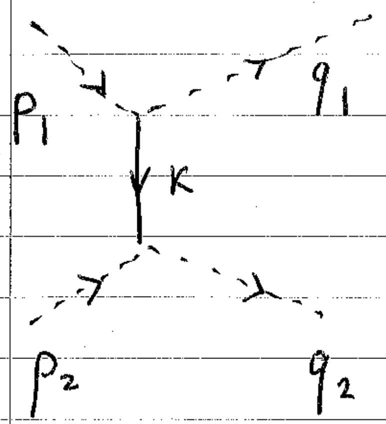
$$T_2 = (-ig)^2 (2\pi)^4 \int d^4k \, \frac{i}{k^2 - m^2 + i\varepsilon} \times$$

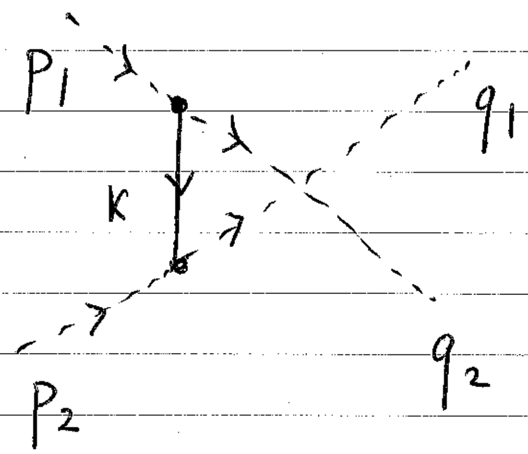
$$\times \Big[\delta^{(4)}(q_1 - p_1 + k)\, \delta^{(4)}(q_2 - p_2 - k) +$$

$$+ \delta^{(4)}(q_2 - p_1 + k)\, \delta^{(4)}(q_1 - p_2 - k) \Big] =$$

$$= i(-ig)^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - q_1 - q_2) \times$$

$$\times \left[\frac{1}{(p_1 - q_1)^2 - m^2 + i\varepsilon} + \frac{1}{(p_1 - q_2)^2 - m^2 + i\varepsilon} \right].$$





$p_1 = k + q_1$ $\qquad\qquad$ $p_1 = k + q_2$

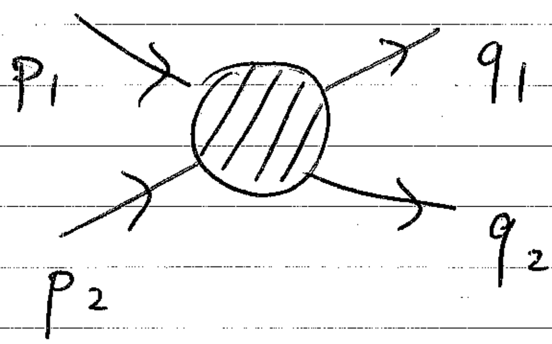
$p_2 + k = q_2$ $\qquad\qquad$ $p_2 + k = q_1$

$$S = \mathbb{1} + iT$$

$$\langle f | S - \mathbb{1} | i \rangle = i \langle f | T | i \rangle \equiv$$

$$\equiv i\,(2\pi)^4 \delta(p_f - p_i)\, \mathcal{M}(i \to f)$$

Mandelstam variables:



$$s = (p_1 + p_2)^2 = (q_1 + q_2)^2$$

$$t = (p_1 - q_1)^2 = (p_2 - q_2)^2$$

$$u = (p_1 - q_2)^2 = (p_2 - q_1)^2$$

$$s + t + u = \sum_{i=1}^{4} m_i^2$$

Our example involved t and u channels.

s, t, u can be expressed via E_{CMF} and scattering angles $\Rightarrow$ phys. data.

General scheme:

$$\langle p_1 \cdots p_n | \hat{S} | k_1 \cdots k_m \rangle \ - ?$$

With $\hat{S} = \mathbb{1} + i\hat{T}$,

$$\langle p_1 \cdots p_n | i\hat{T} | k_1 \cdots k_m \rangle \ - ?$$

with no initial and final momenta coinciding (no particle is a spectator).

