Linear resistivity from hydrodynamics

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Based on: 1311.2451 [hep-th] by RD, K. Schalm, J. Zaanen

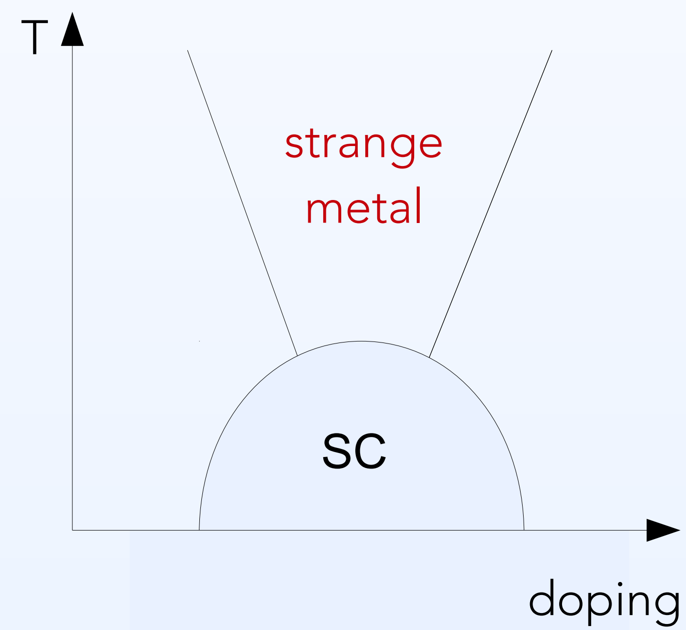
Linear resistivity in the cuprates

- The strange metal state of the high-Tc cuprate superconductors has weird transport properties.

- The most famous is that its resistivity is linear in temperature.

- Why? There is a non-trivial IR fixed point. It is not a Fermi liquid. What is it?

- Taking inspiration from holography, I will describe a very simple mechanism which produces a resistivity like this.



Linear resistivity from holography

- Consider the classical theory of gravity with action

$$S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \left(R - \frac{1}{4} e^{\phi} F_{\mu\nu} F^{\mu\nu} - \frac{3}{2} \partial_\mu \phi \partial^\mu \phi + \frac{6}{L^2} \cosh \phi \right)$$

see e.g. Gubser, Rocha 0911.2898

- This has a charged black brane solution which can be uplifted to a solution of 11D supergravity. Like the cuprate strange metals, it has an entropy linear in T.

- Introducing a random distribution of impurities or a periodic lattice in this state produces a resistivity which is approximately linear in T.

Anantua, Hartnoll, Martin, Ramirez (2012)

How can this be realistic?

- How can they possibly be related? A priori, this field theory looks totally unrelated to the cuprates.
- The mechanism which produces a linear resistivity is independent of many details of the field theory.
- It does not require holography. It can be understood from general principles of strongly interacting quantum critical states.
- The holographic state is just an example of where this mechanism is at work. This is not so dissimilar to the role of holography in understanding the QGP.

Outline of the talk

- Resistivity in states with an almost conserved momentum
- Momentum dissipation rate from dynamics near black brane horizon
- Momentum dissipation rate from hydrodynamics
- Linear resistivity from hydrodynamics

Slow momentum dissipation I

- DC transport properties, like the resistivity, tell us about the late time response of a system to an external source.

- In theories where long-lived quasiparticles carry the current, the quasiparticle decay rate controls the resistivity.

- If there are no long-lived quasiparticles (e.g. in a strongly interacting quantum critical theory), the current intrinsically wants to decay quickly.

- But in a system with perfect translational invariance, momentum is conserved. If the current carries momentum, it cannot decay. Therefore $\rho_{DC} = 0$

Slow momentum dissipation II

- Suppose, in a system like this, translational invariance is broken in a weak way so that momentum dissipates slowly.
- This will cause the current to decay slowly at a rate controlled by the momentum dissipation rate $\rho_{DC} \sim \Gamma$
- As translational invariance is broken weakly, the momentum dissipation rate Γ can be calculated perturbatively.
- Suppose we turn on a lattice i.e. a spatially periodic source for an operator in the IR

$$\delta H = V \int dx e^{ik_L x} \mathcal{O}\left(x\right)$$

Slow momentum dissipation III

- At leading order, the rate at which momentum dissipates into the lattice is determined by the spectral weight in the translationally invariant system

$$\Gamma = \frac{V^2 k_L^2}{\chi_{\vec{P}\vec{P}}} \lim_{\omega \to 0} \frac{\mathrm{Im} G_{\mathcal{O}\mathcal{O}}(\omega, k_L)}{\omega}\Bigg|_{V\to 0}$$

Hartnoll, Hofman (2012)

- This tells us the number of low energy degrees of freedom of the system at the lattice momentum k_L. It is these that will couple to the lattice, once it is turned on.

