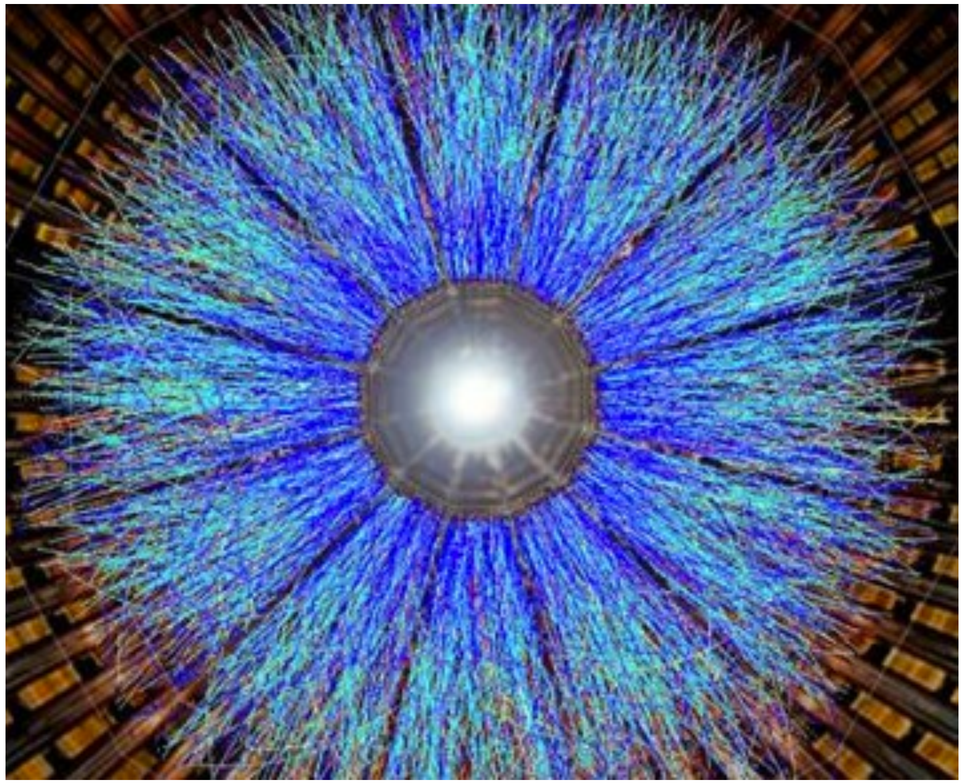
Holographic Transport and the Hall Angle

