Holographic Entanglement Entropy for Interface, Defect or Boundary CFTs

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Based on: work in progress with Kristan Jensen, Andy O'Bannon, Efstratios Tsatis and Timm Wrase

Introduction to Entanglement Entropy:

Classical statistical entropy:

$$S_{\text{Gibbs}} = -\sum_i p_i \ln p_i \qquad (p_i = \text{prob of state } i)$$

Generalization to quantum mechanics:

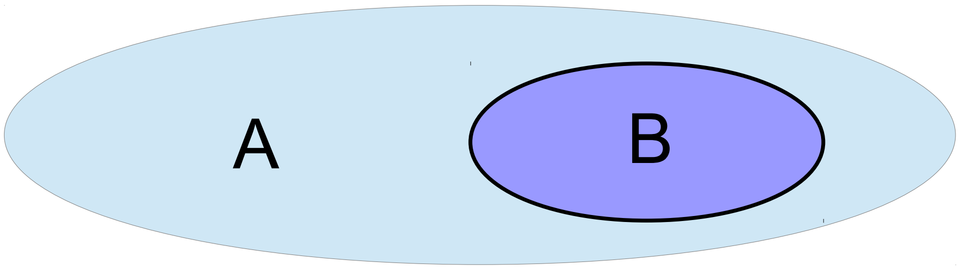
Von Neumann entropy: $S = -\operatorname{tr} \rho_{tot} \ln \rho_{tot}$

Density matrix: $\rho_{tot} = |\Psi\rangle \langle\Psi|$ pure state

$$\rho_{tot} = \sum_i p_i |\Psi_i\rangle \langle\Psi_i|$$ mixed state

$$\rho_{tot} = e^{-\beta H}$$ thermal system

Decompose system into two pieces A and B



$$\mathcal{H}_{tot} = \mathcal{H}_A \otimes \mathcal{H}_B$$

Full Hilbert space

Reduced density matrix:

$$\rho_A = \mathrm{tr}_B\, \rho_{tot}$$ (trace over states in B)

Entanglement entropy:

$$S_B = \mathrm{tr}_A\, \rho_B \ln \rho_B$$ (trace over remaining states A)

Properties (at zero temperature):

Complementarity: $S_A = S_B$

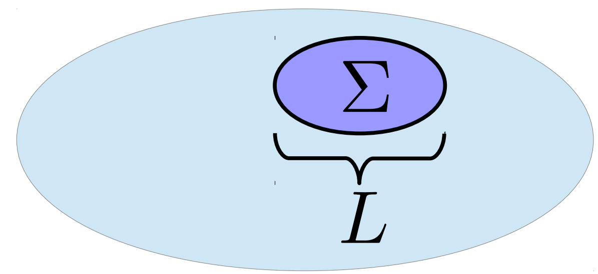
Subadditivity: $\mathcal{H}_B = \mathcal{H}_{B_1} \otimes \mathcal{H}_{B_2}$ $\quad S_{B_1} + S_{B_2} \geq S_B$

1+2-dimensions:

The L expansion of entanglement entropy takes specific form

(assuming rotational and parity symmetry) Grover, Turner, Vishwanath: 1108.4038

d=1+2: $S = \alpha_1 L + \alpha_0 + \beta_1 L^{-1} + \ldots$



- Constant contribution, α_0, gives measure of long range entanglement
- Sources of long range entanglement
    - topological order
    - massless states

$L =$ scale of integration region

$\xi =$ correlation length

$L \gg \xi$

- In the case $\Sigma = \text{disk}$

$$\alpha_0 = -F = \ln |Z| \qquad Z = \text{partition function on } S^3$$

- F-theorem: $F_{UV} \geq F_{IR}$ evidence from entanglement entropy

Casini, Huerta: 1202.5650

1+1-dimensional CFTs:

central charge

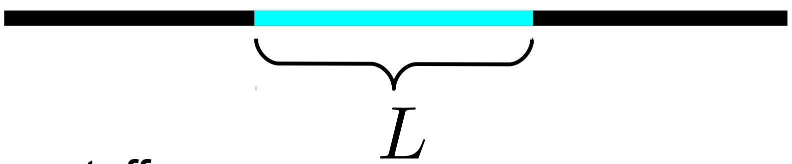
- Conformal anomaly $$\langle T^{\mu}{}_{\mu} \rangle = -\frac{c}{12} R$$

- Entanglement entropy – integrate out a segment of length L

Holzhey, Larsen, Wilczek: hep-th/9403108
Calabrese, Cardy: 0905:4013

$$S_L = \frac{c}{3} \ln \frac{L}{\epsilon} + \alpha_0$$

short distance cutoff



- c-theorem: c decreases monotonically along RG-flows

Zamolodchikov:
JETP Lett. 43, 730-732 (1986)

- Can interpret c as a measure of the number of degrees of freedom

1+3-dimensional CFTs:

central charges

- Conformal anomaly

$$\langle T^{\mu}{}_{\mu}\rangle = \frac{a}{8\pi}E - \frac{c}{72\pi}I$$

Euler characteristic

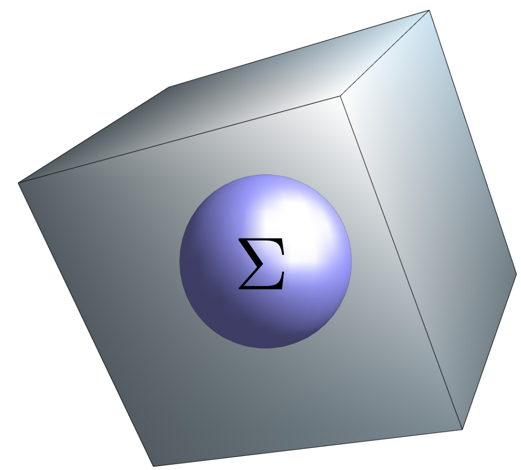
Weyl invariant

- Entanglement entropy – integrate out a volume with surface area A_Σ

$$S_\Sigma = \alpha_2 \frac{A_\Sigma}{\epsilon^2} + \alpha_{log}\ln\epsilon + \alpha_0$$

$$\alpha_{log} = a\int_\Sigma R_\Sigma + c\int_\Sigma \left(K_a^{\mu\nu}K^a_{\nu\mu} - \frac{1}{2}K_a^{\mu\mu}K^a_{\nu\nu}\right)$$

Solodukhin: 0802.3117



- a-theorem: a decreases monotonically along RG-flows

Komargodski, Schwimmer: 1107.3987

Boundary CFT in 1+1-dimensions:

- When system has a boundary, there is a novel contribution to partition function, which is independent of the size of the system

Cardy: Nucl Phys B324 581
Affleck, Ludwig: Phys. Rev. Lett. 67 161

- Interpreted as a "ground state degeneracy", $g_{\mathcal{B}}$, associated with boundary

impose conformally invariant boundary conditions, B

- Boundary entropy $S_{boundary} = \ln(g_{\mathcal{B}})$

- Can view boundary conditions as a boundary state $|\mathcal{B}\rangle$ (by exchanging space and time)

- Degeneracy given by overlap of boundary state and vacuum state $\quad g_{\mathcal{B}} = \langle 0|\mathcal{B}\rangle$

- g-theorem: the value of $g_{\mathcal{B}}$ must decrease under boundary RG-flow

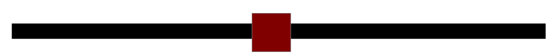
Friedan, Konechny: hep-th/0312197

- Example: 2D Ising model at critical point (free fermions)

