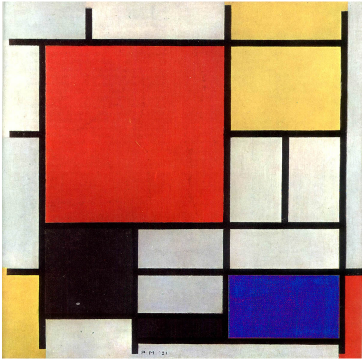
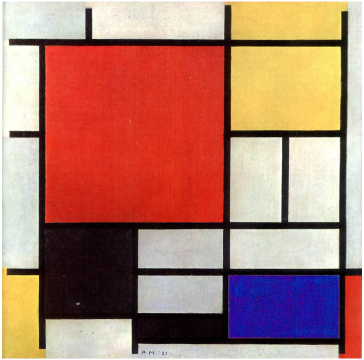
A simple holographic model of momentum relaxation

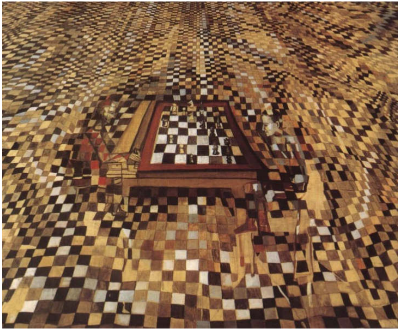
Tomás Andrade (Durham U)

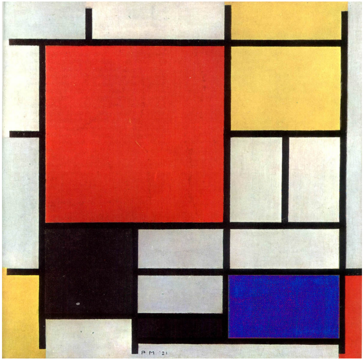
January 28th, 2014, Oxford

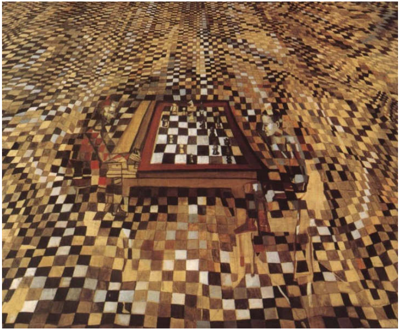
in collaboration with Ben Withers (Southampton)


P M '21


P M '21







"Hasn't it occurred to you to suspect that behind that Mondrian could a Viera da Silva reality start?" Hopscotch, J. Cortázar.

Intro1: AdS/CMT

Use AdS/CFT to understand condensed matter systems.

$$\begin{gathered}\text{Gravity in } M \text{ with AdS boundary conditions} \\ \Updownarrow \\ \text{Field Theory that lives on } \partial M\end{gathered}$$

Intro1: AdS/CMT

Use AdS/CFT to understand condensed matter systems.

$$\begin{gathered}\text{Gravity in } M \text{ with AdS boundary conditions} \\ \Updownarrow \\ \text{Field Theory that lives on } \partial M\end{gathered}$$

- Access to strongly coupled regime, include T, ρ, etc

Intro1: AdS/CMT

Use AdS/CFT to understand condensed matter systems.

$$\begin{gathered}\text{Gravity in } M \text{ with AdS boundary conditions}\\ \Updownarrow \\ \text{Field Theory that lives on } \partial M\end{gathered}$$

- Access to strongly coupled regime, include T, ρ, etc
- Limitations: it's a conjecture, large N limit, only generic features (bottom-up), hard to implement (top-down).

Intro1: AdS/CMT

Use AdS/CFT to understand condensed matter systems.

Gravity in M with AdS boundary conditions
$$\Updownarrow$$
Field Theory that lives on ∂M

- Access to strongly coupled regime, include T, ρ, etc
- Limitations: it's a conjecture, large N limit, only generic features (bottom-up), hard to implement (top-down).
- examples: superconductors, QGP, non-relativistic FT, etc.