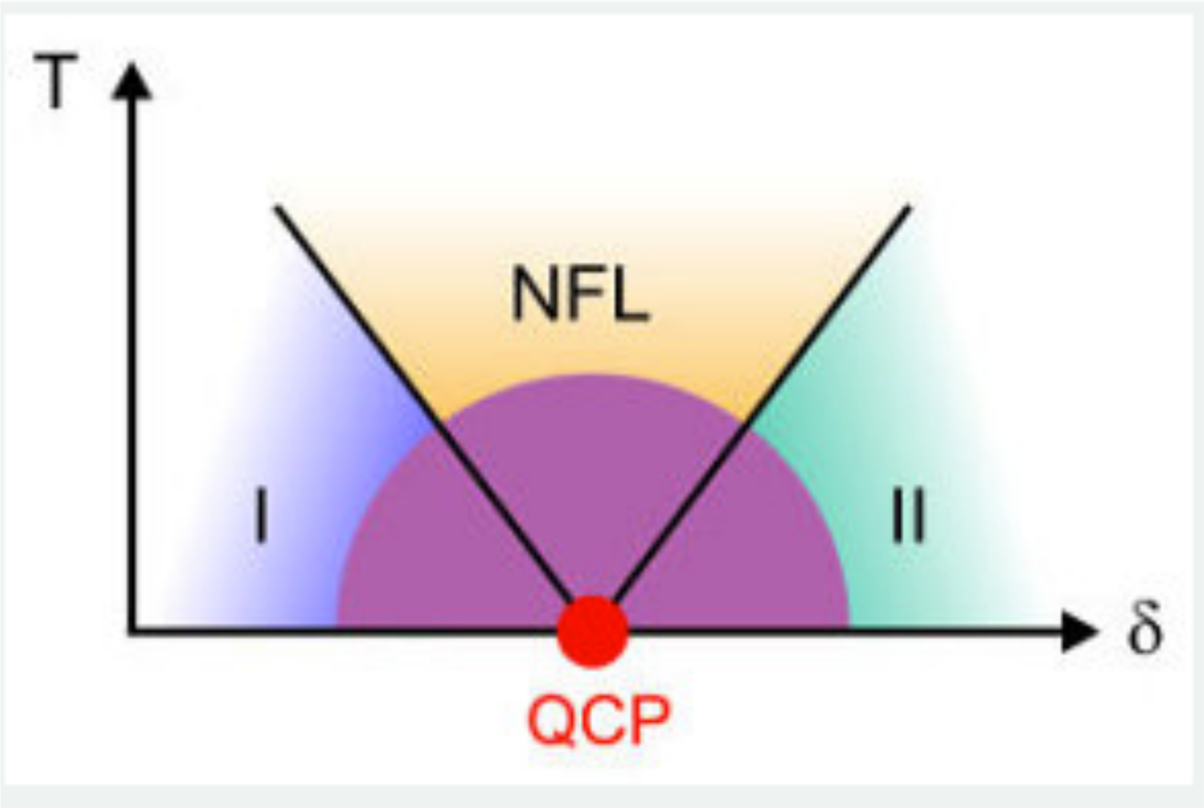
Mike Blake - DAMTP

CHURCHILL COLLEGE
CAMBRIDGE

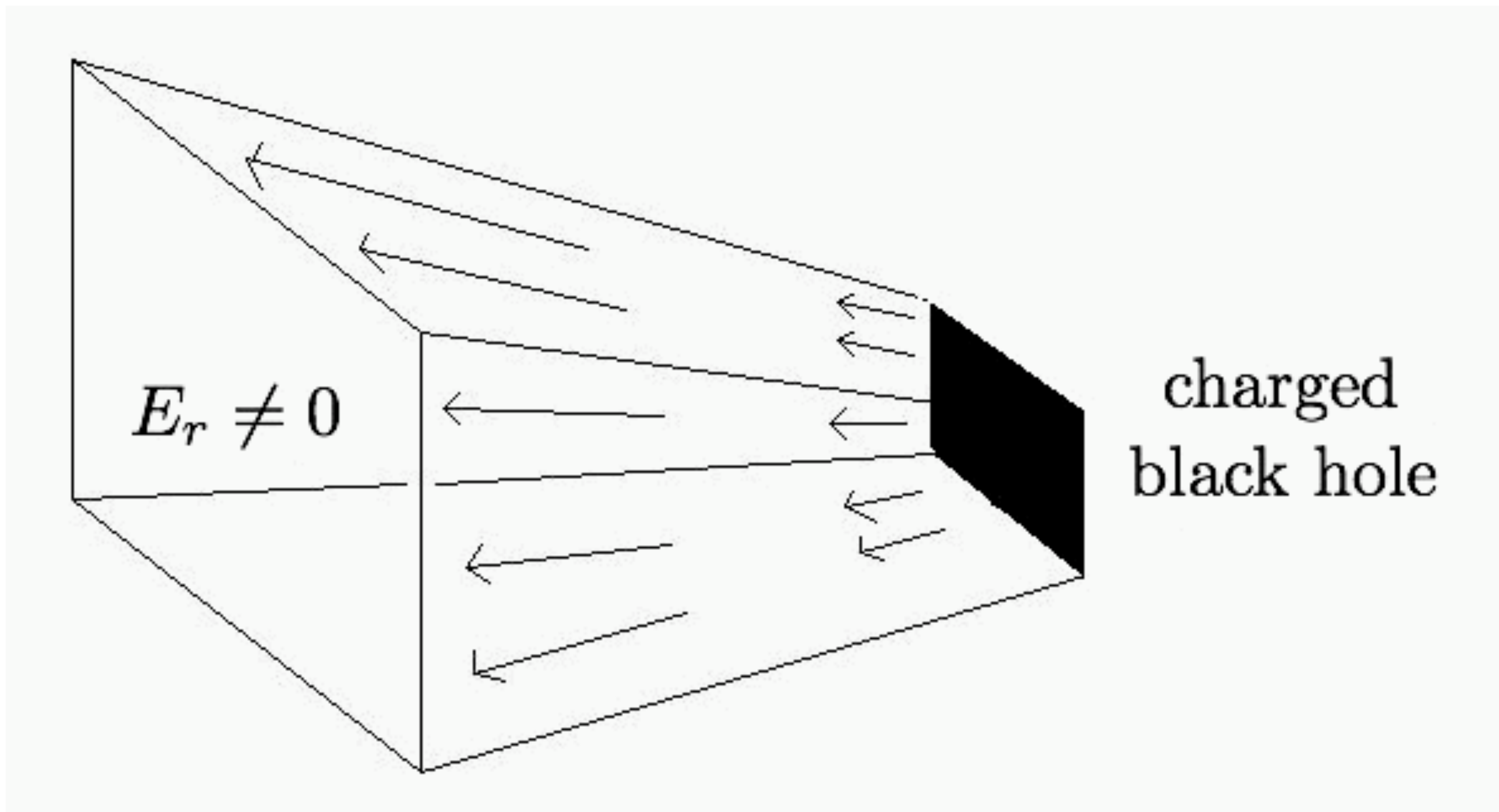
arXiv:1406.1659 with Aristomenis Donos
arXiv:1310.3832 with David Tong and David Vegh
arXiv:1308.4970 with David Tong

