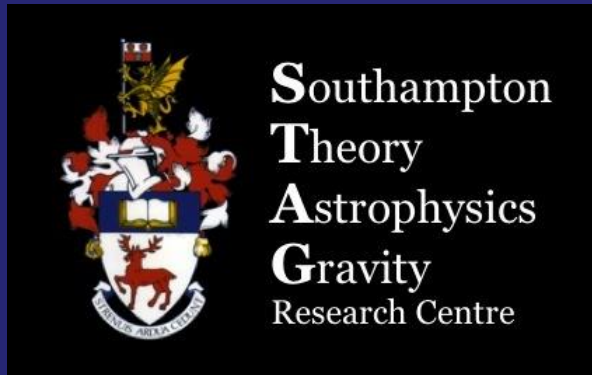


# Dynamic AdS/QCD and the spectra of gauge theories

Nick Evans      University of Southampton



New understanding of holographic descriptions of mesons

A stab at predicting the spectra of arbitrary AF gauge theories

Oxford October 2014

# Back track to 2005

Top down models of chiral symmetry breaking

Bottom up AdS/QCD

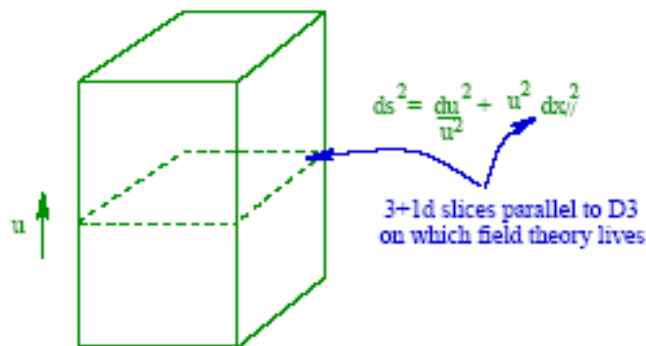
# Building Holographic QCD

## AdS/CFT Correspondence

Maldacena, Witten...

4d strongly coupled  $\mathcal{N}=4$  SYM = IIB strings on  $AdS_5 \times S^5$

Pretty well established by this point!



$u$  corresponds to energy (RG) scale in field theory

The SUGRA fields act as sources

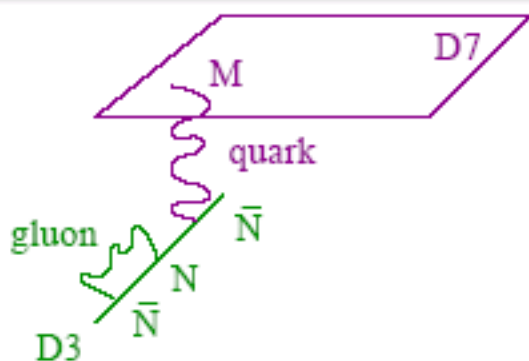
$$\int d^4x \phi_{SUGRA}(u_0) \lambda \lambda$$

eg asymptotic solution ( $u \rightarrow \infty$ ) of scalar

$$\varphi \simeq \frac{m}{u} + \frac{\langle \lambda \lambda \rangle}{u^3}$$

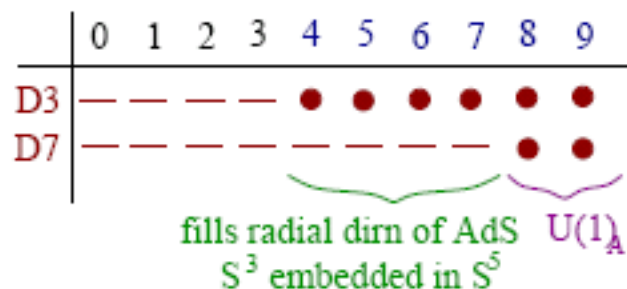
# Adding Quarks

Bertolini, DiVecchia...; Polchinski, Grana; Karch, Katz...



Quarks can be introduced via D7 branes in AdS

The brane set up is



We will treat D7 as a probe - quenching in the gauge theory.

Minimize D7 world volume with DBI action

$$S_{D7} = -T_7 \int d\xi^8 \sqrt{P[G_{ab}]}, \quad P[G_{ab}] = G_{MN} \frac{dx^M}{d\xi^a} \frac{dx^N}{d\xi^b}$$

The D7 lie flat in AdS. We can consider fluctuations that describe R-chargeless mesons

$$W_6 + iW_5 = d + \delta(\rho) e^{ik \cdot x}$$

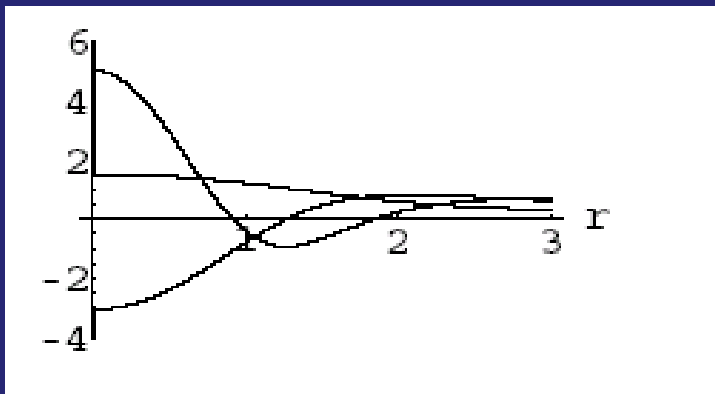
$\delta$  satisfies a linearized EoM

$$\partial_\rho^2 \delta + \frac{3}{\rho} \partial_\rho \delta + \frac{M^2}{(\rho^2 + 1)^2} \delta = 0$$

and the mass spectrum is

$$M = \frac{2d}{R^2} \sqrt{(n+1)(n+2)} \sim \frac{2m}{\sqrt{\lambda_{YM}}}$$

Tightly bound - meson masses suppressed relative to quark mass



Orthonormal wave functions

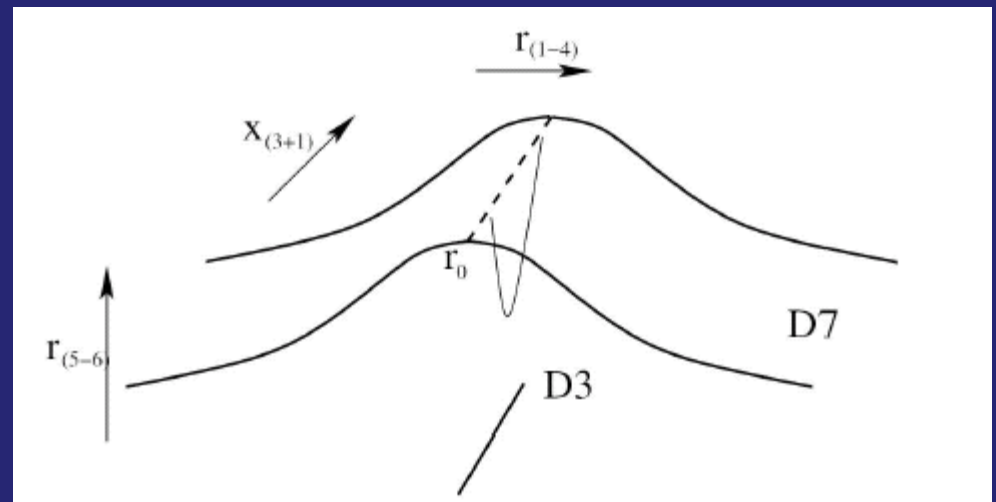
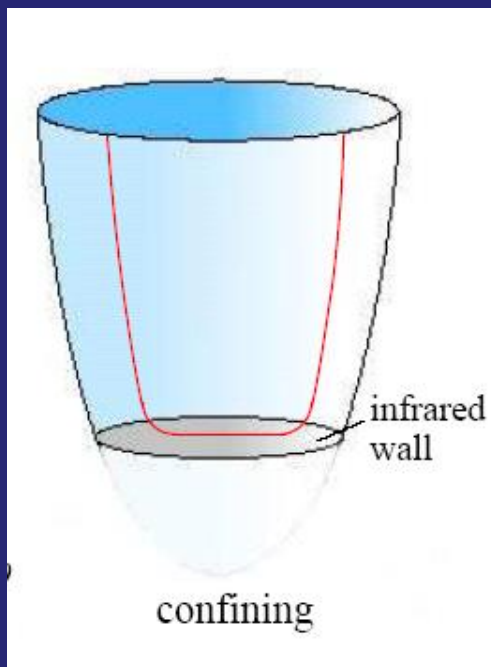
# Add Confinement and Chiral Symmetry Breaking

$$ds^2 = \frac{r^2}{R^2} A^2(r) dx_{3+1}^2 + \frac{R^2}{r^2} dr^2,$$

$$A(r) = \left(1 - \left(\frac{r_w}{r}\right)^8\right)^{1/4}, \quad e^\phi = \left(\frac{1 + (r_w/r)^4}{1 - (r_w/r)^4}\right)^{\sqrt{3/2}}$$

Dilaton Flow Geometry: Gubser, Sfetsos

Here, this is just a simple, back reacted, repulsive, hard wall....



