Gauge-String duality

AdS/CFT corresp $\subset$ Gauge-str. duality $\subset$ Holography

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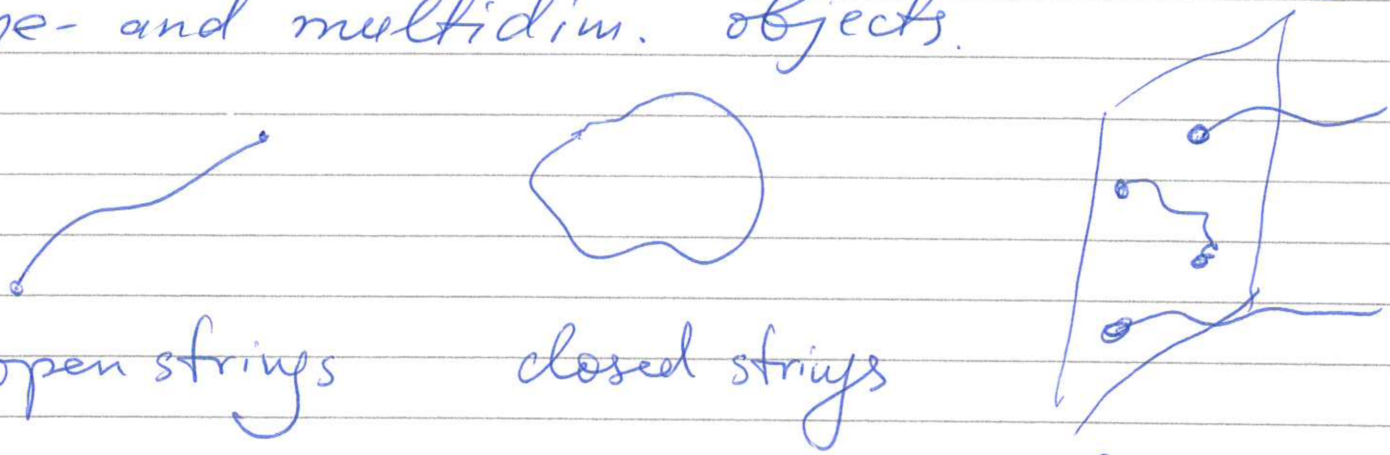
Gauge-grav. duality

String theory - quantum theory of interacting one- and multidim. objects.



open strings closed strings

branes

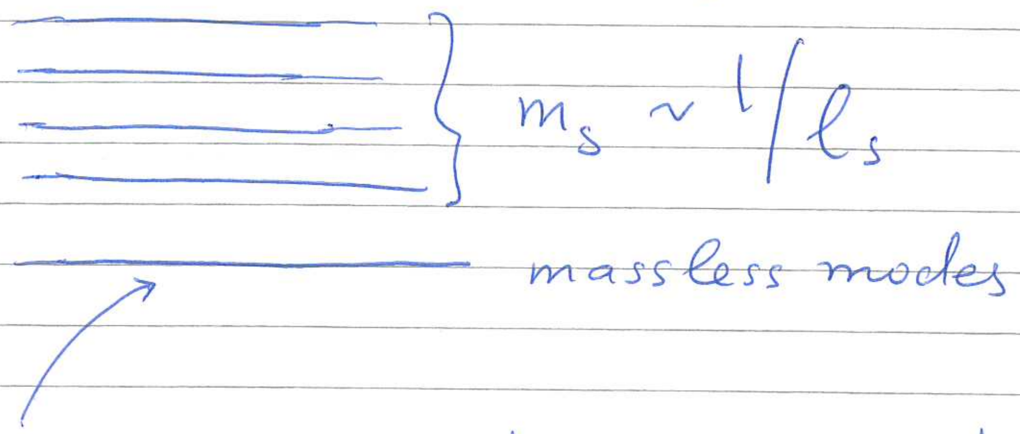
$$S = -T \int d^2\sigma \sqrt{-\det g}$$ Nambu-Goto action

Compare: $S = -m \int ds$ for particle

Here $T = \frac{1}{2\pi\alpha'}$, $\alpha' = l_s^2$.