Part 1: Holographic Transport





AdS/CMT 101



RN Solution

$$ds^2 = \frac{L^2}{r^2}\left(- f(r)dt^2 + \frac{dr^2}{f(r)} + dx^2 + dy^2 \right)$$

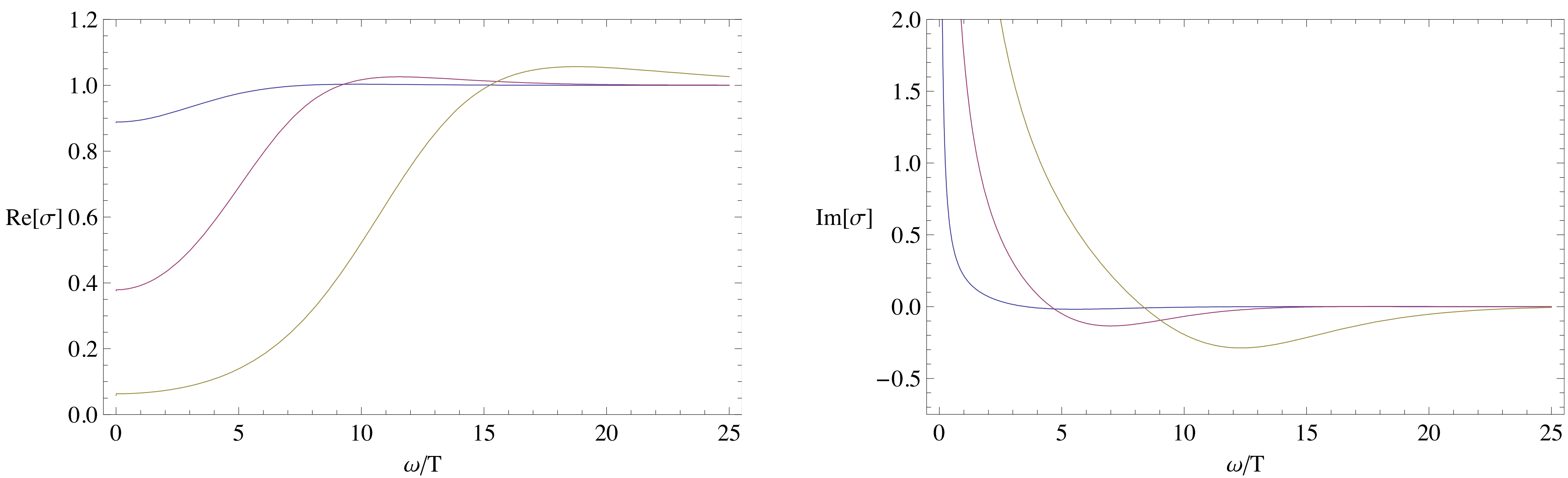
$$A_t = \mu\left(1 - \frac{r}{r_h}\right)$$

$$f(r) = 1 - \left(1 + \frac{\mu^2 r_h^2}{\gamma^2}\right)\frac{r^3}{r_h^3} + \frac{\mu^2 r^4}{\gamma^2 r_h^2}$$

AdS/CMT 102

$$J_x(\omega) = \sigma(\omega) E_x(\omega)$$

$$(f(r)\delta A_x')' + \frac{w^2}{f(r)}\delta A_x = \frac{4\mu^2 r^2}{\gamma^2 r_h^2}\delta A_x \qquad \delta A_x(\omega) = \frac{E_x(\omega)}{i\omega} + \langle J_x(\omega)\rangle r + \ldots$$