Babington et al, Ghoroku..

Erdmenger,  
NE,Kirsch,  
Threlfall  
0711.4467

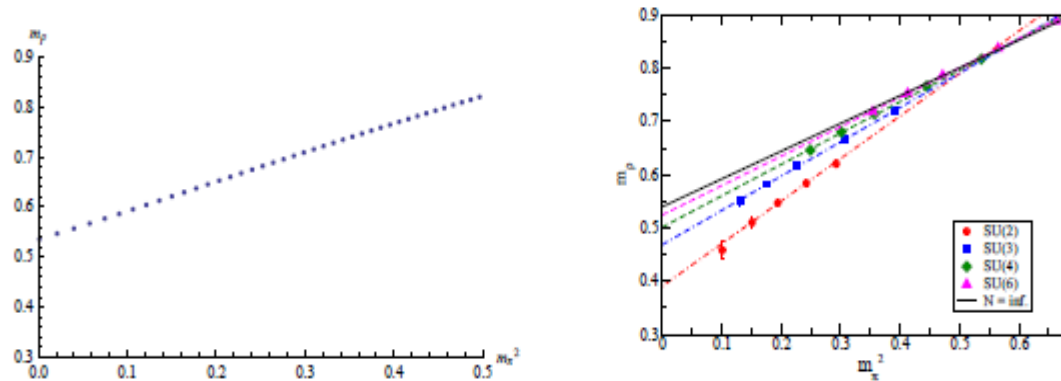


Figure 6.5: A plot of  $m_\rho$  vs  $m_\pi^2$  in the Constable-Myers background on the left (we thank Andrew Tedder for generating this plot). Lattice data [20] (preliminary, quenched and at finite spacing) for the same quantity is also shown on the right.

These models live at very large coupling – QCD is at intermediate coupling strength...

The running coupling is rather different (UV strong fixed point)

Success beyond caricature seems surprising... how does one improve towards QCD?

# Back track to 2005

Top down models of chiral symmetry breaking

Bottom up AdS/QCD



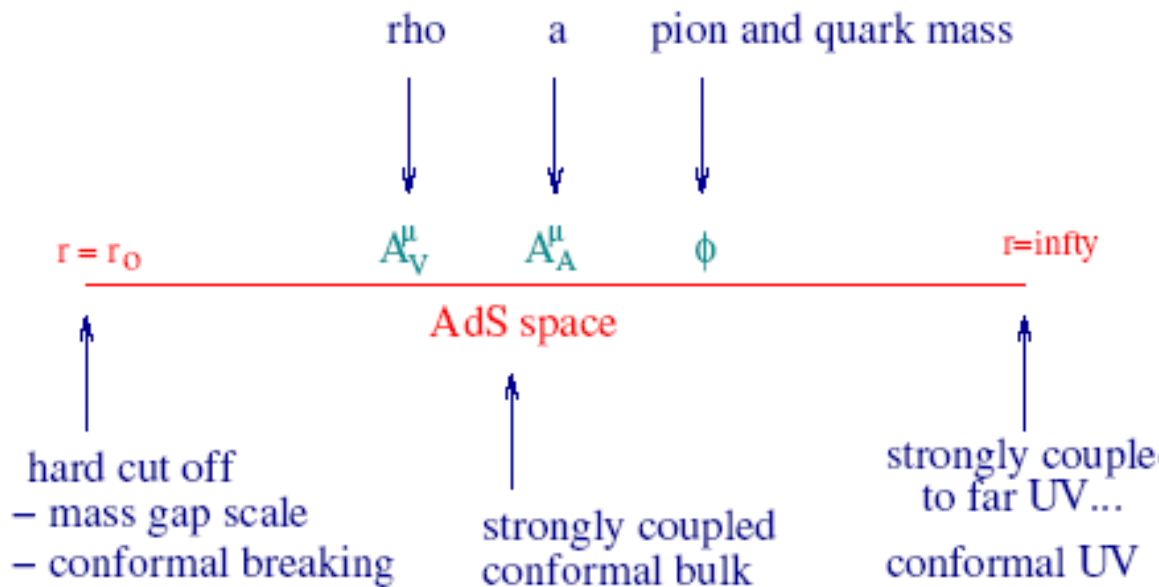
# Traditional AdS/QCD

Son, Stephanov,  
Erlich, Katz

$$S = \int_{r_0}^{\infty} d^5x \sqrt{-g} \text{Tr} \left\{ |DX|^2 + 3|X|^2 - \frac{1}{4g_5^2} (F_L^2 + F_R^2) \right\}$$

Parameter  
count

$r_0$   
 $c = qq$   
 $m$   
 $g_5$



Observable	Measured (MeV)	AdS A (MeV)	AdS B (MeV)
$m_\pi$	$139.6 \pm 0.0004$	139.6*	141
$m_\rho$	$775.8 \pm 0.5$	775.8*	832
$m_{a_1}$	$1230 \pm 40$	1363	1220
$f_\pi$	$92.4 \pm 0.35$	92.4*	84.0
$F_\rho^{1/2}$	$345 \pm 8$	329	353
$F_{a_1}^{1/2}$	$433 \pm 13$	486	440

The basic ideas are remarkably good... but... to systematically move to QCD we would need to:

**IR improve** – include all operators that are non-zero in the vacuum and back react them on each other

**UV perfect** – need to match the running of operators to the true perturbative QCD values in the UV....

**No dynamics** – inputting condensate and hard wall and fitting is only dynamics....

Of course as in any effective description of QCD this is overwhelming and you end up just re-parameterizing the data.... In QCD the **regime of validity** of the gravity description is **very tight**...

# A New Insight in 2012

It had been quite hard to understand how to put in the dynamics of an individual gauge theory...

# Holographic Models for QCD in the Veneziano Limit

Matti Jarvinen, Elias Kiritsis

1112.1261

We model the  $qq$  condensate by a scalar in “AdS”...

## Breitenlohner-Freedman Bound

A scalar in AdS is stable until

$$m^2 < -4$$

$$\text{ie } \Delta < 2$$

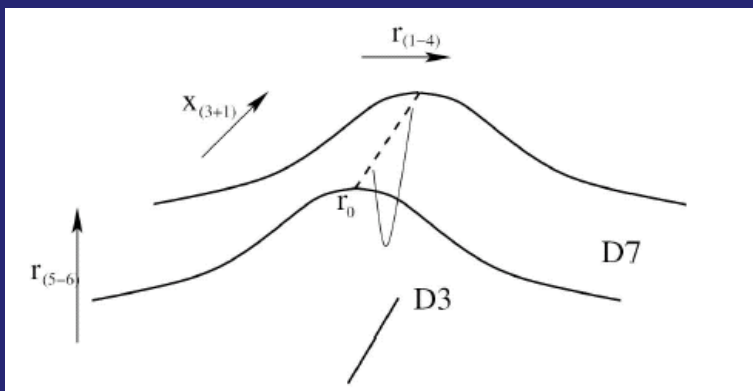
$$m^2 = \Delta(\Delta - 4)$$

A hard prediction that matches gap equation-ology for the on-set of chiral symmetry breaking...

Holographic models work for QCD mesons because they are describing the running of the anomalous dimension of  $qq$  rather than the running of the coupling...

