Black holes in the 1/D expansion

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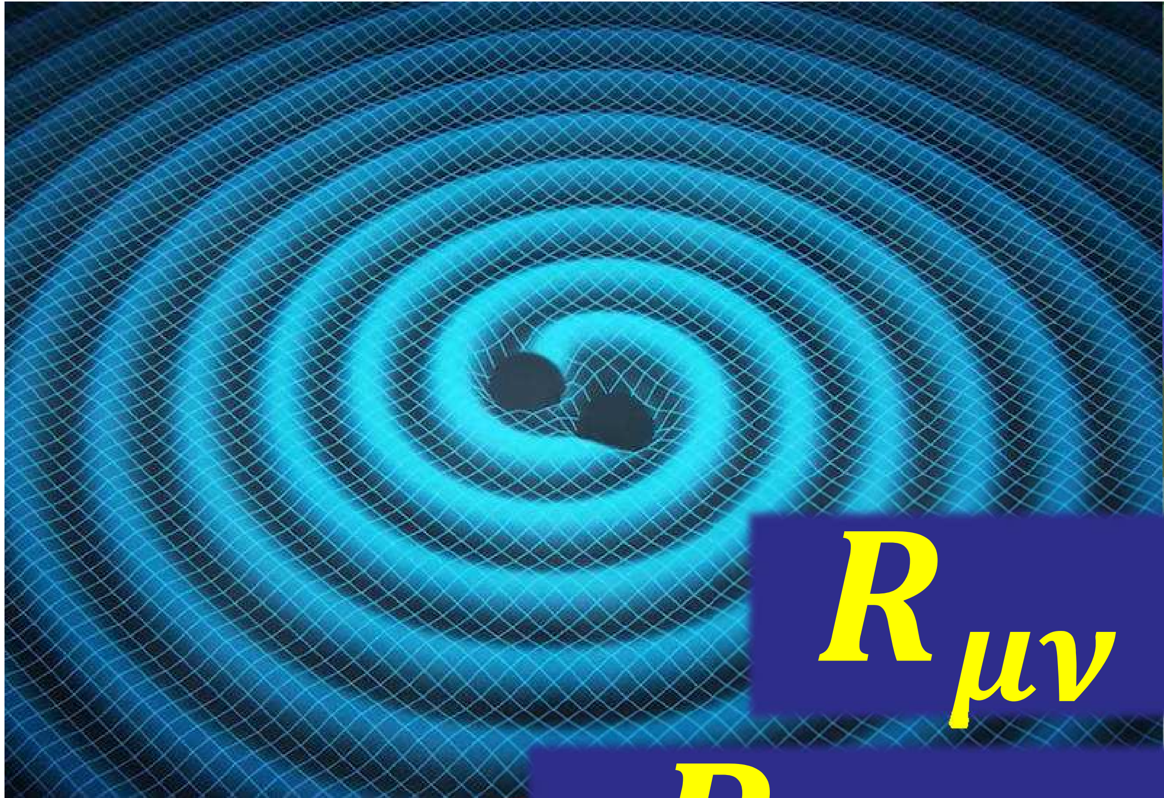
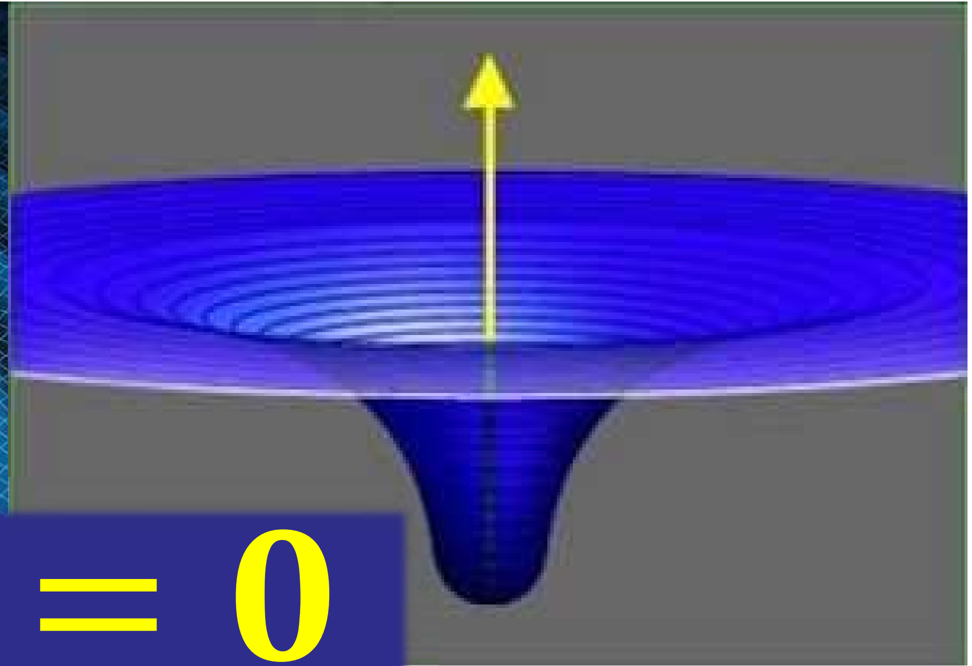