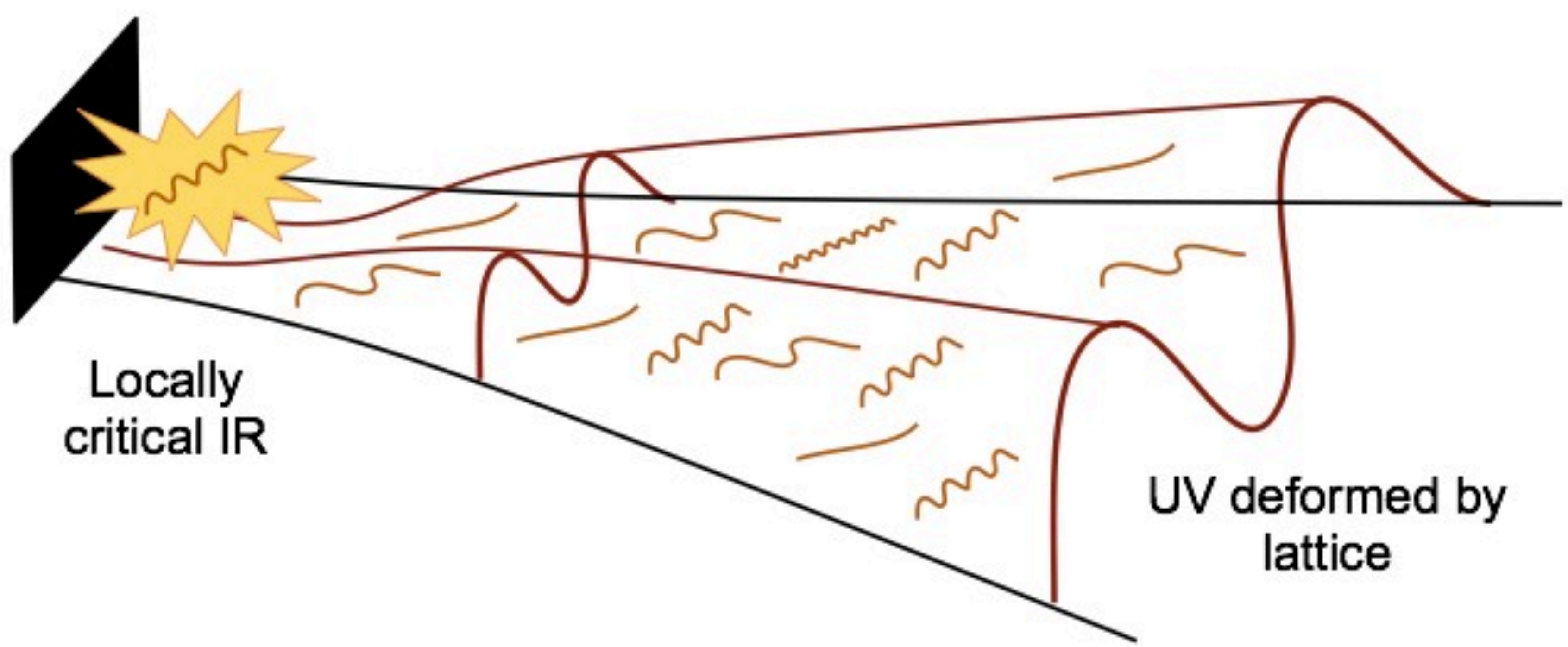
Hartnoll

DC conductivity diverges!

- To get finite DC conductivity need to have momentum dissipation.
- Break translational invariance of boundary theory weakly using irrelevant operator.