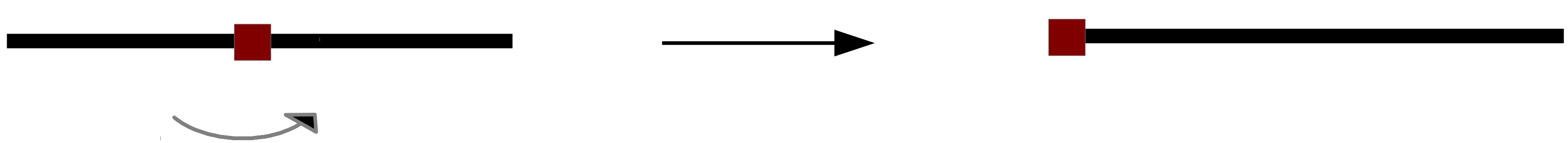
  two invariant boundary conditions

  Free spins: $g = 1$ Fixed spins: $g = 1/\sqrt{2}$

- Can also introduce an entropy associated with a defect or interface

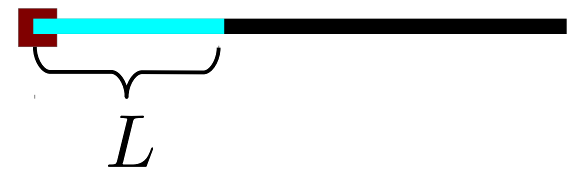


- Related to boundary entropy by folding trick

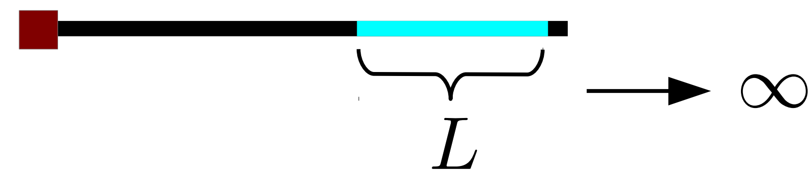


Can use entanglement entropy to compute boundary entropy

$$S_1 = \frac{c}{6}\ln\frac{L}{\epsilon} + \ln(g_{\mathcal{B}}) + \alpha_0$$



$$S_2 = \frac{c}{6}\ln\frac{L}{\epsilon} + \alpha_0$$

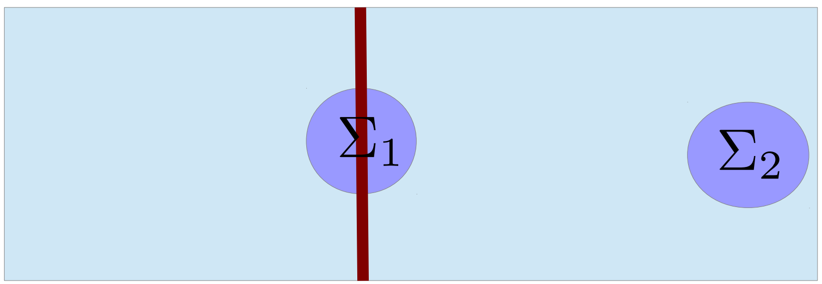


$$S_{boundary} \equiv S_1 - S_2 = \ln(g_{\mathcal{B}})$$

(Alternatively, you can compute S_2 with free boundary conditions)

How to generalize boundary entropy to higher dimensions?

- Cannot swap space and time to interpret boundary conditions as a state
- We can try to use entanglement entropy



$$S_{boundary} \equiv S_1 - \frac{S_2}{2}$$

$$S_{defect} \equiv S_1 - S_2$$

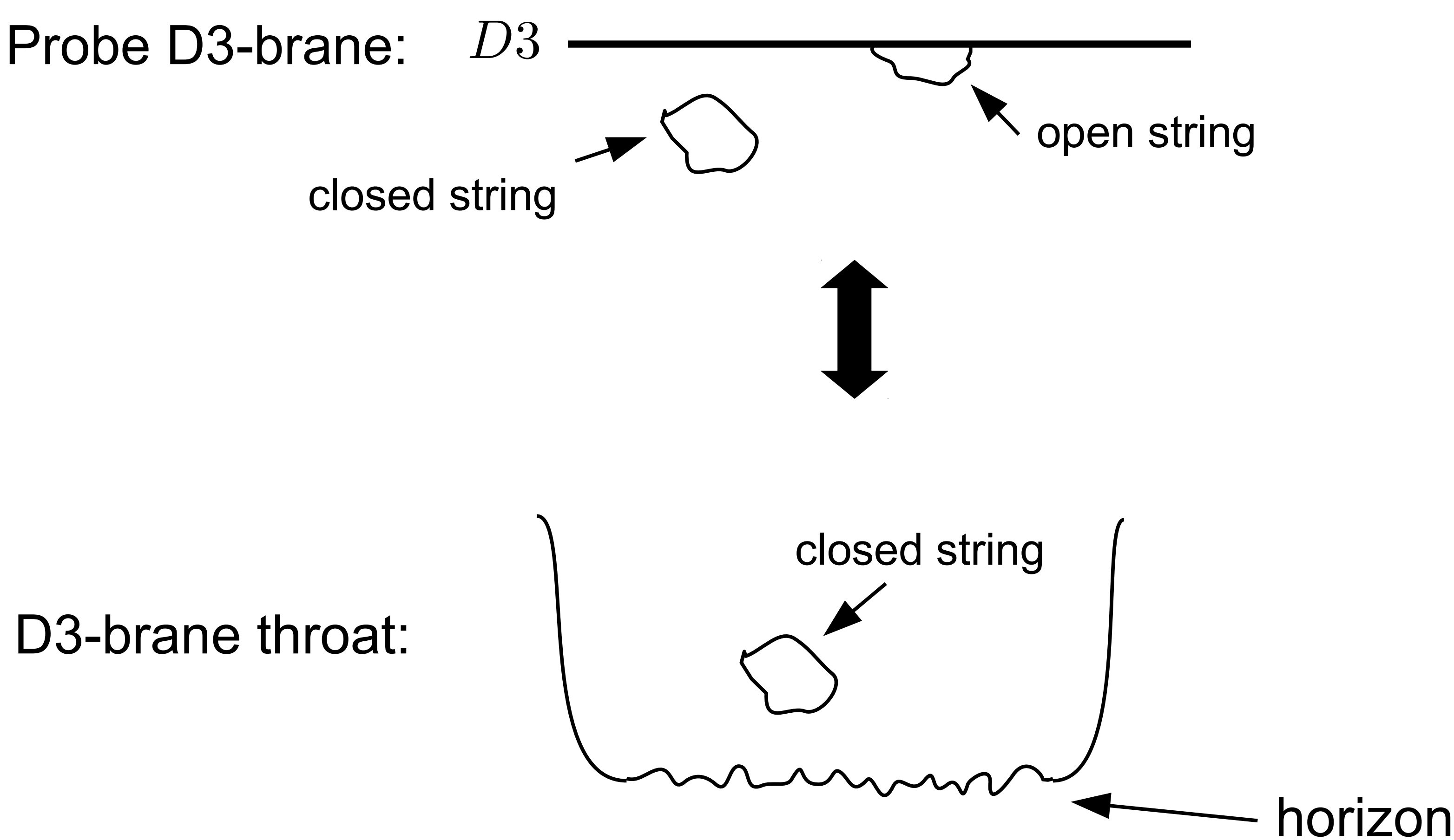
- Do these quantities depend on the regularization scheme?
- Is there an analogue of the g-theorem?
- Is there shape dependence?

Difficult to study entanglement entropy analytically...

Make use of holography

AdS/CFT:

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						



N=4 SYM in 4-dimensions

$$\mathcal{L} = \frac{1}{g_{YM}^2} \operatorname{tr}\left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} D_\mu \Phi_i D^\mu \Phi_i + \frac{1}{4} [\Phi_i, \Phi_j]^2 + \text{fermions} \right) + \frac{\theta g_{YM}^2}{32\pi^2} \operatorname{tr} F \wedge F$$

- Parameters: N, g_{YM}, θ

Closed strings propagating on $AdS_5 \times \mathbb{S}^5$

Metric on $AdS_5 \times \mathbb{S}^5 : ds^2 = \frac{R^2}{z^2} \left(dz^2 - dt^2 + dx_1^2 + dx_2^2 + dx_3^2 \right) + R^2 ds_{\mathbb{S}^5}^2$

- Parameters: $R, \phi, C_{(0)}$

- AdS_5 has a 1+3d boundary at z=0 and the “field theory lives on the boundary”
- Parameter map:

$$e^\phi = g_{YM}^2 / 2\pi = \lambda / N \qquad C_{(0)} = \theta g_{YM}^2 / 2\pi \qquad R^4 = 4\pi\lambda$$

AdS/CFT correspondence:

(Closed strings propagating on $AdS_5 \times \mathbb{S}^5$) = (N=4 SYM in 4-dimensions)

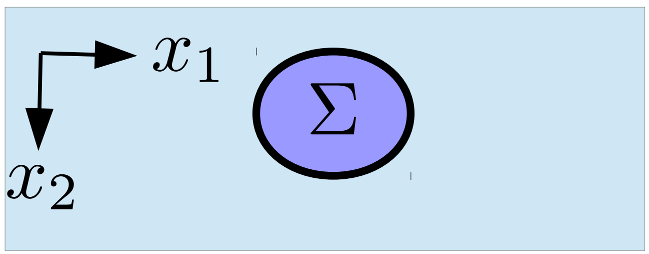
- Symmetry map: isometry of $AdS_5 = SO(2,4) =$ conformal symmetry of N=4 SYM

gravity	Duality map		*gauge theory*
• gravity approximation	$R \gg 1$	$\lambda \gg 1$	• strong 't Hooft coupling
• scalar degree of freedom	$\phi(x)$	$\mathcal{O}_\Delta$	• scalar operator
• scalar mass	$m^2 = \Delta(\Delta - 4)$		• operator dimension
• dilaton	ϕ	$\mathcal{L}_{\mathcal{N}=4}$	• Lagrangian
• axion	$C_{(0)}$	$F \wedge F$	• topological term
• partition function	$Z_{gravity} = Z_{N=4}$		• partition function
• generating functional			• generating functional
• fundamental string			• Wilson loop

