Topics in Gauge-Gravity Duality - II

Lecture 2

Black hole thermodynamics

In Lecture 1 we had seen that for Schwarzschild BH in d-dim. asympt. flat space-time

$$dM = \frac{d-3}{16\pi G_d r_0} \, dA ,$$

where M is the ADM mass, A is the horizon area of the BH. Formally, this can be written as

$$dM = \frac{d-3}{4\pi r_0} \, d\left(\frac{A}{4G_d}\right)$$

i.e. the thermodynamics First law

$$dE = T\,dS$$

with $T = \frac{d-3}{4\pi r_0} \frac{\hbar c}{k_B}$, $S = \frac{k_B c^3 A}{4 G_d \hbar}$

For $d=4$ Schwarzschild BH:

$$T = \frac{1}{4\pi r_0}\frac{\hbar c}{k_B} = \frac{\hbar c^3}{k_B}\frac{1}{8\pi G M}$$

$$S = \frac{k_B}{\hbar c} 4\pi G M^2 \quad \left(A = 4\pi r_0^2 , \quad r_0 = \frac{2GM}{c^2} \right)$$

$$T dS = d(Mc^2) = dE .$$

Note that since $l_p^{d-2} = G_d \hbar / c^3$,

$$S = \frac{k_B A_d}{4 \, l_p^{d-2}} ,$$

i.e. S/k_B is dimensionless as it should.

<u>Four laws of BH mechanics</u>

(Bardeen, Carter, Hawking, 1973)

In $d=4$, a stationary asym. flat BH is uniquely characterized by its mass M, angular momentum J and charge Q

<u>Note</u>: $\exists$ exotic exceptions in $d=4$ and less exotic ones in $d>4$

This is similar to Tδ equilibrium.

<u>The four laws are</u>:

0. The surface gravity $\varkappa$ is constant over the event horizon.

1. $$\delta M = \frac{\varkappa}{8\pi G}\,\delta A + \Omega_H\,\delta J + \Phi_H\,\delta Q,$$

where Ω_H is the angular velocity and Φ_H is the electric potential at the horizon.

2. $\delta A \geq 0$ (the area of the event hor. of a BH never decreases)

3. It is impossible by any procedure to reduce the surface gravity $\varkappa$ to zero in a finite number of steps.

<u>Remark 1</u>: surface gravity $\sim$ local proper acceleration times the grav. redshift

The surface gravity $\varkappa$ of a static Killing horizon is the acceleration (as measured at spatial infinity) necessary to keep an object at the horizon. For a Killing vector K^μ,

$$K^\mu \nabla_\mu K^\nu = \varkappa K^\nu ,$$

with K^μ normalized as $K^\mu K_\mu \to -1$ as $r \to \infty$ in asympt. flat space-time.

<u>Exercise</u>: compute $\varkappa$ for Schwarzschild BH in d dim

<u>Remark 2</u>: for theories more general than the Ei-H gravity, e.g. with

$$S = \int d^d x \sqrt{-g}\, \mathcal{L}(g_{\mu\nu}, R_{\mu\nu\lambda\sigma}, \nabla_\rho R_{\mu\nu\lambda\sigma}, \dots \phi, \nabla\phi)$$

the entropy formula was given by R. Wald (Noether charge entropy):

$$S_W = -2\pi \oint_\Sigma \frac{\delta \mathcal{L}}{\delta R_{abcd}}\, d\Sigma^{abcd}$$

It reduces to S_{BH} for $\mathcal{L} = R$.

Remark 3: there are several ways to derive Hawking radiation. See e.g. Carlip 0807.4520 [gr-qc] for a review.

Lessons: BH behave as TD objects with

$$k_B T_H = \frac{\hbar \varkappa}{2\pi}$$

$$S_{BH}/k_B = \frac{c^3 A}{4 G \hbar}$$

A microscopic theory (quantum gravity) is supposed to account for BH TD. Partial success in counting the BH states has been achieved in string theory (Strominger-Vafa, 1995, see Sen 0708.1270 [hep-th]) and other approaches to QG (Carlip, 2009).

Remark: Hawking rad. reduces BH mass
$\Rightarrow$ area decreases

Generalized 2nd law:

$$S_{TOT} = S_{BH} + S$$

$$d S_{TOT} \geq 0$$

Remark: for non-stationary processes involving grav, identification of S with A is less certain. On the other hand, S is a TD quantity and may not be well defined far from equilibrium.

Remark: more exotic views on quantum nature of BH: the fuzzball picture of S. Mathur, quantum hair of G. Dvali. See also Kerr-CFT correspondence (e.g. J. Simon).