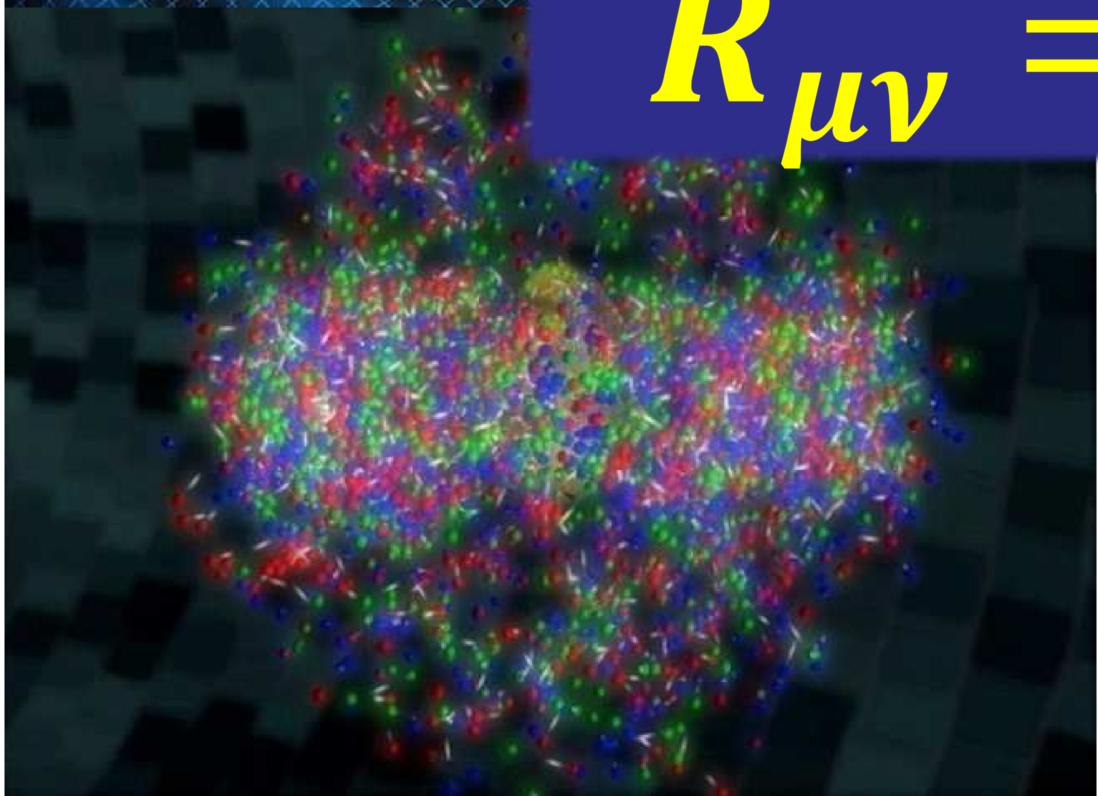


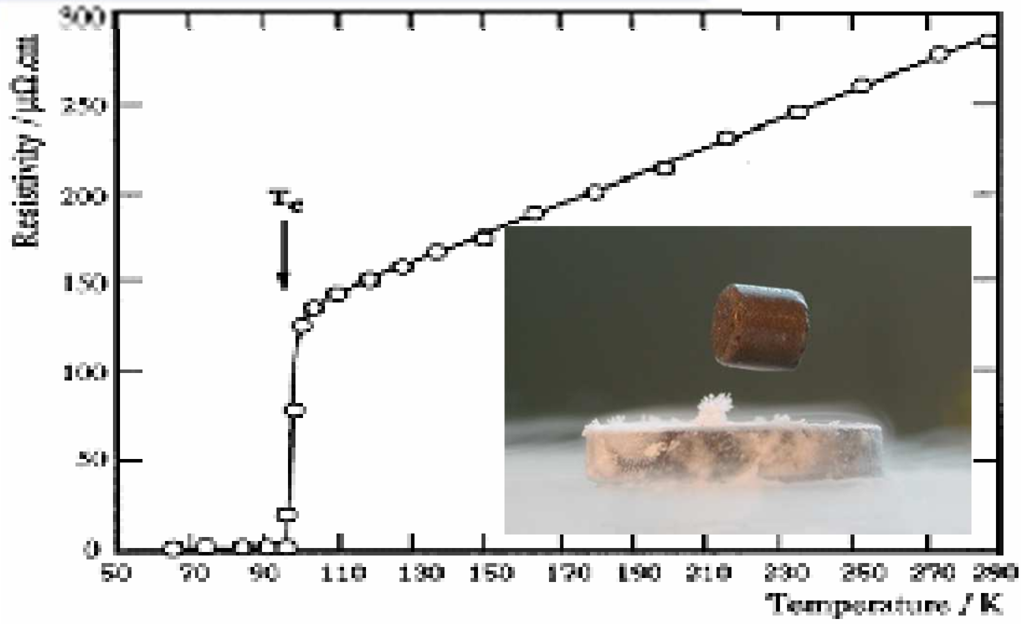
$$R_{\mu\nu} = 0$$

Nov 1915

$$R_{\mu\nu} = -\Lambda g_{\mu\nu}$$

Feb 1917





A dimensionless, adjustable parameter
is a good thing to have
for studying a theory

Quantum ElectroDynamics

Perturb around $e^2 = 0$

Quantum GluoDynamics

SU(3) Yang-Mills theory

No parameter?

Quantum GluoDynamics

SU(**N**) Yang-Mills theory

parameter!

What dimensionless parameter in

$R_{\mu\nu} = \Lambda \, g_{\mu\nu}$?

$$R_{\mu\nu} = \Lambda\, g_{\mu\nu}$$

$$\mu, \nu = 0, \ldots, \boldsymbol{D} - 1$$

YM

$SU(N \to \infty)$

?

$\leftrightsquigarrow$

Quantum GR

$SO\ (D \to \infty, 1)$

Quantum GR: SO(**D**-1,1) local Lorentz group

graviton polarizations grows with D

BUT:

No topological expansion of Feynman diagrams

Strominger 1981

Bjerrum-Bohr 2004

Even worse: UV behavior infinitely bad

YM $\leftrightsquigarrow$ Quantum GR

$SU(N \to \infty)$ $\qquad$ $SO\,(D \to \infty, 1)$

Classical General Relativity

D-diml Einstein's theory

Well-defined for all D

Many problems can be formulated keeping D arbitrary

$\rightarrow$ D = continuous parameter

$\rightarrow$ expand in 1/D

Kol et al

RE+Suzuki+Tanabe

Classical General Relativity

D-diml Einstein’s theory

Large D

keeps essential physics of D=4

∃ black holes

∃ gravitational waves

simplifies the theory

reformulation in terms of other variables?

BH in D dimensions

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right) dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

Localization of interactions

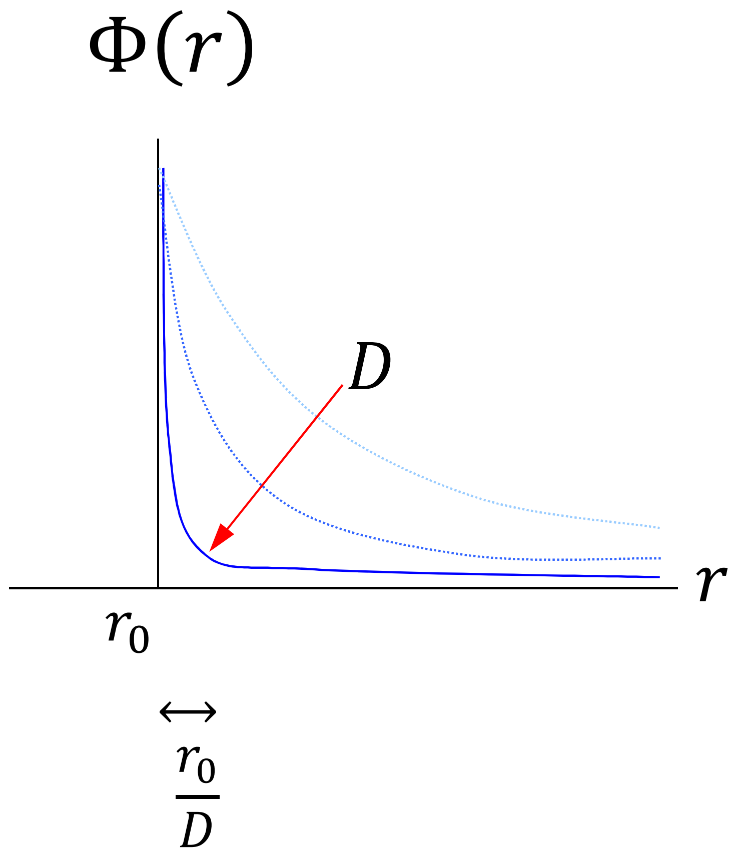
Large potential gradient:

$$\Phi(r) \sim \left(\frac{r_0}{r}\right)^{D-3}$$

$$\nabla\Phi\Big|_{r_0} \sim D/r_0$$

$\Longrightarrow$ Hierarchy of scales

$$\boxed{\frac{r_0}{D} \ll r_0}$$



Fixed $r > r_0$ $\quad D \to \infty$

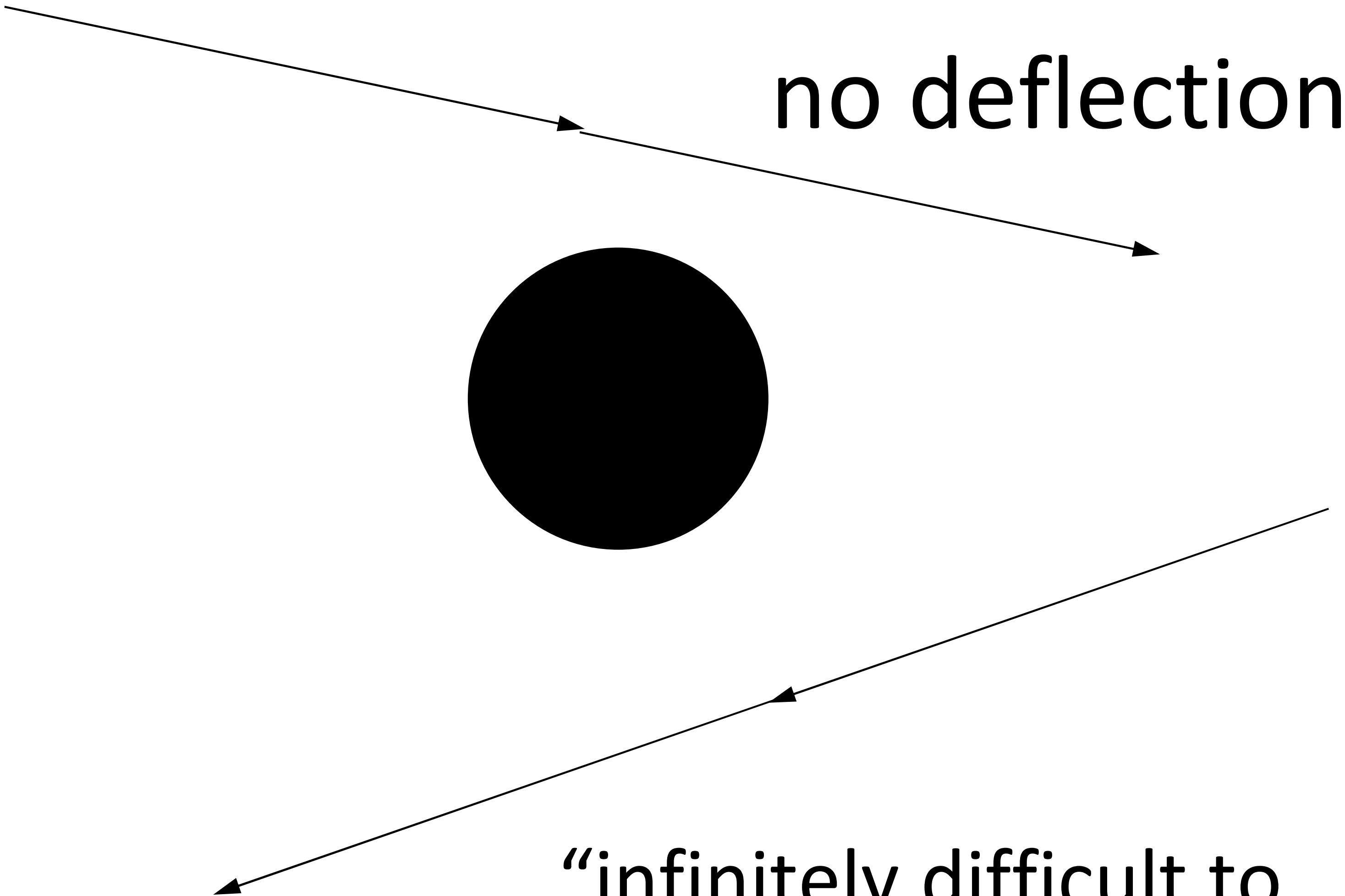
$$1 - \left(\frac{r_0}{r}\right)^{D-3} \to 1$$

$$ds^2 \to -dt^2 + dr^2 + r^2 d\Omega_{D-2}$$

Flat, empty space at $r > r_0$

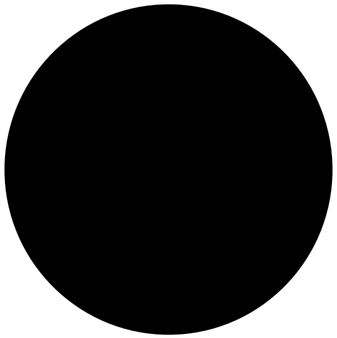
“**Far-zone**” limit

Black Hole scattering:
no deflection



“infinitely difficult to catch a line of force”

