Elements of String theory

1. Strings and Branes

String theory is a quantum theory of interacting, relativistic one- and higher-dimensional objects.

Fundamental parameter:

$$T = \frac{1}{2\pi\alpha'} \qquad \alpha' = l_s^2$$

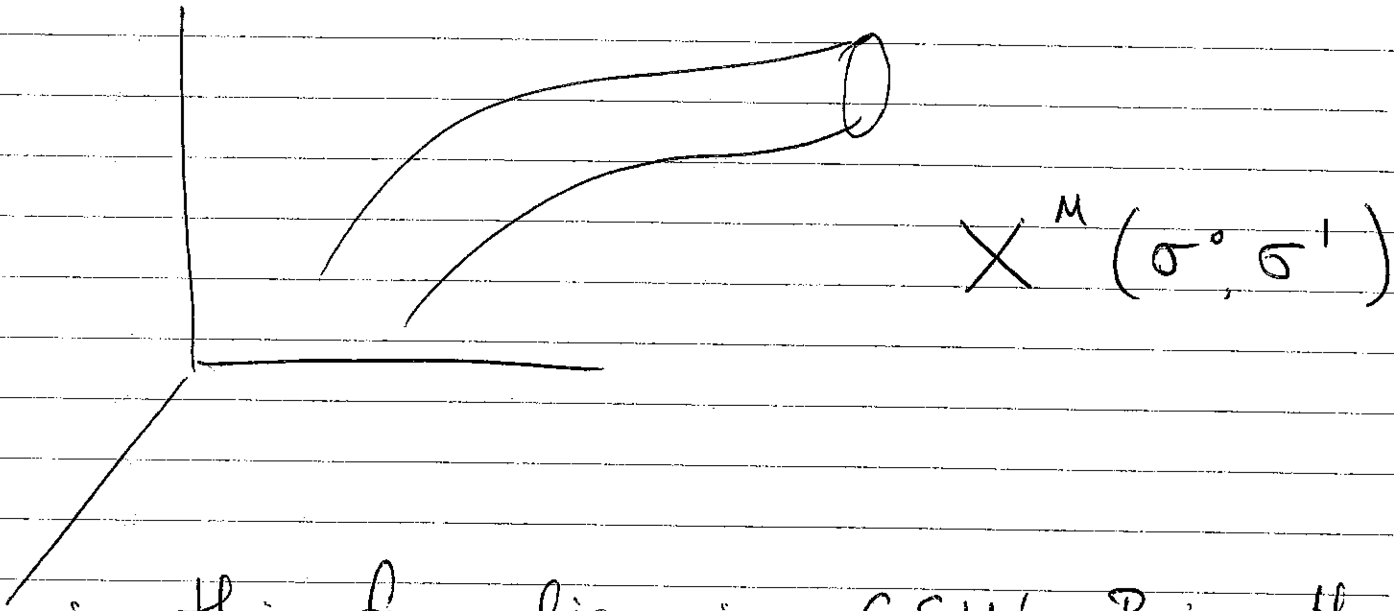
$$S = -T\int d^2\sigma\sqrt{-\det g}, \quad \text{where}$$

$$g_{\alpha\beta} = G_{MN}\frac{\partial X^M}{\partial\sigma^\alpha}\frac{\partial X^N}{\partial\sigma^\beta} \qquad \underline{\text{induced metric}}$$

σ^α world-sheet coord. $\alpha = 0, 1$

X^M ; $M = 0, \ldots D-1$

<u>Remark</u>: we are considering strings in a first-quantized formalism (see treatment of a point particle and supersymm. particle



in this formalism in GSW, Brink-Hennaux and other sources). There exists a formalism known as string field theory.

