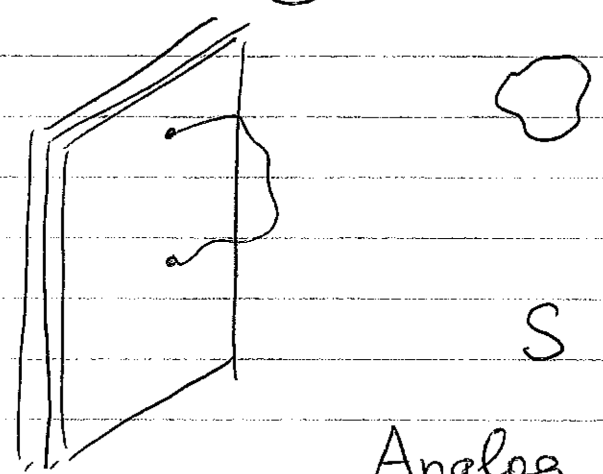
Constructing the gauge-string duality

Basic example: AdS - CFT correspondence

Ingredients: N_c D3 Branes, $\omega \ll 1/l_s$

- open string picture:



$$S = S_{Brane} + S_{Bulk} + S_{int}$$

Analog of

N_c $$S = -m\int ds - e\int A_\mu dx^\mu - \frac{1}{4e^2}\int F_{\mu\nu}^2 dx + \ldots$$

S_{Brane} = $\mathcal{N}=4$ SYM $SU(N_c)$ in $d = 3+1$

+ corrections $O(\omega^2 l_s^2 \ll 1)$, $g_{YM}^2 = 4\pi g_s$

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classical $d=10$ Mink gravity +

$$S_{Bulk} \sim O(\omega^8 G_{10} \sim g_s^2 \omega^8 l_s^8 \ll 1)$$

$$S_{int} \sim O(\omega^8 G_{10} \ll 1)$$

- closed str. picture: $p = 3$ solution to type IIB (sugra) eq. of motion at low energy $(\omega l_s \ll 1)$:

$$ds_{10}^2 = H^{-1/2}\left(-dt^2+dx^2+dy^2+dz^2\right) + H^{1/2}\left(dr^2+r^2 d\Omega_5^2\right)$$

$$H = 1 + L^4/r^4$$

$$F_{(5)} = -\frac{4L^4}{H^2 r^5}(1+*)\, dt\wedge dx\wedge dy\wedge dz\wedge dr$$

$\phi = \text{const}$ $\qquad L^4 = 4\pi g_s N_c l_s^4$ - from

$$\int_{S^{8-p}} * F_{p+2} = Q$$

This particular solution is extremal $(M = Q)$

but one can find also a non-extremal one $(M > Q)$

$$ds_{10}^2 = H^{-1/2}\left(-f\,dt^2+dx^2+dy^2+dz^2\right) + H^{1/2}\left(dr^2+r^2 d\Omega_5^2\right)$$

$$H = 1 + L^4/r^4, \quad f = 1 - r_0^4/r^4$$

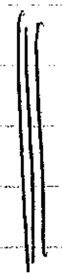
$$F_{(5)} = -\frac{4L^2}{H^2 r^5}\left(r_0^4 + L^4\right)^{1/2}(1+*)\, dt\wedge dx\wedge dy\wedge dz\wedge dr$$

exercise: compute the horizon area and Hawking temperature of the full solution; more generally – the full thermodynamics of the non-extr. and extremal solutions, including the specific heat.

Note: these are asymptotically flat solutions.

