

Superfluid Hydrodynamics, Thermal Partition Function and Lifshitz Scaling

Oxford - Theoretical Physics

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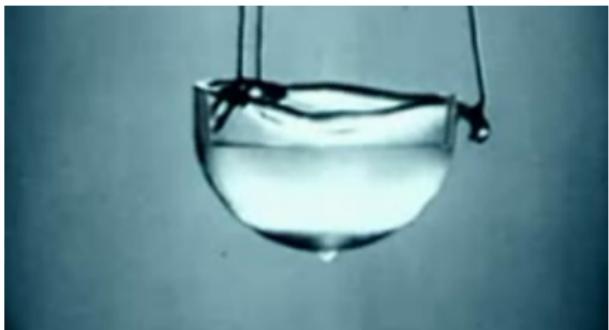
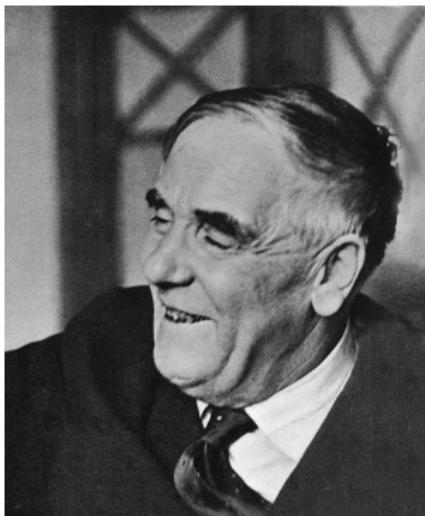
Outline

- ▶ Superfluids and Superconductors.
- ▶ Relativistic Superfluid Dynamics.
- ▶ Chiral Terms in Superfluids.
- ▶ Kubo Formulas From Equilibrium Partition Function.
- ▶ Lifshitz Scaling Symmetry.
- ▶ Quantum Critical Points.
- ▶ Experimental Implications.

Superfluidity and Superconductivity

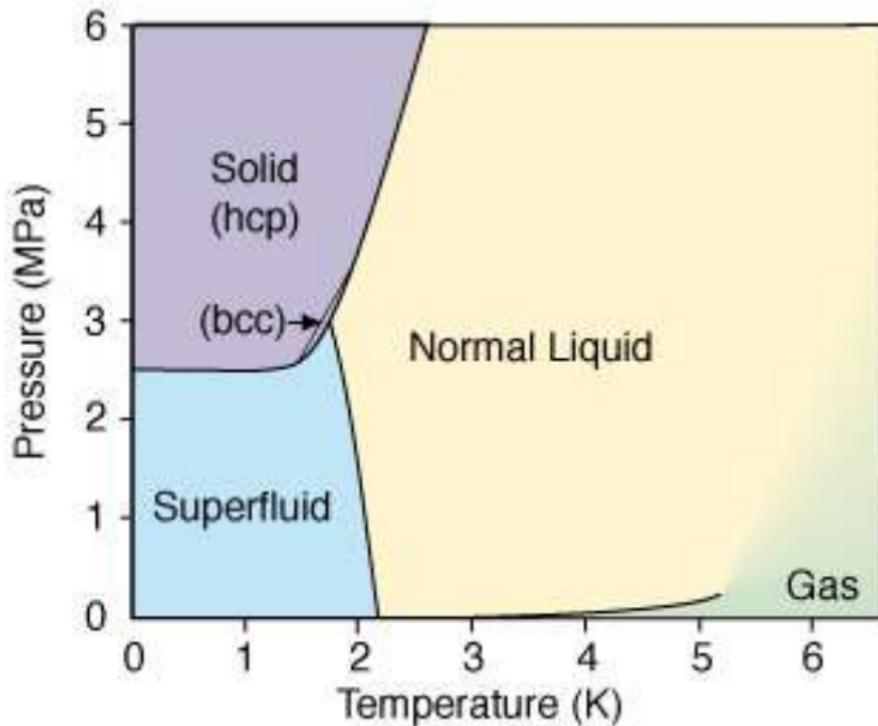
- ▶ **Pyotr Leonidovich Kapitsa - 1938**