$$A_t \to \mu + \lambda \cos(k_L x)$$

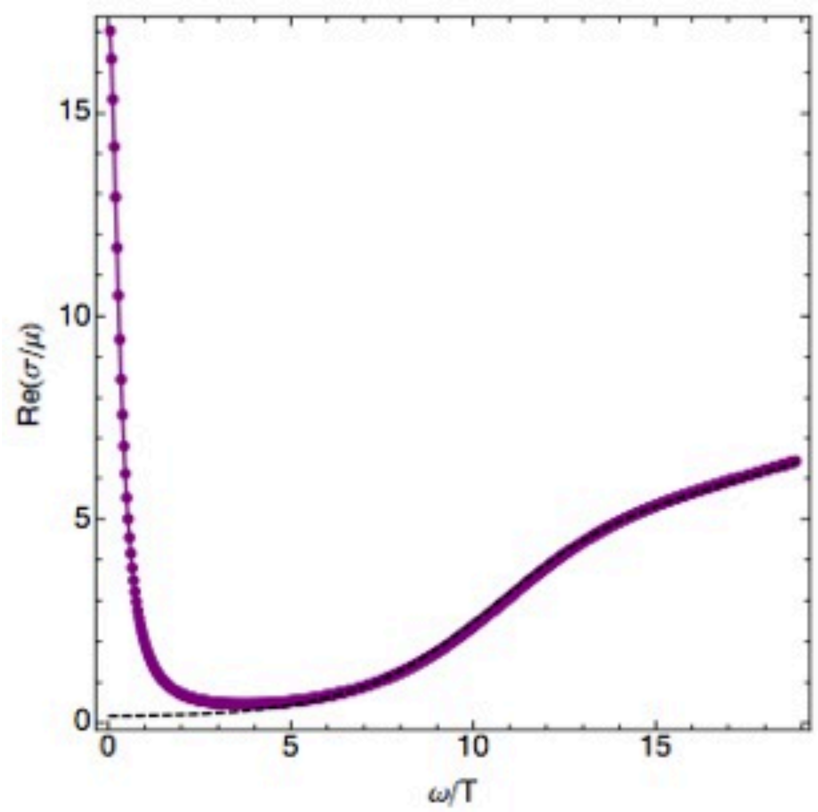
Horowitz, Santos & Tong

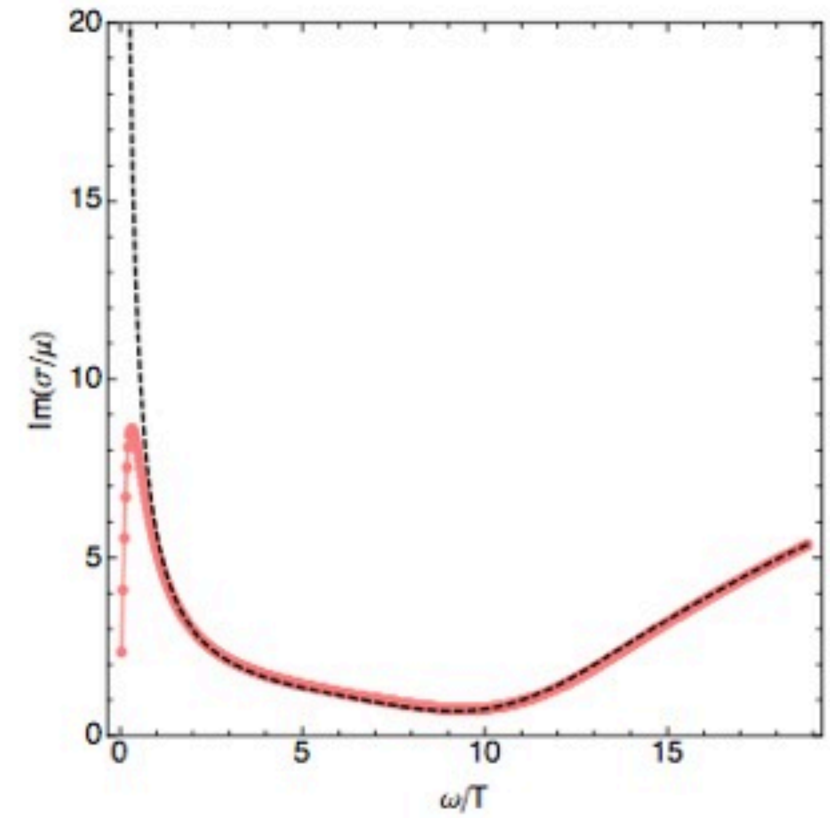


Solve very complicated PDEs!

Numerical Conductivity

Horowitz, Santos & Tong





$$\sigma(\omega) = \frac{\sigma_{DC}}{1 - i\omega\tau}$$

$$\rho \sim T^{2\Delta_{J_t}(k_L)}$$

$$\Delta_{J_t}(k_L) = \frac{1}{2}\sqrt{5 + 2(k_L/\mu)^2 - 4\sqrt{1 + (k_L/\mu)^2}} - \frac{1}{2}$$

Result for resistivity agrees with memory matrix prediction

Hartnoll and Hofman

Analytic Conductivity

- Make progress by working perturbatively in lattice strength

MB, Tong and Vegh

- Simplest thing is to add a background scalar lattice on top of the RN solution

$$\phi_0(r,x,y) \sim \phi_-(x,y)\left(\frac{r}{L}\right)^{\Delta_-} + \phi_+(x,y)\left(\frac{r}{L}\right)^{\Delta_+} + \dots$$

$$\phi_- = \epsilon\cos(k_L x)$$

- To leading order can ignore the backreaction on the metric and gauge field backgrounds.

- At leading order the conductivity calculation simplifies enormously.
- In radial gauge $\delta g_{rx} = 0$ we need only consider

$$(\delta A_x, \delta g_{tx}, \delta\phi)$$

- The novel ingredient is scalar perturbation

$$\delta\phi(r,x,t) = \delta\phi(r,t)\mathrm{sin}(k_L x)$$

- This is simply the phonon mode of the lattice

$$\delta\phi = \epsilon k_L \phi_0(r)\pi(r,t)$$

$$\phi(r,x,t) = \epsilon\phi_0(r)\mathrm{cos}(k_L[x - \pi(r,t)])$$

Connection with Massive Gravity

- After eliminating δg_{tx} equations take the

$$(f\delta A_x')' + \frac{\omega^2}{f}\delta A_x = \frac{\mu^2 r^2}{r_h^2}\delta A_x + \frac{\mu f M^2}{i\omega r_h}\pi'$$

$$\frac{1}{r^2}\left(\frac{r^2 f}{M^2}\left(\frac{fM^2}{r^2}\pi'\right)'\right)' + \frac{\omega^2}{r^2}\pi' = \frac{i\omega\mu}{r_h}\delta A_x + \frac{fM^2}{r^2}\pi'$$

- These are the nothing but the perturbations equations of massive gravity with a radially dependent graviton mass

$$M^2(r) = \frac{1}{2}\epsilon^2 k_L^2 \phi_0^2(r)$$

MB and Tong

Universal Conductivity

- Surprise is the existence of a massless mode even at finite density.