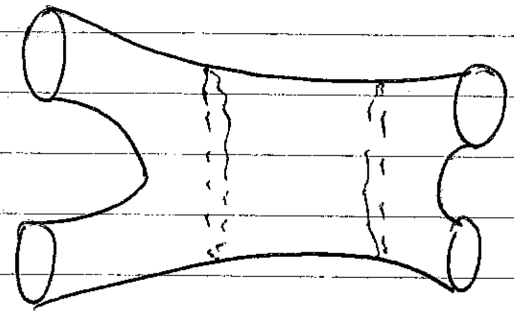
Quantizing the string action (in practice, an equiv. Polyakov action and its SUSY extension is used) one finds that

1) $\exists$ finite number of massless modes and an infinite tower of massive modes $\sim m_s \sim l_s^{-1}$

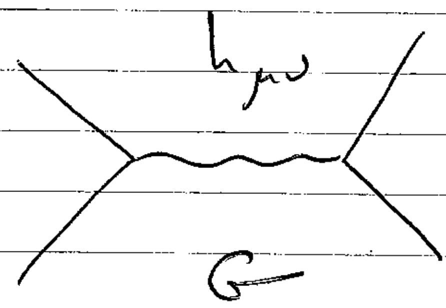
Depending on the b.c., one finds 5 self-consistent theories: type IIA, type IIB, type I, Heterotic $E_8 \times E_8$, Heterotic $SO(32)$: limits of M-th.

2) Absence of negative norm states require $D=10$.

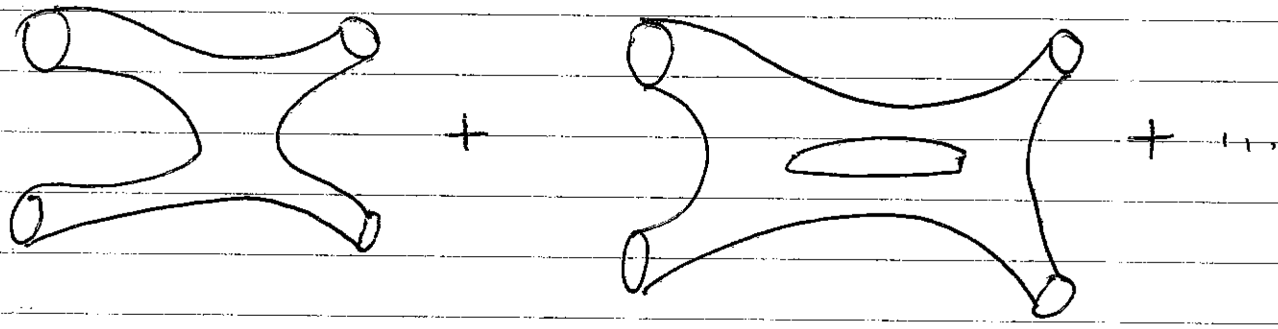
String interactions controlled by g_s



g_s $\quad$ g_s



$$16\pi G_{10} = (2\pi)^7 g_s^2 \ell_s^8$$



$$g_s^{2h-2} \text{ factor}$$

(h = number of holes)

g_s is not a free parameter in str. theory, $g_s = e^{\Phi}$, where Φ is the expectation value of the dilaton, one of the massless modes in the string spectrum. For us, $g_s = e^{\Phi_\infty}$ (asymptotic value).

Effectively, therefore, we have two parameters:

$$l_s \quad \text{and} \quad g_s$$

Approximating gauge-string duality by gauge-gravity duality, we have to keep in mind the conditions

$$L \gg l_s \, , \qquad g_s \ll 1 \, ,$$

(in particular, dilaton should not be too large), where L is the characteristic size of our geometry.

<u>Branes</u> (Polchinski, 1995) enter as non-perturbative (in g_s) higher-dimensional objects with tension (mass per unit volume)

$$T_{Dp} \sim 1/g_s$$

Dp branes are „topological defects" with p+1-dim. worldvolume on which open strings can end (and move freely along the worldvol.)