1978 - Nobel prize for discovery of superfluidity in 4He



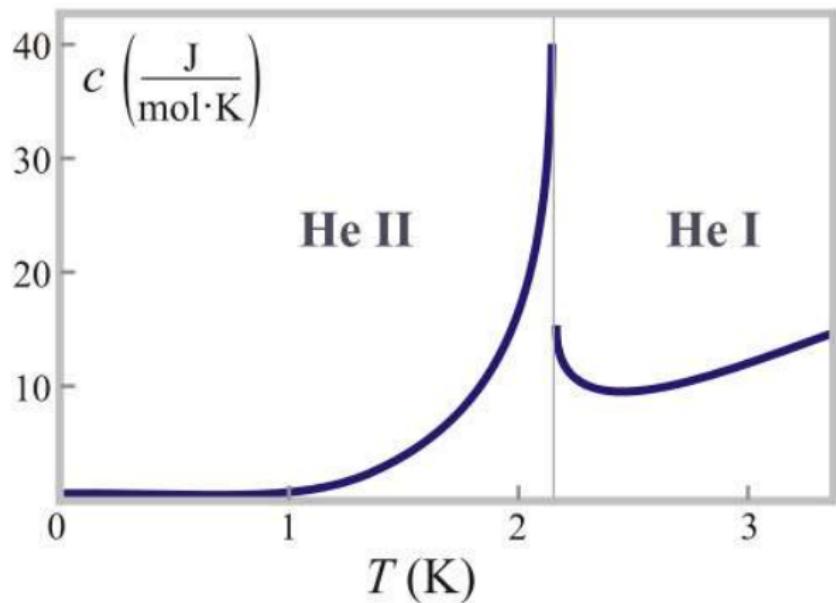
- ▶ Liquifies at $T \approx 4.2^\circ\text{K}$
- ▶ At $T \approx 2.17^\circ\text{K}$ - Second order phase transition.

► Phase diagram of 4He



- Remains liquid at absolute zero
- Condensate of atoms in ground state - Collective mode

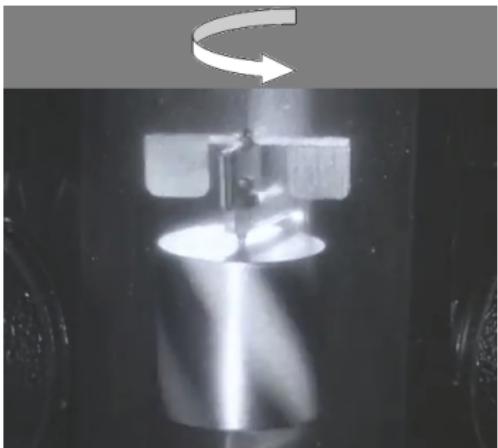
► The λ point



- Heat conductivity increase by a factor of 10^6
- Large part of atoms in ground state - Condensate - collective mode

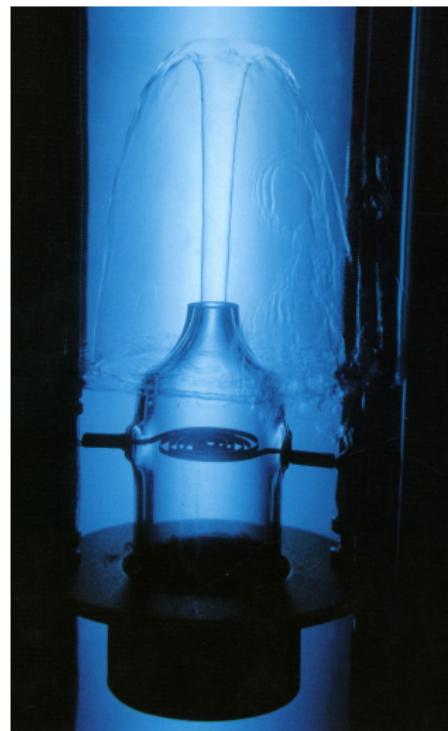
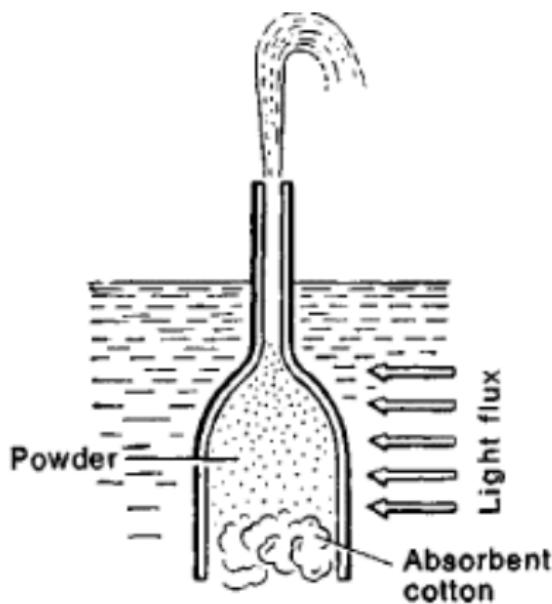
The Two Fluid Picture