Intro1: AdS/CMT

Use AdS/CFT to understand condensed matter systems.

$$\begin{gathered}\text{Gravity in } M \text{ with AdS boundary conditions}\\ \Updownarrow \\ \text{Field Theory that lives on } \partial M\end{gathered}$$

- Access to strongly coupled regime, include T, ρ, etc
- Limitations: it's a conjecture, large N limit, only generic features (bottom-up), hard to implement (top-down).
- examples: superconductors, QGP, non-relativistic FT, etc.
- Motivation from condensed matter to study gravitational systems [AdS, hairy black holes, etc]

Intro2: RN black hole

Gravitational solutions $\Leftrightarrow$ states of the field theory, e.g. pure AdS is the vacuum of the CFT.

Intro2: RN black hole

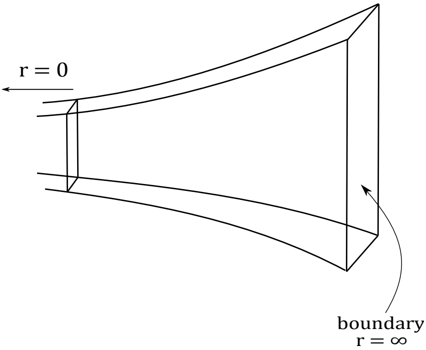
Gravitational solutions $\Leftrightarrow$ states of the field theory, e.g. pure AdS is the vacuum of the CFT.

Finite T: BH solution; finite ρ charged BH

Intro2: RN black hole

Gravitational solutions $\Leftrightarrow$ states of the field theory, e.g. pure AdS is the vacuum of the CFT.

Finite T: BH solution; finite ρ charged BH



$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2)$$

$$f = r^2 - \frac{m_0}{r} + \frac{\mu^2}{4}\frac{r_0^2}{r^2}$$

$$A = \mu\left(1 - \frac{r_0}{r}\right)dt$$

Intro3: Conductivity

Compute conductivity at non-zero charge density: $J = \sigma E$

Intro3: Conductivity

Compute conductivity at non-zero charge density: $J = \sigma E$

Turn on $\delta A_x = a_x(r)e^{-i\omega t}$, couples to δg_{tx} (but can be eliminated)

Intro3: Conductivity

Compute conductivity at non-zero charge density: $J = \sigma E$

Turn on $\delta A_x = a_x(r) e^{-i\omega t}$, couples to δg_{tx} (but can be eliminated)

$$a_x(r) = a_x^{(0)} + \frac{1}{r} a_x^{(1)} + \ldots \qquad E_x = i\omega a_x^{(0)} \qquad \langle J^x \rangle = a_x^{(1)}$$

$$\sigma(\omega) = \frac{a_x^{(1)}}{i\omega a_x^{(0)}}$$

Intro3: Conductivity

Compute conductivity at non-zero charge density: $J = \sigma E$

Turn on $\delta A_x = a_x(r) e^{-i\omega t}$, couples to δg_{tx} (but can be eliminated)

$$a_x(r) = a_x^{(0)} + \frac{1}{r} a_x^{(1)} + \dots \qquad E_x = i\omega a_x^{(0)} \qquad \langle J^x \rangle = a_x^{(1)}$$

$$\sigma(\omega) = \frac{a_x^{(1)}}{i\omega a_x^{(0)}}$$

Ingoing bc's for retarded 2-pt $a_x \approx (r - r_0)^{-i\omega/4\pi T}$

Intro3: Conductivity

Compute conductivity at non-zero charge density: $J = \sigma E$

Turn on $\delta A_x = a_x(r) e^{-i\omega t}$, couples to δg_{tx} (but can be eliminated)

$$a_x(r) = a_x^{(0)} + \frac{1}{r} a_x^{(1)} + \dots \qquad E_x = i\omega a_x^{(0)} \qquad \langle J^x \rangle = a_x^{(1)}$$

$$\sigma(\omega) = \frac{a_x^{(1)}}{i\omega a_x^{(0)}}$$

Ingoing bc's for retarded 2-pt $a_x \approx (r - r_0)^{-i\omega/4\pi T}$

For small ω,

$$\sigma(\omega) \approx \frac{\mu^2}{r_0} \left(\delta(\omega) + \frac{i}{\omega} \right)$$

Intro3: Conductivity cont'd

$$\sigma(\omega) \approx \frac{\mu^2}{r_0}\left(\delta(\omega) + \frac{i}{\omega}\right)$$

Consequence of translational invariance of the background.