Black Hole scattering

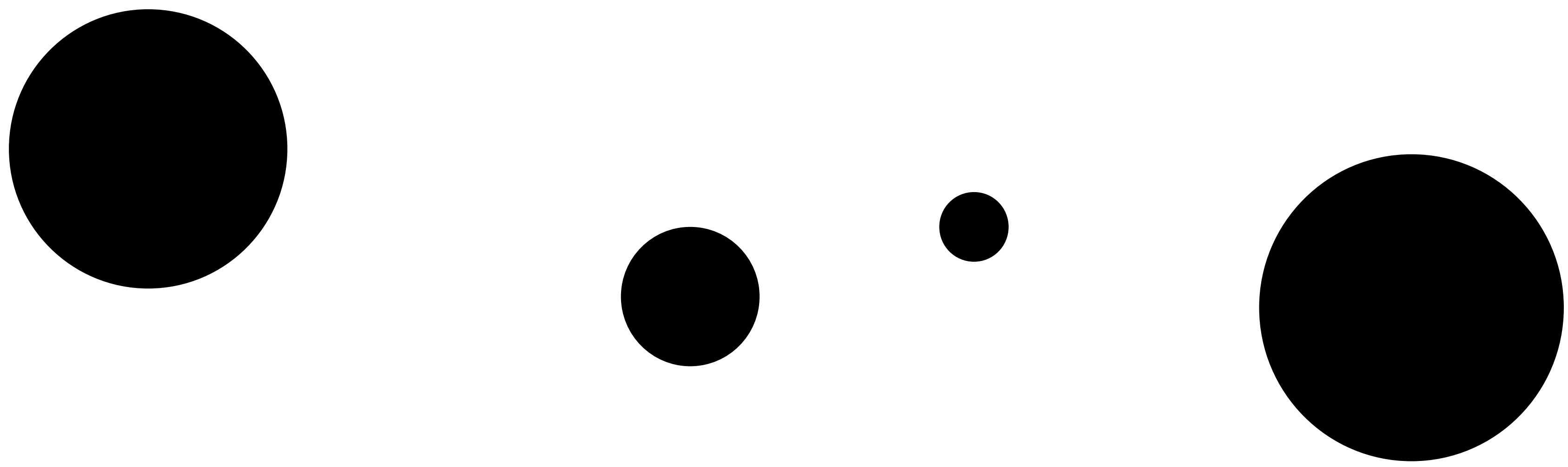


No absorption of waves

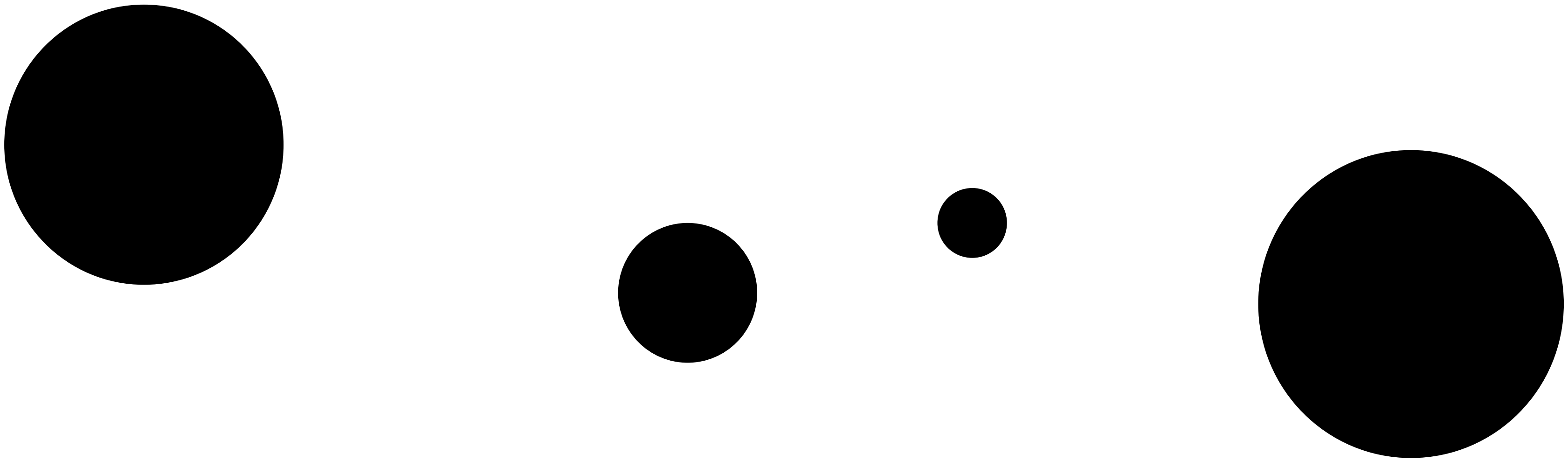
with wavelength

$$\lambda \sim r_0$$

No interaction



Holes cut out in Minkowski space

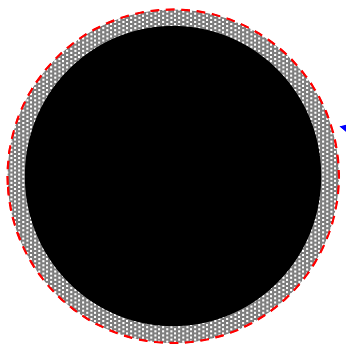


We are keeping length scales $\sim r_0$ finite as we send $D \to \infty$

"Far-zone" limit

Now take a limit that does *not* *trivialize* the gravitational field

$$\left(\frac{r_0}{r}\right)^{D-3} = \mathcal{O}(1) \Leftrightarrow r - r_0 < \frac{r_0}{D}$$



$$r - r_0 \sim \frac{r_0}{D}$$

"**Near-horizon**" limit

Near-horizon geometry

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right)dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

$$\left.\begin{aligned} \left(\frac{r}{r_0}\right)^{D-3} &= \cosh^2\rho \\ t_{near} &= \frac{D}{2r_0}t \end{aligned}\right\} \text{ finite as } D \to \infty$$

Near-horizon geometry

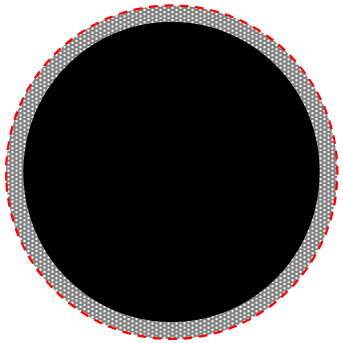
2d string bh

$$ds^2_{nh} \to \frac{4r_0^2}{D^2}\left(-\tanh^2\rho \; dt^2_{near} + d\rho^2\right)$$

$$+\, r_0^2 \,(\cosh\rho)^{4/D}\, d\Omega^2_{D-2}$$

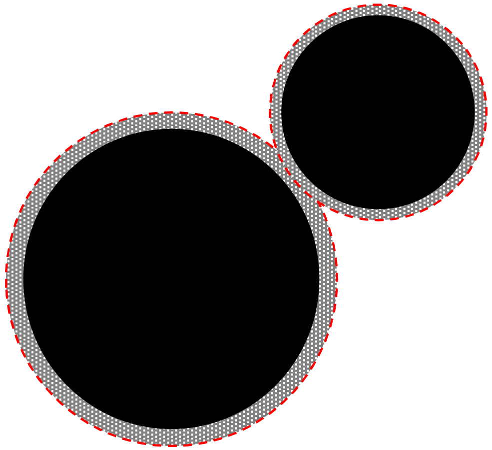
Soda 1993
Grumiller et al 2002

Physics at $\sim r_0/D$ close to the horizon is *not* trivial



Perfect absorption
of waves with
$\lambda \sim r_0/D$
$\omega \sim D/r_0$

"**Near-horizon**" dynamics



Not an exact solution

Non-trivial interaction

“**Near-horizon**” dynamics

Near-horizon universality

2d string bh = near-horizon geometry
of **all neutral non-extremal bhs**

rotation = local boost
(along horizon)

cosmo const = 2d bh mass-shift

Large D Effective Theory

Solve near-horizon equations

integrate-out short-distance dynamics

→ Boundary conds for far-zone fields

Long-distance effective theory

Black hole perturbations ✓ *all analytic*

Scattering

Quasinormal modes

Ultraspinning instability

Holographic superconductors

Full non-linear GR ✓

General theory of static black holes: ***Soap-film*** theory

Black droplets

simple ODE

Non-uniform black strings

BH perturbations: How accurate?

Small expansion parameter: $\frac{1}{D-3}$

not quite good for $D = 4$ ...

BH perturbations: How accurate?

Small expansion parameter: $\frac{1}{D-3}$

not quite good for $D = 4$ ...

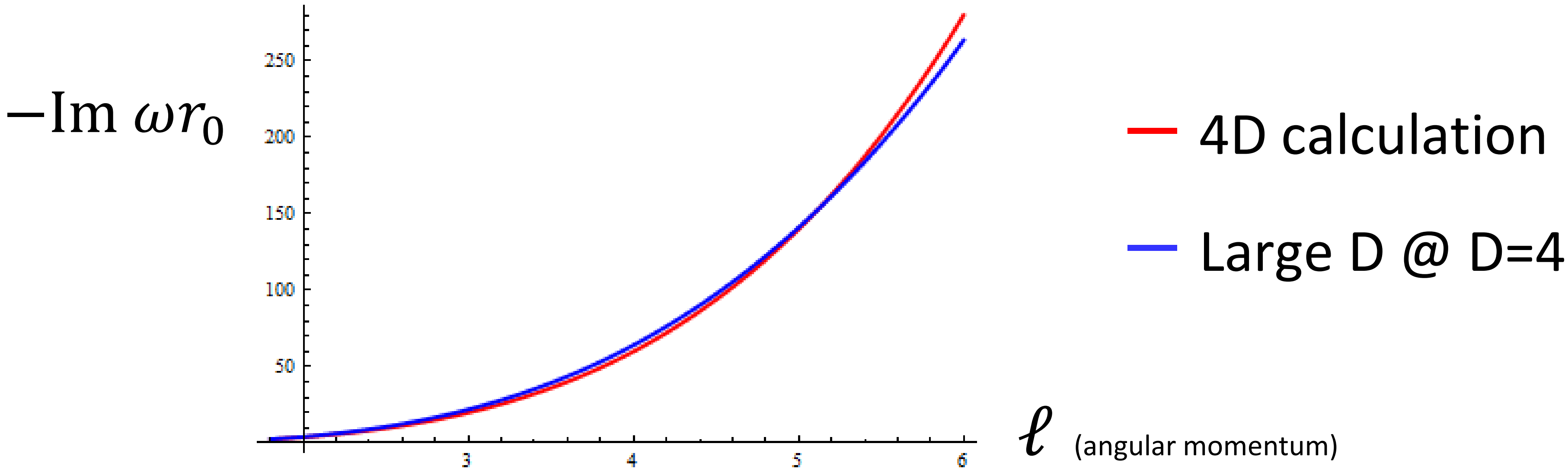
But it seems to be $\frac{\mathbf{1}}{\mathbf{2}(D-3)}$

not *so* bad in $D = 4$, if we can compute higher orders

(in AdS: $\frac{1}{2(D-1)}$)