Gravitational dressing of D3 branes

Consider again the open string picture:



N_c

Recall: corrections to flat metric in $d=4$: $\sim \frac{GM}{r}$

(becomes relevant for $\frac{GM}{r} \sim 1$)

For an extended object ($p+1$-dim) in d dim:

$$\sim \frac{G M_{TOT}}{r^{d-p-3}} \qquad \left(\sim \frac{GM}{r^{D-2}} \text{ in } D \text{ spatial dim} \right).$$

For N_c D3 branes: $\sim \frac{G_{10} N_c T_3}{r^4}$

$$G_{10} \sim g_s^2 \, l_s^8$$

$$T_3 \sim \frac{1}{g_s \, l_s^4}$$

Grav. correction : $\sim \dfrac{g_s N_c \, l_s^4}{r^4}$

- for $g_s N_c \ll 1$, correction ~ 1 for $r \ll l_s$
- for $g_s N_c \gg 1$, correction important for $r \gg l_s$

$\Rightarrow$ cannot use the <u>open string picture</u>

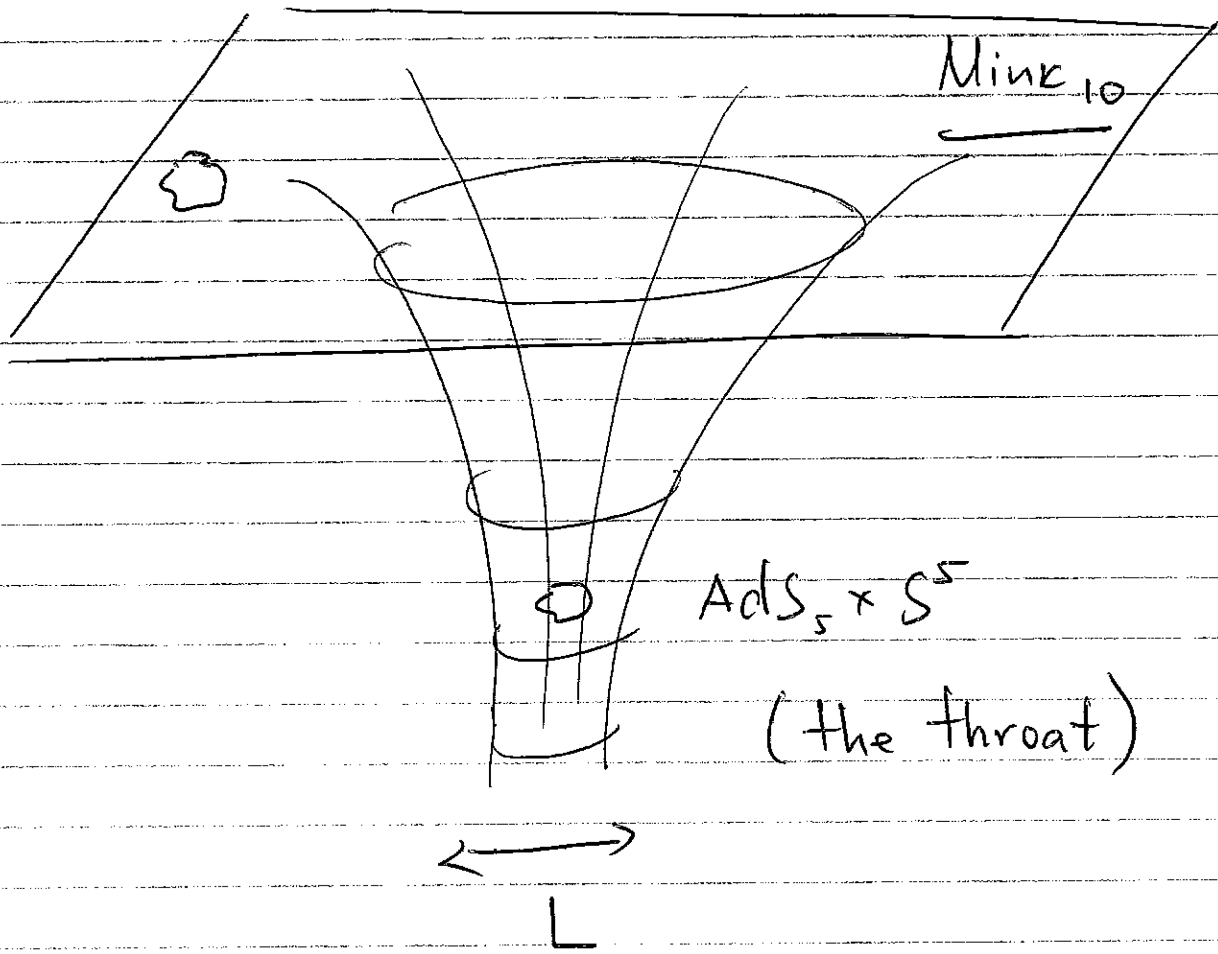
In the closed string picture, the relev. parameter is L^4/r^4 : for $L/r \gg 1$ $(r \ll L)$ the metric becomes