Self-consistency $\Rightarrow$ $d = 10$ dim.

- Spectrum of closed strings:



$g_{\mu\nu}$, φ, C, $B_{\mu\nu}$, $C^{+}_{\mu\nu\lambda\sigma}$, $\psi_{\alpha,\alpha}$, λ_α

- Closed str. low-energy e.o.m. ($E \ll 1/\ell_s$) can be derived – they are e.o.m. for these massless modes, e.g.

$$R_{\mu\nu} = \frac{1}{2}\partial_\mu \phi \, \partial_\nu \phi + \dots + \frac{1}{96}\tilde{F}_{\mu\lambda\rho\sigma\varkappa}\tilde{F}_\nu^{\ \lambda\rho\sigma\varkappa}$$

etc.

RR fields

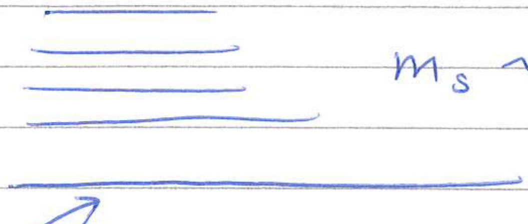
Solution: black 3-brane

$$ds^2_{10} = H^{-1/2}(r)\left[-f\,dt^2 + dx^2 + dy^2 + dz^2\right] + H^{1/2}(r)\left[\frac{dr^2}{f} + r^2 d\Omega_5^2\right]$$

$$H = 1 + L^4/r^4 \qquad f = 1 - r_0^4/r^4$$

L is related to mass (charge), r_0 to T_H.

- Open string spectrum



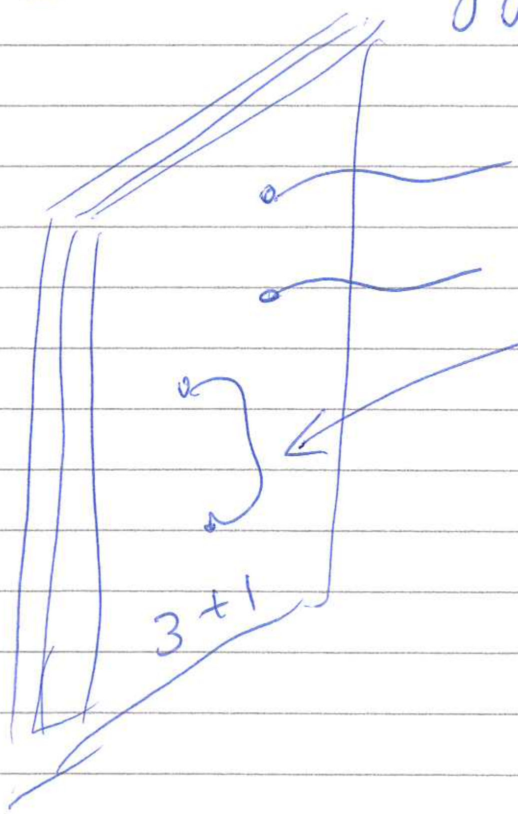
$m_s \sim 1/\ell_s$

$m = 0$

$A_\mu, \ \phi_I, \ \lambda^a_\alpha$

$I = 1, \dots 6$
$\alpha = 1, 2$
$a = 1, \dots 4$

- <u>At low energy</u> $(E \ll 1/\ell_s)$:

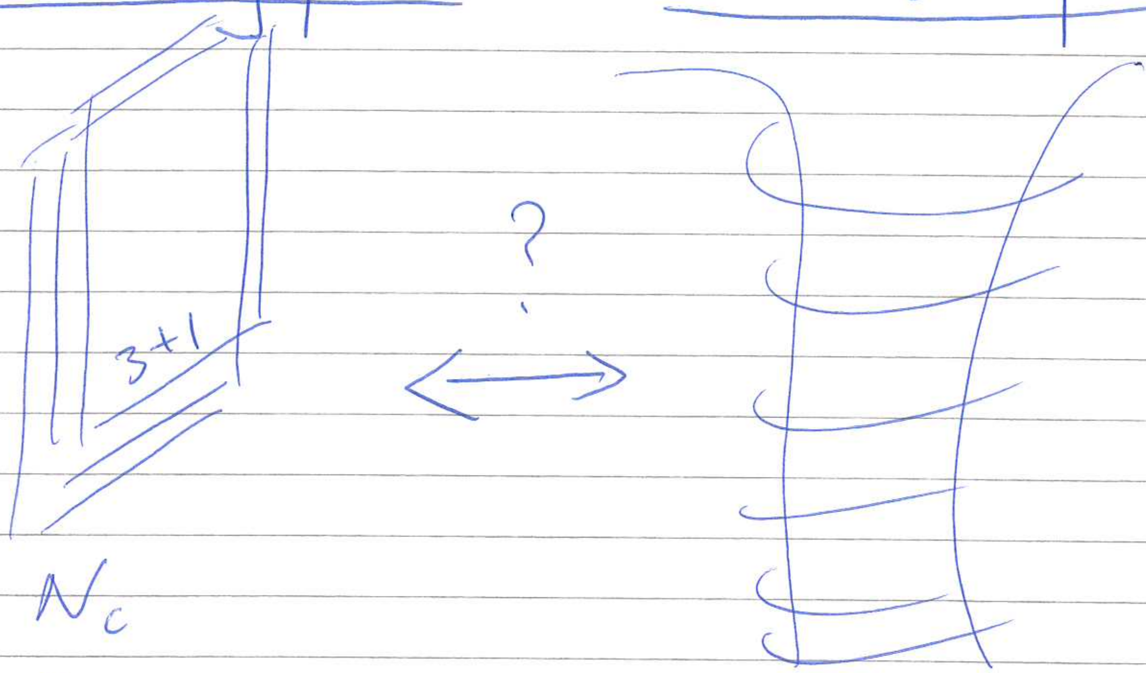


N_c branes
D3

$$\mathcal{L}_{YM} = -\frac{1}{g_{YM}^2} \operatorname{tr}\Big(\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} D_\mu \phi^I D^\mu \phi^I + [\phi^I, \phi^J]^2\Big) + \text{fermions} + O(E \ell_s)$$

$SU(N_c)$ Yang-Mills theory $(\lambda \equiv g_{YM}^2 N_c)$

Open string picture | Closed str. picture



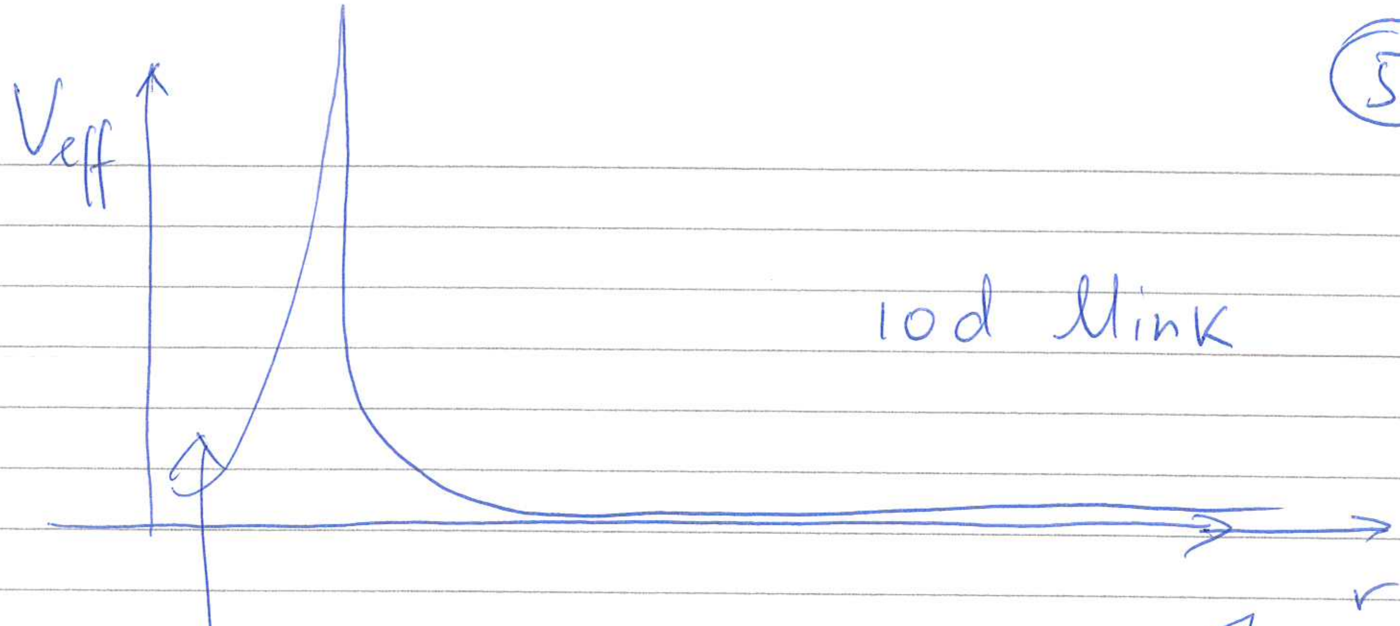
N_c	
D3 Branes	3+1 dim horizon
YM $SU(N_c)$	L
g_{YM}	l_s, g_s
10 dim	10 dim