Quite accurate

Quasinormal frequency in $D = 4$ (vector-type)

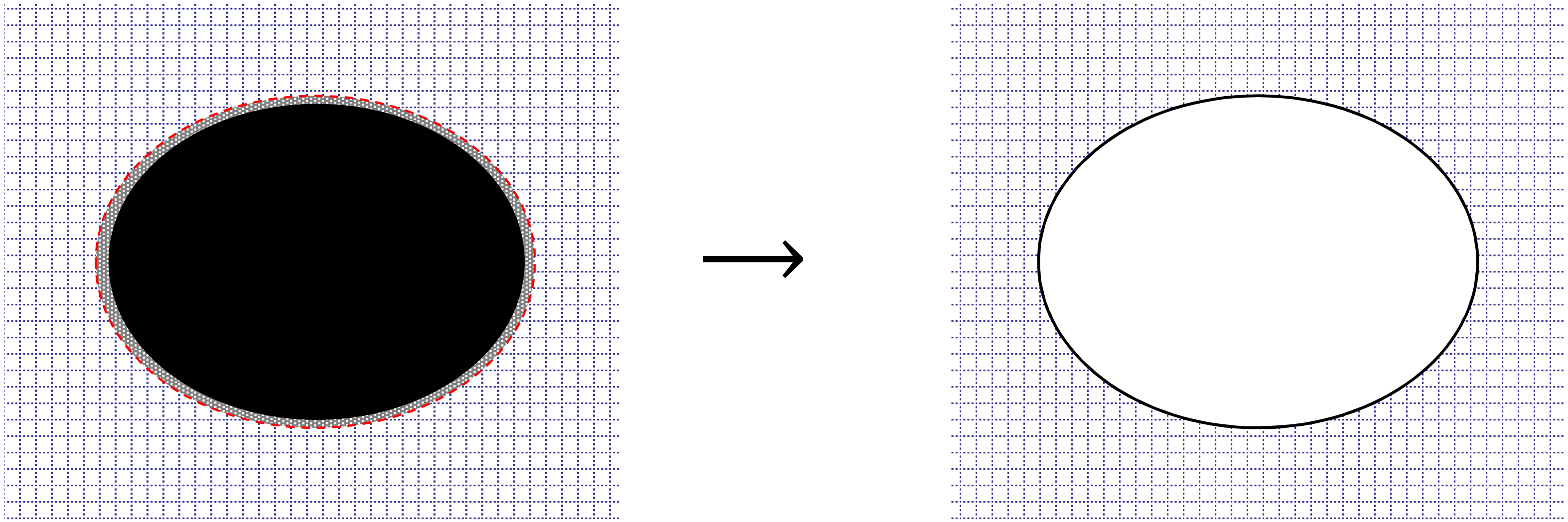


Calculation up to $\frac{1}{D^3}$ yields 6% accuracy in $D = 4$

$$6\% = \frac{1}{\left(2(D-3)\right)^4}\Big|_{D=4}$$

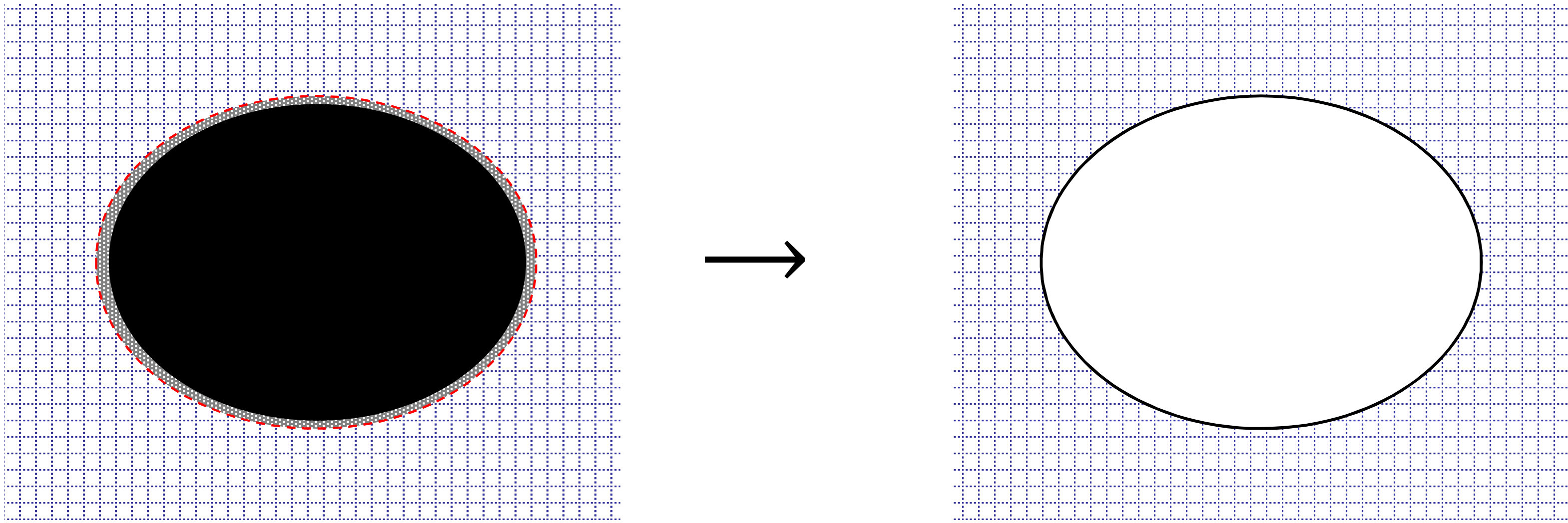
Fully non-linear GR @ large D

Large-D $\Rightarrow$ neat separation bh / background



Replace bh $\longrightarrow$ surface in background

What eqs determine this surface?



