A universal correction to higher spin entanglement entropy

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AdS/CFT as a strong/weak duality

Study a different regime where the CFT side is not strongly coupled

- Vasiliev’s higher spin gravity on AdS$_4$ and large $N$ limit of $O(N)$ vector models in 3D (Klebanov, Polyakov 2002)

- Vasiliev’s higher spin gravity on AdS$_3$ and large $N$ limit of $W_N$ minimal models (Gaberdiel, Gopakumar 2010)
• Truncate massless higher spin modes to $s \leq N$ and $SL_N(\mathbb{R}) \times SL_N(\mathbb{R})$ CS theory (Blencowe 1989)

• Universal results in 2D CFTs

  ▶ Cardy’s entropy formula reproduces BTZ thermal entropy

  $$S = 2\pi \sqrt{\frac{c}{6} h} + 2\pi \sqrt{\frac{c}{6} \bar{h}}$$

  ▶ Entanglement entropy for single interval at finite temperature $T = \beta^{-1}$ reproduced by holographic prescription Ryu + Takayanagi

  $$S_{EE}(\Delta) = \frac{c}{3} \ln \left[ \frac{\beta}{\pi \epsilon} \sinh \left( \frac{\pi \Delta}{\beta} \right) \right]$$
No results for CFTs deformed by higher spin operators

- Generalise Cardy's formula in the presence of higher spin charges

- Compute RE/EE for finite temperature CFTs with extended $W$ symmetries deformed by higher spin operators

- How do we holographically compute EE in higher spin theories?
CFTs deformed by \((3, 0) + (0, 3)\) higher spin operators

\[ \mathcal{L}_{\text{CFT}} \rightarrow \mathcal{L}_{\text{CFT}} - \mu W(z) - \mu \bar{W}(\bar{z}) \]

Compute partition function in conformal perturbation theory

\[
\frac{Z}{Z_{\text{CFT}}} = 1 + \frac{\mu^2}{2} \int d^2 z_1 d^2 z_2 \langle W(z_1)W(z_2) \rangle_{\text{CFT}} + \cdots
\]

\[
\langle W(z_1)W(z_2) \rangle_{\text{CFT}} = -\frac{5\pi^3 c}{6\beta^6} \sinh^{-6} \left[ \frac{\pi}{\beta} (z_1 - z_2) \right]
\]
Explicitly evaluating the integral

\[
\frac{\ln(Z)}{L} = \frac{\pi c}{6\beta} + \mu^2 \frac{8\pi^3 c}{9\beta^3} + \cdots
\]

Match with expansion for higher spin black holes originally computed using CFT modular invariance (Gaberdiel, Hartman, Jin 2012)

Generalised Cardy’s formula in presence of higher spin chemical potential \( \mu \)
Rényi and entanglement entropies

- 2D system divided in $A \oplus A^c$ where $A$ is a spatial segment on the plane ($z = x + it$). EE for the subsystem $A$

$$S_{EE}(A) = - \text{tr}[\rho_A \ln(\rho_A)]$$

- EE computed indirectly through Rényi entropies

$$S_n(A) = \frac{1}{1-n} \ln[\text{tr}(\rho_A^n)]$$

$$S_{EE}(A) = \lim_{n \to 1} S_n(A)$$
Replica trick and twist fields

- Use the replica trick: take $n$ copies of the original system and glue the copies together along the cuts which define subsystem $A$ (Calabrese, Cardy 2009)

Such construction creates the Riemann surface $R_n$. 

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REs by means of twist fields (Calabrese, Cardy 2004)

$$\text{tr}(\rho^n_A) = \frac{Z^{(n)}}{Z^n} = \frac{\langle \bar{\sigma}_n(y_1, \bar{y}_1)\sigma_n(y_2, \bar{y}_2) \rangle_{\text{pla}}}{Z^n}$$

$$h(\sigma_n) = \bar{h}(\sigma_n) = \frac{c}{24}(n - n^{-1})$$

Correlators on $R_n$

$$\langle U_s(z) \rangle_{R_n} = \frac{\langle \bar{\sigma}_n(y_1, \bar{y}_1)U_s(z)\sigma_n(y_2, \bar{y}_2) \rangle_{\text{pla}}}{\langle \bar{\sigma}_n(y_1, \bar{y}_1)\sigma_n(y_2, \bar{y}_2) \rangle_{\text{pla}}}$$