Intro3: Conductivity cont'd

$$\sigma(\omega) \approx \frac{\mu^2}{r_0}\left(\delta(\omega) + \frac{i}{\omega}\right)$$

Consequence of translational invariance of the background.

Finite ρ, apply a constant E, charge carriers can't dissipate p.

Intro3: Conductivity cont'd

$$\sigma(\omega) \approx \frac{\mu^2}{r_0}\left(\delta(\omega) + \frac{i}{\omega}\right)$$

Consequence of translational invariance of the background.

Finite ρ, apply a constant E, charge carriers can't dissipate p.

In more realistic situations, p dissipates due to break translation invariance (lattice).

Intro3: Conductivity cont'd

$$\sigma(\omega) \approx \frac{\mu^2}{r_0}\left(\delta(\omega) + \frac{i}{\omega}\right)$$

Consequence of translational invariance of the background.

Finite ρ, apply a constant E, charge carriers can't dissipate p.

In more realistic situations, p dissipates due to break translation invariance (lattice).

Studied in holography introducing a hol. lattice [Horowitz, Santos, Tong] and breaking diff inv in the bulk (MG) [Vegh]

Intro3: Conductivity cont'd

$$\sigma(\omega) \approx \frac{\mu^2}{r_0}\left(\delta(\omega) + \frac{i}{\omega}\right)$$

Consequence of translational invariance of the background.

Finite ρ, apply a constant E, charge carriers can't dissipate p.

In more realistic situations, p dissipates due to break translation invariance (lattice).

Studied in holography introducing a hol. lattice [Horowitz, Santos, Tong] and breaking diff inv in the bulk (MG) [Vegh]

Goal here: present a simple model of momentum relaxation in the holographic setup.

Outline

Outline

- Ward identity for $\nabla_i \langle T^{ij} \rangle$

Outline

- Ward identity for $\nabla_i \langle T^{ij} \rangle$
- The model

Outline

- Ward identity for $\nabla_i \langle T^{ij} \rangle$
- The model
- (Finite) DC conductivity

Outline

- Ward identity for $\nabla_i \langle T^{ij} \rangle$
- The model
- (Finite) DC conductivity
- Comparison with Massive Gravity

Outline

- Ward identity for $\nabla_i \langle T^{ij} \rangle$
- The model
- (Finite) DC conductivity
- Comparison with Massive Gravity
- Conclusions

Ward identity

Theory with scalar operator O and $U(1)$ current

$$\nabla_i \langle T^{ij} \rangle = \nabla^j \psi^{(0)} \langle O \rangle + F^{ij} \langle J_i \rangle$$

Basic idea: turn on sources (provided vevs are non-zero)

Ward identity

Theory with scalar operator O and $U(1)$ current

$$\nabla_i \langle T^{ij} \rangle = \nabla^j \psi^{(0)} \langle O \rangle + F^{ij} \langle J_i \rangle$$

Basic idea: turn on sources (provided vevs are non-zero)

Holographically, consider $g_{\mu\nu}$, ψ_I, A_μ,

$$ds^2 = \frac{d\rho^2}{\rho^2} + \frac{1}{\rho^2}(g_{ij}^{(0)} + \ldots + \rho^d \tau_{ij} + \ldots) dx^i dx^j$$

$$A = (A_i^{(0)} + \ldots + \rho^{d-2} \tilde{A}_i + \ldots) dx^i$$

$$\psi_I = \rho^{\Delta_-} \psi_I^{(0)} + \ldots + \rho^{\Delta_+} \tilde{\psi}_I + \ldots$$

Ward identity cont'd

Then, the variation of the on-shell action reads

$$\delta S_{ren} = \int_{\partial M} \sqrt{-g^{(0)}} \left[\frac{1}{2} \langle T^{ij} \rangle \delta g^{(0)}_{ij} + \langle O_I \rangle \delta \psi^{(0)}_I + \langle J^i \rangle \delta A^{(0)}_i \right]$$

Ward identity cont'd

Then, the variation of the on-shell action reads

$$\delta S_{ren} = \int_{\partial M} \sqrt{-g^{(0)}} \left[\frac{1}{2} \langle T^{ij} \rangle \delta g^{(0)}_{ij} + \langle O_I \rangle \delta \psi^{(0)}_I + \langle J^i \rangle \delta A^{(0)}_i \right]$$

$$\langle T^{ij} \rangle \propto \tau_{ij} \qquad \langle O_I \rangle \propto \tilde{\psi}_I \qquad \langle J^i \rangle \propto \tilde{A}_i$$