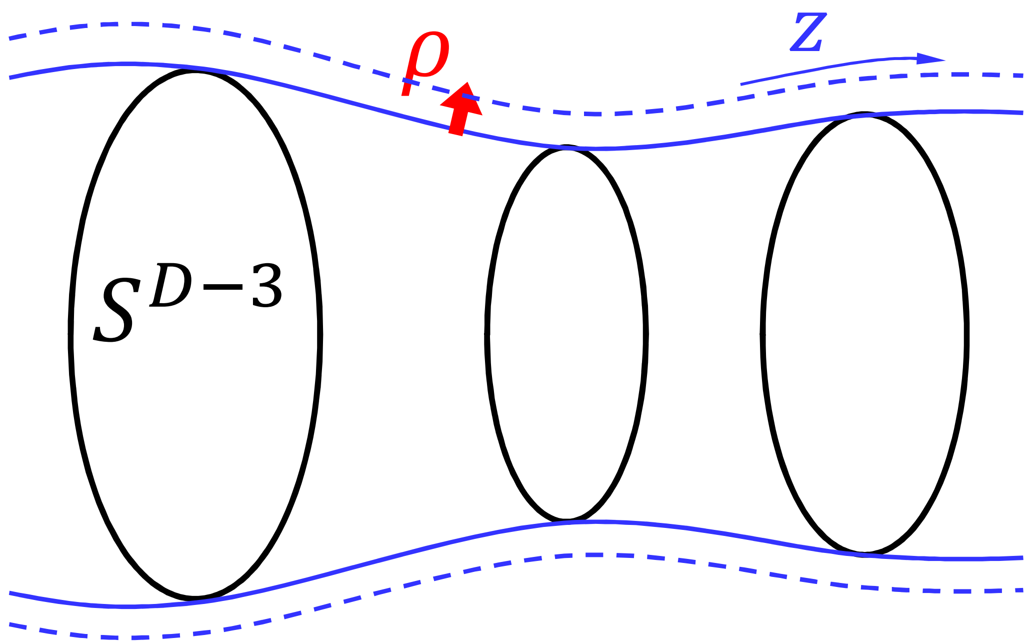
Derive them by solving Einstein's eqs

in near-horizon zone

Gradient hierarchy

$\perp$ Horizon: $\partial_\rho \sim D$

$\parallel$ Horizon: $\partial_z \sim 1$

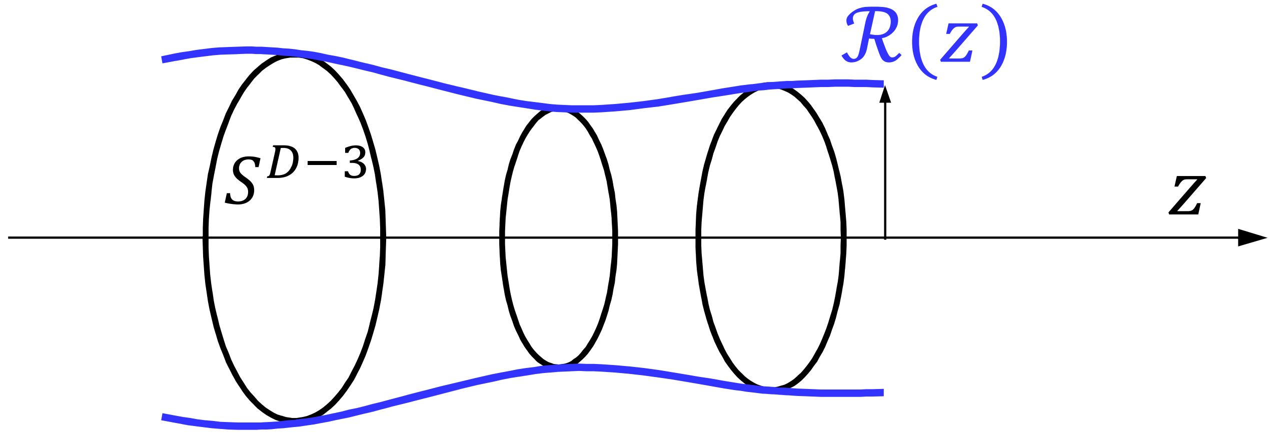


Einstein 'momentum-constraint' in ρ:

$$\sqrt{-g_{tt}}K = \text{const}$$

K = mean curvature of 'horizon surface'

$$ds^2\Big|_h = g_{tt}(z)dt^2 + dz^2 + \mathcal{R}^2(z)d\Omega_{D-3}$$



Soap-film equation (redshifted)

$$\sqrt{-g_{tt}}K = \mathrm{const}$$

Valid up to NLO in 1/D (but *not* at NNLO)

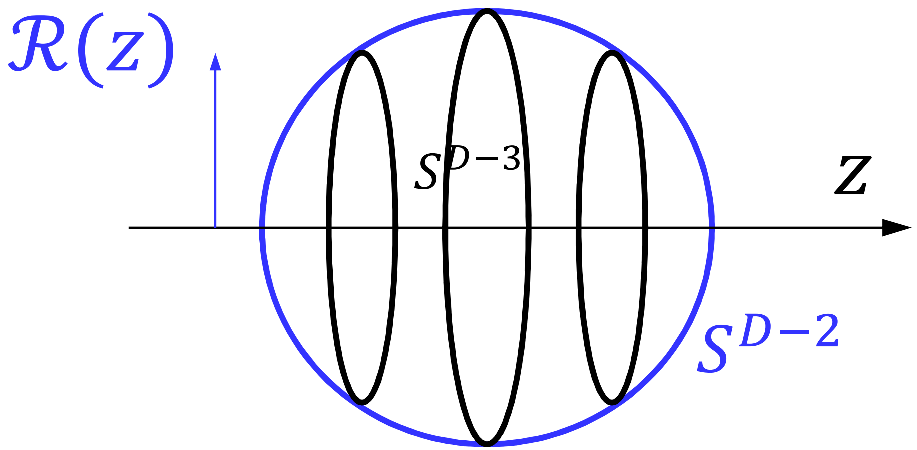
Some applications

Soap bubble in Minkowski = Schw BH

$$ds^2 = -dt^2 + dz^2 + dr^2 + r^2 d\Omega_{D-3}$$

$$r = \mathcal{R}(z)$$

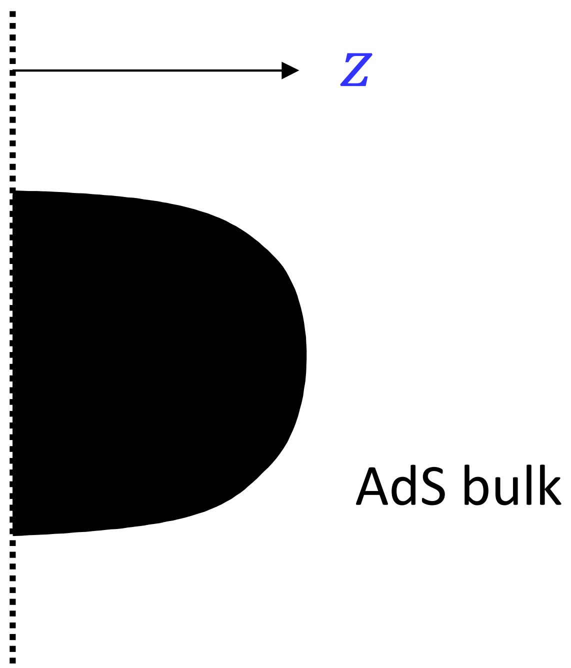
$$\sqrt{-g_{tt}}K = \text{const} \;\Rightarrow\; \mathcal{R}'^2 + \mathcal{R}^2 = 1$$