MB and Tong

$$\delta\lambda_1 = \left(1 + \frac{\mu^2 r^2}{M^2 r_h^2}\right)^{-1} \left[\delta A_x - \frac{\mu f}{i\omega r_h}\pi'\right]$$

- Whenever you have massless mode you can use the Iqbal/Liu trick to show the membrane conductivity $\sigma_{DC}(r)$ is constant.
- Evaluating at the horizon gives

$$\sigma_{DC} = \frac{Q^2 r_h^2}{M^2(r_h)}$$

Locally Critical Scaling

- Key result is that resistivity is determined by value of the graviton mass at the horizon

$$\rho \sim M^2(r_h) \sim \epsilon^2 k_L^2 \phi_0(r_h)^2$$

- Zero temperature near horizon geometry is $\mathrm{AdS}_2 \times R^2$

$$\phi_0 \sim \xi^{-\Delta_O(k_L)}$$

- At finite temperature the geometry terminates at $\xi_h \sim T^{-1}$ giving a resistivity

$$\rho \sim T^{2\Delta_O(k_L)}$$

MB, Tong and Vegh

Related Work

- Can break momentum conservation in other (simpler) ways:

Massive Gravity — Vegh, Davison

Linear Axions $\chi \sim kx$ — Andrade and Withers, Gouteraux

Q-lattices $\phi_1 \sim \sin(k_L x)$ $\phi_2 \sim \cos(k_L x)$ — Donos and Gauntlett

- In these models our method gives the exact DC conductivity in terms of horizon data.

- Can use a similar approach to calculate thermal and electrothermal conductivity.

Donos and Gauntlett

Part II: The Hall Angle

VOLUME 67, NUMBER 15 PHYSICAL REVIEW LETTERS 7 OCTOBER 1991

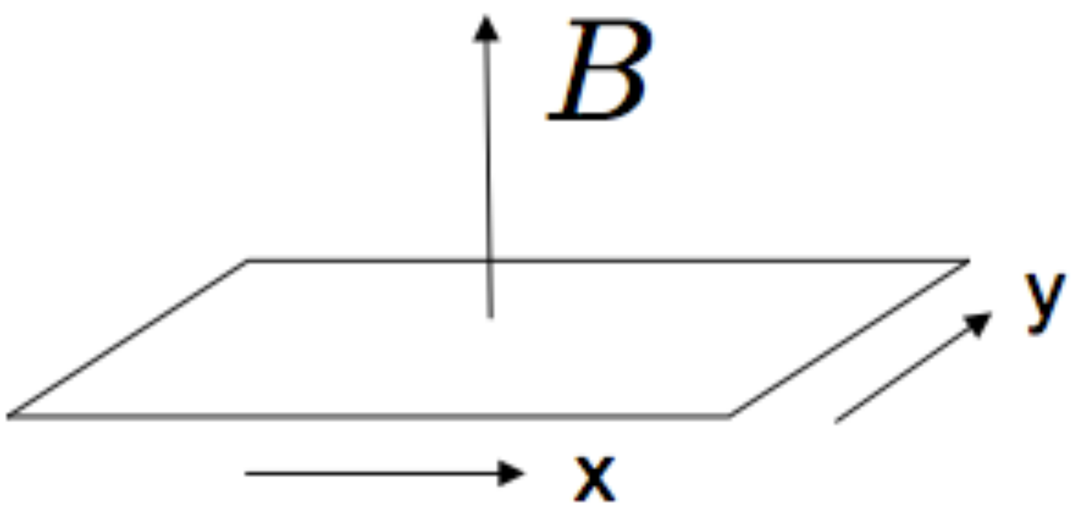
Effect of Zn Impurities on the Normal-State Hall Angle in Single-Crystal $YBa_2Cu_{3-x}Zn_xO_{7-\delta}$

T. R. Chien, Z. Z. Wang,[a] and N. P. Ong
Joseph Henry Laboratories of Physics, Princeton University, Princeton, New Jersey 08544
(Received 17 July 1991)

In Zn-doped single-crystal $YBa_2Cu_{3-x}Zn_xO_{7-\delta}$ we show that the normal-state Hall angle varies as $\cot\theta_H = \alpha T^2 + \beta x$, as predicted by Anderson (T is temperature). The existence of two distinct relaxation time scales ($\tau_H \sim 1/T^2$ and $\tau_{tr} \sim 1/T$), required by experiment, precludes all multiband Drude-type models. A number of puzzling features of the Hall effect in the cuprates are resolved with the new analysis. We also report an improved measurement of the scattering cross section of Zn in the CuO_2 planes.

PACS numbers: 74.70.Vy, 72.15.Gd, 72.15.Qm

Drude model



$$\vec{j}(\omega) = \sigma(\omega)\vec{E}(\omega)$$

$$m\frac{d\vec{v}}{dt} + \frac{m}{\tau}\vec{v} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{j} = nq\vec{v}$$

A puzzle...

	$B = 0$	$B \neq 0$
Drude	$\sigma_{DC} = \frac{nq^2\tau}{m}$	$\theta_H = \frac{\sigma_{xy}}{\sigma_{xx}} = \frac{qB\tau}{m}$
Strange Metal	$\sigma_{DC} \sim \frac{1}{T}$	$\theta_H \sim \frac{1}{T^2}$

The strange metal experiments seem to imply different scattering times for electric and Hall currents.