# D3/ Probe D7 Model

Alvares, NE, Kim,  
1204.2474



$$S_{D7} = -T \int d^4x d\rho \rho^3 e^\phi \sqrt{1 + (\partial_\rho L)^2}$$

$$S = \int d\rho \lambda(r) \rho^3 \sqrt{1 + L'^2} \quad \text{We expand for small } L$$

$$S = \int d\rho \left( \frac{1}{2} \lambda(r) \Big|_{L=0} \rho^3 L'^2 + \rho^3 \frac{d\lambda}{dL^2} \Big|_{L=0} L^2 \right)$$

we can now make a coordinate transformation

$$\lambda(\rho) \rho^3 \frac{d}{d\rho} = \tilde{\rho}^3 \frac{d}{d\tilde{\rho}}, \quad \tilde{\rho} = \sqrt{\frac{1}{2} \frac{1}{\int_\rho^\infty \frac{d\rho}{\lambda \rho^3}}}$$

$$L = \tilde{\rho} \phi$$

$$S = \int d\tilde{\rho} \frac{1}{2} \left( \tilde{\rho}^5 \phi'^2 - 3\tilde{\rho}^3 \phi^2 \right) + \int d\tilde{\rho} \frac{1}{2} \lambda \frac{\rho^5}{\tilde{\rho}} \frac{d\lambda}{d\rho} \phi^2$$

This is the action of a scalar in AdS with a mass squared of -3 +  $\rho$  dependent correction from the gradient of  $\lambda$

For example if we try to (very naively) input the two loop QCD running of the coupling...

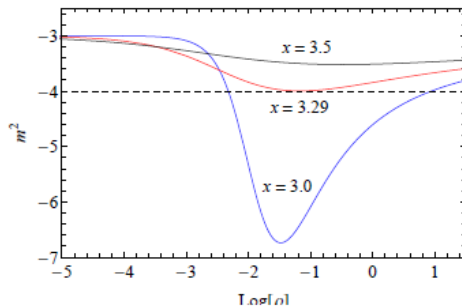
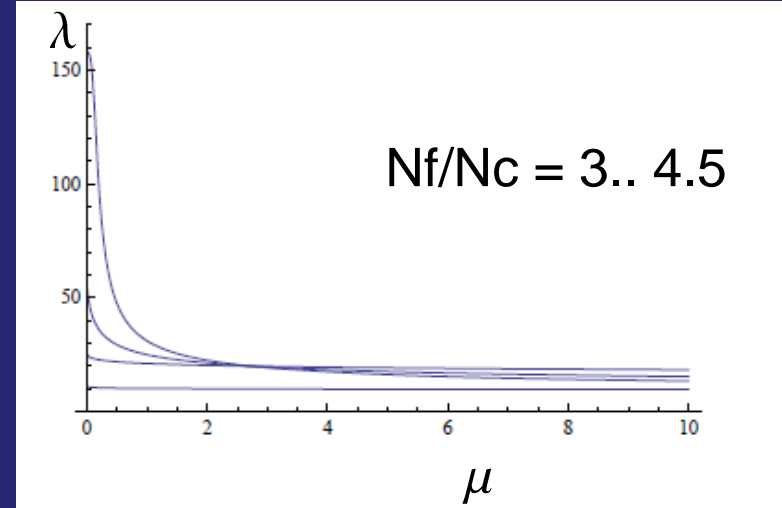
$$\beta(g) = -\frac{g^3}{(4\pi)^2} \left\{ \frac{11}{3}N_c - \frac{2}{3}N_f \right\} - \frac{g^5}{(4\pi)^4} \left\{ \frac{34}{3}N_c^2 - \frac{N_f}{N_c} \left[ \frac{13}{3}N_c^2 - 1 \right] \right\} + \dots$$

Using the 't Hooft coupling, and setting  $\frac{N_f}{N_c} \rightarrow x$  we obtain

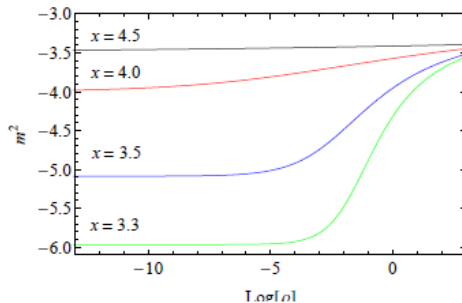
$$\lambda \equiv g^2 N_c, \quad \dot{\lambda} = -b_0 \lambda^2 + b_1 \lambda^3 + \mathcal{O}(\lambda^4)$$

with

$$b_0 = \frac{2(11-2x)}{3(4\pi)^2}, \quad b_1 = -\frac{3(34-13x)}{2(11-2x)^2}$$



(a) The model with the QCD running imposed in section III ( $x = 3.5, 3.29, 3.0$ ).

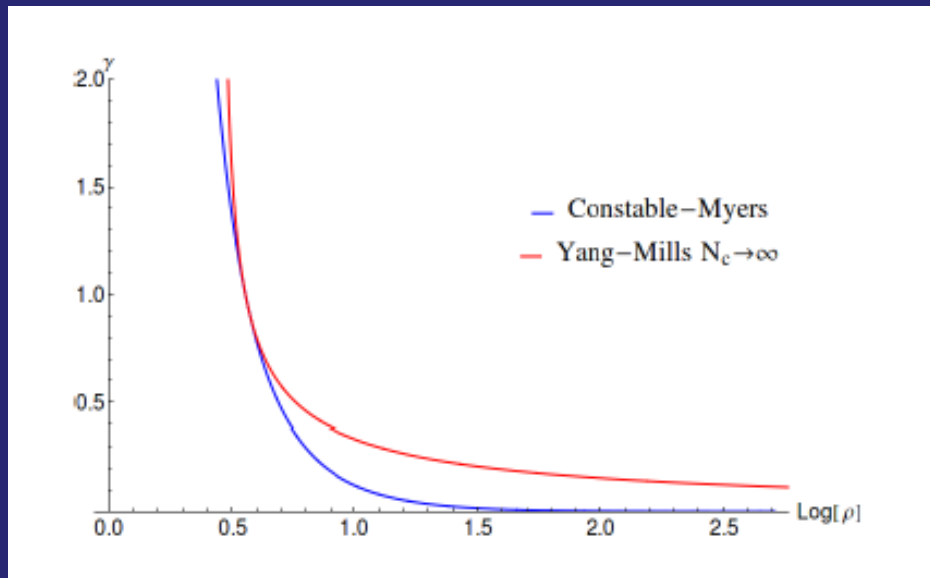


(b) The model of section IVa where the QCD anomalous dimension is imposed in the IR ( $x = 4.5, 4, 3.5, 3.3$ ).

We output these running masses... to be compared with the perturbative expectation below...

$$\gamma_m^{(1)} = \mu \frac{d \ln m_q}{d\mu} = \frac{3(N_c^2 - 1)}{4N_c \pi} \alpha$$

$$m^2 = \Delta(\Delta - 4)$$



Top-down probe-brane models of QCD are just AdS/QCD with the background providing a running  $\gamma$ ...

# Dynamic AdS/QCD

Timo Alho, NE, Kimmo Tuominen  
1307.4896

$$S = \int d^4x d\rho \text{Tr} \rho^3 \left[ \frac{1}{\rho^2 + |X|^2} |DX|^2 + \frac{\Delta m^2}{\rho^2} |X|^2 + \frac{1}{2\kappa^2} (F_V^2 + F_A^2) \right]$$

$$X = L(\rho) e^{2i\pi^a T^a}.$$

$$ds^2 = \frac{d\rho^2}{(\rho^2 + |X|^2)} + (\rho^2 + |X|^2) dx^2,$$

D7 probe action in AdS expanded to quadratic order

X is now a dynamical field **whose solution will determine the condensate** as a function of m

We use the top-down IR boundary condition on mass-shell:  $X'(\rho=X) = 0$

X enters into the AdS metric to cut off the radial scale at the value of m or the condensate – no hard wall