Ward identity is asympt. eom (bulk diff inv)

Ward identity cont'd

Then, the variation of the on-shell action reads

$$\delta S_{ren} = \int_{\partial M} \sqrt{-g^{(0)}} \left[\frac{1}{2} \langle T^{ij} \rangle \delta g^{(0)}_{ij} + \langle O_I \rangle \delta \psi^{(0)}_I + \langle J^i \rangle \delta A^{(0)}_i \right]$$

$$\langle T^{ij} \rangle \propto \tau_{ij} \qquad \langle O_I \rangle \propto \tilde{\psi}_I \qquad \langle J^i \rangle \propto \tilde{A}_i$$

Ward identity is asympt. eom (bulk diff inv)

Generically, spatially dependent sources introduce explicit anisotropies and non-homogeneities (solve PDE's)

Ward identity cont'd

Then, the variation of the on-shell action reads

$$\delta S_{ren} = \int_{\partial M} \sqrt{-g^{(0)}} \left[\frac{1}{2} \langle T^{ij} \rangle \delta g^{(0)}_{ij} + \langle O_I \rangle \delta \psi^{(0)}_I + \langle J^i \rangle \delta A^{(0)}_i \right]$$

$$\langle T^{ij} \rangle \propto \tau_{ij} \qquad \langle O_I \rangle \propto \tilde{\psi}_I \qquad \langle J^i \rangle \propto \tilde{A}_i$$

Ward identity is asympt. eom (bulk diff inv)

Generically, spatially dependent sources introduce explicit anisotropies and non-homogeneities (solve PDE's)

Take $\psi \propto x$ with $m^2_\psi = 0$ makes bulk geometry homogeneous, can arrange more than one scalar to have isotropy. Makes use of the shift symmetry $\psi_I \to \psi_I + c_I$.

The model

$$S_0 = \int_M \sqrt{-g}\left[R - 2\Lambda - \frac{1}{2}\sum_I^{d-1}(\partial\psi_I)^2 - \frac{1}{4}F^2\right] d^{d+1}x$$

The model

$$S_0 = \int_M \sqrt{-g}\left[R - 2\Lambda - \frac{1}{2}\sum_I^{d-1}(\partial\psi_I)^2 - \frac{1}{4}F^2\right] d^{d+1}x$$

Take the ansatz

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\delta_{ab}dx^a dx^b, \quad A = \mu\left(1 - \frac{r_0^{d-2}}{r^{d-2}}\right)dt, \quad \psi_I = \alpha_{Ia}x^a,$$

The model

$$S_0 = \int_M \sqrt{-g} \left[R - 2\Lambda - \frac{1}{2} \sum_I^{d-1} (\partial \psi_I)^2 - \frac{1}{4} F^2 \right] d^{d+1}x$$

Take the ansatz

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 \delta_{ab} dx^a dx^b, \quad A = \mu \left(1 - \frac{r_0^{d-2}}{r^{d-2}} \right) dt, \quad \psi_I = \alpha_{Ia} x^a,$$

Find the solution [Bardoux, Caldarelli, Charmousis, '12]

$$f = r^2 - \frac{\alpha^2}{2(d-2)} - \frac{m_0}{r^{d-2}} + \frac{(d-2)\mu^2}{2(d-1)} \frac{r_0^{2(d-2)}}{r^{2(d-2)}}, \qquad \alpha^2 \equiv \frac{1}{d-1} \sum_{a=1}^{d-1} \vec{\alpha}_a \cdot \vec{\alpha}_a,$$

provided

$$\vec{\alpha}_a \cdot \vec{\alpha}_b = \alpha^2 \delta_{ab} \qquad \forall a, b. \tag{1}$$

The model cont'd

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\delta_{ab}dx^a dx^b, \quad A = \mu\left(1 - \frac{r_0^{d-2}}{r^{d-2}}\right)dt, \quad \psi_I = \alpha_{Ia}x^a,$$

Geometry is isotropic and homogenous but solution is not.

The model cont'd

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\delta_{ab}dx^a dx^b, \quad A = \mu\left(1 - \frac{r_0^{d-2}}{r^{d-2}}\right)dt, \quad \psi_I = \alpha_{Ia}x^a,$$

Geometry is isotropic and homogenous but solution is not.

