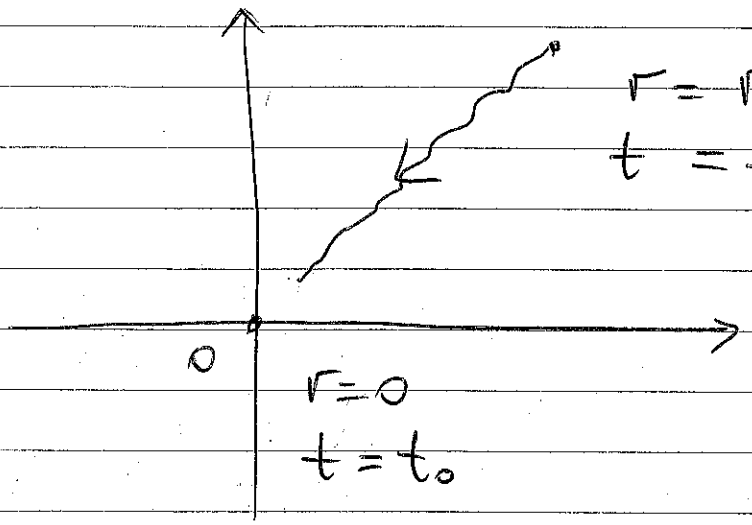
Cosmological redshift

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}$$



$$c\int_{t_1}^{t_0}\frac{dt}{a} = \int_0^{r_1}\frac{dx}{\sqrt{1-kx^2}}$$

Second pulse:

$$c\int_{t_1+\delta t_1}^{t_0+\delta t_0}\frac{dt}{a} = \int_0^{r_1}\frac{dx}{\sqrt{1-kx^2}}$$

$$\Rightarrow \int_{t_1+\delta t_1}^{t_0+\delta t_0}\frac{dt}{a} = \int_{t_1}^{t_0}\frac{dt}{a}$$

$$\Rightarrow \int_{t_0}^{t_0+\delta t_0}\frac{dt}{a} = \int_{t_1}^{t_1+\delta t_1}\frac{dt}{a}$$

$$\delta t_1 = T_1 = 1/\nu_{em}$$

$$\delta t_0 = T_0 = 1/\nu_{obs} \qquad a \sim \text{const}$$

$$\Rightarrow \quad \frac{\delta t_0}{a(t_0)} = \frac{\delta t_1}{a(t_1)}$$

$$\Rightarrow \quad \frac{\nu_{obs}}{\nu_{em}} = \frac{a(t_{em})}{a(t_{obs})}$$

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} = \frac{a(t_{obs})}{a(t_{em})} - 1$$

$$z + 1 = \frac{1}{a} \qquad \text{with} \quad a(t_0) = 1.$$