The gauge DYNAMICS is input through  $\Delta m$

$$\Delta m^2 = -2\gamma = -\frac{3(N_c^2 - 1)}{2N_c \pi} \alpha$$

$$m^2 = \Delta(\Delta - 4)$$

The only free parameters are  $N_c, N_f, m, \Lambda$



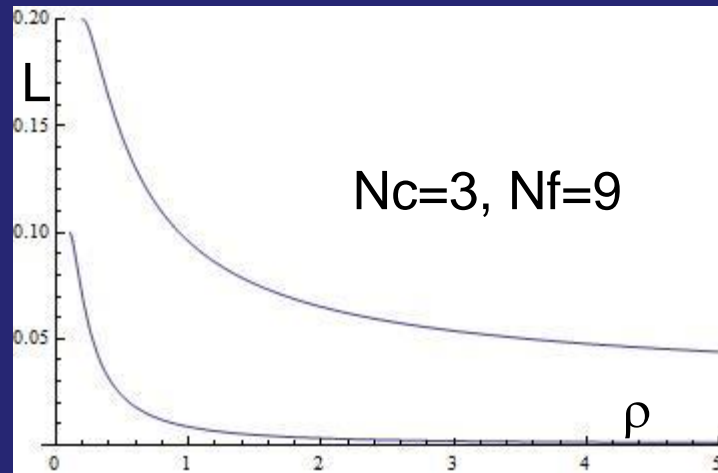
# Formation of the Chiral Condensate

We solve for the vacuum configuration of  $L$

$$\partial_\rho[\rho^3 \partial_\rho L] - \rho \Delta m^2 L = 0.$$

Shoot out  
with

$$L'(\rho=L) = 0$$

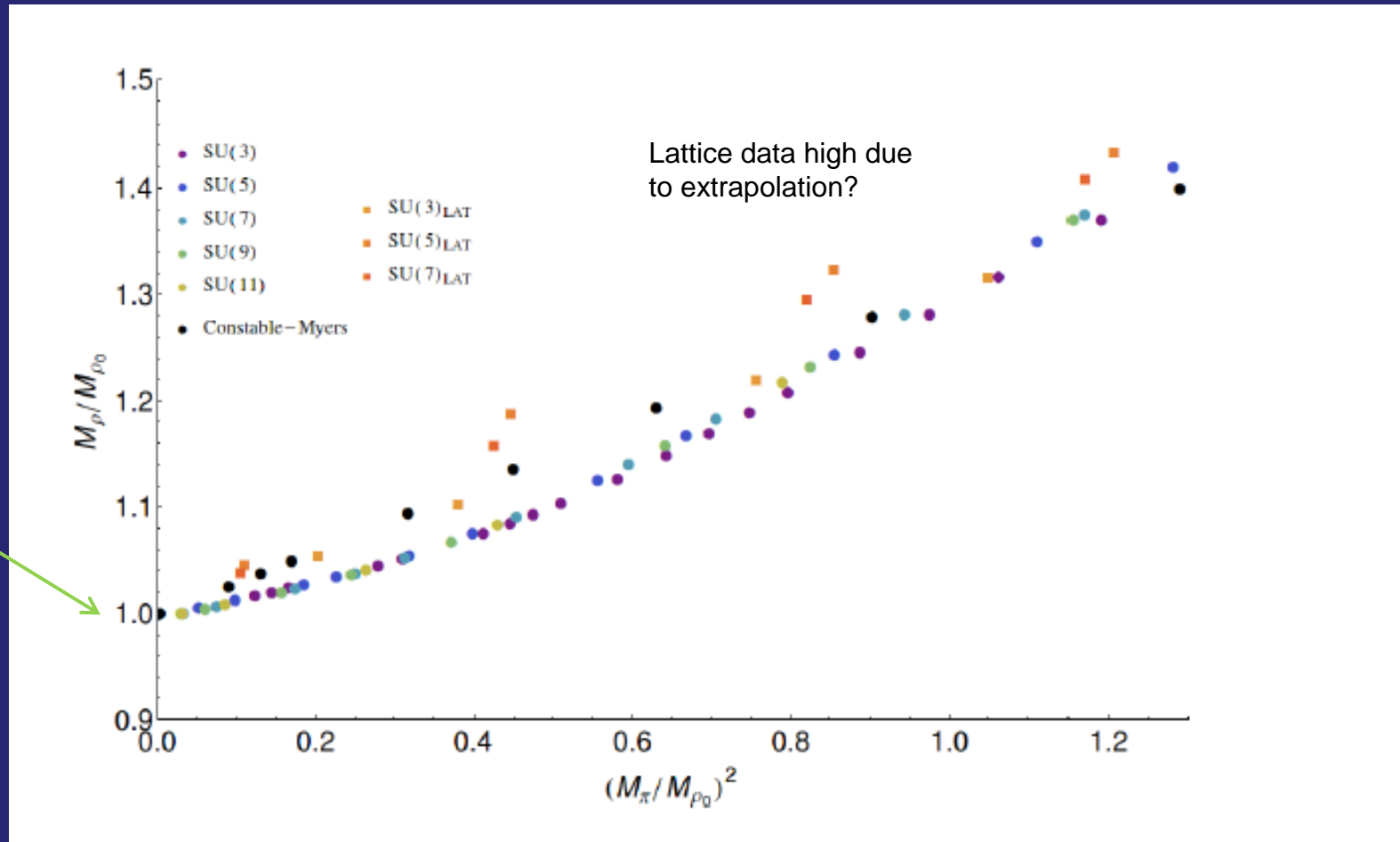


Read off  $m$   
and  $qq$  in  
the UV...

Now solve for meson masses by looking at linearized fluctuations about this vacuum...

# Quenched SU(Nc) gauge

NE, Erdmenger & Mark Scott

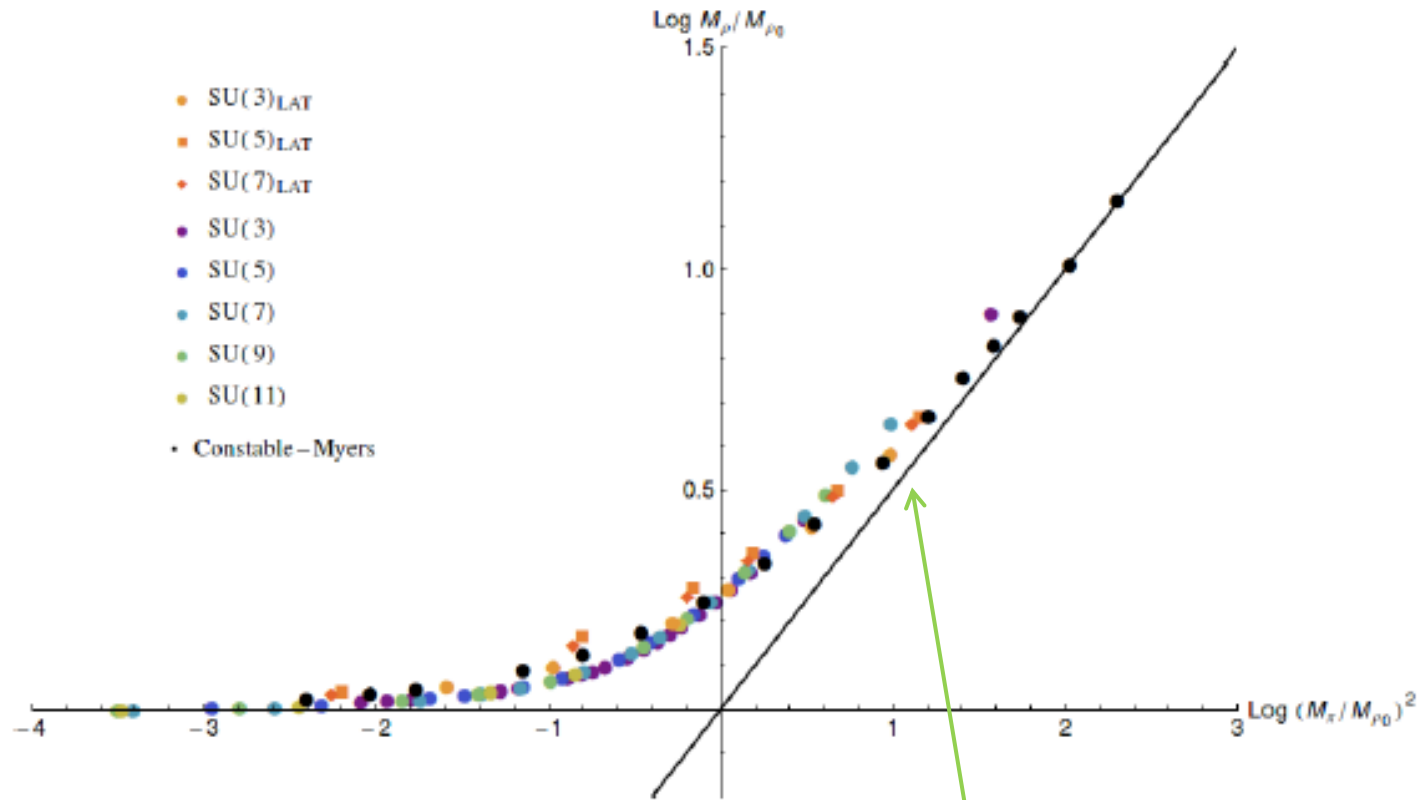


There is very little  $N_c$  dependence

Comparison to quenched lattice data (Bali et al... arXiv1304.4437)