Use rotational residual symmetry to set $\alpha_{Ia} = \delta_{Ia}\alpha$. Solution is fully characterized by μ, α and

$$T = \frac{1}{4\pi}\left(dr_0 - \frac{\alpha^2}{2r_0} - \frac{(d-2)^2\mu^2}{2(d-1)r_0}\right).$$

The model cont'd

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\delta_{ab}dx^a dx^b, \quad A = \mu\left(1 - \frac{r_0^{d-2}}{r^{d-2}}\right)dt, \quad \psi_I = \alpha_{Ia}x^a,$$

Geometry is isotropic and homogenous but solution is not.

Use rotational residual symmetry to set $\alpha_{Ia} = \delta_{Ia}\alpha$. Solution is fully characterized by μ, α and

$$T = \frac{1}{4\pi}\left(dr_0 - \frac{\alpha^2}{2r_0} - \frac{(d-2)^2\mu^2}{2(d-1)r_0}\right).$$

Mechanism for dissipation? solution has $\langle O_I \rangle = 0$ and $F_{ij}^{(0)} = 0$, so $\nabla_i \langle T^{ij} \rangle = 0$. Linearized fluctuations

$$\partial_t \delta\langle P_a \rangle = \alpha_{aI}\delta\langle O_I \rangle + \delta F_{at}^{(0)}\langle J_t \rangle \tag{2}$$

Holographic Q-lattices

Similar construction by [Donos+Gauntlett], which uses $U(1)$ of a complex scalar, $\phi \to e^{ikx}\phi$.

Break translational invariance by $\phi = e^{ikx}\varphi(r)$, but $T_{\mu\nu}$ is indep. of x so the problem reduces to ODE's.

DC conductivity

Key idea: massless mode $\Rightarrow$ conserved quantity, express the DC conductivity in terms of r_0.

DC conductivity

Key idea: massless mode $\Rightarrow$ conserved quantity, express the DC conductivity in terms of r_0.

$$\delta A_x = e^{-i\omega t} a_x(r), \qquad \delta g_{tx} = e^{-i\omega t} h_{tx}(r) \qquad \delta\psi_1 = e^{-i\omega t}\chi(r)$$

DC conductivity

Key idea: massless mode $\Rightarrow$ conserved quantity, express the DC conductivity in terms of r_0.

$$\delta A_x = e^{-i\omega t} a_x(r), \qquad \delta g_{tx} = e^{-i\omega t} h_{tx}(r) \qquad \delta\psi_1 = e^{-i\omega t}\chi(r)$$

$$L_2 \begin{pmatrix} a_x \\ \chi' \end{pmatrix} + \omega^2 \begin{pmatrix} a_x \\ \chi' \end{pmatrix} = M \begin{pmatrix} a_x \\ \chi' \end{pmatrix}$$

DC conductivity

Key idea: massless mode $\Rightarrow$ conserved quantity, express the DC conductivity in terms of r_0.

$$\delta A_x = e^{-i\omega t} a_x(r), \qquad \delta g_{tx} = e^{-i\omega t} h_{tx}(r) \qquad \delta\psi_1 = e^{-i\omega t}\chi(r)$$

$$L_2 \begin{pmatrix} a_x \\ \chi' \end{pmatrix} + \omega^2 \begin{pmatrix} a_x \\ \chi' \end{pmatrix} = M \begin{pmatrix} a_x \\ \chi' \end{pmatrix}$$

One finds $\det M = 0$. Diagonalize mass matrix by λ_1, λ_2.

$$\Pi' + \omega^2 \lambda_1 = 0 \qquad \Rightarrow \qquad \Pi' = 0 \quad \text{at} \quad \omega = 0$$

DC conductivity

Key idea: massless mode $\Rightarrow$ conserved quantity, express the DC conductivity in terms of r_0.

$$\delta A_x = e^{-i\omega t} a_x(r), \qquad \delta g_{tx} = e^{-i\omega t} h_{tx}(r) \qquad \delta\psi_1 = e^{-i\omega t}\chi(r)$$

$$L_2 \begin{pmatrix} a_x \\ \chi' \end{pmatrix} + \omega^2 \begin{pmatrix} a_x \\ \chi' \end{pmatrix} = M \begin{pmatrix} a_x \\ \chi' \end{pmatrix}$$

