Cosmology

Alan
AQM

① Homogeneous and isotropic Universe:

$$ds^2 = -c^2 dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$$

$k = 0$: flat (Euclidean) 3d space

$k = \pm 1$: curved 3d space
spherical
hyperbolic

(r here dimensionless, $[a] = L$).

② Modeling matter / rhs of Es eqs by perfect (non-visc.) fluid

$$T_{\mu\nu} = \frac{P}{c^2} g_{\mu\nu} + \left(\rho + \frac{P}{c^2}\right) u_\mu u_\nu$$

pressure   energy density

Eqs:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

give :

$$\frac{\dot{a}^2}{a^2} \equiv H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{c^2\Lambda}{3}$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + P/c^2\right) = 0$$

For $a(t)$, $\rho(t)$, $P(t)$. Also, need eos: $P = P(\rho)$, e.g.

$P = \rho c^2/3$ for rad. or $P = 0$ (dust)

Parametrized as $P = w\rho c^2$.

($w = 1/3$ rad, $w = 0$ dust)

W/o Λ:

$$\begin{cases} H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} \\ \dot{\rho} + 3H(1+w)\rho = 0 \end{cases}$$ for a, ρ

$$\Rightarrow \quad 1 = \frac{8\pi G}{3H^2}\rho - \frac{kc^2}{a^2H^2}$$

or $$\Omega - 1 = \frac{kc^2}{a^2H^2}$$

$$\Omega \equiv \frac{8\pi G}{3H^2}\rho \qquad \Omega = 1 : \text{flat Universe}$$

Second eq:

$$\dot{\rho} a + 3\rho \dot{a}(1+w) = 0$$

$$\Rightarrow \frac{d}{dt}\left(\rho a^{3(1+w)}\right) = 0$$

$$\Rightarrow \rho = \rho_0 / a^{3(1+w)}$$

$$H^2 = \frac{8\pi G}{3}\frac{\rho_0}{a^{3+3w}} - \frac{kc^2}{a^2}$$: Note:

$$a^2H^2 = \frac{8\pi G}{3}\frac{\rho_0}{a^{3w+1}} - kc^2$$

$$\Omega - 1 = \frac{kc^2}{\frac{8\pi G}{3}\rho_0 a^{-1-3w} - kc^2}$$

3

Another question: $a(t)$ parametrized by W.

Explore $a(t)$ for various W.

Expands forever? $a(t \to \infty) \to \infty$

BB? Big Crunch? Big Rip?

$a(t_{in}) = 0$ $\quad$ $a(t_f) = 0$ $\quad$ $a(t_f) \to \infty$.

4 Horizons: particle hor.
event hor.

Particle hor. of a given observer: all points emitting light since the beginning of time

$$d_H = c\,a(t_0) \int_0^{t_0} \frac{dt}{a(t)}$$

Event horizon: all pts the signal from

which will be able to reach us $\underbrace{}_{\text{located at } r=0 \text{ at } t=t_*}$ during the remaining life-time of the Universe.

$$d_{EH} = c\, a(t_*) \int_{t_*}^{T_{fin}} \frac{dt}{a(t)}$$

5 Deceleration parameter

$$\boxed{q = -\frac{a\ddot{a}}{\dot{a}^2}}$$

Recall Es eq:

$$2a\ddot{a} + \dot{a}^2 + Kc^2 - c^2\Lambda a^2 = -\frac{8\pi G}{c^2} P a^2$$

$\Rightarrow$

$$q = \frac{1}{2}\left[\frac{8\pi G}{3H^2}\left(\frac{3P}{c^2} + \rho\right) - \frac{2}{3}\frac{c^2\Lambda}{H^2}\right]$$

$P = w\rho c^2 \Rightarrow$

6

$$q(t) = \frac{1}{2}\frac{8\pi G\rho}{3H^2}(1+3w) - \frac{\Lambda c^2}{3H^2}$$

$$q_0 = \frac{1}{2}\sum_i \Omega_i (1+3w_i) - \Omega_\Lambda$$

$$\Omega_\Lambda \equiv \frac{\Lambda c^2}{3H_0^2}.$$

6 $$\frac{\dot{a}^2}{a^2} = H_0^2\left(\frac{\Omega_{rad}}{a^4} + \frac{\Omega_M}{a^3} + \frac{\Omega_k}{a^2}\right)$$

$$\sum_i \Omega_i = 1$$

$$\downarrow$$

$$\frac{\mu\dot{a}^2}{2} + V(a) = E = 0.$$