- ▶ **Viscosity vs. no viscosity**



- ▶ Effective picture - two fluid flows \vec{v}_n , \vec{v}_s
- ▶ Due to Landau

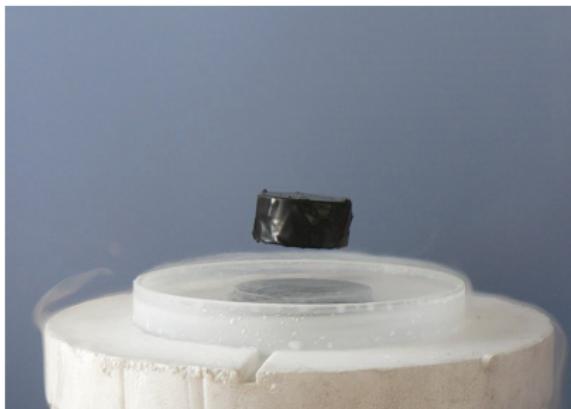
- ▶ Superfluid component carries no entropy



Superconductivity

- ▶ **Heike Kamerlingh Onnes - 1911**

1913 - Nobel prize for liquifying helium



- ▶ Zero resistance to DC current.
- ▶ Meissner effect (1933).
- ▶ Condensation of Cooper pairs.
- ▶ Two fluid picture - paired electrons form superfluid.

Relativistic Superfluids

- ▶ Spontaneous symmetry breaking
- ▶ S-wave superfluid - condensate - complex scalar operator
- ▶ Phase of scalar ϕ - Goldstone mode - participates in the hydrodynamics.
- ▶ Fluid variables:
 - T - temperature,
 - μ - chemical potential,
 - u^μ - normal fluid 4-velocity,
 - $\xi^\mu = -\partial_\mu \phi$ - Goldstone phase gradient,
 - $u_s^\mu = -\xi^\mu/\xi$ - Superfluid velocity.
- ▶ Superconductors - broken gauge symmetry - $\xi^\mu \equiv -\partial_\mu \phi + \mathcal{A}_\mu$.

Relativistic Superfluids

- ▶ New thermodynamic parameters ξ^μ .
- ▶ Thermodynamic relations:

$$\varepsilon_n + P = sT + q_n\mu$$

$$dP = sdT + q_nd\mu + \frac{f}{2}d\xi^2$$

- ▶ Stress tensor and current:

$$T^{\mu\nu} = \varepsilon_n u^\mu u^\nu + P(\eta^{\mu\nu} + u^\mu u^\nu) + \varepsilon_s u_s^\mu u_s^\nu + \pi^{\mu\nu}$$

$$J^\mu = q_n u^\mu + q_s u_s^\mu + j_{diss}^\mu$$

- ▶ Josephson relation:

$$u^\mu \xi_\mu = \mu + \mu_{diss}.$$

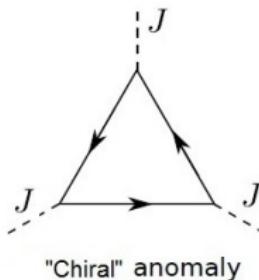
- ▶ Hydrodynamic equations - conservation law:

$$\partial_\mu T^{\mu\nu} = \mathcal{F}^{\nu\mu} J_\mu$$

$$\partial_\mu J^\mu = CE_\mu B^\mu$$

$$\partial_\mu \xi_\nu - \partial_\nu \xi_\mu = \mathcal{F}_{\mu\nu}$$

- ▶ Electric field: $E^\mu = \mathcal{F}^{\mu\nu} u_\nu$
- ▶ Magnetic field: $B^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} u_\nu \mathcal{F}_{\rho\sigma}$
- ▶ C - triangular anomaly of three currents.