$$\Rightarrow \mathcal{R}(z) = \sin z$$

Black droplets

Black hole at boundary of AdS

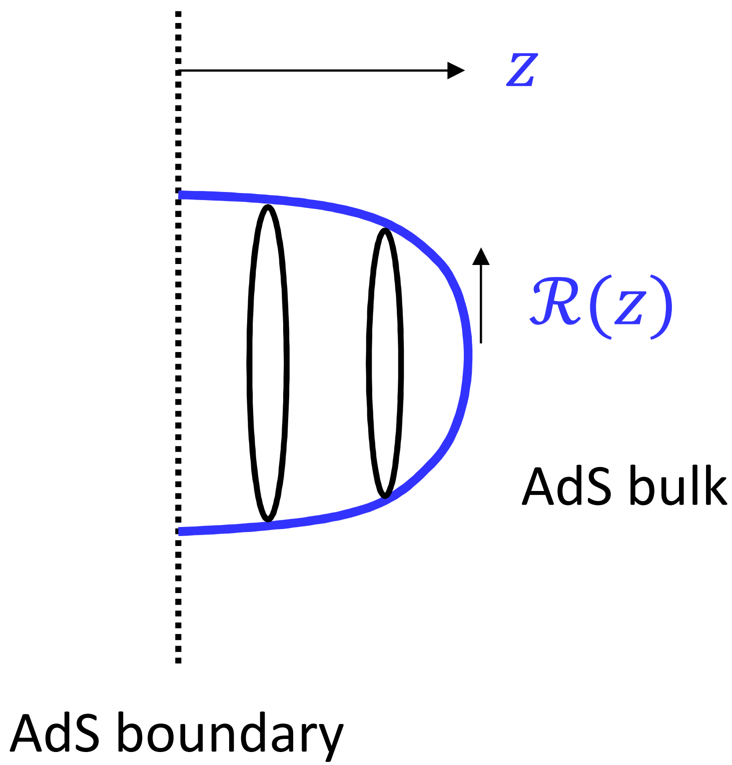


AdS boundary

dual to CFT in BH background

Numerical solution:

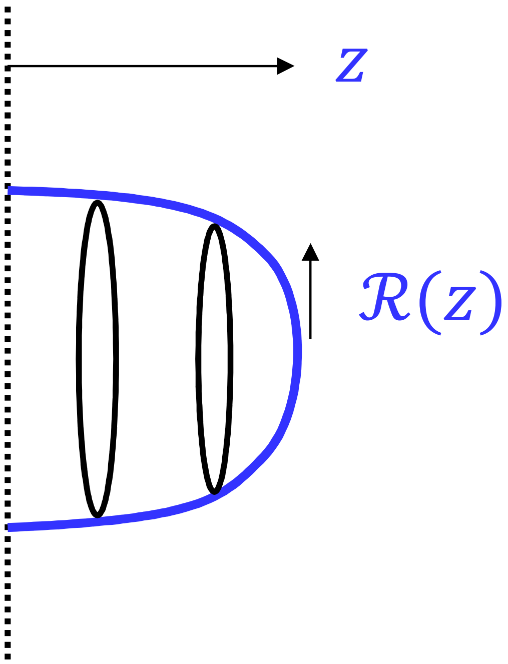
Figueras+Lucietti+Wiseman



z
$\mathcal{R}(z)$
AdS bulk
AdS boundary

$$\sqrt{-g_{tt}}K = \text{const}$$

$$\Rightarrow \ \mathcal{R}(z)' = -\frac{z}{\mathcal{R}(z)}\frac{1 \pm \sqrt{z^2 + \mathcal{R}(z)^2(1-z^2)}}{1-z^2}$$

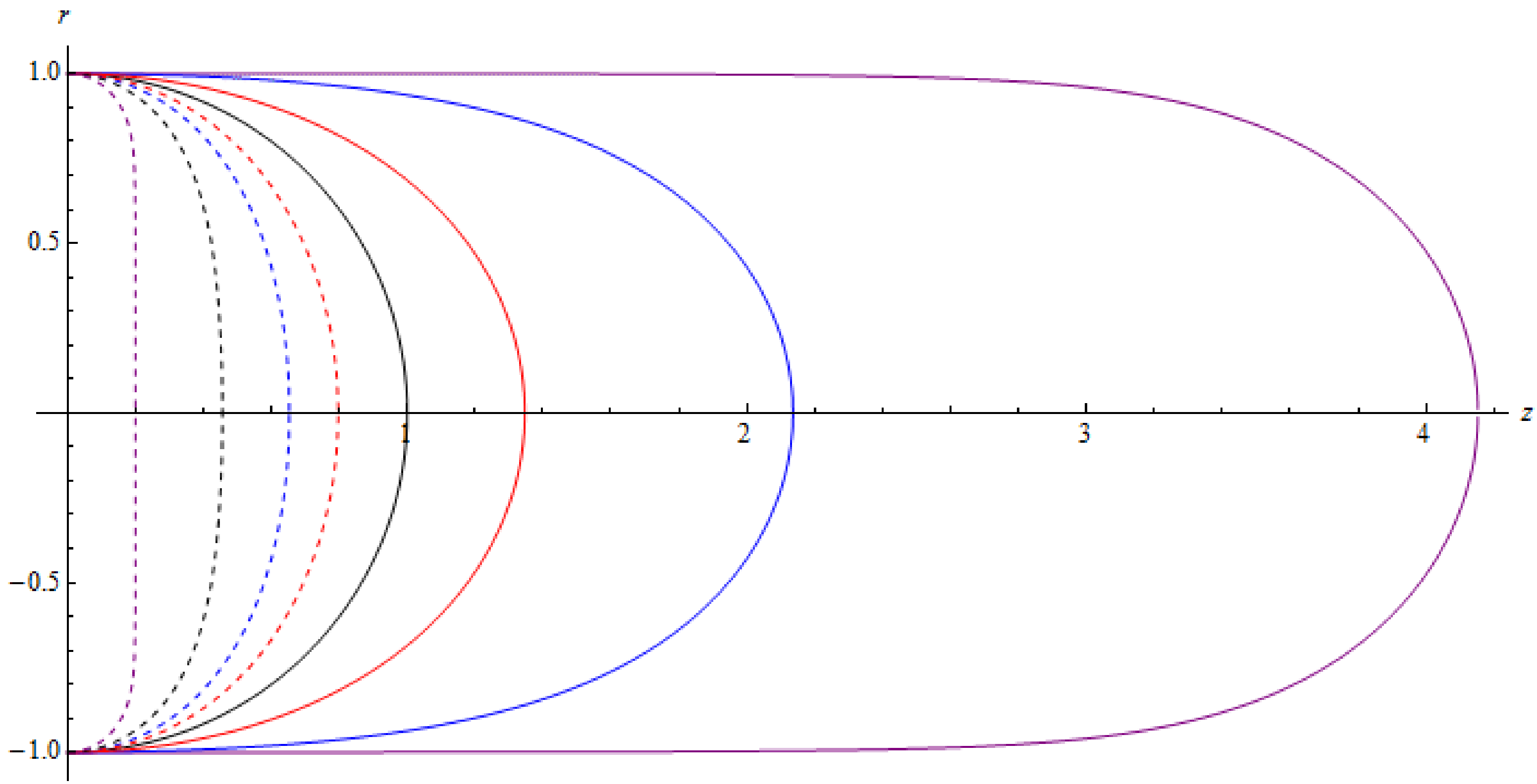


Numerical code

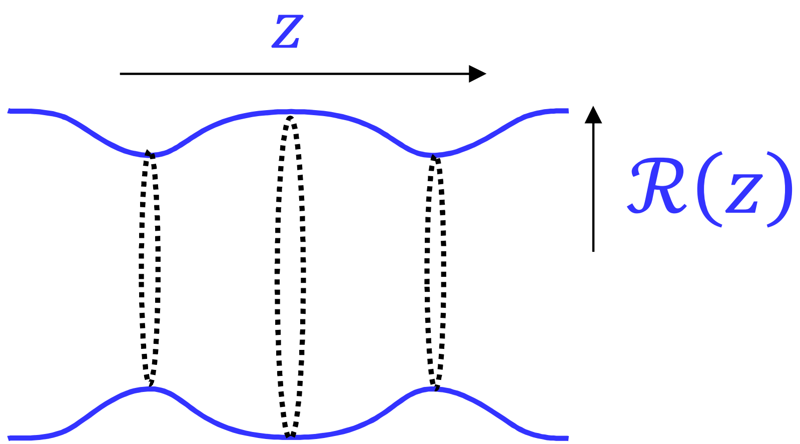
```
zmin=0.000001;
zmax=0.67;
r0=.5;
```