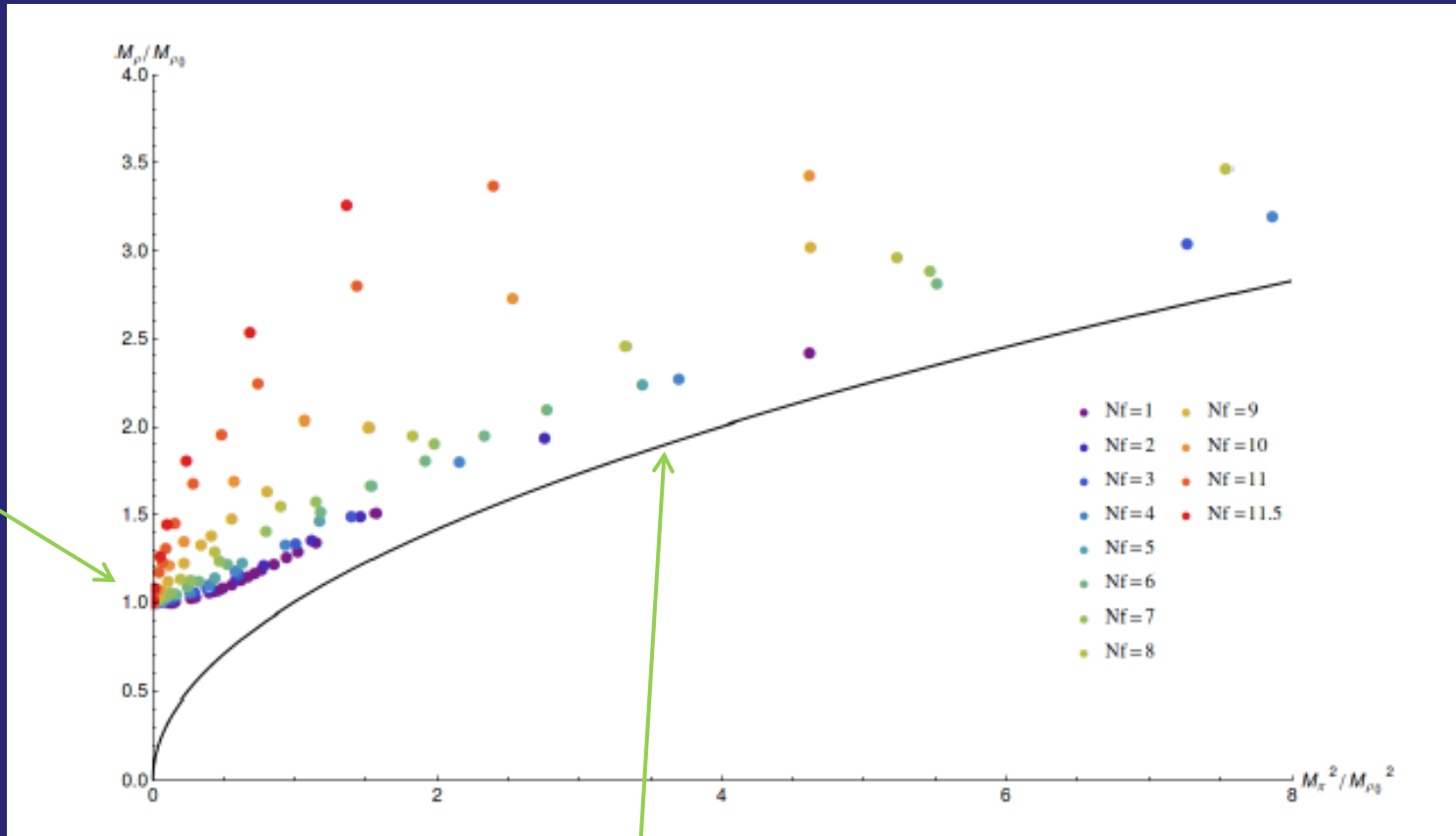
All of these models lie within 10% on any point....

# Quenched SU(Nc) gauge



$M_\rho = M_\pi$

# SU(3) gauge theory + Nf quarks

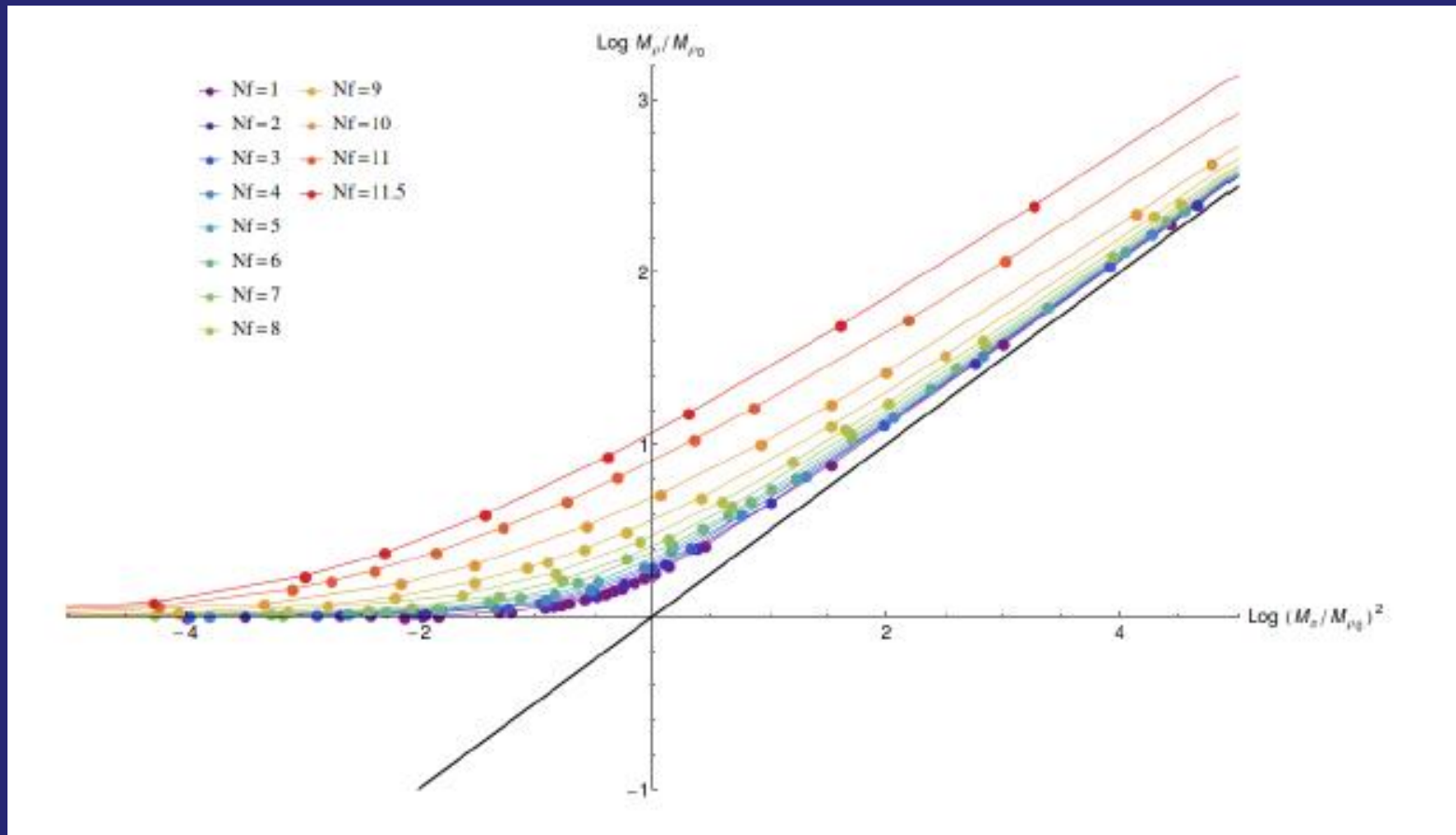


$M_\rho = 1$   
defines  $\Lambda$

Real QCD  
lies here

$M_\rho = M_\pi$

# SU(3) gauge theory + Nf quarks



We do see new behaviour as  $N_f$  heads towards 12....