Is it the same object?

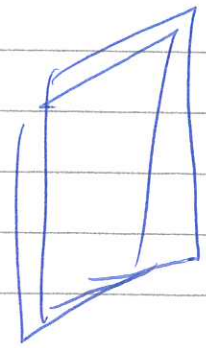
Do "scattering experiments" to check.

I. Klebanov: hep-th/9702076

Scatter e.g. $h_{\mu\nu}$ in 2 pictures



near-horiz. region of ds^2_{10} $= AdS_5 \times S^5$ $\Longleftrightarrow$



YM in $d=3+1$

$\downarrow$ $\quad\quad$ $\downarrow$

Symm. $SO(4,2)$ $\quad\quad$ $SO(6) \simeq SU(4)$

This YM theory is superconformal:

$\beta_{YM} = 0$

Symmetry: $SO(4,2)$

$SU(4)_R$

$$\begin{cases} \dfrac{L^4}{\ell_s^4} = g_{YM}^2 N_c = \lambda & \text{t'Hooft coupling} \\[2ex] g_s = g_{YM}^2 / 4\pi \end{cases}$$

$$Z_{YM}[J] = \tilde{Z}[\tilde{J}]$$

$d=3+1$ | IIB str. theory on $AdS_5 \times S^5$ | Maldacena, 1997

$N_c \to \infty$ .... $\frac{L}{l_p} \to \infty$

$\lambda \to \infty$ .... $L/l_s \to \infty$

$$Z_{YM}[J] = e^{-S_{grav}[\tilde{J}]} + \dots$$

in $N \to \infty, \lambda \to \infty$ limit

(gauge — gravity duality)

Near-horizon metric ($u = r_0^2/r^2$):

$$ds_{10}^2 = \underline{\underline{ds_5^2}} \oplus L^2 d\Omega_5^2$$

$$ds_5^2 = \frac{(\pi T L)^2}{u}\left(-f dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{L^2}{4 f u^2} du^2$$

$$f = 1 - u^2$$

This solution = thermal state of YM at $\beta = 1/T$

(in $N_c \to \infty$, $\lambda \to \infty$ limit)

1) Entropy density: $s = S/V_3 = \frac{1}{4} A_H \Rightarrow$

$$s_{\lambda=\infty} = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s_{\lambda=0}.$$

2) Correlators: $S \to S + \int J \hat{O}$

$\Rightarrow \langle \hat{O} \rangle$, $\langle \hat{O} \hat{O} \rangle$ etc

Here: $g^{BG}_{\mu\nu} \rightarrow S^{BG}_{grav} \equiv S_{grav}[g^{BG}_{\mu\nu}]$