$$\text{NDSolve}\left[\left\{r'[z]==-\frac{z}{r[z]}\,\frac{1-\sqrt{r[z]^2+z^2\,\left(1-r[z]^2\right)}}{1-z^2},\ r[\text{zmin}]==\text{r0}\right\},\ r,\ \{z,\text{zmin},\text{zmax}\}\right]$$

Black droplets

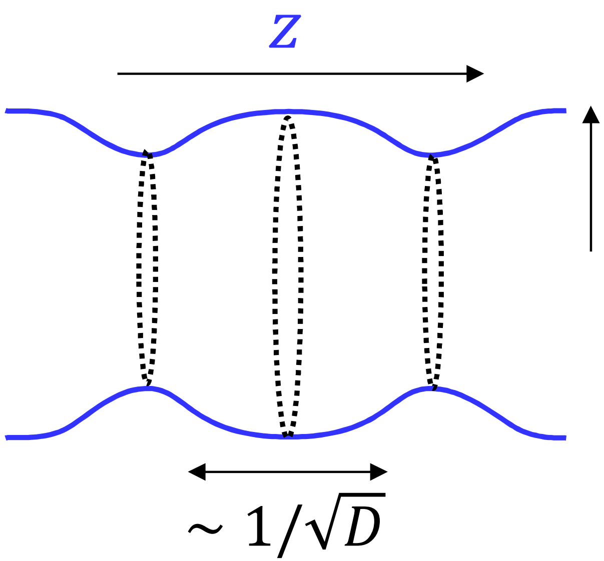


Non-uniform black strings



Numerical solution: *Wiseman*

Non-uniform black strings



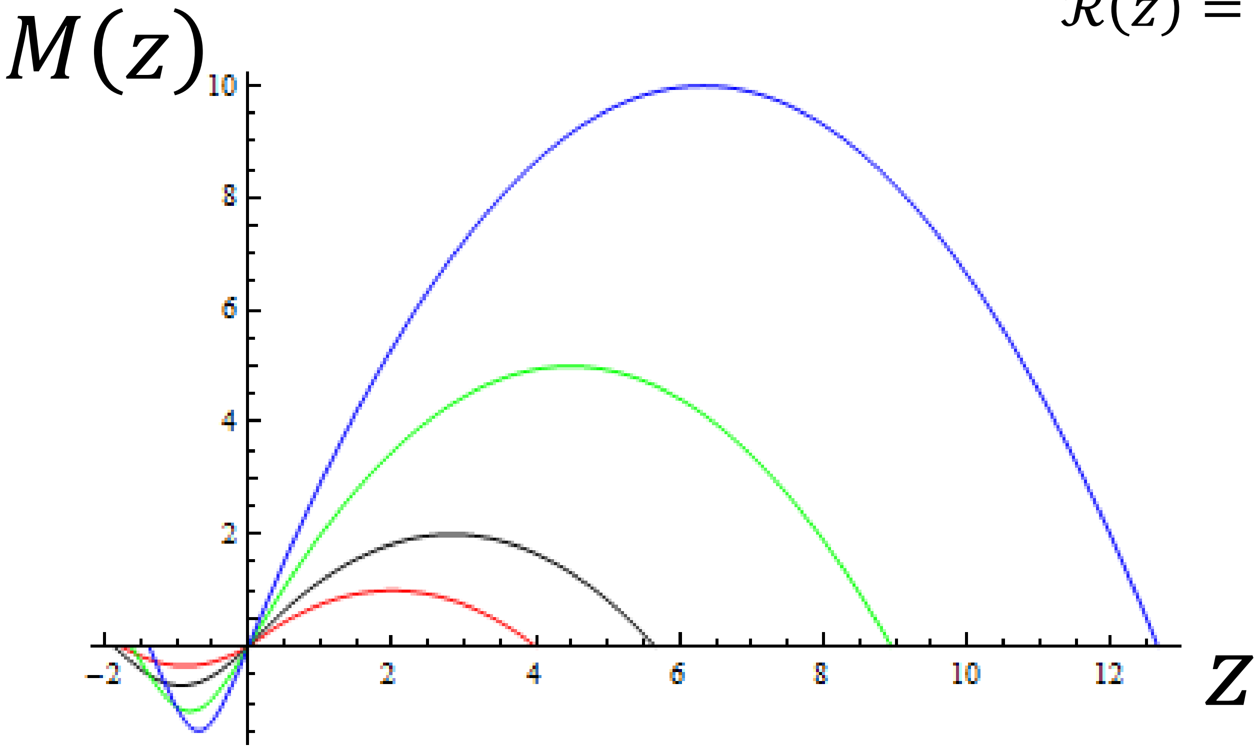
$$\mathcal{R}(z) = 1 + \frac{2M(z)}{D}$$

$$K = \text{const}$$

$$\Longrightarrow \ M''(z) + M'(z)^2 + M(z) = \text{const}$$

Non-uniform black strings

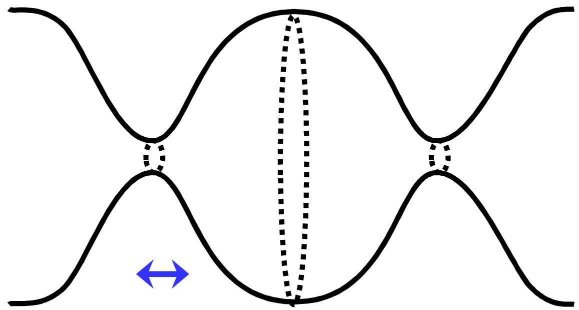
$$\mathcal{R}(z) = 1 + \frac{2M(z)}{D}$$



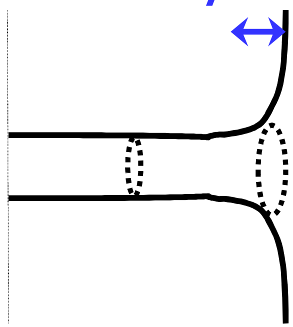
Limitations

1/D expansion breaks down when $\partial_z \sim D$

- Highly non-uniform black strings



- AdS black funnels



In progress

Extensions of $\sqrt{-g_{tt}}\,K = \mathrm{const}$

Charged black holes

Rotating black holes

(Time-evolving black holes)

Conclusions

1/D: it works

(not obvious beforehand!)

Static black holes are soap bubbles

at large D (up to NLO)

Can we reformulate GR

around $D \to \infty$,

with black holes as

basic (extended) objects?