta^2 \qquad \eta = x + x^{-1} - 2$$

$$F_1 = \frac{n^2 - 1}{4n} \qquad F_2 = \frac{(n^2-1)^2}{120 n^3} - \frac{n^2 - 1}{40 n^3}$$

General result $\forall \lambda$

- Cylinder with coordinates $u = \sigma + i\tau$

$$S_n(\Delta) = \frac{c(n+1)}{6n} \ln\left|\frac{\beta}{\pi}\sinh\left(\frac{\pi\Delta}{\beta}\right)\right| + \frac{5\pi^4 c\mu^2}{6\beta^6(n-1)} S_n^{(2)} + \cdots$$

$$S_n^{(2)} = \int\limits_{\mathbb{R}\times S^1} d^2u_1 d^2u_2 \, \sinh^{-6}\left[\frac{\pi(u_1-u_2)}{\beta}\right] (F_1\eta_{cyl} + F_2\eta_{cyl}^2)$$

$$S_{\rm EE}(\Delta) = +\,\frac{c}{3}\ln\left|\frac{\beta}{\pi}\sinh\left(\frac{\pi\Delta}{\beta}\right)\right| + \frac{c\mu^2}{\beta^2}\left[\frac{32\pi^2}{9}\frac{\pi\Delta}{\beta}\coth\left(\frac{\pi\Delta}{\beta}\right) - \frac{20\pi^2}{9}\right]$$
$$- \frac{4\pi^2 c\mu^2}{3\beta^2}\operatorname{cosech}^2\left(\frac{\pi\Delta}{\beta}\right)\left[\left(\frac{\pi\Delta}{\beta}\right)^2 + \left(\frac{\pi\Delta}{\beta}\coth\left(\frac{\pi\Delta}{\beta}\right) - 1\right)^2\right]$$
$$+\cdots$$

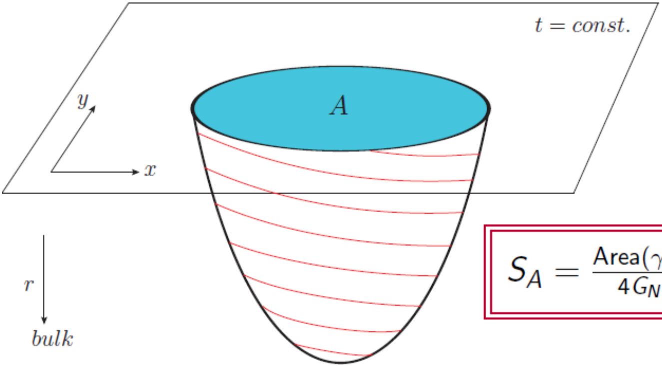
- Leading correction to EE is universal (λ independent)
- Extensive limit $\Delta \gg \beta$ it reduces to the thermal entropy of a higher spin black hole (at order μ^2)

$$\frac{2\pi S_{\rm EE}}{\Delta} = S_{\rm BH}$$

- Subleading corrections to EE will be λ dependent

Holographic EE and Wilson lines

- In AdS/CFT, EE is holographically computed with prescription of Ryu and Takayanagi (Ryu, Takayanagi 2006)



$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N}$$

- Einstein gravity on AdS_3 can be cast as $\mathrm{SL}_2(\mathbb{R}) \times \mathrm{SL}_2(\mathbb{R})$ CS gauge theory (Achucarro, Townsend 1986. Witten 1988)

- Triad e and dualised spin connection ω in $\mathrm{sl}_2(\mathbb{R})$ gauge connections $A = \omega + e$ and $\bar{A} = \omega - e$

$$I_{\mathrm{CS}}[A] = \frac{k}{4\pi} \int \mathrm{tr}\left(A \wedge \mathrm{d}A + \frac{2}{3} A \wedge A \wedge A \right) \qquad k = \frac{1}{4G_{\mathrm{N}}}$$