- ▶ Goal: constrain expressions for $\pi^{\mu\nu}$, j_{diss}^μ , μ_{diss} .

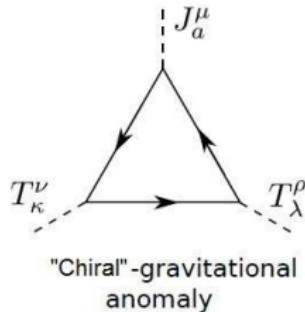
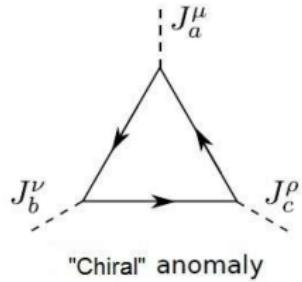
Chiral Effects in Superfluid

- ▶ Local second law - Constrain current and conductivities

$$\begin{aligned} j_{diss}^\mu &= \sigma \left(E^\mu - T P^{\mu\nu} \partial_\nu \left(\frac{\mu}{T} \right) \right) \\ &\quad + B^\mu (C\mu + 2Tg_1) + \omega^\mu (C\mu^2 + 4g_1\mu T - 2g_2 T^2) + \dots \\ \pi^{\mu\nu} &= \eta \sigma_{\mu\nu} + \zeta (\partial_\alpha u^\alpha) P^{\mu\nu} + \dots \end{aligned}$$

- ▶ $g_1 = g_1(T, \mu, \xi^2); \quad g_2 = g_2(T, \mu, \xi^2).$
- ▶ Vorticity: $\omega^\mu = \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$
- ▶ Shear tensor: $\sigma_{\mu\nu} = P_\mu^\rho P_\nu^\sigma \partial_{(\rho} u_{\sigma)} - \frac{1}{3} P_{\mu\nu} \partial_\rho u^\rho.$
- ▶ Comparison to normal fluid

- ▶ C - triangular anomaly of three currents.
- ▶ In the normal fluid g_2 integration constant - related to mixed chiral-gravitational anomaly. [Yarom: 1207.5824]



$$\partial_\mu J^\mu \sim \epsilon^{\mu\nu\rho\sigma} \left(\frac{C}{8} F_{\mu\nu} F_{\rho\sigma} + \frac{\beta}{32\pi^2} \mathcal{R}_{\beta\mu\nu}^\alpha \mathcal{R}_{\alpha\rho\sigma}^\beta \right).$$

- ▶ Numerical evidence that at $T \rightarrow 0$ one restore the normal fluid values [Amado: 1401.5795]
- ▶ Chiral effects - 3He , Neutron stars.

Equilibrium Partition Function

- ▶ Alternative method to derive hydrodynamic current
- ▶ Minwalla et al. - 1203.3544, Yarom et al. - 1203.3556
- ▶ Consider equilibrated fluid on a curved manifold with non-trivial gauge fields

$$ds^2 = -e^{2\sigma(\vec{x})} (dt + a_i(\vec{x})dx^i)^2 + g_{ij}(\vec{x})dx^i dx^j,$$
$$\mathcal{A} = \mathcal{A}_0(\vec{x})dx^0 + \mathcal{A}_i(\vec{x})dx^i,$$

- ▶ KK invariant gauge field:

$$A_0 \equiv \mathcal{A}_0 + \mu_0 ,$$

$$A_i \equiv \mathcal{A}_i - A_0 a_i ,$$

- ▶ Local temperature $T(\vec{x}) = T_0 e^{-\sigma}$
- ▶ Local chemical potential $\mu(\vec{x}) = A_0 e^{-\sigma}$

Equilibrium Partition Function

- ▶ Build the most general equilibrium partition function [effective action]

$$S = S_0 + S_1 ,$$

$$S_0 = \int d^3x \frac{1}{T} P(T, \mu, \hat{\zeta}^2) ,$$

$$S_1 = \int d^3x \hat{\zeta} \cdot (g_1 \partial \times A + T g_2 \partial \times a)$$

$$+ C \int d^3x A \cdot \left(\frac{\mu}{3T} \partial \times A + \frac{\mu^2}{6T} \partial \times a \right) + \dots$$