- If a spatially random source for an operator is turned on,

$$\Gamma = \frac{\bar{V}^2}{\chi_{\vec{P}\vec{P}}} \int d^2k \, k^2 \lim_{\omega \to 0} \frac{\mathrm{Im} G_{\mathcal{O}\mathcal{O}}(\omega, k)}{\omega}\Bigg|_{\bar{V}\to 0}$$

Hartnoll et. al. (2007)
Hartnoll, Herzog (2008)

Slow momentum dissipation: summary

- If we have a charged state in which the only long-lived quantity is the momentum, the resistivity is proportional to the momentum dissipation rate.
- At leading order, this is determined by properties of the translationally invariant state.
- Although this is independent of holography, it is applicable to some of the field theory states described by holography.
- In these cases, we can use holography to calculate the response functions that control the momentum dissipation rate and resistivity.

Holography: gravitational solution

- Using these tools, it was found that the state dual to the charged black brane solution to the Einstein-Maxwell-Dilaton theory

$$S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \left(R - \frac{1}{4} e^{\phi} F_{\mu\nu} F^{\mu\nu} - \frac{3}{2} \partial_\mu \phi \partial^\mu \phi + \frac{6}{L^2} \cosh \phi \right)$$

has $\rho_{DC} \sim T$ when coupled to periodic, or spatially random, sources of charge density or energy density.

Anantua, Hartnoll, Martin, Ramirez (2012)

- The relevant gravitational solution is

$$ds^2 = \frac{r^2 g(r)}{L^2} \left(-h(r) dt^2 + dx^2 + dy^2 \right) + \frac{L^2}{r^2 g(r) h(r)} dr^2, \qquad A_t(r) = \mu \left(1 - \frac{Q + r_0}{Q + r} \right),$$

$$\phi(r) = \frac{1}{3} \log(g(r)), \qquad g(r) = \left(1 + \frac{Q}{r} \right)^{\frac{3}{2}}, \qquad h(r) = 1 - \frac{(Q + r_0)^3}{(Q + r)^3},$$

IR geometry of charged black brane

- The near horizon geometry is conformal to $\mathrm{AdS}_2 \times \mathrm{R}^2$. In the usual classification of near-horizon geometries, it has

$$z \to \infty \quad \text{with} \quad \theta/z = -1$$

- It is similar to the near-horizon $\mathrm{AdS}_2 \times \mathrm{R}^2$ geometry of $\mathrm{RN\text{-}AdS}_4$. The main difference is that this state has entropy

$$s \sim T$$

- $z \to \infty$ means local quantum criticality in the field theory: the low energy physics is approximately momentum-independent.

- Greens functions of fields in this IR geometry have the generic form $$\lim_{\omega \to 0} \mathrm{Im}\mathcal{G}^{IR}(\omega, k) \sim \omega\, T^{2\nu(k)-1}$$

Spectral functions from gravity

- Linear perturbations of the energy density T^{tt} and charge density J^t are irrelevant in the IR: spatially periodic or random sources will cause momentum to dissipate slowly.

- The Greens functions can, in principle, be obtained from a matching calculation

$$G\left(\omega,k\right)=\frac{A(k)+B(k)\mathcal{G}^{IR}\left(\omega,k\right)+\ldots}{C(k)+D(k)\mathcal{G}^{IR}\left(\omega,k\right)+\ldots}$$

- The matching does not have to be done explicitly. At low T, the leading dissipative term is proportional to the IR Greens function

$$\mathrm{Im}G\left(\omega\to 0,k\right)\sim H(k)\,\mathrm{Im}\mathcal{G}^{IR}\left(\omega,k\right)\sim\omega\,T^{2\nu(k)-1}\;H(k)$$

Linear resistivity from disorder

- The momentum dissipation rate due to neutral or charged disorder is:

$$\Gamma \sim \int d^2k\, k^2 \lim_{\omega\to 0} \frac{\mathrm{Im} G\,(\omega,k)}{\omega} \sim \int d^2k\, k^2\, H(k)\, T^{2\nu(k)-1} \sim T^{2\nu(0)-1}$$

- The homogeneous (k=0) mode dominates the integral at low temperatures.