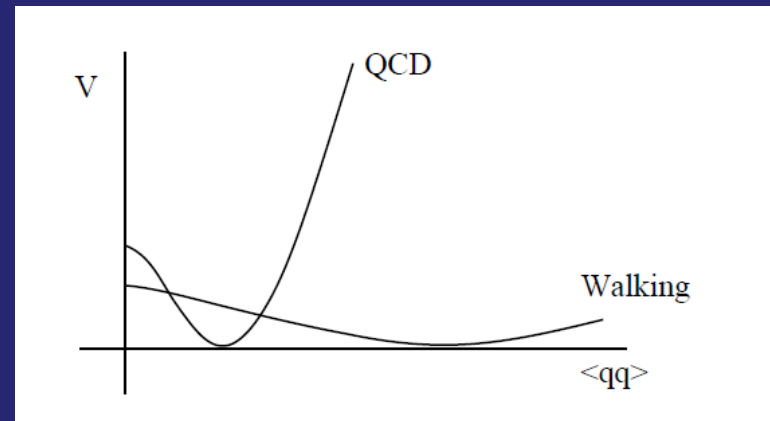
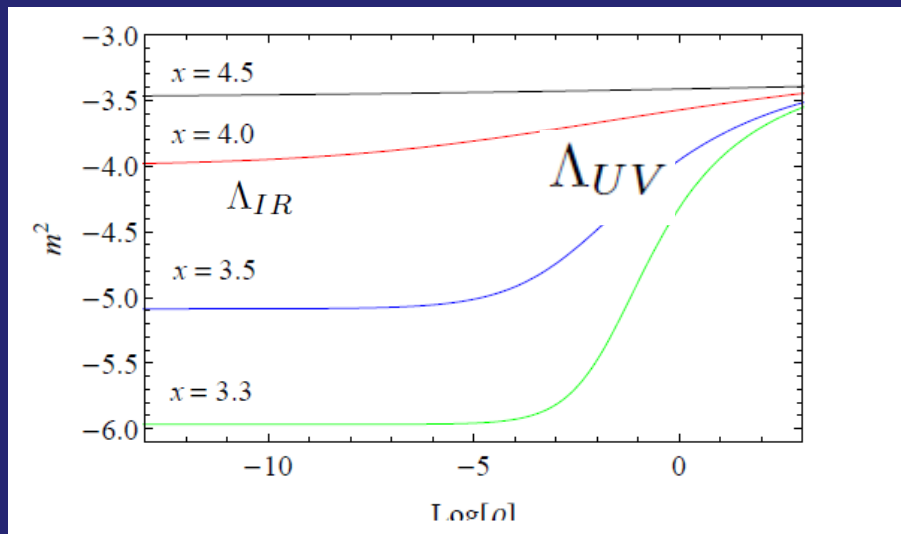


# Walking Dynamics Holdom

Just above the CW regime theories have an enhanced UV quark condensate

$$\langle \bar{q}q \rangle_{UV} \sim \Lambda_{UV} \langle \bar{q}q \rangle_{IR} \sim \Lambda_{UV} \Lambda_{IR}^2$$

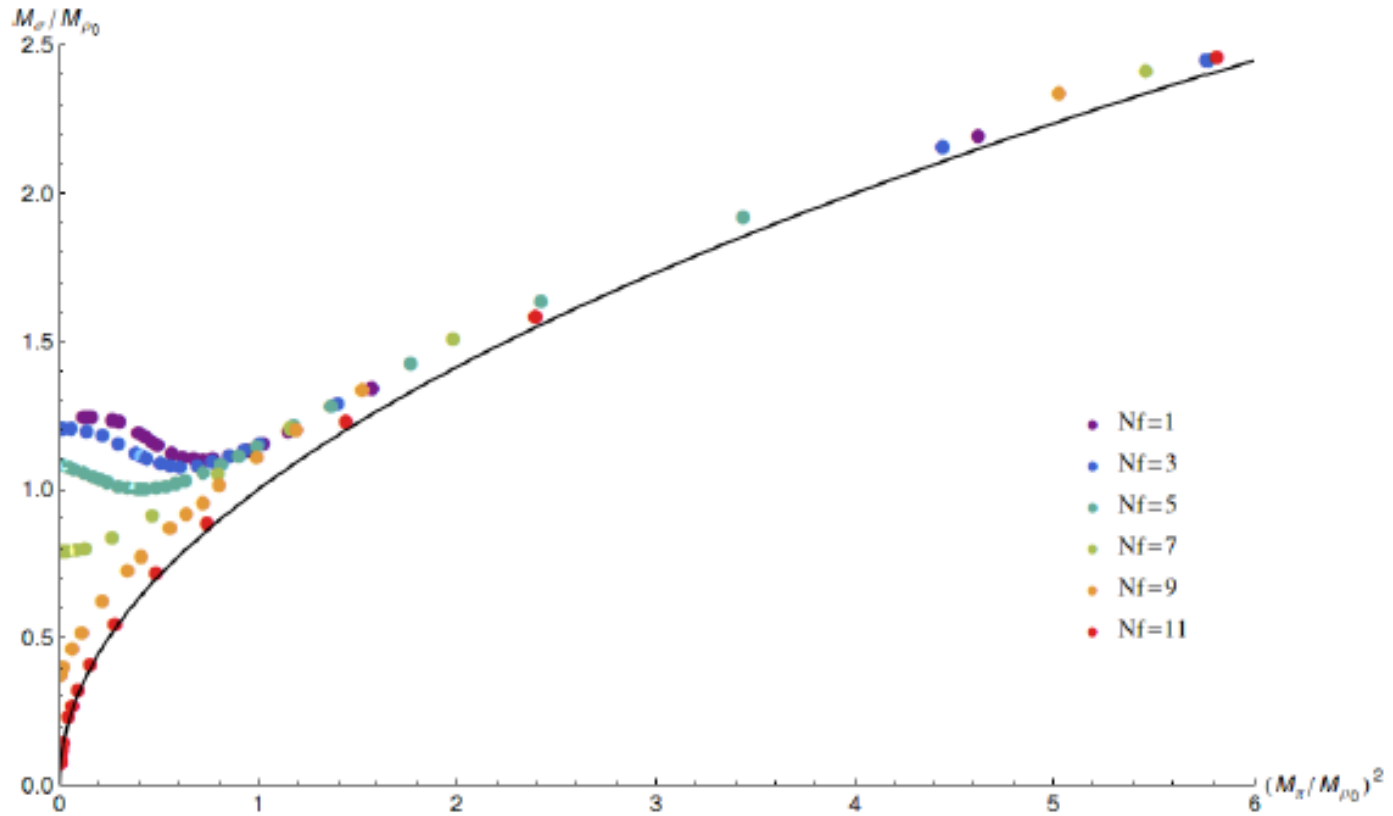
$$f_\pi \sim \Lambda_{IR}$$



- Is the sigma particle light – a techni-dilaton?
- Is the higgs such a technicolor state?

# SU(3) gauge theory + Nf quarks

The QCD point is not right for the  $f_0(500)$  but about right for the  $f_0(980)$  – is the  $f_0(500)$  odd eg a molecule ???



We indeed see a light sigma relative to the rho...