- One finds

$$I[A, \bar{A}] = I_{\mathrm{CS}}[A] - I_{\mathrm{CS}}[\bar{A}] = I_{\mathrm{EH}}[e, \omega] + \text{boundary term}$$

- EOM $\to \mathrm{d}A + A \wedge A = 0$ and $\mathrm{d}\bar{A} + \bar{A} \wedge \bar{A} = 0$

- Generalise $\mathrm{SL}_2(\mathbb{R}) \to \mathrm{SL}_N(\mathbb{R})$: gravity interacting with tower $s = 3, 4, \ldots, N$

- Asymptotic symmetry algebra is semiclassical $(c \gg 1)$ W_N algebra (Campoleoni, Fredenhagen, Pfenninger, Theisen 2010)

- Proposals, based on Wilson line functionals, to compute EE in $\mathrm{SL}_N(\mathbb{R}) \times \mathrm{SL}_N(\mathbb{R})$ higher spin gravities (Ammon, Castro, Iqbal 2013. de Boer, Jottar 2013)

- CFT picture is most naturally compatible with the following proposal (de Boer, Jottar 2013)

$$S_{\mathrm{EE}}(\Delta) = \frac{c}{24} \ln W_{\mathrm{L}}(P, Q)$$

$$W_{\mathrm{L}}(P, Q) = \lim_{\rho \to \infty} \mathrm{tr} \left[\mathrm{P} \exp \int_P^Q \bar{A}_{\bar{z}} \, \mathrm{d}\bar{z} \; \mathrm{P} \exp \int_P^Q A_z \, \mathrm{d}z \right]$$

where (P, Q) are the endpoints of the entangling interval

- Recover known results for $\mathrm{SL}_2(\mathbb{R}) \times \mathrm{SL}_2(\mathbb{R})$

- CFTs with extended symmetries are dual to higher spin black holes

- Apply prescription to $\mathrm{SL}_3(\mathbb{R}) \times \mathrm{SL}_3(\mathbb{R})$ black hole built by Gutperle and Kraus (Gutperle, Kraus 2011)

- Extracting the leading large ρ limit and expanding to order μ^2: agreement with CFT result

$$S_{\mathrm{EE}}(\Delta) = +\frac{c}{3}\ln\left|\frac{\beta}{\pi\epsilon}\sinh\left(\frac{\pi\Delta}{\beta}\right)\right| + \frac{c\mu^2}{\beta^2}\left[\frac{32\pi^2}{9}\frac{\pi\Delta}{\beta}\coth\left(\frac{\pi\Delta}{\beta}\right) - \frac{20\pi^2}{9}\right] - \frac{4\pi^2 c\mu^2}{3\beta^2}\mathrm{cosech}^2\left(\frac{\pi\Delta}{\beta}\right)\left[\left(\frac{\pi\Delta}{\beta}\right)^2 + \left(\frac{\pi\Delta}{\beta}\coth\left(\frac{\pi\Delta}{\beta}\right) - 1\right)^2\right] + \cdots$$

Conclusions

- Computed leading correction to RE/EE for CFTs with $W_\infty(\lambda)$ symmetries deformed by higher spin chemical potential μ
- CFT result is universal (λ independent)
- Match holographic computation performed in the $\mathrm{SL}_3(\mathbb{R}) \times \mathrm{SL}_3(\mathbb{R})$ CS theory
  - Gravity computation probes boundary CFTs with W_3 symmetries with $c \to \infty$ whereas the CFT results are obtained at finite c

Future directions

- Analyse entropies for free field theories defined on the torus
- Compute REs from the $\mathrm{SL}_N(\mathbb{R}) \times \mathrm{SL}_N(\mathbb{R})$ gravitational theory and match with CFT universal results
- Generalise the holographic prescription to compute RE/EE in Vasiliev's theories with $\mathrm{hs}(\lambda)$ gauge symmetry