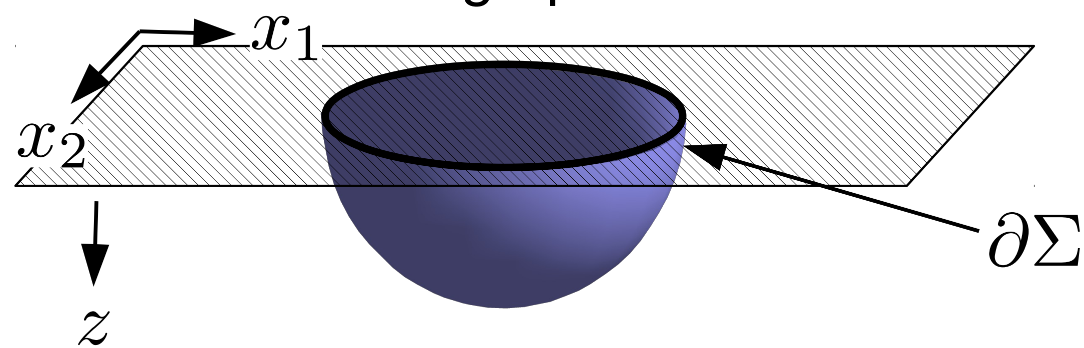
Holographic Entanglement Entropy:

Ryu and Takayangi proposal: Ryu, Takayangi – hep-th/0603001

Field theory



Holographic dual



Entanglement entropy is given by the area of a minimal surface (co-dimension-2) whose boundary is fixed to be the entangling surface

$$S_\Sigma = \frac{\mathcal{A}}{4G_N}$$

Area of minimal surface

Newton's constant

Inspired by Bekenstein-Hawking entropy formula for black holes:

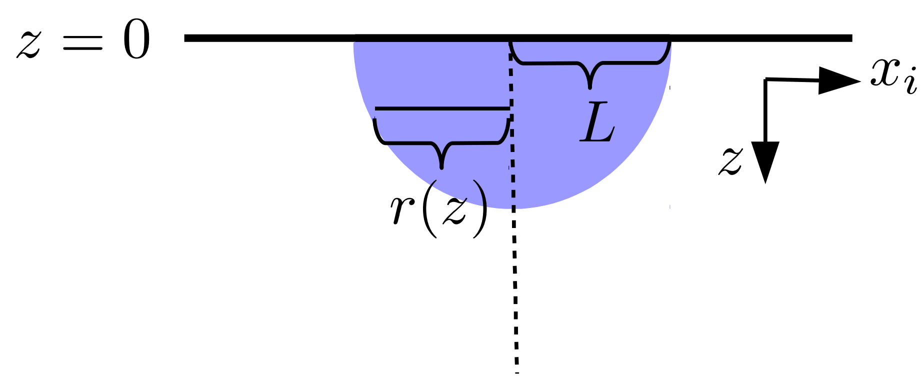
$$S_{BH} = \frac{\mathcal{A}_{\text{BH horizon}}}{4G_N}$$

Evidence for conjecture given in: Casini, Huerta, Myers – 1102.0040
Lewkowycz, Maldacena – 1304.4926

Example: $AdS_5 \times \mathbb{S}^5$

Focus on AdS_5: $$ds^2_{AdS_5} = \frac{R^2}{z^2}\left(dz^2 - dt^2 + dr^2 + r^2(d\theta^2 + \sin^2(\theta)d\phi^2\right)$$

- Consider spherical entangling surface
- Minimal area whose boundary is entangling surface
- Parameterize by z, θ, ϕ
- Problem has spherical symmetry
- Need to determine $r(z)$



$$\mathcal{A} = \int d^3x \sqrt{|\det g|} = (4\pi)\int dz \left(\frac{R^3 r^2}{z^3}\sqrt{1 + (\partial_z r)^2}\right)$$

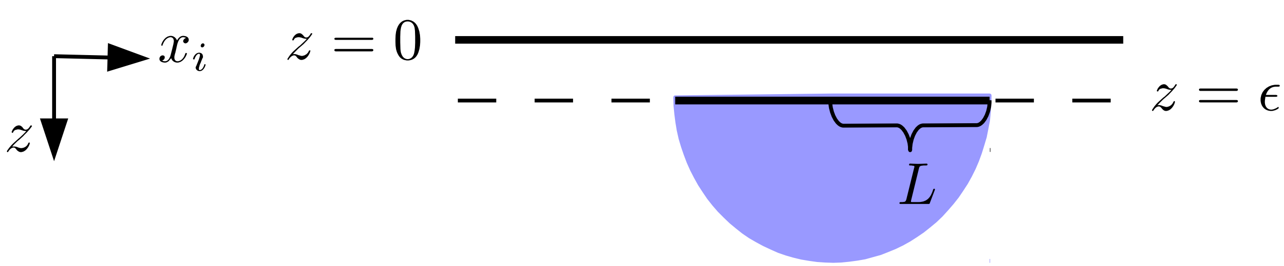
Minimize area: $$\frac{d}{dz}\left(\frac{\partial \mathcal{A}}{\partial_z r}\right) - \frac{\partial \mathcal{A}}{\partial r} = 0$$

Perturbative solution: $$r(z) = L - \frac{z^2}{2L} - \frac{z^4}{12L^3} + \ldots$$

Example: $AdS_5 \times \mathbb{S}^5$

$$ds^2_{AdS_5} = \frac{R^2}{z^2}\left(dz^2 - dt^2 + dr^2 + r^2(d\theta^2 + \sin^2(\theta)d\phi^2\right)$$

- Area is divergent
- Introduce cutoff surface as a regulator
- Natural cutoff surface defined by $z = \epsilon$



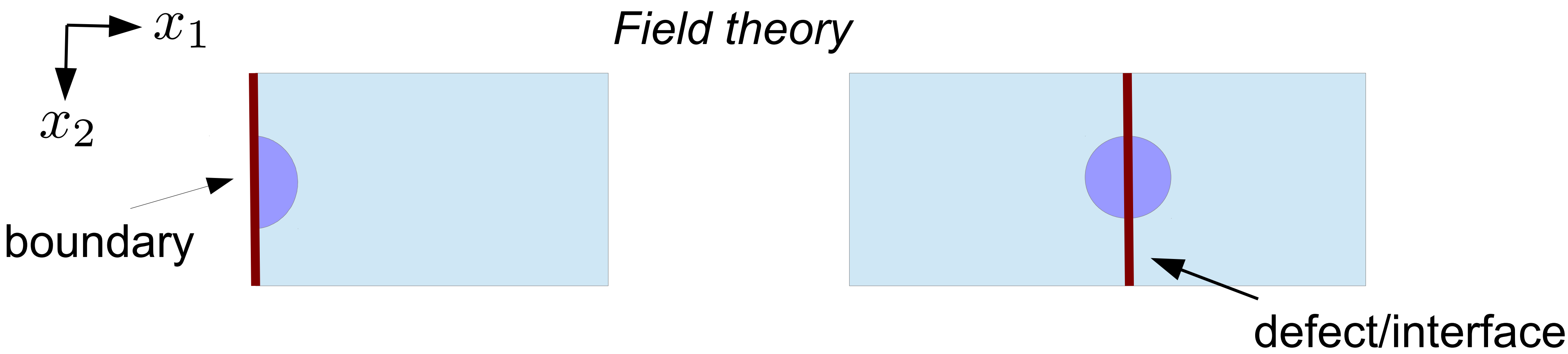
$$\mathcal{A} = (4\pi)R^3 \int_\epsilon dz \left(\frac{R^2}{z^3} - \frac{1}{2z} + ...\right) = (2\pi)R^3 \left[\frac{L^2}{\epsilon^2} + \ln\left(\frac{\epsilon}{L}\right) + ...\right]$$

$$S_{S^2} = N^2 \frac{L^2}{\epsilon^2} - N^2 \ln\left(\frac{2L}{\epsilon}\right) - \alpha_0$$

Extract central charge, agrees with
field theory computation!