Zero temperature EE: $\langle \bar{\sigma}_n(y_1, \bar{y}_1)\sigma_n(y_2, \bar{y}_2) \rangle_{\text{pla}} = \Delta^{-4}h(\sigma_n)$ (Holzhey, Larsen, Wilczek 1994. Calabrese, Cardy 2004)

$$S_{\text{EE}}(\Delta) = \frac{c}{3} \ln\left(\frac{\Delta}{\epsilon}\right) \quad \Delta = |y_1 - y_2|$$
- Compute RE/EE for deformed CFTs with extended symmetries

\[ Z^{(n)} = \int e^{-I^{(n)} - \delta I^{(n)}} \delta I^{(n)} = -\mu \int (W + \bar{W}) \]

- Series expand

\[ Z^{(n)} = + \langle \bar{\sigma}_n(y_1, \bar{y}_1)\sigma_n(y_2, \bar{y}_2) \rangle_{\text{pla}} \]
\[ + \mu \int d^2 z \langle \bar{\sigma}_n(y_1, \bar{y}_1)\sigma_n(y_2, \bar{y}_2)W(z) \rangle_{\text{pla}} \]
\[ + \frac{1}{2} \mu^2 \int d^2 z_1 d^2 z_2 \langle \bar{\sigma}_n(y_1, \bar{y}_1)W(z_1)W(z_2)\sigma_n(y_2, \bar{y}_2) \rangle_{\text{pla}} \]
\[ + \cdots \]
- $N$ free complex fermions/bosons have extended $W$ symmetries (Bergshoeff, Pope, Romans, Sezgin, Shen 1990. Bakas, Kiritsis 1990)

- Explicitly compute correlators for free field theories (Datta, David, MF, Kumar 2014)

- Consider $W_\infty(\lambda)$ CFTs generated by infinite tower of conserved currents $s \geq 2$ and recover free theories at $\lambda = 0, 1$ (Kraus, Perlmutter 2011. Gaberdiel, Gopakumar 2012)
Leading $\mu^2$ correction

\[ G^{(4)} = \langle \bar{\sigma}_n(y_1, \bar{y}_1)W(z_1)W(z_2)\sigma_n(y_2, \bar{y}_2) \rangle_{\text{pla}} \]

By conformal symmetry

\[ G^{(4)} = -\frac{5c}{6\pi^2(z_1 - z_2)^6|y_1 - y_2|^{4h(\sigma_n)}} F(x) \quad x = \frac{(z_1 - y_2)(z_2 - y_1)}{(z_1 - y_1)(z_2 - y_2)} \]

Can express $F$ as

\[ F = 1 + F_1\eta + F_2\eta^2 \quad \eta = x + x^{-1} - 2 \]

\[ F_1 = \frac{n^2 - 1}{4n} \quad F_2 = \frac{(n^2 - 1)^2}{120n^3} - \frac{n^2 - 1}{40n^3} \]

General result $\forall \lambda$
Cylinder with coordinates $u = \sigma + i\tau$

$$S_n(\Delta) = \frac{c(n+1)}{6n} \ln \left| \frac{\beta}{\pi} \sinh \left( \frac{\pi \Delta}{\beta} \right) \right| + \frac{5\pi^4 c \mu^2}{6\beta^6 (n-1)} S_n^{(2)} + \cdots$$

$$S_n^{(2)} = \int_{\mathbb{R} \times S^1} d^2 u_1 d^2 u_2 \sinh^{-6} \left[ \frac{\pi (u_1 - u_2)}{\beta} \right] \left( F_1 \eta_{cyl} + F_2 \eta_{cyl}^2 \right)$$
$S_{EE}(\Delta) = + \frac{c}{3} \ln \frac{\beta}{\pi} \sinh \left( \frac{\pi \Delta}{\beta} \right) + \frac{c\mu^2}{\beta^2} \left[ \frac{32\pi^2}{9} \frac{\pi \Delta}{\beta} \coth \left( \frac{\pi \Delta}{\beta} \right) - \frac{20\pi^2}{9} \right]$

$- \frac{4\pi^2 c\mu^2}{3\beta^2} \text{cosech}^2 \left( \frac{\pi \Delta}{\beta} \right) \left[ \left( \frac{\pi \Delta}{\beta} \right)^2 + \left( \frac{\pi \Delta}{\beta} \coth \left( \frac{\pi \Delta}{\beta} \right) - 1 \right)^2 \right]$

$+ \cdots$

- **Leading correction to EE is universal ($\lambda$ independent)**

- **Extensive limit $\Delta \gg \beta$** it reduces to the thermal entropy of a higher spin black hole (at order $\mu^2$)

\[
\frac{2\pi S_{EE}}{\Delta} = S_{BH}
\]