One finds $\det M = 0$. Diagonalize mass matrix by λ_1, λ_2.

$$\Pi' + \omega^2 \lambda_1 = 0 \qquad \Rightarrow \quad \Pi' = 0 \quad \text{at} \quad \omega = 0$$

$$\sigma_{DC}(r) = \lim_{\omega \to 0} \left. \frac{-\Pi}{i\omega\lambda_1} \right|_r \qquad \sigma_{DC}(\infty) = \sigma_{DC}$$

DC conductivity

Key idea: massless mode $\Rightarrow$ conserved quantity, express the DC conductivity in terms of r_0.

$$\delta A_x = e^{-i\omega t} a_x(r), \qquad \delta g_{tx} = e^{-i\omega t} h_{tx}(r) \qquad \delta\psi_1 = e^{-i\omega t}\chi(r)$$

$$L_2 \begin{pmatrix} a_x \\ \chi' \end{pmatrix} + \omega^2 \begin{pmatrix} a_x \\ \chi' \end{pmatrix} = M \begin{pmatrix} a_x \\ \chi' \end{pmatrix}$$

One finds $\det M = 0$. Diagonalize mass matrix by λ_1, λ_2.

$$\Pi' + \omega^2 \lambda_1 = 0 \qquad \Rightarrow \qquad \Pi' = 0 \quad \text{at} \quad \omega = 0$$

$$\sigma_{DC}(r) = \lim_{\omega\to 0} \left.\frac{-\Pi}{i\omega\lambda_1}\right|_r \qquad \sigma_{DC}(\infty) = \sigma_{DC}$$

can show $\sigma'_{DC}(r) = 0$! So

$$\sigma_{DC} = \sigma_{DC}(r_0) = r_0^{d-3}\left(1 + (d-2)^2 \frac{\mu^2}{\alpha^2}\right).$$

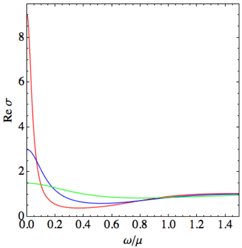
Optical conductivity

Can also compute $\sigma = \sigma(\omega)$ numerically.

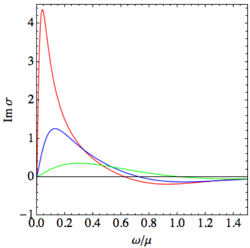
Optical conductivity

Can also compute $\sigma = \sigma(\omega)$ numerically. Drude physics for small ω

$$\sigma = \frac{\sigma_{DC}}{1 - i\omega\tau}$$



(a)



(b)

Figure : The different curves correspond to, from top to bottom, $\alpha/\mu = 0.1, 1.0, 2.0$.

Massive gravity

Break bulk diffeo inv. to break translational inv. on ∂M.

Massive gravity

Break bulk diffeo inv. to break translational inv. on ∂M.

Generically yields ghosts, but [de Rham, Gabadadze, Tolley, '10] argues that it's ok

$$I_{MG} = \int_M \sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}F^2 + \beta m^2([\mathcal{K}]^2 - [\mathcal{K}^2]) + \cancel{\alpha m^2[\mathcal{K}]}\right] d^4x.$$

$$\mathcal{K}^\mu{}_\alpha \mathcal{K}^\alpha{}_\nu = g^{\mu\alpha} f_{\alpha\nu} \qquad f_{\mu\nu} = \mathrm{diag}(0, 0, F, F)$$

Massive gravity

Break bulk diffeo inv. to break translational inv. on ∂M.

Generically yields ghosts, but [de Rham, Gabadadze, Tolley, '10] argues that it's ok

$$I_{MG} = \int_M \sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}F^2 + \beta m^2([\mathcal{K}]^2 - [\mathcal{K}^2]) + \cancel{\alpha m^2[\mathcal{K}]}\right] d^4x.$$

$$\mathcal{K}^\mu{}_\alpha \mathcal{K}^\alpha{}_\nu = g^{\mu\alpha} f_{\alpha\nu} \qquad f_{\mu\nu} = \mathrm{diag}(0,0,F,F)$$

$$ds^2 = -f_{MG}(r)dt^2 + \frac{dr^2}{f_{MG}(r)} + r^2(dx^2+dy^2), \qquad A = \mu\left(1 - \frac{r_0}{r}\right)dt,$$

$$f_{MG}(r) = r^2 - m_\beta^2 - \frac{m_0}{r} + \frac{\mu^2 r_0^2}{4r^2},$$

Massive gravity

Break bulk diffeo inv. to break translational inv. on ∂M.