$$ds_{10}^2 = \frac{r^2}{L^2}\left(-dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{L^2}{r^2} dr^2 +$$

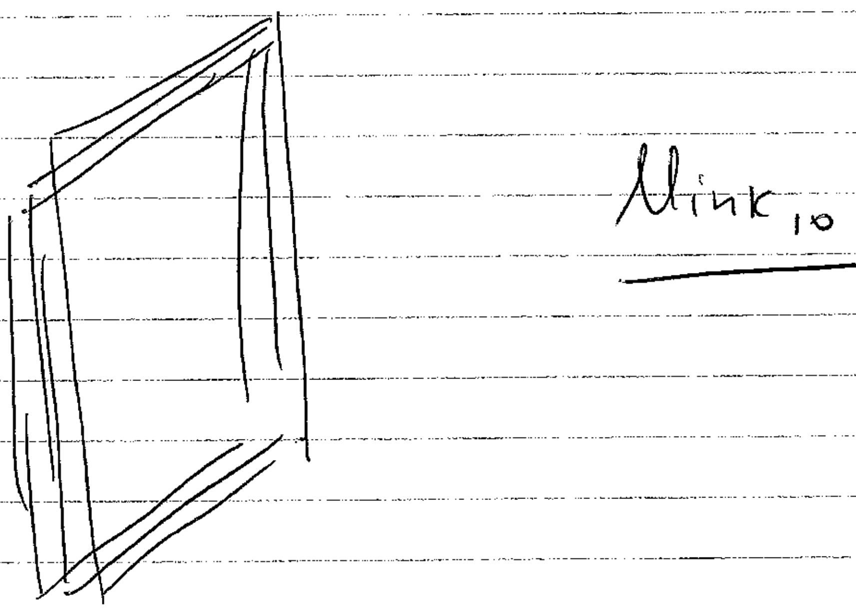
$$+ L^2 d\Omega_5^2$$

This is $AdS_5 \times S^5$.

Closed str. picture:



Open str. picture:



N_c
D3 branes : $\mathcal{N}=4$ SYM $SU(N_c)$ $d=3+1$

<u>Remark</u>: in the closed str. picture far away from the throat only the decoupled excitations of the $d=10$ graviton supermultiplet are relevant at low energies ($\omega \ll 1/\ell_s$), but arbitrarily large excitations of type IIB strings are relevant deep down the throat since they are <u>redshifted</u> from the point of view of an asympt. observer.

<u>Remark</u>: computing the <u>curvature invariants</u> (e.g. Kretschmann inv - exercise) for $p=3$ brane solutions in type IIB, <u>we find they are valid for $g_s N_c \gg 1$.</u>

Absence of quantum grav. corrections:

$$\frac{L^4}{\ell_p^4} \gg 1 . \qquad \text{Recall} \quad \left[\ell_p^{(10)}\right]^8 = \frac{\hbar G_{10}}{c^3},$$

$$G_{10} \sim g_s^2 \ell_s^8 \quad \Longrightarrow$$

$$\frac{L^4}{l_p^4} \sim \frac{4\pi g_s N_c l_s^4}{G_{10}^{1/2}} \sim \frac{g_s N_c l_s^4}{g_s l_s^4} \sim N_c \gg 1$$

Remark: in the open string picture, $g_s N_c$ controls string loop corrections $\Rightarrow$ convenient description (perturbative) for $g_s N_c \ll 1$, not convenient for $g_s N_c \gg 1$.