- ▶ $\hat{\zeta}_i \equiv -\partial_i \phi + A_i$, transverse [spatial] part of goldstone field
- ▶ Differentiate with respect to the gauge field to obtain the current
- ▶ Advantage - algebraic rather than differential
- ▶ Disadvantage - only captures equilibrium properties

Linear Response Theory

- ▶ Relates transport coefficients to retarded correlation function of stress tensors and currents in terms of **Kubo formulas**
- ▶ Allow for a microscopic calculation e.g. Feynmann diagrams
- ▶ Deriving Kubo formulas - normally requires to solve the the equations of motion for a particular source of perturbation
- ▶ Alternative shorter algebraic method - from variations of the equilibrium partition function
- ▶ Reproduces known Kubo formulas for various fluid cases
- ▶ New Kubo formulas for superfluids

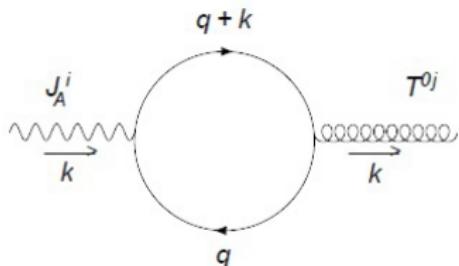
Results

- ▶ Kubo formulas -

$$g_1(T, \mu, \zeta^2) = - \lim_{k_n \rightarrow 0} \sum_{ij} \frac{i}{4T k_n} \epsilon_{ijn} \langle J^i(k_n) J^j(-k_n) \rangle \Big|_{\substack{\omega=0 \\ k \parallel \zeta}} - \frac{C}{2} \left(\frac{\mu}{T} \right),$$

$$g_2(T, \mu, \zeta^2) = \lim_{k_n \rightarrow 0} \sum_{ij} \frac{i}{2T^2 k_n} \epsilon_{ijn} \left[\langle J^i T^{0j} \rangle - \mu \langle J^i J^j \rangle \right] \Big|_{\substack{\omega=0 \\ k \parallel \zeta}} - \frac{C}{2} \left(\frac{\mu}{T} \right)^2$$

- ▶ Spatial superfluid velocity - new thermal parameter - ζ^i .
- ▶ Similar role to chemical potential in the spatial direction.
- ▶ Substitution rules in propagators - $q^\mu \rightarrow (i\omega_n + \mu, \vec{q} + \vec{\zeta})$.



- ▶ Holographic calculation also possible.

Lifshitz Superfluids - Quantum Critical Points

- ▶ Anisotropic Weyl - Lifshitz scaling symmetry:

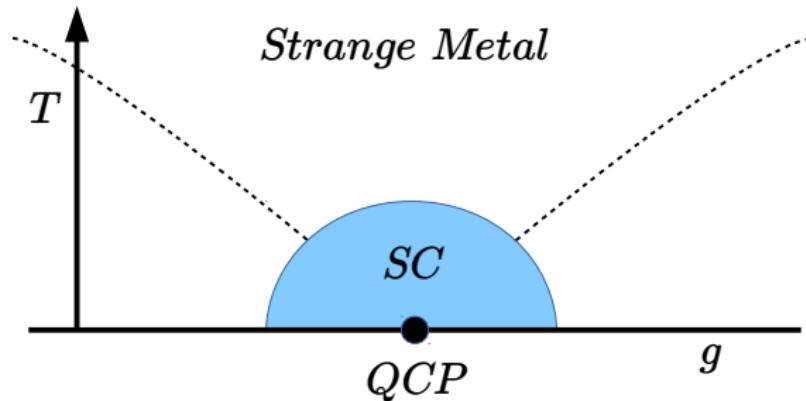
$$t \rightarrow \Omega^z t$$
$$x^i \rightarrow \Omega x^i$$

z - dynamical critical exponent

- ▶ Must be accompanied by broken boost invariance
- ▶ Phase transition at zero temperature
 - ▶ Driven by quantum fluctuations
 - ▶ Quantum tuning parameter [B , doping, pressure]
- ▶ First and Second order transition
- ▶ Infinite correlation length - scale invariance
- ▶ Hydrodynamic regime - $l_c \gg L \gg l_T$

Quantum Criticality

- Influence of quantum critical point felt way above $T = 0$.