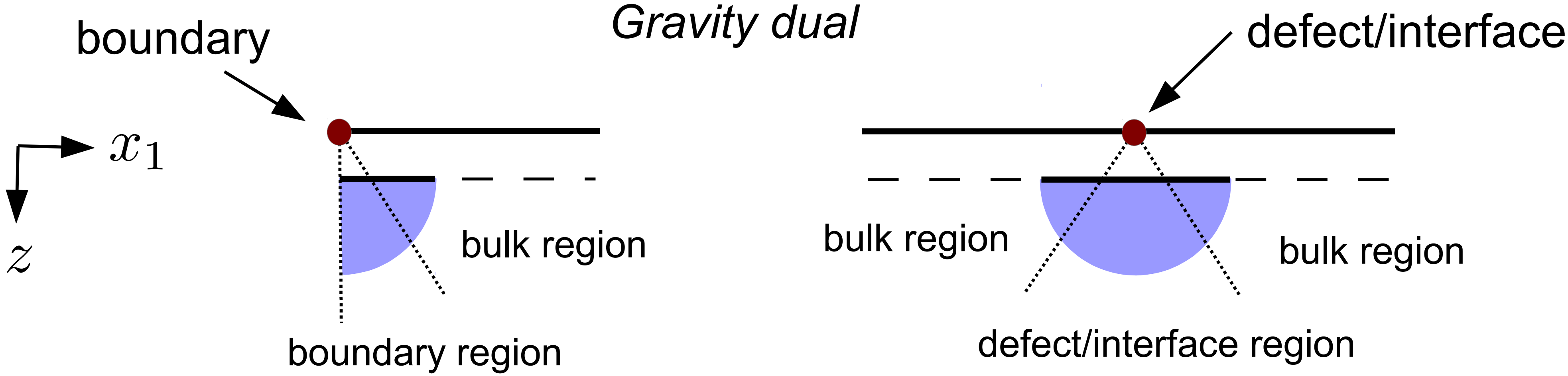
Generalization to interfaces, defects and boundaries:

Field theory



Strategy:
- Consider spherical entangling surface
- Universal solution which minimizes area
- Cutoff prescription
- Determine universal terms

Gravity dual



Slicing coordinates:

In general a conformal interface will reduce the symmetry:

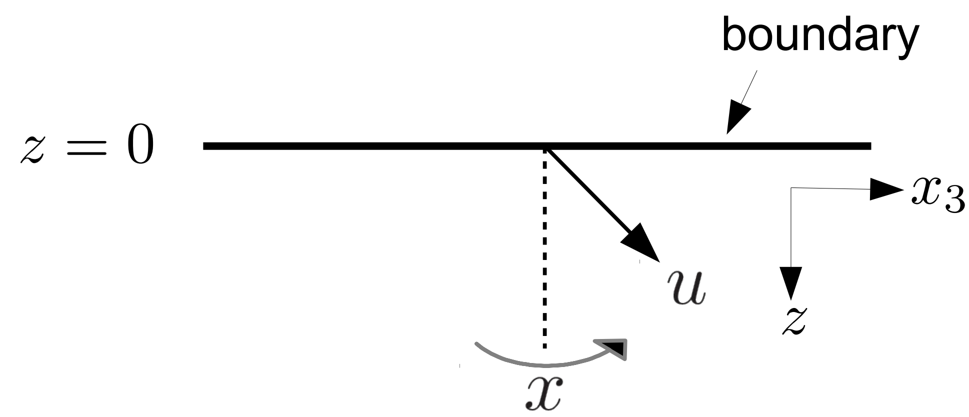
$SO(2,4) \to SO(2,3)$ ➡ AdS_4 slicing of AdS_5

boundary is decomposed into three pieces:

left: $x = -\infty$

middle: $u = 0$

right: $x = +\infty$



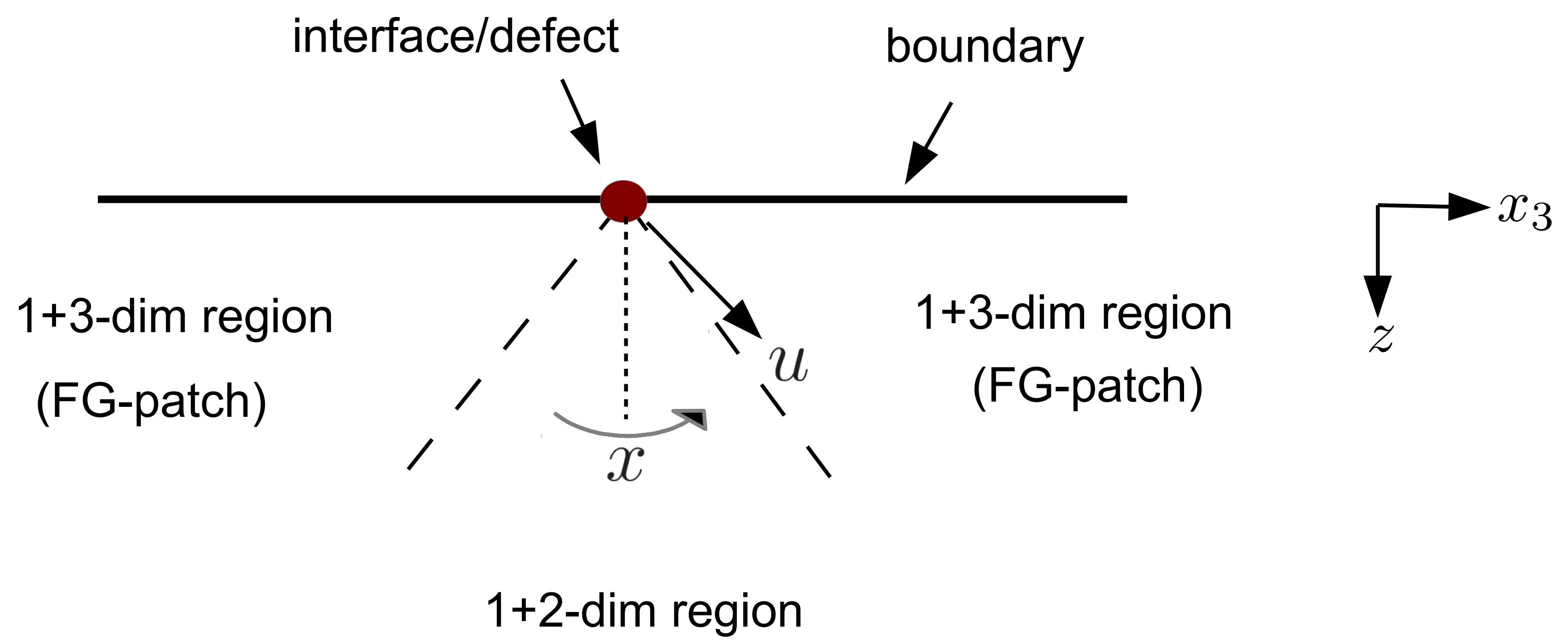
$$ds^2_{AdS_5} = \frac{1}{z^2}\left(dz^2 - dt^2 + dx_1^2 + dx_2^2 + dx_3^3\right)$$

$$z = u/\cosh(x)$$

$$x_3 = u\tanh(x)$$

$$ds^2_{AdS_5} = \cosh^2(x)\overbrace{\left(\frac{du^2 - dt^2 + dx_1^2 + dx_2^2}{u^2}\right)}^{AdS_4} + dx^2$$

General structure:



- Interface reduces conformal symmetry:

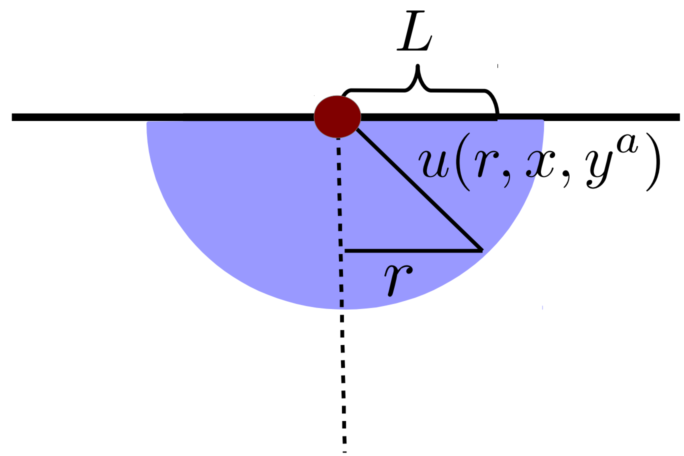
$$SO(2,4) \to SO(2,3) \quad \Longrightarrow \quad AdS_4 \text{ slicing}$$

$$ds^2 = f^2(x,y)ds^2_{AdS_4} + \rho^2(x,y)dx^2 + G_{ab}dy^a dy^b$$

$$ds^2_{AdS_4} = \frac{1}{u^2}\left(du^2 - dt^2 + dx_1^2 + dx_2^2\right)$$

- Metric is required to be asymptotically AdS_5 as $x \to \pm\infty$

Minimal area surface:



- Parameterize surface by: $r, x, y^a, \mathbb{S}^1$
- For spherical entangling surface, problem has spherical symmetry
- Need to determine: $u(r,x,y^a)$

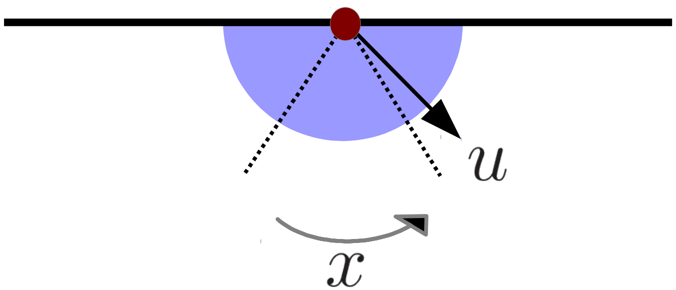
$$ds^2 = f^2(x,y)ds^2_{AdS_4} + \rho^2(x,y)dx^2 + G_{ab}dy^a dy^b$$

$$\mathcal{A} = \mathrm{vol}(\mathbb{S}^1)\int dy\,dx\,dr\,r\rho\left(\frac{f}{u}\right)^2\sqrt{\det G}\sqrt{1+(\partial_r u)^2+\frac{f^2}{u^2}\left[\rho^{-2}(\partial_x u)^2+G^{ab}\partial_a u\partial_b u\right]}$$

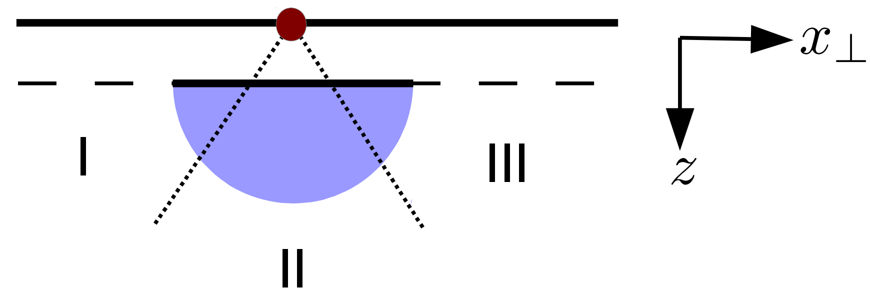
universal solution: $u^2 + r^2 = L^2$ $\longleftarrow$ integration constant

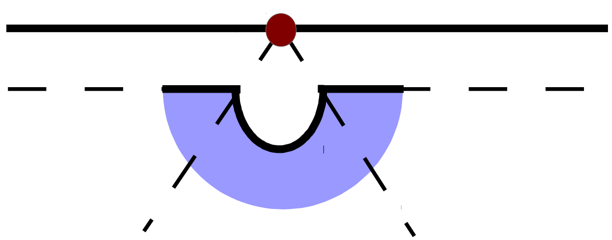
Regularization:

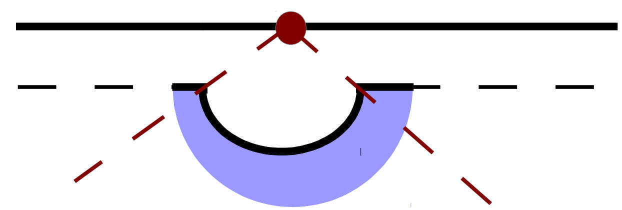
start with divergent area



introduce
cutoff surface





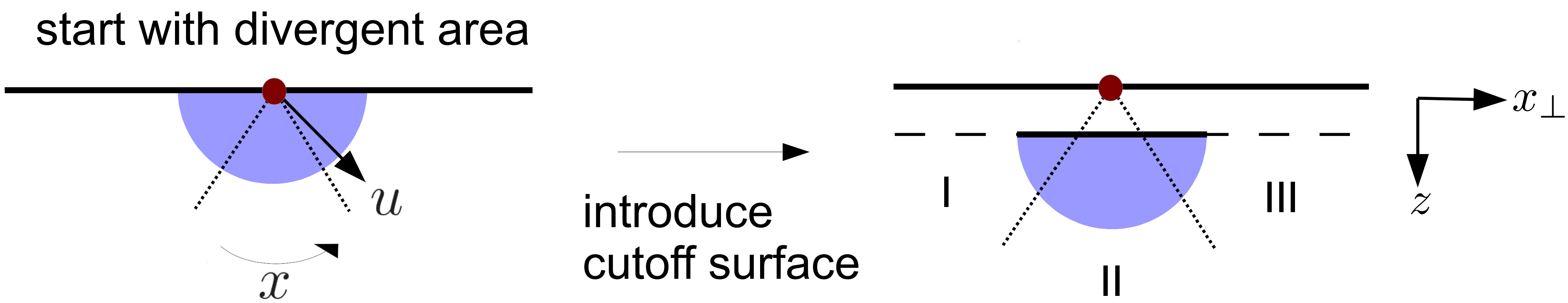


choice of cutoff surface is not unique!

Are there terms which do not depend on the regularization scheme?

Background subtraction:

$$ds^2 = f(x,y)ds^2_{AdS_d} + \rho^2(x,y)dx^2$$



- Background subtraction to define boundary/defect entropy
- Use same regularization scheme as background
  - recall for $AdS_5 \times S^5$ we used $z = \epsilon$
- In regions I and III we can use FG-coodinates

FG-coordinates: $z = u\,k_1^{(\pm)}(x)$ $\quad x_\perp = u\,k_2^{(\pm)}(x)$

- In regions I and III, impose cutoff: $z = \epsilon \Rightarrow x_c^{(\pm)} = K_1^{(\pm)}\left(\frac{\epsilon}{u}\right)$
- In region II impose cutoff $u = u_c(x)$ with $u_c(x)$ chosen so that cutoff surface is continuous

Choice of cutoff surface is not unique:

two alternative regularization schemes

- We show the existence of universal terms, which do not depend on the regularization scheme

- simple regularization prescription:

$$x_c^{(\pm)} = \pm \ln\left(\frac{2u}{\epsilon}\right) - c_0^{(\pm)} + c_1^{(\pm)}\left(\frac{\epsilon}{u}\right) + c_2^{(\pm)}\left(\frac{z}{\epsilon}\right)^2$$

$$u_c = \epsilon$$

the c_i are determined by FG-transformation

Result:

- Entanglement entropy:

$$S_{EE} = \frac{\mathrm{vol}(\mathbb{S}^{d-3})R}{4G} \int dy \int_{u_c}^{R} du \int_{x_c^{(-)}}^{x_c^{(+)}} dx \, \sqrt{\det G} \rho f^{d-2} \frac{(R^2 - u^2)^{(d-4)/2}}{u^{d-2}}$$

$$\boxed{= \sum_{n=1}^{d-2} D_n \left(\frac{L}{\epsilon}\right)^n + D_{log} \ln\left(\frac{L}{\epsilon}\right) + D_0}$$

- For even d, both D_{log} and D_0 can be computed unambiguously

$$d = 1+3 : S_{defect} = S_{EE} - S_{background} = \alpha_1 \frac{L}{\epsilon} - \alpha_0$$

characterizes 1+2-dimensional defect

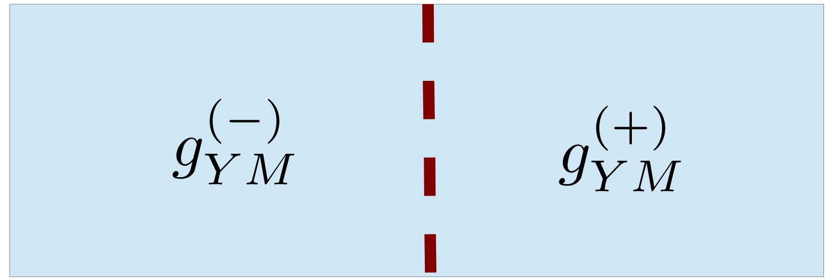
- For odd d, D_{log} can be computed unambiguously, while D_0 depends on the choice of regulator

$$d = 1+2 : S_{defect} = S_{EE} - S_{background} = \alpha_{log} \ln\left(\frac{L}{\epsilon}\right) - \alpha_0$$