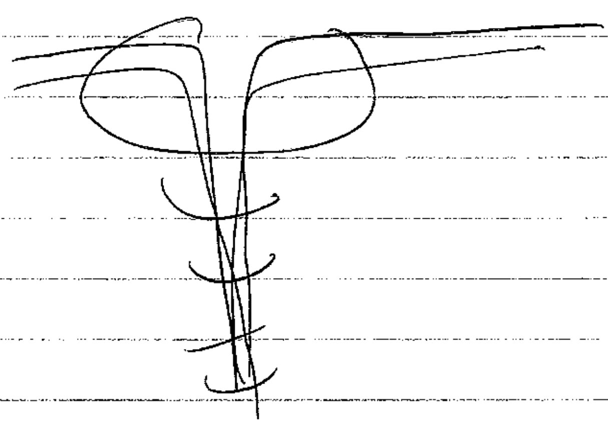
$g_s N_c \ll 1$:

+ 10-dim Mink

$\mathcal{N}=4$ SYM
$SU(N_c)$
$3+1$

$g_s N_c \gg 1$

$N_c \gg 1$



+ 10-dim Mink

Important remark: both open and closed str. descriptions are in principle valid for any value of $g_s N_c$, N_c but we don't know how to extend them for all values of $g_s N_c$, N_c.

I : Open str. $\lambda \ll 1$

II : Closed str. $\lambda \gg 1$, $N_c \gg 1$

$\lambda = g_{YM}^2 N_c$ (recall $g_s = 4\pi g_{YM}^2$)

<u>Conjecture</u> (Maldacena, 1997) :

I and II describe the same object.

$\mathcal{N}=4$ $SU(N_c)$ SYM in $d=3+1$ $=$ type IIB superstr. theory on $AdS_5 \times S^5$.

<u>Remark</u> : $\mathcal{N}=4$ SYM is a highly constrained theory, some quantities are the same for weak and strong coupling (e.g. scaling dim. of some

operators). One may expect that computing those quantities in pictures I, II, one gets the same result. This is indeed the case, see I. Klebanov, „World volume approach to absorption ~~by~~ by non-dilatonic branes",
hep-th/9702076.

Remark: we have weak-strong duality, with relevant d.o.f. (very different ones!) used at weak (strong) coupling
(compare with Thirring – SG, electr.-magn. duality, QCD)

Remark:
the duality is holographic: one side involves grav. d.o.f. (closed str.) and a theory without grav. d.o.f. (in $d=3+1$) is equivalent (conjecture!)
to theory of (quantum) grav. in $d=10$ ($d=5(+5)$ with internal dim. of S^5).
This explicitly realizes the hologr. principle

of 't Hooft and Susskind.

Sanity check: symmetries

$\mathcal{N}=4$ SYM in $d=3+1$: conformal $SO(4,2)$ + $SO(6)_R$ R-symmetry

isometries of $AdS_5 \times S^5$: $SO(4,2) \times SO(6)$

Also, $\mathcal{N}=4$ SYM has $SL(2,\mathbb{Z})$ duality group ($g_{YM} \to 4\pi/g_{YM}$), type IIB str. theory has $SL(2,\mathbb{Z})$ duality as well.

Declaration:

$$Z_{\substack{\mathcal{N}=4 \\ SYM \\ d=4}}[J] = Z_{\substack{\text{type IIB} \\ \text{string} \\ AdS_5 \times S^5}}[J]$$

Some immediate consequences: <u>entropy of $\mathcal{N}=4$ SYM</u>