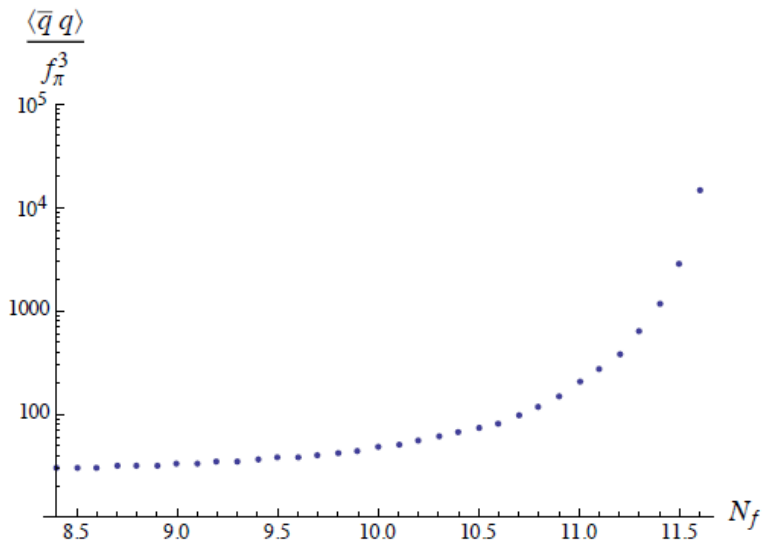


FIG. 5: The quark condensate normalized by  $f_\pi^3$  vs  $N_f$ .

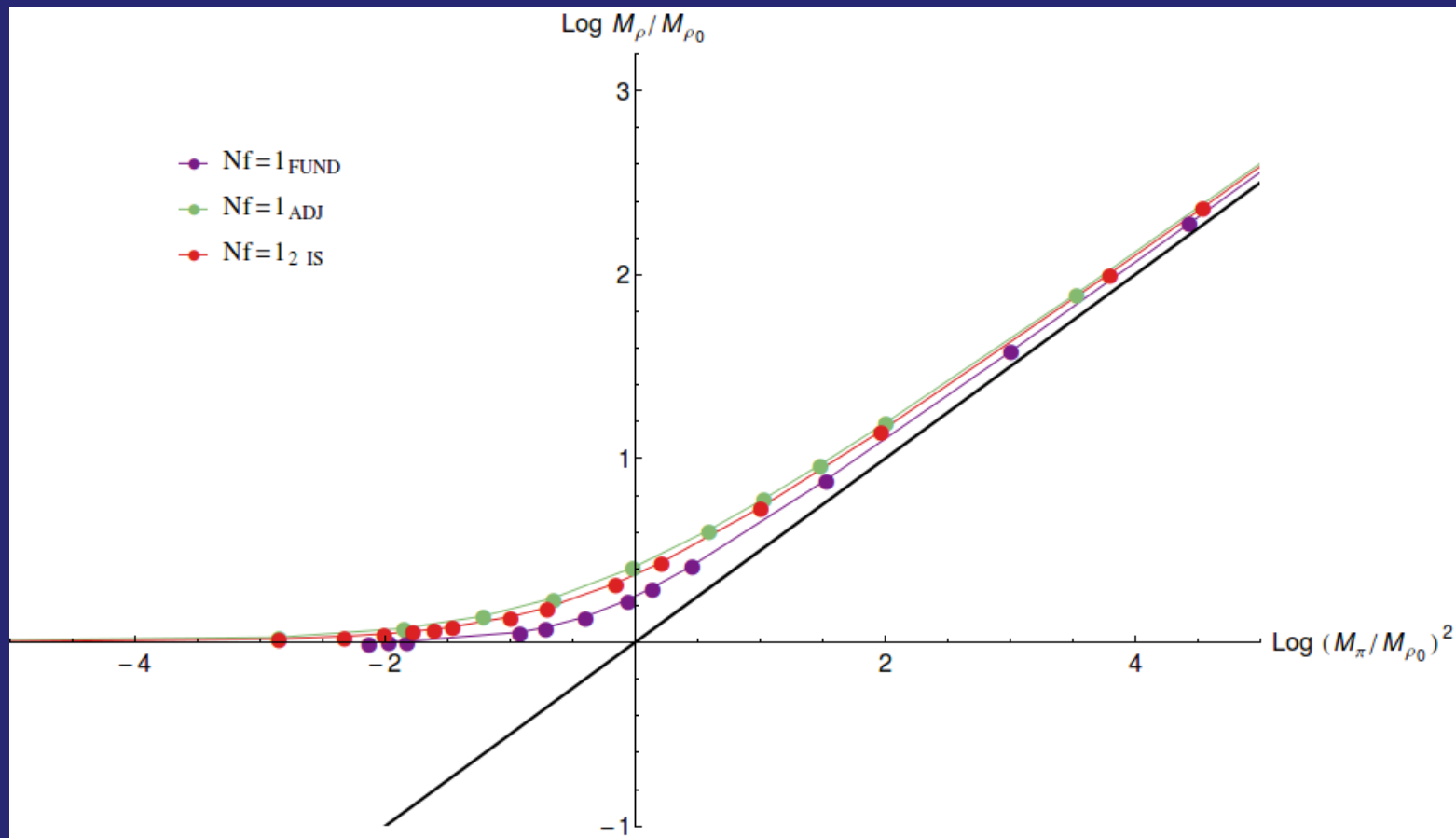
qq enhancement  
reproduced....

Cf FCNC problem in  
TC

# Pick your theory...

Input (assumed) running of  $\gamma$  for any theory you fancy...