Generically yields ghosts, but [de Rham, Gabadadze, Tolley, '10] argues that it's ok

$$I_{MG} = \int_M \sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}F^2 + \beta m^2([\mathcal{K}]^2 - [\mathcal{K}^2]) + \cancel{\alpha m^2[\mathcal{K}]}\right] d^4x.$$

$$\mathcal{K}^\mu{}_\alpha \mathcal{K}^\alpha{}_\nu = g^{\mu\alpha} f_{\alpha\nu} \qquad f_{\mu\nu} = \mathrm{diag}(0, 0, F, F)$$

$$ds^2 = -f_{MG}(r)dt^2 + \frac{dr^2}{f_{MG}(r)} + r^2(dx^2 + dy^2), \qquad A = \mu\left(1 - \frac{r_0}{r}\right)dt,$$

$$f_{MG}(r) = r^2 - m_\beta^2 - \frac{m_0}{r} + \frac{\mu^2 r_0^2}{4r^2},$$

same as our model with $\alpha^2 = 2m_\beta^2$.

Massive gravity

Break bulk diffeo inv. to break translational inv. on ∂M.

Generically yields ghosts, but [de Rham, Gabadadze, Tolley, '10] argues that it's ok

$$I_{MG} = \int_M \sqrt{-g} \left[R - 2\Lambda - \frac{1}{4} F^2 + \beta m^2 ([\mathcal{K}]^2 - [\mathcal{K}^2]) + \cancel{\alpha m^2 [\mathcal{K}]} \right] d^4 x.$$

$$\mathcal{K}^\mu{}_\alpha \mathcal{K}^\alpha{}_\nu = g^{\mu\alpha} f_{\alpha\nu} \qquad f_{\mu\nu} = \mathrm{diag}(0, 0, F, F)$$

$$ds^2 = -f_{MG}(r) dt^2 + \frac{dr^2}{f_{MG}(r)} + r^2 (dx^2 + dy^2), \qquad A = \mu \left(1 - \frac{r_0}{r}\right) dt,$$

$$f_{MG}(r) = r^2 - m_\beta^2 - \frac{m_0}{r} + \frac{\mu^2 r_0^2}{4r^2},$$

same as our model with $\alpha^2 = 2m_\beta^2$.

Drude at small ω, $\sigma_{DC}^{MG} = \sigma_{DC}^{ours}$

Massive gravity cont'd

Consider shear modes $\sim e^{-i\omega t+ikx}\Phi(r)$

$$\delta g_{ry}\,, \quad \delta g_{ty}\,, \quad \delta g_{xy}\,, \quad \delta A_y$$

Massive gravity cont'd

Consider shear modes $\sim e^{-i\omega t+ikx}\Phi(r)$

$$\delta g_{ry}\,, \quad \delta g_{ty}\,, \quad \delta g_{xy}\,, \quad \delta A_y$$

Can find master fields

$$\delta g_{ry} \sim \Phi_\pm, \Phi_1', \quad \delta g_{ty} \sim \Phi_0, \quad \delta g_{xy} \sim \Phi_1, \quad \delta A_y \sim \Phi_\pm$$

Massive gravity cont'd

Consider shear modes $\sim e^{-i\omega t+ikx}\Phi(r)$

$$\delta g_{ry}\,, \quad \delta g_{ty}\,, \quad \delta g_{xy}\,, \quad \delta A_y$$

Can find master fields

$$\delta g_{ry} \sim \Phi_\pm, \Phi_1', \quad \delta g_{ty} \sim \Phi_0, \quad \delta g_{xy} \sim \Phi_1, \quad \delta A_y \sim \Phi_\pm$$

$$\begin{aligned} r^2(f\Phi_\pm')' + \left(\frac{r^2\omega^2}{f} - k^2 - \frac{\mu^2 r_0^2}{r^2} + \frac{\mu r_0}{r}c_\pm\right)\Phi_\pm = 0, \\ \frac{1}{r^2 f}(r^2 f\Phi_1')' + \frac{\omega^2}{f^2}\Phi_1 = 0, \\ \Phi_0 + \frac{f}{r^2}(c_+ r\Phi_+ + c_- r\Phi_-)' - \frac{k\omega}{k^2+2m_\beta^2}\Phi_1 = 0. \end{aligned}$$