Anderson

Coleman, Schofield & Tsvelik

Q-Lattices

- Can use 'Q-lattices' to obtain analytic expression for transport even when momentum dissipation is strong.

- Build lattices out of two complex scalar fields

$$\psi_1 \sim \phi e^{i\chi_1} \qquad \psi_2 \sim \phi e^{i\chi_2}$$

Donos and Gauntlett

$$\chi_1 \to kx \qquad \chi_2 \to ky$$

$$S = \int d^4x \sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 + \Phi(\phi)((\partial\chi_1)^2 + (\partial\chi_2)^2) + V(\phi) - \frac{Z(\phi)}{4}F^2\right]$$

- Stress tensor is homogeneous: can study exactly using ODEs.

DC Transport

$$\sigma_{DC} = \left[Z(\phi) + \frac{4\pi \mathcal{Q}^2}{k^2 \Phi(\phi) s} \right]_{r_h}$$

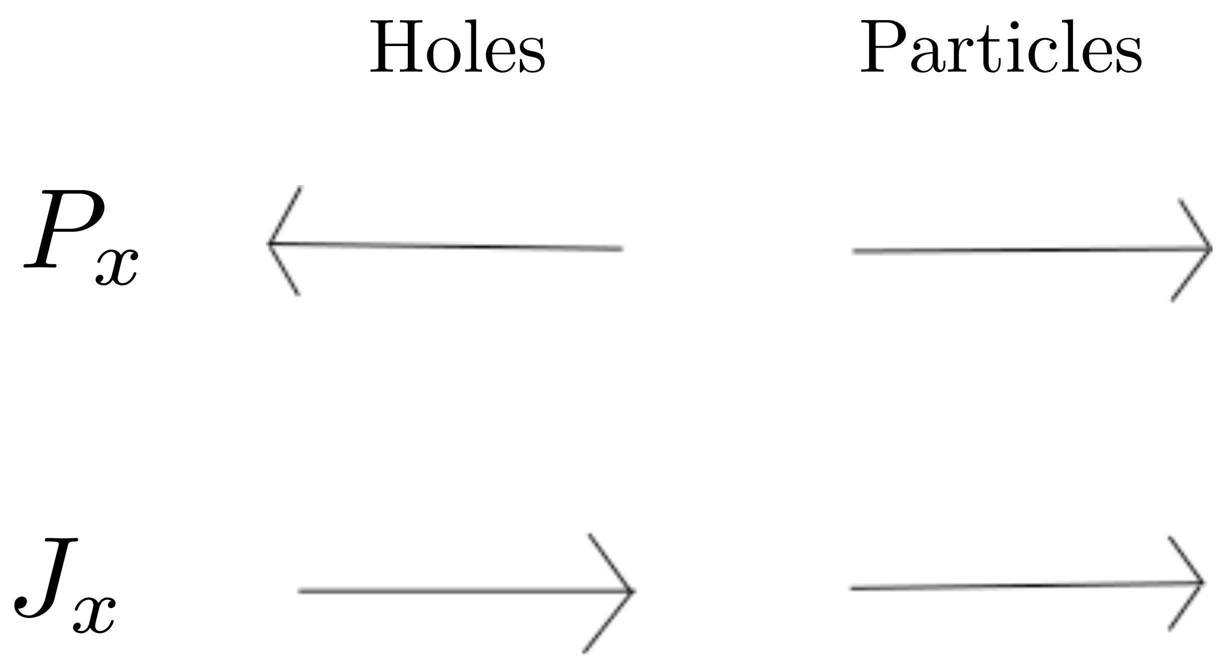
- $Z(\phi)|_{r_h}$ is a new term that did not appear in our perturbative lattice calculation.
- Compare with the electrothermal conductivity

$$\bar{\alpha} = \left[\frac{4\pi \mathcal{Q}}{k^2 \Phi(\phi)} \right]_{r_h}$$

Donos and Gauntlett

- Hence $Z(\phi)|_{r_h}$ corresponds to excitations that carry a current but no momentum.

Weak Coupling Intuition



Sachdev and Damle

- This is analogous to what happens at `charge-conjugation symmetric' critical points.
- Hence we define

$$\sigma_{ccs} = Z(\phi)|_{r_h}$$

DC Conductivity

$$\sigma_{DC} = \sigma_{ccs} + \frac{\mathcal{Q}^2}{\mathcal{E} + \mathcal{P}} \tau_L$$

$$\sigma_{ccs} = Z(\phi)|_{r_h} \qquad \tau_L^{-1} = \frac{s}{4\pi} \frac{k^2 \Phi(\phi)}{\mathcal{E} + \mathcal{P}}\bigg|_{r_h}$$

- At finite density there are two additive contributions to the conductivity - `Inverse Matthiessen Law'.
- In holography, σ_{ccs} is present even at low energies. This is not true for weakly coupled particles at finite density.