- Example: anti-ferromagnetic \rightarrow heavy fermion metal transition.
- Strange metal behavior $\rho \sim T$ ($\sim T^2$ in normal metals)
- Characteristic of high T_c superconductors in the non-superconducting regime.

Hydrodynamics with broken boost invariance

- ▶ Under lorentz transformations

$$\delta\mathcal{L} = T_{\mu\nu}\omega^{\mu\nu}$$

$\omega^{\mu\nu}$ antisymmetric parameter of Lorentz transformation

- ▶ Asymmetric stress tensor in time direction $\sim T^{[0i]}$.
- ▶ Assumption - fluid can be described using former variables.
- ▶ No need of external time vector [phonons]
- ▶ Fluid velocity in the local rest frame points in the time direction
- ▶ Antisymmetric part of stress tensor:

$$T^{[\mu\nu]} = u^{[\mu} V_A^{\nu]}$$

Constitutive relations

- ▶ Stress-tensor:

$$T^{\mu\nu} = (\varepsilon_n + p)u^\mu u^\nu + p\eta^{\mu\nu} + \varepsilon_s u_s^\mu u_s^\nu + \pi^{(\mu\nu)} + \pi_A^{[\mu\nu]} .$$

- ▶ Choice of frame - removing a redundancy by shift of thermal variables:

$$u^\mu \rightarrow u^\mu + \delta u^\mu; \quad T \rightarrow T + \delta T; \quad \mu \rightarrow \mu + \delta \mu .$$

- ▶ Clark Putterman frame - no current corrections,

$$j_{diss}^\mu = 0$$

$$\pi^{\mu\nu} u_\mu u_\nu = 0$$

- Decompose:

$$\pi^{(\mu\nu)} = (Q^\mu u^\nu + Q^\nu u^\mu) + \Pi P^{\mu\nu} + \Pi_t^{\mu\nu} ,$$

where

$$Q^\mu u_\mu = 0, \quad \Pi_t^{\mu\nu} u_\nu = 0, \quad \Pi_t^{\mu\nu} P_{\mu\nu} = 0 .$$

Q^μ represent the heat flow.

Entropy Increase

- ▶ Entropy current:

$$J_s^\mu = s u^\mu - \frac{u_\nu}{T} \pi^{\mu\nu} + \frac{f}{T} \mu_{diss} \zeta^\mu .$$

- ▶ Entropy production rate

$$\begin{aligned}\partial_\mu J_s^\mu = & - \frac{[\Pi(\partial_\mu u^\mu) + \Pi_t^{\mu\nu} \sigma_{\mu\nu}]}{T} - \frac{Q_\mu}{T} \left[a^\mu + P^{\mu\nu} \frac{\partial_\nu T}{T} \right] \\ & + \mu_{diss} P^{\mu\nu} \partial_\mu \left(\frac{f \zeta_\nu}{T} \right) - \frac{V_{A\mu}}{2T} \left[a^\mu - P^{\mu\nu} \frac{\partial_\nu T}{T} \right] ,\end{aligned}$$

- ▶ Has to be positive sum of quadratic forms.
- ▶ Constraint dissipative corrections: $\Pi_t^{\mu\nu}, \Pi, Q^\mu, V_{A\mu}, \mu_{diss}$
- ▶ New vector - acceleration $a^\mu \equiv u^\nu \partial_\nu u^\mu$. Two projections - in the direction and in the transverse direction to the superfluid velocity

- ▶ Number of transport terms in a superfluid

	T – preserving	T – breaking
non – Lifshitz	14	7
Lifshitz	22	13

- ▶ More detailed results in the NR limit
- ▶ Only included parity preserving effects