- **Subleading corrections to EE will be $\lambda$ dependent**
In AdS/CFT, EE is holographically computed with prescription of Ryu and Takayanagi (Ryu, Takayanagi 2006)

\[ S_A = \frac{\text{Area}(\gamma_A)}{4G_N} \]
• Einstein gravity on AdS_3 can be cast as SL_2(\mathbb{R}) \times SL_2(\mathbb{R}) CS gauge theory (Achucarro, Townsend 1986. Witten 1988)

• Triad e and dualised spin connection \omega in sl_2(\mathbb{R}) gauge connections \( A = \omega + e \) and \( \tilde{A} = \omega - e \)

\[
I_{CS}[A] = \frac{k}{4\pi} \int \text{tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) \quad k = \frac{1}{4G_N}
\]

• One finds

\[
I[A, \tilde{A}] = I_{CS}[A] - I_{CS}[\tilde{A}] = I_{EH}[e, \omega] + \text{boundary term}
\]
- EOM $\rightarrow dA + A \wedge A = 0$ and $d\bar{A} + \bar{A} \wedge \bar{A} = 0$

- Generalise $\text{SL}_2(\mathbb{R}) \rightarrow \text{SL}_N(\mathbb{R})$: gravity interacting with tower $s = 3, 4, \ldots, N$

- Asymptotic symmetry algebra is semiclassical ($c \gg 1$) $W_N$ algebra (Campoleoni, Fredenhagen, Pfenninger, Theisen 2010)

- Proposals, based on Wilson line functionals, to compute EE in $\text{SL}_N(\mathbb{R}) \times \text{SL}_N(\mathbb{R})$ higher spin gravities (Ammon, Castro, Iqbal 2013. de Boer, Jottar 2013)
CFT picture is most naturally compatible with the following proposal (de Boer, Jottar 2013)

\[ S_{EE}(\Delta) = \frac{c}{24} \ln W_L(P, Q) \]

\[ W_L(P, Q) = \lim_{\rho \to \infty} \text{tr} \left[ P \exp \int_P^Q \bar{A}_z \, d\bar{z} \, P \exp \int_P^Q A_z \, dz \right] \]

where \((P, Q)\) are the endpoints of the entangling interval

Recover known results for \(SL_2(\mathbb{R}) \times SL_2(\mathbb{R})\)
CFTs with extended symmetries are dual to higher spin black holes

Apply prescription to $\text{SL}_3(\mathbb{R}) \times \text{SL}_3(\mathbb{R})$ black hole built by Gutperle and Kraus (Gutperle, Kraus 2011)

Extracting the leading large $\rho$ limit and expanding to order $\mu^2$: agreement with CFT result

$$S_{\text{EE}}(\Delta) = + \frac{c}{3} \ln \left| \frac{\beta}{\pi \epsilon} \sinh \left( \frac{\pi \Delta}{\beta} \right) \right| + \frac{c \mu^2}{\beta^2} \left[ \frac{32 \pi^2}{9} \frac{\pi \Delta}{\beta} \coth \left( \frac{\pi \Delta}{\beta} \right) - \frac{20 \pi^2}{9} \right]$$

$$- \frac{4 \pi^2 c \mu^2}{3 \beta^2} \text{cosech}^2 \left( \frac{\pi \Delta}{\beta} \right) \left[ \left( \frac{\pi \Delta}{\beta} \right)^2 + \left( \frac{\pi \Delta}{\beta} \coth \left( \frac{\pi \Delta}{\beta} \right) - 1 \right)^2 \right]$$

$$+ \cdots$$
Computed leading correction to RE/EE for CFTs with $W_\infty(\lambda)$ symmetries deformed by higher spin chemical potential $\mu$

CFT result is universal ($\lambda$ independent)

Match holographic computation performed in the $SL_3(\mathbb{R}) \times SL_3(\mathbb{R})$ CS theory

Gravity computation probes boundary CFTs with $W_3$ symmetries with $c \to \infty$ whereas the CFT results are obtained at finite $c$
Future directions

- Analyse entropies for free field theories defined on the torus

- Compute REs from the $SL_N(\mathbb{R}) \times SL_N(\mathbb{R})$ gravitational theory and match with CFT universal results

- Generalise the holographic prescription to compute RE/EE in Vasiliev’s theories with $h_s(\lambda)$ gauge symmetry