Hall angle

$$\theta_H = \frac{4\pi B\mathcal{Q}}{k^2\Phi s}\left[\frac{B^2Z^2+\mathcal{Q}^2+8\pi Z k^2\Phi/s}{B^2Z^2+\mathcal{Q}^2+4\pi Z k^2\Phi/s}\right]\Bigg|_{r_h}$$

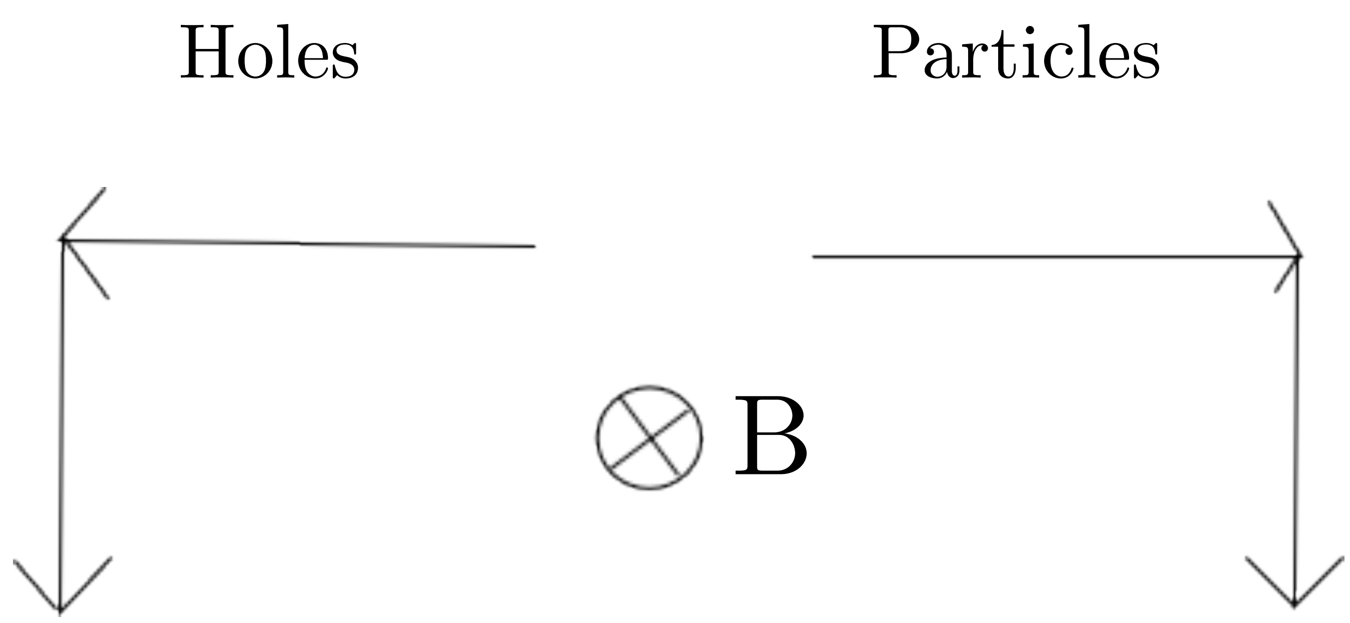
Hall angle

$$\theta_H \sim \frac{B\mathcal{Q}}{\mathcal{E}+\mathcal{P}}\tau_L$$

Hall angle

$$\theta_H \sim \frac{B\mathcal{Q}}{\mathcal{E}+\mathcal{P}}\tau_L$$

No analogous term to σ_{ccs}



MB and Donos

- Weak momentum dissipation - $\tau_L \to \infty$

$$\sigma_{DC} = \frac{\mathcal{Q}^2}{\mathcal{E}+\mathcal{P}}\tau_L \qquad \theta_H = \frac{B\mathcal{Q}}{\mathcal{E}+\mathcal{P}}\tau_L$$

reproduces Drude-like results.

c.f. Hartnoll & Hofman etc

- Strong momentum dissipation - $\tau_L \to 0$

$$\sigma_{DC} = \sigma_{ccs} \qquad \theta_H = \frac{2B\mathcal{Q}}{\mathcal{E}+\mathcal{P}}\tau_L$$

can now get different scalings!

Comments

- Story can be applied more generally than to the specific lattice models studied here e.g. to hydro, probe branes.
  Karch

- Would be exciting to understand whether mechanism can be applied to the cuprates or other experimental systems.

$$\sigma_{ccs} \sim 1/T \qquad \sigma_{diss} \sim 1/T^2 \quad \Longrightarrow \quad \sigma_{DC} \sim 1/T + 1/T^2 \qquad \theta_H \sim 1/T^2$$

- Supports recent suggestion that strange metals are governed by incoherent transport.
  Hartnoll

`` Over broad regions of doping, the two kinds of relaxation rates, the one for the conductivity and the one for the Hall rotation, seem to add as inverses: Conductivity is proportional to $1/T + 1/T^2$—that is, it obeys an anti-Matthiessen law.”

P.W. Anderson - Physics Today

Thank you!