The non-relativistic limit

- ▶ Fluid variables:
 - ▶ ρ_n, ρ_s - mass densities,
 - ▶ \vec{v}_n, \vec{v}_s - velocities,
 - ▶ $\vec{\omega} = \vec{v}_s - \vec{v}_n$ - counterflow
 - ▶ projector $P_w^{ij} = \delta^{ij} - \frac{w^i w^j}{w^2}$.
- ▶ expansion in powers of c :
 - ▶ Expand thermal parameters:
 - ▶ $u^\mu = (1, \frac{\vec{v}_n}{c})$
 - ▶ $\xi^\mu = -c(1, \frac{\vec{v}_s}{c})$
 - ▶ $\mu_{rel} = c + \frac{1}{c}(\mu + \omega^2/2)$
 - ▶ $\epsilon_n = \rho_n c^2 + U_n - \rho_n \frac{v_n^2}{2}$
 - ▶ Expand constitutive relations

$$\pi^{\mu\nu} = \sum_n \frac{1}{c^n} \pi_{(n)}^{\mu\nu}$$

- ▶ equations of motion:

- ▶ mass conservation:

$$\partial_t(\rho_n + \rho_s) + \partial_i(\rho_n v_n^i + \rho_s v_s^i) = 0$$

- ▶ Navier-Stokes:

$$\partial_t(\rho_n v_n^i + \rho_s v_s^i) + \partial_k(\rho_n v_n^i v_n^k + \rho_s v_s^i v_s^k) + \partial^i p + \nu^i = 0$$

- ▶ Energy conservation:

$$\partial_t E + \partial_i [Q^i + Q'^i] + \nu_e = 0$$

- Dissipative corrections:

$$\begin{aligned}
 Q'^i &\sim \pi_{(1)}^{i0} = \\
 &= \omega^i(Q_1\omega^j\partial_j T + Q_2\omega^j D_t v_{nj}) + Q_3 P_\omega^{ij}\partial_j T + Q_4 P_\omega^{ij} D_t v_{nj} + \dots \\
 \nu^j &= \partial_t \pi_{(-1)}^{0i} + \partial_k \pi_{T(0)}^{(ki)} + \dots \\
 \pi_{T(0)}^{ij} &= -\eta \sigma^{ij} - \zeta \partial_k v^k \delta_{ij} - t_1 \partial_k (\rho_s \omega^k) \\
 &\quad - w^i w^j (t_2 \omega^i \partial_i T + t_3 \omega^i D_t v_i) + \dots \\
 \pi_{(-1)}^{0i} &= -\omega^i(A_1\omega^j\partial_j T + A_2\omega^j D_t v_j) - A_3 P_\omega^{ij}\partial_j T - A_4 P_\omega^{ij} D_t v_j + \dots
 \end{aligned}$$

- scaling of transport coefficients

$$\sim T^{\frac{\Delta}{z}} F \left(\frac{\mu}{T^{\frac{2(z-1)}{z}}}, \frac{w^2}{T^{\frac{2(z-1)}{z}}} \right).$$

$$\begin{aligned}
 [Q_1] &= d - z; & [Q_2] &= d - 2(z - 1); & [Q_3] &= z + d - 2; & [Q_4] &= d \\
 [A_1] &= d + 2 - 3z; & [A_2] &= d - 4(z - 1); & [A_3] &= d - z; & [A_4] &= d - 2(z - 1)
 \end{aligned}$$

Experimental Implications

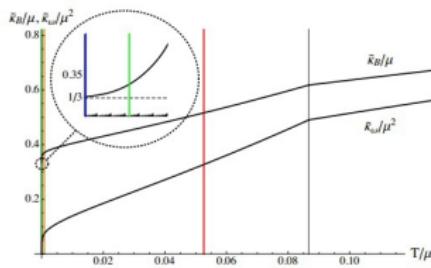
- ▶ Part of heat flow proportional to acceleration
- ▶ Can be related to chemical potential:

$$D_t v_n^i \simeq \partial^i \left(\mu + \frac{w^2}{2} \right) + \dots$$

- ▶ Anisotropy between the direction of the counterflow and the transverse direction.
- ▶ Hard to disentangle from effect of shear viscosity
- ▶ In superconductors alternating current create phase between normal and super components
- ▶ Perhaps some unusual frequency dependence would reveal the effect in superconductors

Outlook

- ▶ Evaluate g_1 and g_2 using the Kubo formulas
 - ▶ Weakly coupled field theory
 - ▶ Holographic model
 - ▶ Perhaps explain temperature dependence of Amado [1401.5795]:



- ▶ Lifshitz Superfluid
 - ▶ Suggest a measurement in superconductors.
 - ▶ Include parity violating effects.
 - ▶ Construct holographic model



Thank you