$g^{BG}_{\mu\nu} + h_{\mu\nu} \rightarrow S^{\cancel{BG}}_{grav}[g^{BG}_{\mu\nu} + h_{\mu\nu}]$

$$Z_{YM}[J] = e^{-S_{grav}(g^{BG}_{\mu\nu} + h_{\mu\nu}(h^{0}_{\mu\nu}))}$$

$$J = \frac{1}{2} h^{0\,\nu}_{\mu\alpha} = \frac{1}{2} h^{\nu}_{\mu\alpha}\Big|_{\partial(AdS_5)}$$

$$S_{grav} = \frac{\pi^3 L^5}{2\varkappa_{10}^2}\left[\int_0^1 du\, d^4x \sqrt{-g}\,(R - 2\Lambda) + \right.$$

$$\left. + 2\int d^4x \sqrt{-h}\, K + a \int d^4x \sqrt{-h}\,\right]$$

Gibbons-Hawking term

$a = -6/L$

renormalisation

$\varkappa_{10} = 2\pi^2 \sqrt{\pi}\, L^4 / N_c$

$S_{grav}[h^{0}_{\mu\nu}]$ on shell:

$$S_{grav} = \frac{\pi^2 N_c^2 T^4}{8} \int d^4x \left[-1 + \frac{1}{2}\left(3H^{0}_{tt} + H^{0}_{xx} + H^{0}_{yy} + H^{0}_{zz}\right) + O(H_0^2) \right]$$

$$\langle T^{00} \rangle_T = 2 \frac{\delta S_{grav}}{\delta H^{0}_{tt}} = \frac{3}{8}\pi^2 N_c^2 T^4 \equiv \epsilon = \frac{3}{4}\epsilon_{\lambda=0}$$

$$\langle T^{zz} \rangle_T = \frac{1}{8}\pi^2 N_c^2 T^4 \equiv P = \frac{3}{4} P_{\lambda=0}$$

$\Rightarrow$ eos : $\epsilon = 3P$

as expected for conformal theory:

$$\begin{pmatrix} \epsilon & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix} \Rightarrow T^{\mu}_{\ \mu} = 0 \Rightarrow \epsilon - 3p = 0$$

Now compute $\langle T_{xy}(\omega,q)\, T_{xy}(-\omega,-q)\rangle_R$.

Need solution $H_{xy} \equiv h^x_y\,(h^x_{y\,0})$ of

e.o.m. $Z_3 \equiv H_{xy}$ (gauge-inv. var)

$$Z_3'' - \frac{1+u^2}{uf} Z_3' + \frac{\bar{\omega}^2 - \bar{q}^2 f}{u f^2} Z_3 = 0$$

$$\bar{\omega} \equiv \omega/2\pi T \quad , \quad \bar{q} = q/2\pi T$$

$$S_{grav} = -\frac{\pi^2 N_c^2 T^4}{8} \lim_{u\to 0} \int \frac{d\omega\, dq}{(2\pi)^2} \frac{f(u)}{u} Z_3'(k) Z_3(-k)$$

$$G^R_{T_{xy}T_{xy}} = -\frac{\pi^2 N_c^2 T^4}{2} \frac{B(\omega,q)}{A(\omega,q)} ,$$

from $Z_3 = A(1+\cdots) + B u^2 (1+\cdots)$.

$$G^R_{T_{xy}T_{xy}}(\omega,q) = \frac{\pi^2 N_c^2}{8} T^4 - i \frac{\pi N_c^2 T^3}{8} \omega + O(\omega^2, q^2)$$