Massive gravity cont'd

In our model: $\psi_1 = \alpha x$, $\psi_2 = \alpha y$

$$\delta g_{ry}\,, \quad \delta g_{ty}\,, \quad \delta g_{xy}\,, \quad \delta A_y\,, \quad \delta\psi_2$$

Massive gravity cont'd

In our model: $\psi_1 = \alpha x$, $\psi_2 = \alpha y$

$$\delta g_{ry}, \quad \delta g_{ty}, \quad \delta g_{xy}, \quad \delta A_y, \quad \delta\psi_2$$

Introduce master fields as before and get

$$r^2(f\Phi_\pm')' + \left(\frac{r^2\omega^2}{f} - k^2 - \frac{\mu^2 r_0^2}{r^2} + \frac{\mu r_0}{r}c_\pm\right)\Phi_\pm = 0,$$

$$\frac{1}{r^2 f}(r^2 f\Phi_1')' + \left(\frac{\omega^2}{f^2} - \frac{(k^2+\alpha^2)}{r^2 f}\right)\Phi_1 = 0,$$

$$\Phi_0 + \frac{f}{r^2}(c_+ r\Phi_+ + c_- r\Phi_-)' - \frac{k\omega}{k^2+\alpha^2}\Phi_1 = 0.$$

Massive gravity cont'd

In our model: $\psi_1 = \alpha x$, $\psi_2 = \alpha y$

$$\delta g_{ry}\,, \quad \delta g_{ty}\,, \quad \delta g_{xy}\,, \quad \delta A_y\,, \quad \delta\psi_2$$

Introduce master fields as before and get

$$\begin{aligned} r^2(f\Phi_\pm')' + \left(\frac{r^2\omega^2}{f} - k^2 - \frac{\mu^2 r_0^2}{r^2} + \frac{\mu r_0}{r}c_\pm\right)\Phi_\pm &= 0, \\ \frac{1}{r^2 f}(r^2 f\Phi_1')' + \left(\frac{\omega^2}{f^2} - \frac{(k^2+\alpha^2)}{r^2 f}\right)\Phi_1 &= 0, \\ \Phi_0 + \frac{f}{r^2}(c_+ r\Phi_+ + c_- r\Phi_-)' - \frac{k\omega}{k^2+\alpha^2}\Phi_1 &= 0. \end{aligned}$$

Same equation for $\Phi_\pm$ so the electrical conductivities coincide!

The thermal conductivity $\sim \delta g_{ty}$ differs.

Conclusions

Conclusions

- model of momentum relaxation with a diff invariant $\mathcal{L}$ and simple matter content. diff invariance broken by bc's.

Conclusions

- model of momentum relaxation with a diff invariant $\mathcal{L}$ and simple matter content. diff invariance broken by bc's.
- relevant BH is analytic and σ can be computed using ODE's.

Conclusions

- model of momentum relaxation with a diff invariant $\mathcal{L}$ and simple matter content. diff invariance broken by bc's.
- relevant BH is analytic and σ can be computed using ODE's.
- $\langle jj \rangle = \langle jj \rangle_{MG}$, which suggests that MG is not related to lattice physics. Intuition for this agreement? Stuckelberg fields.

Conclusions

- model of momentum relaxation with a diff invariant $\mathcal{L}$ and simple matter content. diff invariance broken by bc's.
- relevant BH is analytic and σ can be computed using ODE's.
- $\langle jj \rangle = \langle jj \rangle_{MG}$, which suggests that MG is not related to lattice physics. Intuition for this agreement? Stuckelberg fields.
- embedding in string theory?

Conclusions

- model of momentum relaxation with a diff invariant $\mathcal{L}$ and simple matter content. diff invariance broken by bc's.
- relevant BH is analytic and σ can be computed using ODE's.
- $\langle jj \rangle = \langle jj \rangle_{MG}$, which suggests that MG is not related to lattice physics. Intuition for this agreement? Stuckelberg fields.
- embedding in string theory?
- include HSC, spatially modulated phases, ...