Generically, all these d.o.f. (open, closed strings and various Branes) are excited.
However, at low energy $E \ll m_s \sim 1/l_s$ only the lowest states are of interest.
Consider type IIB (closed) str. th. massless spectrum:

$g_{\mu\nu}$	graviton
$\phi, \quad C$	dilaton, axion
$B_{\mu\nu}, \quad A_{\mu\nu}$	rank 2 antisymm
$A^{+}_{\mu\nu\lambda\sigma}$	rank 4 antisymm (self-dual)
$\psi^{I=1,2}_{\mu,\alpha}$	Majorana-Weyl gravitini
$\lambda^{I=1,2}_{\alpha}$	Maj-Weyl dilatini

This is $\mathcal{N}=2$ SUSY $d=10$ supergravity!
Preserving symmetries of a string action

$$S_{Polyakov} = -\frac{1}{4\pi\alpha'}\int d^2x \sqrt{-h}\; h^{ab}\,\partial_a X^{\mu}_{(x)}\,\partial_b X^{\nu}_{(x)}\, g_{\mu\nu}(X) + \ldots$$

at quantum level leads to certain conditions (β - functionals equal to zero) such as:

$$\beta_{\mu\nu}(X^\rho) = R_{\mu\nu} + \frac{1}{4} H_\mu^{\ \lambda\sigma} H_{\nu\lambda\sigma} - 2 D_\mu D_\nu \phi + O(\alpha') = 0$$

on low-energy modes.

Here $H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\rho B_{\mu\nu} + \partial_\nu B_{\rho\mu}$ is a generalization of $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

More precisely, type IIB low-energy e.o.m.

$$\boxed{\begin{aligned} R_{\mu\nu} = {} & \frac{1}{2} \partial_\mu \phi \partial_\nu \phi + \frac{1}{4} e^{-\phi} \left(H_{\mu\alpha\beta} H_\nu^{\ \alpha\beta} - \frac{1}{2} g_{\mu\nu} H^2 \right) \\ & + e^{2\phi} \frac{1}{2} \partial_\mu C \partial_\nu C \\ & + e^{\phi} \frac{1}{4} \left(\tilde{F}_{\mu\lambda\sigma} F_\nu^{\ \lambda\sigma} - \frac{1}{2} g_{\mu\nu} \tilde{F}^2_{(3)} \right) + \frac{1}{96} \tilde{F}_{\mu\lambda\rho\sigma\varkappa} F_\nu^{\ \lambda\rho\sigma\varkappa} \end{aligned}}$$

$$\boxed{\nabla^2 \phi = e^{2\phi} \partial_\mu C \partial^\mu C - \frac{1}{12} e^{-\phi} H_3^2 + \frac{1}{12} e^{\phi} \tilde{F}_3^2}$$

$$\boxed{\partial_\mu \left(\sqrt{-g}\, g^{\mu\nu} e^{2\phi} \partial_\nu C \right) = -\frac{1}{6} e^{\phi} H_{\mu\nu\sigma} \tilde{F}^{\mu\nu\sigma}}$$

$$d * \tilde{F}_5 = H_3 \wedge F_3$$

$$d * \left(e^{\phi} \tilde{F}_3\right) = \tilde{F}_5 \wedge H_3$$

$$d * \left(C \tilde{F}_3 e^{\phi} - H_3 e^{-\phi} \right) = \tilde{F}_3 \wedge F_3$$

$$\tilde{F}_5 = * \tilde{F}_5$$

$$\left.\begin{aligned} d\tilde{F}_3 &= - dC \wedge H_3 \\ d\tilde{F}_5 &= H_3 \wedge F_3 \end{aligned}\right\} \text{Bianchi}$$

Here $\tilde{F}_3 = F_3 - C H_3$, $F_3 = dA_2$

$H_3 = dB_2$

$$\tilde{F}_5 = F_5 - \frac{1}{2} A_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3$$

$$F_p^2 \equiv F_{M_1 \dots M_p} F^{M_1 \dots M_p}$$

For $\Phi = \frac{1}{p!} \phi_{i_1 \dots i_p} dx^{i_1} \wedge \dots \wedge dx^{i_p}$

$$*\Phi = \frac{1}{(d-p)!} \frac{1}{p!} \varepsilon_{i_1 \dots i_d} \sqrt{|g|} \, \phi_{j_1 \dots j_p} g^{i_1 j_1} \dots g^{i_p j_p} dx^{i_{p+1}} \wedge \dots \wedge dx^{i_d}$$

$$\boxed{\frac{L}{l_s} = \left(g_{YM}^2 N_c\right)^{1/4} , \quad g_s = \frac{g_{YM}^2}{4\pi}}$$