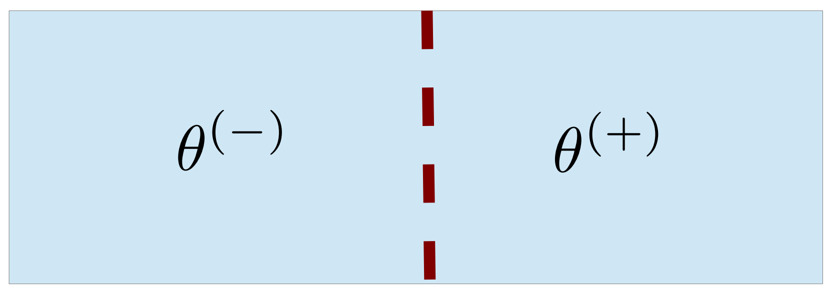
characterizes 1+1-dimensional defect

Janus: a simple interface

Dielectric interface:



Topological interface:



Supergravity solutions constructed for both cases:

Bak, Gutperle, Hirano: hep-th/0304129
D'Hoker, JE, Gutperle: 0705.0022

Janus solution:

Weierstrass function: $(\partial_v \wp)^2 = 4\wp^3 - g_2 \wp - g_3$

Metric: $$ds^2 = R^2 \left(\gamma^{-1} h(v)^2 dv^2 + h(v)\, ds^2_{AdS_4}\right) + R^2 ds^2_{\mathbb{S}^5}$$

Dilaton: $$\phi(v) = \phi_0 + \sqrt{6(1-\gamma)} \left(v + \frac{4\gamma - 3}{\wp'(v_1)} \left(\ln \frac{\sigma(v+v_1)}{\sigma(v-v_1)} - 2\zeta(v_1) v \right) \right)$$

$$h(v) = \left(1 + \frac{4-3}{\wp(v) + 1 - 2} \right) \qquad \wp(v_1) = 2(1-\gamma) \qquad \begin{aligned} g_2 &= 16\gamma(1-\gamma) \\ g_3 &= 4(\gamma - 1) \end{aligned}$$

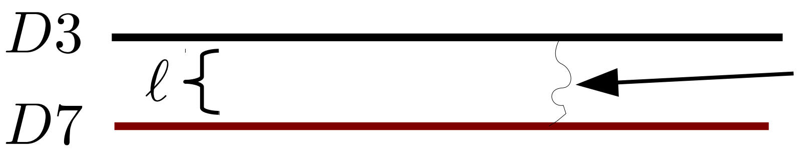
One parameter deformation of $AdS_5 \times \mathbb{S}^5$: $\quad \frac{3}{4} \leq \gamma \leq 1$

Dilaton takes different values at $v = \pm\infty$ $\Longrightarrow$ dielectric interface

Use $SL(2,\mathbb{Z})$ symmetry to map solution to one where the axion takes different values at $v = \pm\infty$ $\Longrightarrow$ topological interface

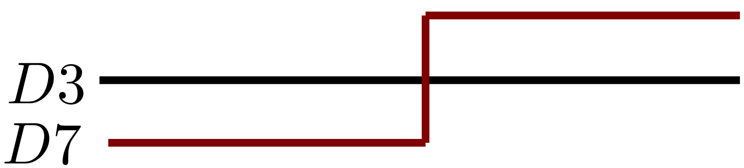
Janus: brane construction

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						
D7	X	X	X	X	X	X	X	X		



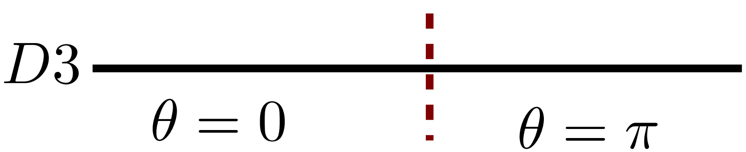
open strings lead to massive matter

$M \propto \ell$



Fractional topological insulator, with massless edge states

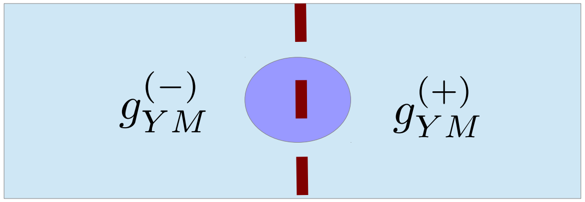
Maciejko, Qi, Karch, Zhang: 1004.3628
Hoyos-Badajoz, Jensen, Karch: 1007.3253



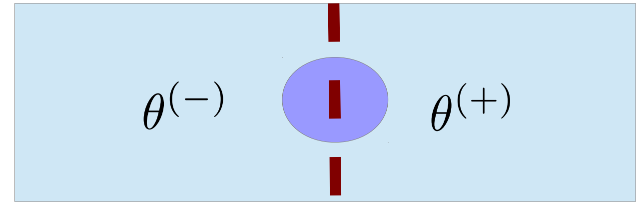
Lift D7-brane out of page, integrate out massive fermions, flow to non-trivial infrared fixed point JE, O'Bannon, Tsatis, Wrase: 1210.0534

$$\mathcal{L} = \frac{1}{g_{YM}^2} \operatorname{tr} F_{\mu\nu} F^{\mu\nu} + \frac{g_{YM}^2}{32\pi^2} \theta(x) \operatorname{tr} F \wedge F + \ldots$$

step function

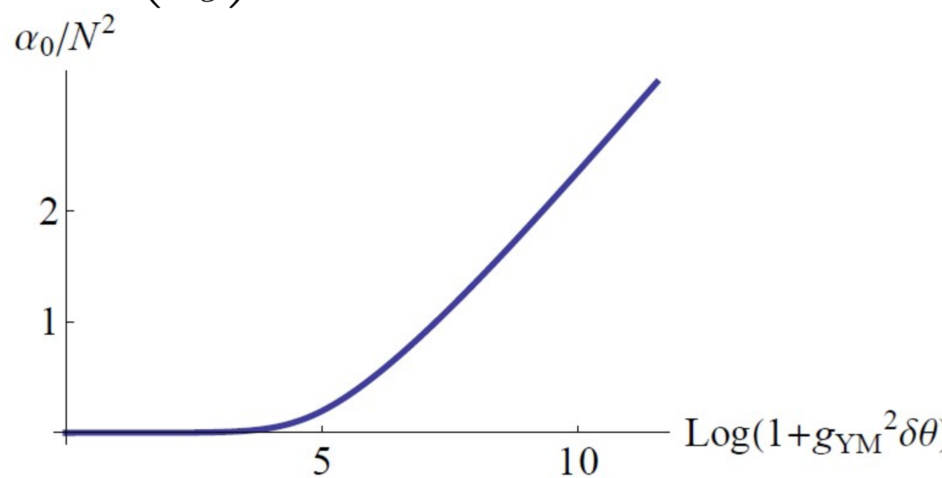


Dielectric interface:



Topological interface:

Non-supersymmetric case: $$S_{defect} = \alpha_1 \left(\frac{L}{z_c}\right) - \alpha_0$$

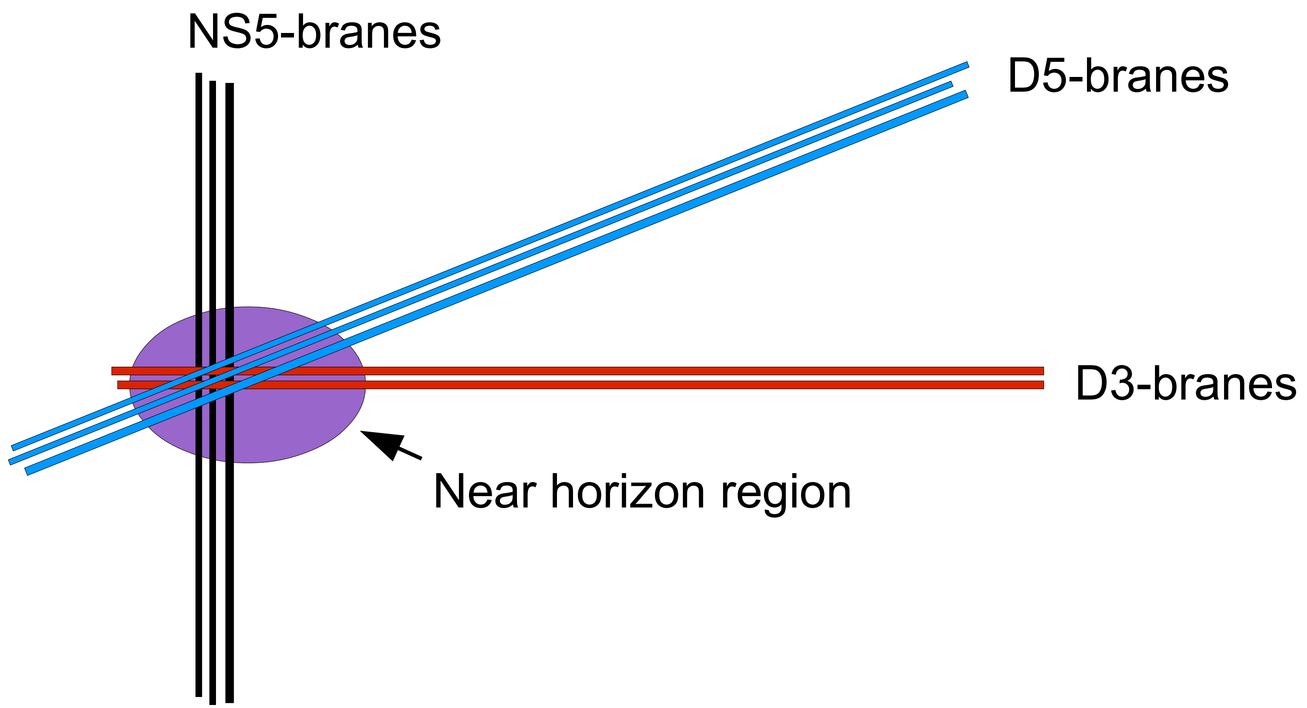


Supersymmetric case:

$$S_{defect} = \alpha_1 \left(\frac{L}{z_c}\right) - \frac{N^2}{2} \ln \left(1 + \frac{\left(g_{YM}^{(+)} - g_{YM}^{(-)}\right)^2}{2\, g_{YM}^{(-)}\, g_{YM}^{(+)}}\right) \qquad S_{defect} = \alpha_1 \left(\frac{L}{z_c}\right) - \frac{N^2}{4} \ln \left(1 + \frac{\delta\theta^2 g_{YM}^4}{64\pi^4}\right)$$

Half-BPS defects:

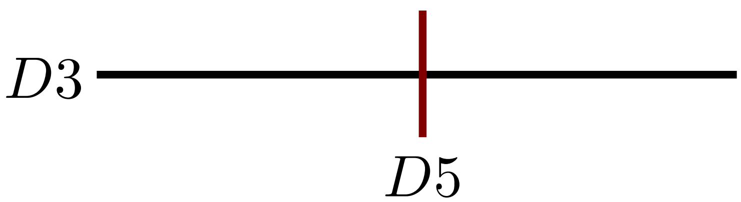
Half-BPS defects can be constructed by introducing D5 and NS5 branes.

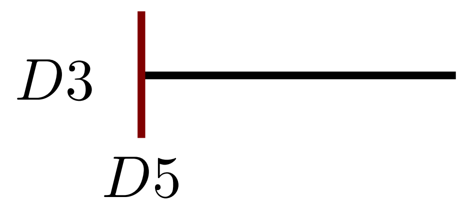


	0	1	2	3	4	5	6	7	8	9
D3	x	x	x	x						
D5	x	x	x		x	x	x			
NS5	x	x	x					x	x	x

D5-branes: a conformal defect

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						
D5	X	X	X		X	X	X			





3-5 strings lead to defect degrees of freedom

- N=4 SYM coupled to a 1+2d defect

Ending D3-branes on D5-branes leads to a boundary CFT

- N=4 SYM coupled to a 1+2d boundary

Dual supergravity solutions are known

D'Hoker, JE, Gutperle: 0705.0022, 0705.0024

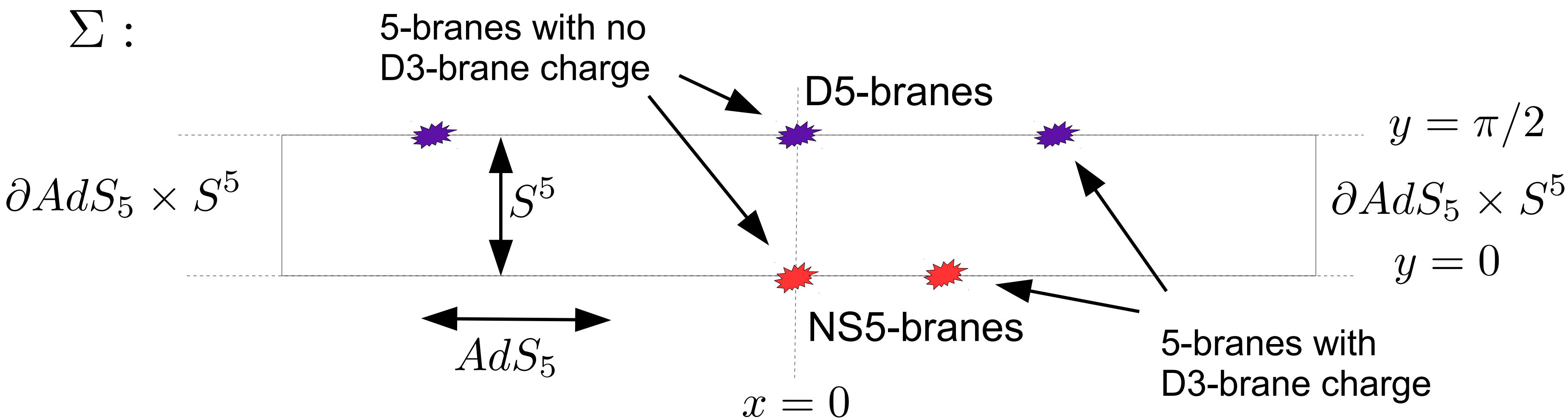
Aharony, Berdichevsky, Berkooz, Shamir:1106.1870

Probe description:

- 5-branes preserve OSp(4|4,R) symmetry and therefore wrap $AdS_4 \times S^2$ cycles
- Slice $AdS_5 \times S^5$ into $AdS_4 \times S^2 \times S^2$ slices which are fibered over a 2d base space Σ :

$$ds^2 = \cosh^2(x) ds^2_{AdS_4} + \sin^2(y) ds^2_{S^2} + \cos^2(y) ds^2_{S^2} + (dx^2 + dy^2)$$

Σ :



- D5-branes and NS5-branes are orthogonal in the directions transverse to the D3-branes and therefore wrap different S^2's
- To preserve full SO(3) x SO(3), the transverse S^2 must vanish at the probe locations
- In general 5-branes can have D3-brane charge dissolved into them
  - D3-branch charge determines the value of x they sit at

Backreacted solutions:

- Strategy: solve BPS equations after imposing SO(2,3) x SO(3) x SO(3) symmetry
- General solutions are parametrized by the choice of a Reimann surface Σ, possibly with boundary, and two functions $h_1(\Sigma)$ and $h_2(\Sigma)$ which are harmonic on Σ

Introduce auxiliary functions: $W = \partial\bar{\partial}(h_1 h_2) \qquad N_{1(2)} = 2h_1 h_2 |\partial h_{1(2)}|^2 - h_{1(2)}^2 W$

<u>metric:</u> $$ds^2 = f_4^2 ds^2_{AdS_4} + f_1^2 ds^2_{S^2} + f_2^2 ds^2_{S^2} + 4\rho^2 dz d\bar{z}$$

$$f_4^8 = 16 \frac{N_1 N_2}{W^2} \qquad f_1^8 = 16 h_1^8 \frac{N_2 W^2}{N_1^3} \qquad f_2^8 = 16 h_2^8 \frac{N_1 W^2}{N_2^3}$$

<u>dilaton:</u> $e^{4\phi} = \frac{N_2}{N_1}$

<u>three forms:</u> $H_3 = db_1 \wedge \hat{e}^{45} \qquad F_3 = db_2 \wedge \hat{e}^{67}$

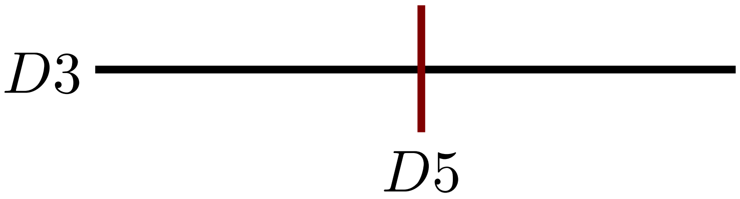
$$b_{1(2)} = 2ih_{1(2)} \frac{h_1 h_2(\partial h_1 \bar{\partial} h_2 - \bar{\partial} h_1 \partial h_2)}{N_{1(2)}} + 2h^D_{2(1)}$$

dual harmonic function

Regularity conditions: $f_{1(2)} = 0 \Leftrightarrow h_{1(2)} = 0$

$h_{1(2)} = 0 \text{ and } f_4 \neq 0 \Rightarrow W = 0$

D5-brane defect:



N_3 = number of D3-branes

N_5 = number of D5-branes

Geometry given by:

$$h_1 = \alpha' \left[-i\,\alpha \sinh(v) - \frac{N_5}{4} \ln \left(\tanh \left(\frac{i\pi}{4} - \frac{v}{2} \right) \right) \right] + \text{c.c.} \qquad g_s = \frac{\hat{\alpha}}{\alpha}$$

$$h_2 = \alpha' \hat{\alpha} \cosh(v) + \text{c.c.} \qquad R^4 = 8\hat{\alpha}(N_5 + 2\alpha)(\alpha')^2 = 4\pi N_3$$

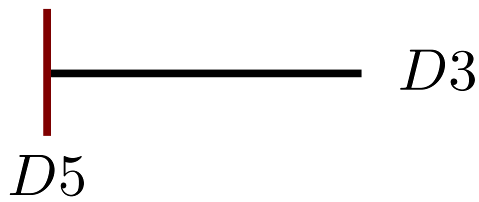
Defect entropy:

$$S_{defect} = \alpha_1 \left(\frac{L}{\varepsilon} \right) + \frac{N_3^2}{2} \ln \left(\frac{(\xi - g_{YM}^2 N_5)^2}{4\pi g_{YM}^2 N_3} \right) - \frac{N_5 (\xi - g_{YM}^2 N_5)^2 (5\xi + 4 g_{YM}^2 N_5)}{48 g_{YM}^2 \pi^2}$$

$$\xi^2 = 4\pi g_{YM}^2 N_3 + (g_{YM}^2 N_5)^2$$

Agrees with probe computation in the limit: $N_5 \ll N_3$ Jensen, O'Bannon:1309.4523

D5-brane boundary:



$$N_3 = \text{number of D3-branes}$$

$$N_5 = \text{number of D5-branes}$$

Geometry given by:

$$h_1 = \alpha' \left[-\frac{i\alpha}{2} e^{v} - \frac{N_5}{4} \ln\left(\tanh\left(\frac{i\pi}{4} - \frac{v}{2} \right) \right) \right] + \text{c.c.} \qquad g_s = \frac{\hat{\alpha}}{\alpha}$$

$$h_2 = \alpha' \frac{\hat{\alpha}}{2} e^{v} + \text{c.c.} \qquad R^4 = 8 N_5 \hat{\alpha} (\alpha')^2 = 4\pi N_3$$

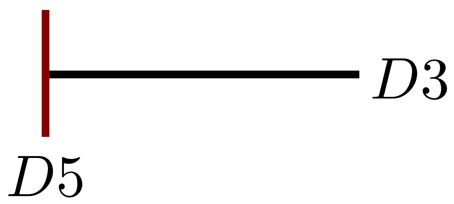
Defect entropy:

$$S_{boundary} = \alpha_1 \left(\frac{L}{\varepsilon} \right) + \frac{N_3^2}{8} \left(2 \ln\left(\frac{4\pi N_3}{g_{YM}^2 N_5^2} \right) - 3 \right) + \frac{\pi N_3^3}{12 g_{YM}^2 N_5^2}$$

Monotonicity:

In 1+1 dimensions, the boundary entropy obeys a monotonicity condition under boundary RG-flows. Does a similar condition hold for our 1+3 dimensional boundary entropy?

Since we are studying theories at their conformal fixed points, we cannot directly test monotonicity. However we can compare two conformal fixed points, which are connected by an RG-flow.



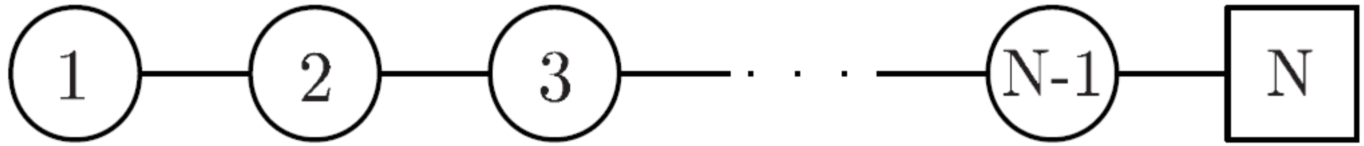
- We consider moving ΔN_5 D5-branes out of the page
- This gives masses to ΔN_5 of the defect fields
- The conformal fixed point is then the same defect theory, but with $N_5 - \Delta N_5$ defect fields

Writing the boundary entropy as $S_{boundary} = \alpha_1 \frac{L}{\epsilon} - \alpha_0$, we find $\frac{\partial \alpha_0}{\partial N_5} < 0$

- We can also consider going onto the Higgs branch of the theory.
- This corresponds to separating the D3-branes.
- The conformal fixed point is the same defect theory, with a reduced gauge group.

In this case, the RG-flow is not a boundary RG-flow and we find that $\frac{\partial \alpha_0}{\partial N_3}$ can take either sign

T[SU(N)]:



- Conjecture that these theories flow to non-trivial 3d CFT — Gaiotto, Witten: 0807.3720

- Dual supergravity solutions constructed, supporting conjecture — Assel, Bachas, JE, Gomis: 1106.4253

$$h_1 = \alpha' \left[-\frac{N}{4} \ln \left(\tanh \left(\frac{i\pi}{4} - \frac{v-\delta}{2} \right) \right) \right] + c.c.$$

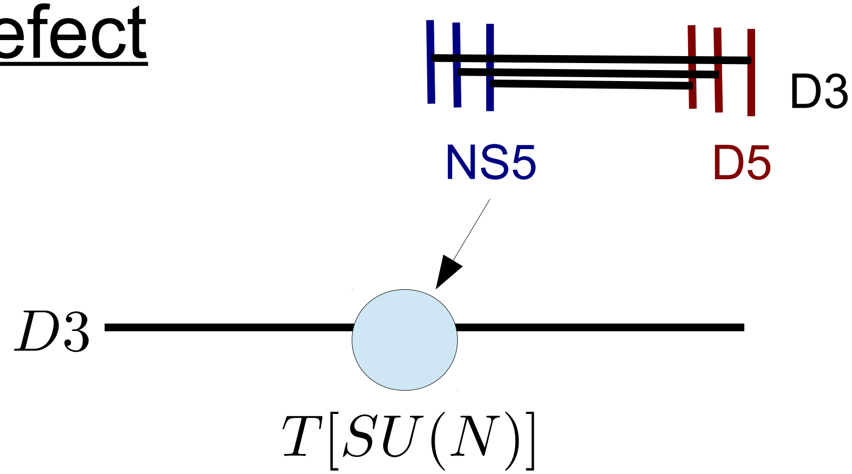
$$h_2 = \alpha' \left[-\frac{N}{4} \ln \left(\tanh \left(\frac{v-\hat{\delta}}{2} \right) \right) \right] + c.c.$$

$$\frac{\pi N}{2} = \arctan(e^{\hat{\delta}-\delta})$$

- Agreement between CFT and gravity partition functions in large N limit — Assel, JE, Yamazaki: 1206.2920

$$F = \frac{1}{2} N^2 \ln(N) + \mathcal{O}(N^2)$$

T[SU(N)] as a defect



$$N_3 = \text{number of D3-branes} \qquad N = \text{size of } T[SU(N)]$$

reproduces: $F = -\frac{1}{2}N^2\ln(N) + \mathcal{O}(N^2)$

$$N \gg N_3 \gg 1 : S_{defect} = \alpha_1 \frac{L}{z_c} - \frac{1}{2}N^2 \ln N + \mathcal{O}(N^2)$$

$$N_3 \sim N \gg 1 : S_{defect} = \alpha_1 \frac{L}{z_c} - \frac{1}{2}N^2 \left(1 + 2\frac{N_3}{N} + 2\frac{N_3^2}{N^2}\right) \ln N + \mathcal{O}(N^2)$$

$$N^2 \gg N_3 \gg N \gg 1 : S_{defect} = \alpha_1 \frac{L}{z_c} - N_3^2 \ln\left(\frac{N^2}{N_3}\right) + \mathcal{O}(N_3^2)$$

$$N_3 \gg N^2 : S_{defect} = \alpha_1 \frac{L}{z_c} + \mathcal{O}(N_3^2)$$

Outlook:

- Shape dependence?
  - We worked with the special case of a spherical entangling surface
  - How does the boundary entropy depend on the choice of surface?
  - Do deformations of the entangling surface away from the defect/boundary modify the boundary entropy?

- g-theorem?
  - Are the universal terms we identified monotonic under RG-flow?
  - Extend holographic proofs of c-theorem, F-theorem to the case of interfaces/defects/boundaries?

- Surface superconductivity
  - Holographic systems exhibit superconductivity
  - Constant term indicates presence of long range entanglement
  - Do the defect/interfaces exhibit surface superconductivity?
  - Relation to localized gravity...

In progress with J. Indekeu