Important remark: the structure of $\mathcal{L}$ and scaling dimensions of fields

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \underbrace{\frac{m^2}{2}\phi^2 + O(\phi^3)}_{V(\phi)}$$

kinetic term
$\Rightarrow$ 2nd order eom

e.g. $(\partial_\mu\phi\partial^\mu\phi)^2 \Rightarrow$
higher der. terms

coupling „constants"

$$V(\phi) = \frac{m^2}{2}\phi^2 + \sum_{n\geq 3}\frac{\lambda_n}{n!}\phi^n$$

Dimension of ϕ: the action S is dimensionless in units $\hbar = 1$ (recall $\sim e^{iS/\hbar}$)

$$S \sim \int d^Dx\; m^2\phi^2 \quad (\text{in } D\text{-dim})$$

$$[m] = 1/L \Rightarrow L^D L^{-2}[\phi]^2 = 1$$

$$\Rightarrow [\phi] = L^{1-D/2} = M^{D/2-1}$$

In $D=4$: $[\phi] = M = 1/L$. $\boxed{\text{(Mass) dim. one.}}$

$$[V] = L^{-D} = M^{D}$$

$$\Rightarrow \quad [\lambda_n]\, L^{n - nD/2} = L^{-D}$$

$$\Rightarrow \quad [\lambda_n] = L^{nD/2 - D - n} = M^{D + n - nD/2}$$

In $D = 4$: $\quad [\lambda_n] = L^{n-4} = M^{4-n}$

The coupling λ_n with $n = 4$ $\left(n_* = \frac{D}{D/2 - 1}\right)$ is dimensionless.

For a given term $\lambda_n \phi^n$, its contribution

$$\mathcal{L} \sim m^2 \phi^2 + \lambda_n \phi^n$$

$$\mathcal{L} \sim M^D + \lambda_n M^{\frac{Dn}{2} - n}$$

For a solution (process) with $E \sim M$

$$\mathcal{L} \sim E^D \left(1 + \lambda_n E^{\frac{nD}{2} - n - D} \right)$$

Let $\quad \lambda_n = \bar{\lambda}_n m_*^{D + n - nD/2}$,

where m_* is a scale associated with ϕ^n (possibly a fund. scale of the theory)

Then:

$$\mathcal{L} \sim E^D \left(1 + \bar{\lambda}_n \left(\frac{E}{m_*} \right)^{\frac{n-n_*}{2}} \right)$$

List of n_*

$D = 1+1$	∞
$D = 2+1$	6
$D = 3+1$	4
$D = 4+1$	10/3
$D = 5+1$	3
$D \to \infty$	2

1) $n > n_*$ $\begin{cases} E \gg m_* & \text{term } \lambda_n \phi^n \text{ contributes} \\ E \ll m_* & \underline{\text{irrelevant}} \end{cases}$

2) $n < n_*$ $\begin{cases} E \gg m_* & \text{can be ignored} \\ E \ll m_* & \underline{\text{relevant}} \end{cases}$

3) $n = n_*$ contrib. of $\lambda_n \phi^n$ is E-independent (marginal)

E.g. in $D=4$: ϕ^2, ϕ^3 are relevant, ϕ^4 marginal, ϕ^5, ϕ^6 etc irrelevant (at low E).

<u>Remark:</u> similar analysis can be done for other fields (A_μ, ψ etc)

<u>Remark:</u> classical scaling dimensions can be modified by quantum corrections

<u>Remark:</u> for low-energy physics, $E \ll m_*$, need only relevant and marginal terms. Presence of irrelev. terms in QFT makes the theory „non-renormalizable" $\Rightarrow$ need „UV completion" (e.g. - electroweak theory is a UV completion w.r.t. Fermi theory of weak interactions, with $m_* \sim m_W$).

Ex 4) Massive vector field (Proca field)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu$$

e.o.m. $\quad \partial_\mu F^{\mu\nu} + m^2 A^\nu = 0$ (Proca)

Note: $A_\mu A^\mu$ term is not gauge-invar.
($A_\mu \to A_\mu - \partial_\mu f$ is not a symm.)

Ex 5) Complex scalar field (free)

$$\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi - m^2 \varphi^* \varphi$$

e.o.m. $$\begin{cases} \partial_\mu \partial^\mu \varphi + m^2 \varphi = 0 \\ \partial_\mu \partial^\mu \varphi^* + m^2 \varphi^* = 0 \end{cases}$$

Can introduce $\varphi = \frac{1}{\sqrt{2}} (\varphi_1 + i \varphi_2)$

$$\Rightarrow \mathcal{L} = \sum_{i=1,2} \left(\frac{1}{2} \partial_\mu \varphi_i \partial^\mu \varphi^i - \frac{m^2}{2} \varphi_i \varphi_i \right)$$

New element: can construct conserved current

$$j_\mu = -i \left(\varphi^* \partial_\mu \varphi - \varphi \partial_\mu \varphi^* \right)$$

with $\partial_\mu j^\mu = 0$ <u>on shell</u> (on e.o.m.)

(exercise - check this).

With $Q = \int j^0 d^3x$ we have

$$\partial_0 Q = \int \partial_0 j^0 d^3x = - \int d^3x \, \partial_i j^i =$$

$$= - \int_\Sigma d\Sigma_i j^i = 0$$ for φ vanishing suffic. fast at Σ at infinity.

$$\Rightarrow \dot{Q} = 0$$