Anantua, Hartnoll, Martin, Ramirez (2012)

- This gives a DC resistivity $\rho_{DC} \sim T$ because an analysis of mass terms in the near horizon geometry shows that the scaling dimension of T^{tt} and J^t is $\nu(k) = \sqrt{\frac{11}{3} + 2\hat{k}^2 - \frac{8}{3}\sqrt{1+\frac{3}{2}\hat{k}^2}} = 1 + O(\hat{k}^4)$
- Finite momentum contributions to $\nu(k)$ are small and give logarithmic corrections to ρ_{DC} : $\rho_{DC}(T) \sim T/\log\left(\mu/T\right)$

Linear resistivity from a lattice

- The momentum dissipation rate due to a neutral or charged lattice is:

$$\Gamma \sim k_L^2 \lim_{\omega \to 0} \frac{\mathrm{Im} G\,(\omega, k_L)}{\omega} \sim k_L^2 H(k_L)\, T^{2\nu(k_L)-1} \sim T^{2\nu(k_L)-1}$$

- Provided the lattice momentum is of the order of the chemical potential (or less), there is an approximately linear DC resistivity

$$\rho_{DC} \sim T$$

Anantua, Hartnoll, Martin, Ramirez (2012)

- Again, it is because the finite k corrections to the dimension are small e.g. $\nu(\hat{k} = 0.5) \sim 1.02$

$$\nu(\hat{k} = 1) \sim 1.2$$

Brief summary of these results

- Without reference to holography, we can summarise why this state has a linear resistivity:

- A lattice or random disorder causes momentum to dissipate slowly.

- The dissipation rate is determined by the two-point functions of T^{tt} and J^t in the translationally invariant, locally critical state.

- At low T, these are approximately proportional to T because T^{tt} and J^t have dimension $\nu(k) = 1 + O(k^4)$.

- Generally, one finds power laws for locally critical states $\rho_{DC} \sim T^\eta$

A different perspective

- Why do these correlators have a term which is approximately linear in T??? There is another way to understand it.

RD, Schalm, Zaanen, 1311.2451

- We have learned a lot about the general principles of how charge and momentum are transported in holographic theories with translational invariance.

- These general principles appear to be true in real strongly interacting systems: they do not require the existence of a dual classical gravity description.

- This highlights a simple mechanism that can produce linear resistivity and which may be at work in real systems.

Some history

- The simplest case: a black brane dual to a neutral, thermal state.

- At long distances and low energies $\omega, k \ll T$, these behave like hydrodynamic fluids with a minimal viscosity

$$\eta = \frac{\hbar}{k_B}\frac{s}{4\pi}$$

Kovtun, Son, Starinets (2004)
Iqbal, Liu (2008)

- A small viscosity means that a fluid thermalises very quickly. e.g. in a kinetic theory of quasiparticles, $\eta \sim \text{mean free time}$

- It is not so surprising that a state with a holographic dual forms a hydrodynamic state in a short time.

Hydrodynamics

- Hydrodynamics is an effective theory, telling us what the collective properties of the system are at long distances and low energies.

- For a relativistic fluid with $\epsilon = 2P$,

$$T^{\mu\nu} = (\epsilon + P)\, u^\mu u^\nu + P g^{\mu\nu} - \eta \Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - g_{\alpha\beta}\partial_\gamma u^\gamma\right) + \ldots$$
$$J^\mu = \rho u^\mu - \sigma_Q T \Delta^{\mu\nu}\partial_\nu\left(\mu/T\right) + \ldots$$

- At leading order in spatial derivatives, dissipation is controlled by two transport coefficients: shear viscosity η and "universal conductivity" σ_Q .

- Their values depend upon the specific microscopic theory

Greens functions from hydrodynamics

- These hydrodynamic equations tell us how the state will respond to small perturbations.

- They fix the form of the Greens functions at long distances and low energies e.g.

$$G_{\vec{P}_\perp \vec{P}_\perp}(\omega, k) = \frac{\eta k^2 + \dots}{i\omega - D k^2 + \dots} \qquad\qquad D = \frac{\eta}{\epsilon + P}$$

- The shear viscosity controls the rate at which momentum diffuses and the universal conductivity controls the rate at which charge diffuses.

Hydrodynamics of locally critical states I

- At long distances and low energies, hydrodynamics is a good approximate description of locally critical holographic states.

- Greens functions can be calculated by matching the IR Greens functions to the asymptotically AdS UV region.

$$G\left(\omega,k\right)=\frac{A(k)+B(k)\mathcal{G}^{IR}\left(\omega,k\right)+\dots}{C(k)+D(k)\mathcal{G}^{IR}\left(\omega,k\right)+\dots}$$

- This can be done numerically or, in some cases, analytically.