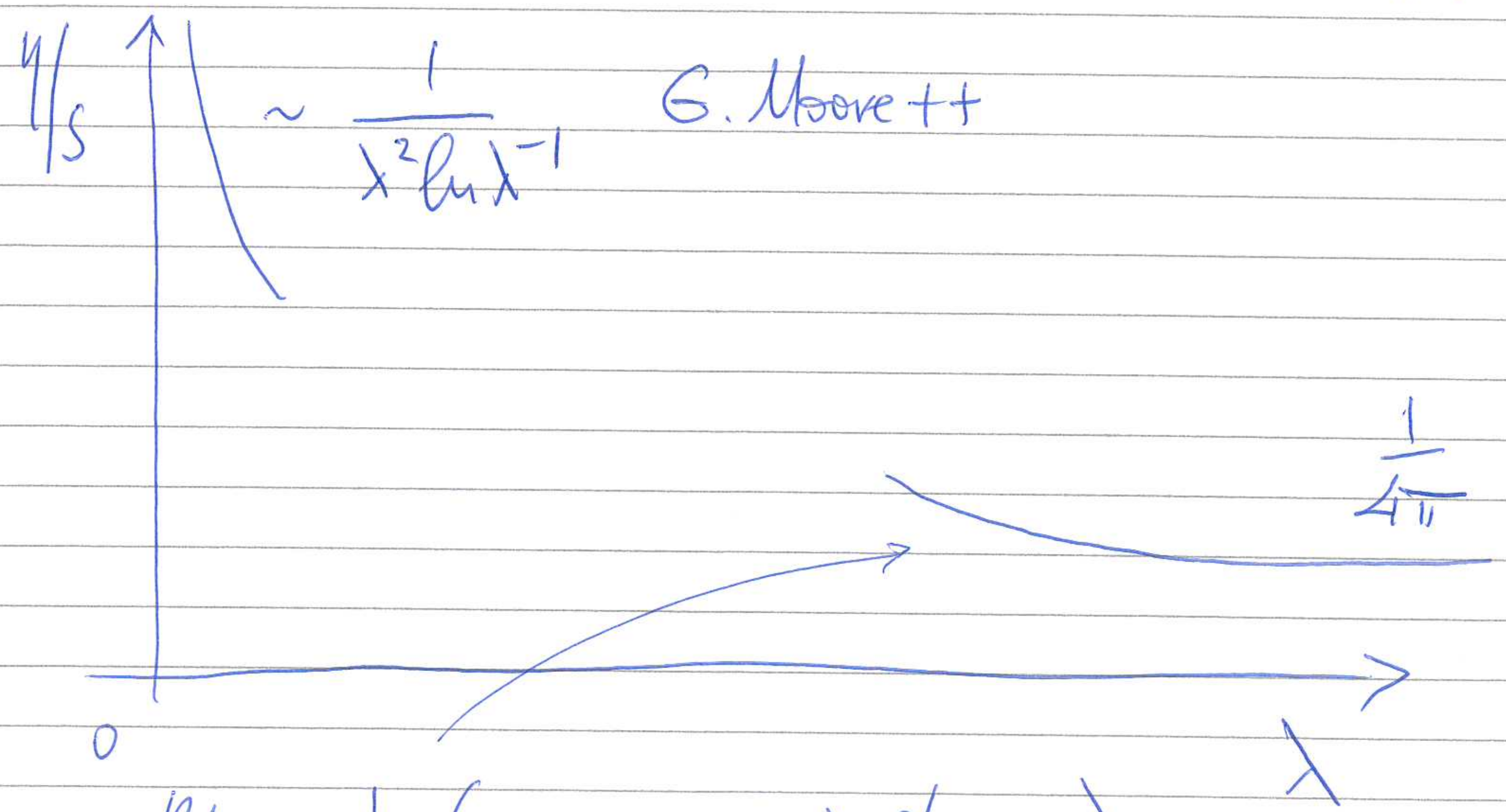
Compare with $G^R_{T_{xy}T_{xy}}$ from linear response:

$$G^R_{T_{xy}T_{xy}}(\omega, 0) = P - i\omega\eta + O(\omega^2)$$

$$\Rightarrow P = \frac{1}{8}\pi^2 N_c^2 T^4 \qquad \checkmark \text{ OK}$$

$$\Rightarrow \eta = \frac{\pi N_c^2 T^3}{8} \qquad \left(s = \frac{\pi^2}{2} N_c^2 T^3\right)$$

Note: $\eta/s = 1/4\pi$. $\left(\text{or } \eta/s = \frac{\hbar}{4\pi k_B}\right)$.



$$\eta/s = \frac{1}{4\pi}\left(1 + 15\zeta(3)\lambda^{-3/2} + \dots\right)$$