eg  $SU(3)$  gauge +

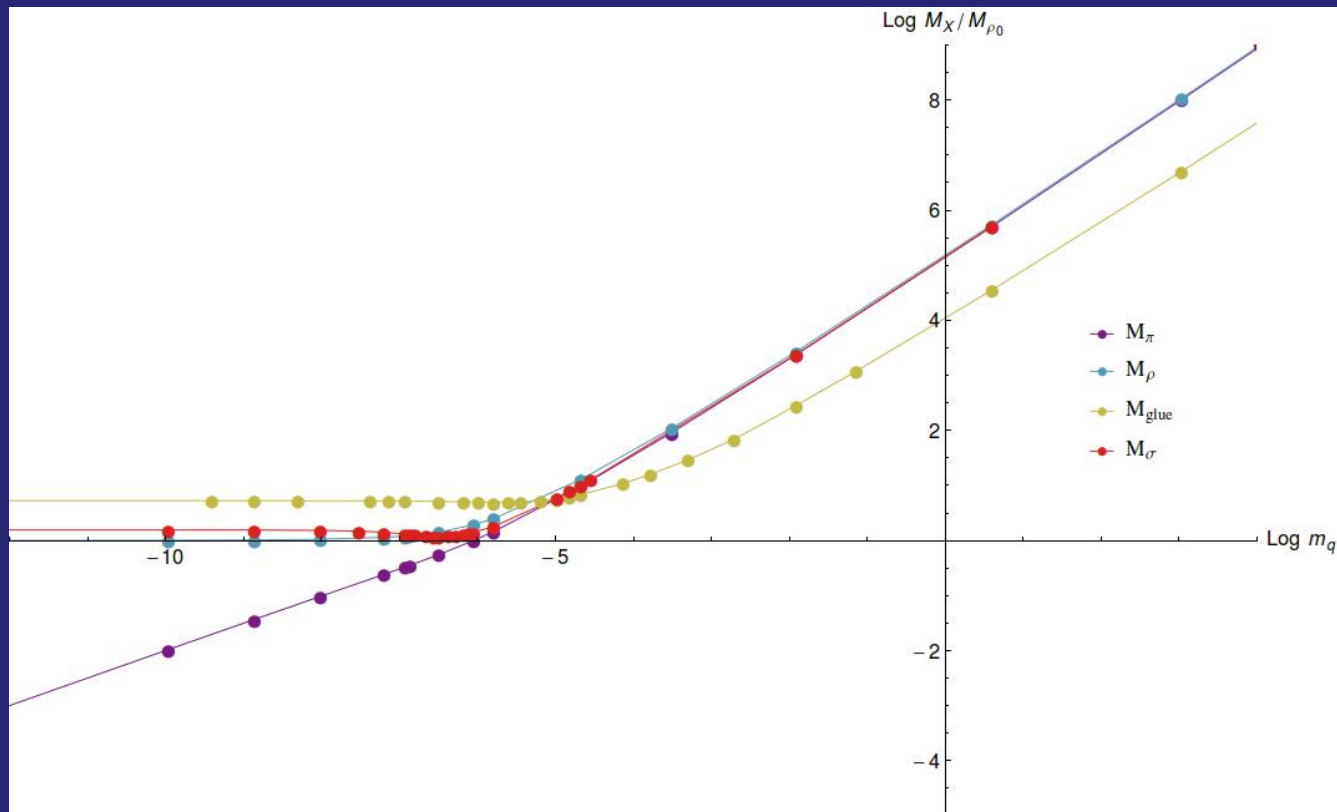


# Glueballs

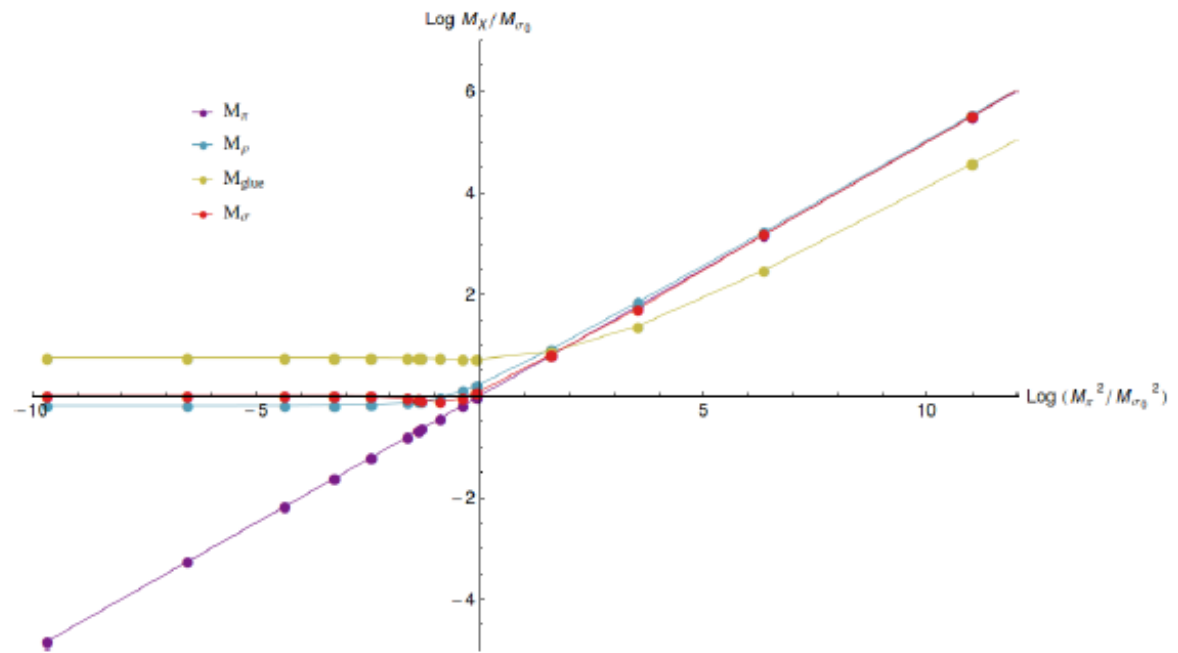
The model concentrates on the quark states... we're not trying to describe the running of the coupling or  $\text{Tr } F^2$ ... however for us roughly...

Find the dynamical IR quark mass... below that scale run the coupling as pure YM.. Find the IR pole... multiply by 8 and that's the glueball mass!

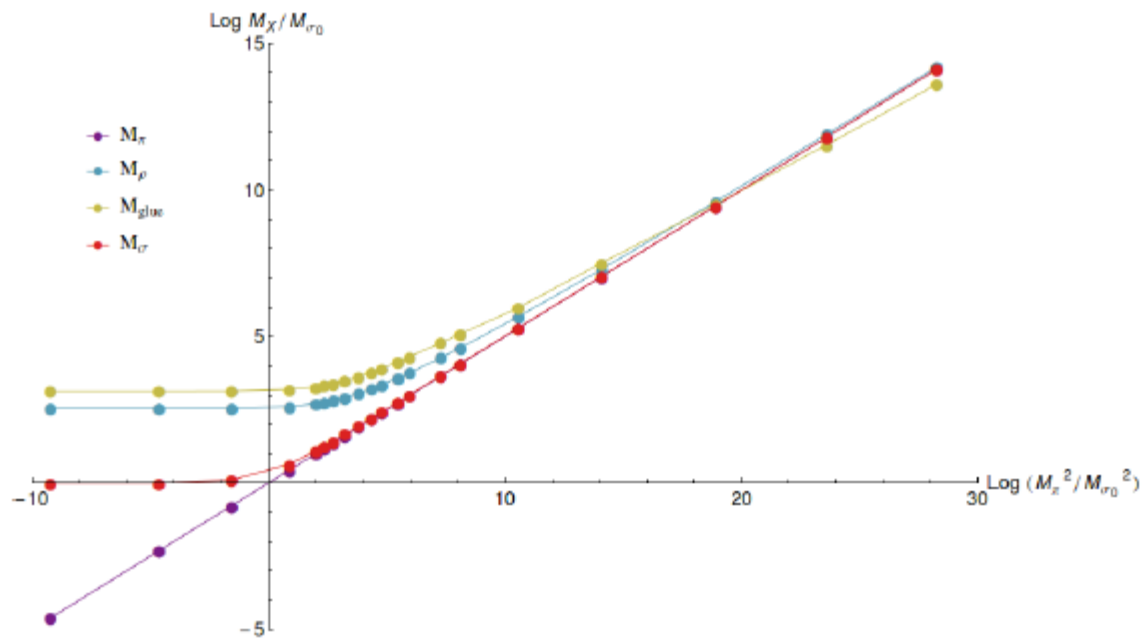
SU(3)  
Nf=3



SU(3)  
Nf=3



SU(3)  
Nf=11

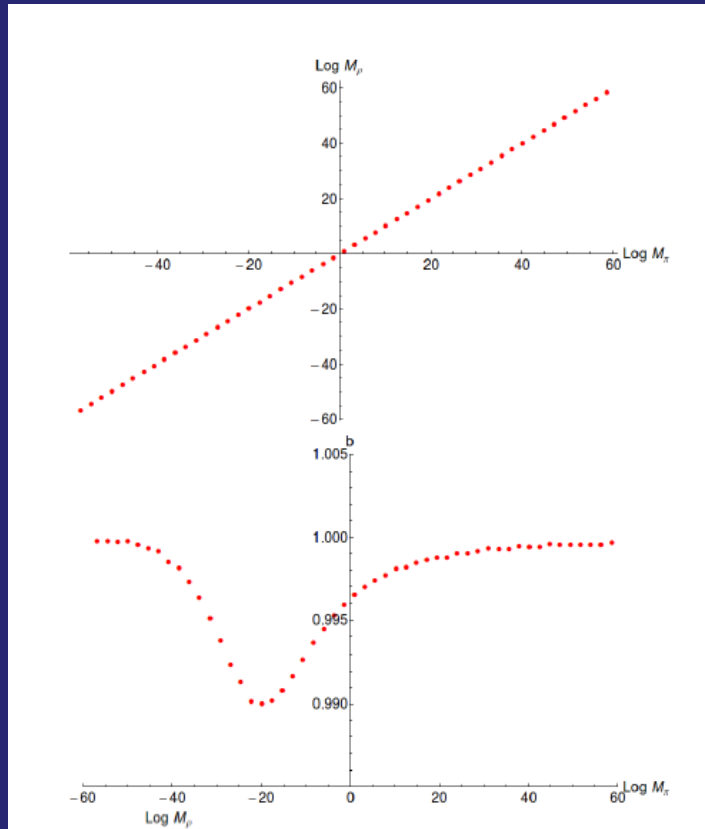


# Study the Conformal Window

NE, Scott 1405.5373

For  $N_f > 4 N_c$  there is no dynamical scale so study the theory at non-zero quark mass...

eg SU(3) gauge + 13 flavours



$$M_\rho \propto M_\pi^b$$

$b=1$  for the conformal IR and UV regimes but deviates at the intermediate running scale...

# Conclusions

Holographic models of QCD continue to improve... running of  $\gamma$  is crucial...one gets a good description of the lowest lying spectra at better than 10% and you can see generic behaviours with  $N_c N_f$  easily...

Holography is a remarkably simple method to get a ball park answer for behaviour... but it still can't be systematically improved...

Hopefully Models are useful to guide lattice practitioners...

On going work:  $T$   $\mu$  phase structure... pomeron physics... enlarging to the full QCD spectra... why can I ignore other operators?