- Unlike the neutral case, hydrodynamics is a good approximate description even at low temperatures, provided that $\omega, k \ll \mu$

see e.g. Edalati, Jottar, Leigh (2010), RD, Parnachev (2013) Tarrio (2013) and others

Hydrodynamics of locally critical states II

- In the simplest case of RN-AdS, the matching can be done explicitly and analytically for some operators.

RD, Parnachev (2013)

- Ignoring finite k corrections to $\nu(k)$ in the IR geometry, the correlation functions are just those of hydrodynamics, with certain values of the transport coefficients.
- These corrections are not important for the leading order resistivity in the presence of disorder or a lattice.
- The key point is that if a theory obeys hydrodynamics, the IR dimensions of operators are not random numbers: they are related to the transport coefficients.

Hydrodynamics of locally critical states III

- The T dependence of Greens functions in a hydro theory are controlled by the T dependence of the transport coefficients.

- We have replaced one aspect of microscopic physics (operator dimensions) with another: values of transport coefficients.

- This is a complimentary view of the same situation.

- It is advantageous for one reason: we can make an informed estimate of the size of one of these transport coefficients in general

$$\eta \sim \frac{\hbar}{k_B} s$$

Viscous contribution to resistivity

- There are many hydrodynamic contributions to the resistivity which will depend upon microscopic details of the theory.

neutral

$$\rho_{DC} \sim \frac{\mathcal{V}^2_{T^{tt}}}{\sigma^2} \int dk k \left(\eta k^2 + \ldots\right) ,$$

charged

$$\rho_{DC} \sim \frac{\mathcal{V}^2_{J^t}}{\sigma^2} \int dk k \left(\frac{1}{\sigma_Q} \left[2 \frac{\sigma^2}{\epsilon + P} - \left(\frac{d\sigma}{d\mu} \right)_T \right]^2 + k^2 \frac{\sigma^2}{(\epsilon + P)^2} \eta + \ldots \right)$$

+ analogous expressions for lattice deformations

- We will concentrate on the viscous term.

- A simple argument of why it exists is that momentum diffuses in a hydrodynamic liquid with diffusion constant $D = \eta / m_e n_e$.

- If translational invariance is broken over a length scale l, the time it takes for the momentum to dissipate is $\tau^{-1} = D/l^2$

Resistivity = entropy

- The memory matrix calculation confirms this. It has also been observed by other methods e.g.

$$\rho_{2D} = \frac{1}{2e^2} \left\langle \frac{T}{\kappa} (\delta s_0)^2 + (\eta + \zeta) \left(\nabla \frac{1}{n_0} \right)^2 \right\rangle .$$

From 1011.3068 [cond-mat.mes-hall] by A. Andreev, S. Kivelson, B. Spivak

- If a theory behaves like a hydrodynamic liquid with minimal viscosity down to the length scale over which impurities/the lattice are present, it will have a viscous contribution to its resistivity

$$\rho_{DC}(T) \sim \eta(T) \sim s(T)$$

provided that momentum is almost conserved.

- The locally critical states of holography obey this "entropy law".

Fermi liquids

- Why do conventional metals not have $\rho_{DC}(T) \sim s(T)$?

- These do not behave hydrodynamically at long times. The quasiparticle interaction rate is small: $\Gamma_{ee} \sim T^2$

- The corresponding viscosity is large: $\eta_{FL} \sim \frac{1}{T^2}$

- This means it takes a long time $\tau \sim T^{-2}$ for a Fermi liquid to equilibrate via interactions and form a hydrodynamic state.

- The electrons lose their momentum via interactions with the ionic lattice before the hydrodynamic state forms.

Cuprates

- Strong electronic interactions cause the formation of a hydrodynamic state with a minimal viscosity over a short time scale. This hydro description applies at distances ~ μ^{-1} .

- Slow momentum-dissipating interactions then produce a resistivity $$\rho_{DC}(T) = \frac{A\hbar}{\omega_p^2 m_e l^2}\frac{S(T)}{k_B} \sim T$$

- This requires a small length scale ~ $l \sim 10^{-9}\mathrm{m}$

- But there is no residual (T=0) resistivity as, in this limit, the electrons behave as a perfect fluid.

- This is radically different from FL theory: it should be testable.

work in progress....

Conclusions

- Strongly interacting quantum critical systems are highly collective states without long-lived quasiparticles.
- Holography gives us examples of quantum critical states which behave like hydrodynamic fluids with a minimal viscosity.
- If a charged hydrodynamic state with minimal viscosity is weakly coupled to disorder/lattice, it will get a viscous contribution to its resistivity $\rho_{DC}(T) \sim s(T)$.
- This mechanism does not require holography. It may explain some of the strange transport properties of the strange metal phase of the high Tc cuprate superconductors.