LSZ (Lehmann - Symanzik - Zimmermann) reduction formula

$$\langle p_1 \cdots p_n | i\hat{T} | k_1 \cdots k_m \rangle =$$

$$= \left(i Z^{-1/2}\right)^{n+m} \int d^4x_1 \cdots d^4x_m \int d^4y_1 \cdots d^4y_n \ \times$$

$$\times \, e^{ip_1 y_1 + \cdots ip_n y_n - ik_1 x_1 - \cdots - ik_m x_m}$$

$$\times \left(\Box_{x_1} + m^2\right) \cdots \left(\Box_{y_n} + m^2\right) \langle 0 | T\left(\phi(x_1) \cdots \phi(y_n)\right) | 0 \rangle$$

Here: $\phi(x) \longrightarrow Z^{1/2} \phi_{in}(x) \qquad t \to -\infty$

$\phi(x) \to Z^{1/2} \phi_{out}(x) \qquad t \to +\infty$

Z is known as wave-function renormalization.
(See e.g. Maggiore „A modern Intro to QFT", § 5.2.)

=> Attention shifts to N-point correlation functions

$$ {}_H\langle 0 | T \phi_H(x_1) \cdots \phi_H(x_N) | 0 \rangle_H \equiv G^{(N)}(x_1 \cdots x_N) $$

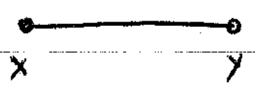
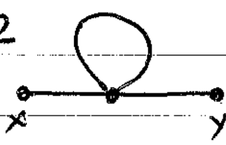
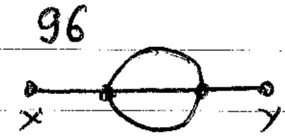
(time-ordered products of <u>interacting</u> fields)

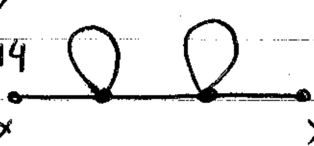
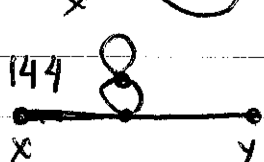
$$ {}_H\langle 0 | T \phi_H(x_1) \cdots \phi_H(x_N) | 0 \rangle_H = $$

$$ = \frac{\langle 0 | T \phi_{ip}(x_1) \cdots \phi_{ip}(x_N)\, e^{-i\int d^4x\, \mathcal{H}^{ip}_{int}} | 0 \rangle}{\langle 0 | T e^{-i\int d^4x\, \mathcal{H}^{ip}_{int}} | 0 \rangle} $$

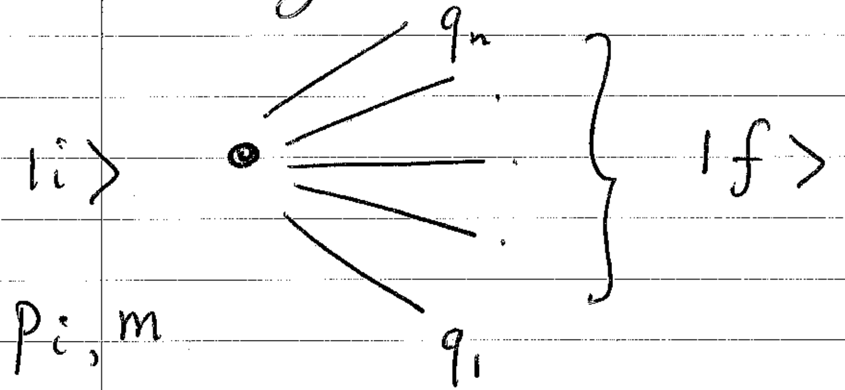
Gell-Mann and Low, 1951

E.g. in $\lambda\phi^4$ theory:

$G^{(2)}(x,y) =$  $+ \; 12$  $+ \; 96$  $+$

$+ \; 144$  $+ \; 144$  $+ \, O(\lambda^3)$

Decay rates



$$p_f = \sum_{j=1}^{n} q_j$$

$$P_n = \frac{|\langle f|\hat{S}|i\rangle|^2}{\langle i|i\rangle\langle f|f\rangle}$$

Recall: $\langle i|j\rangle = (2\pi)^3\, 2\omega_{p_i}\, \delta^{(3)}(p_i - p_j)$

$\langle i|i\rangle = \cancel{(2\pi)^3}\, 2\omega_{p_i}\, V$, since $(2\pi)^3 \delta^{(3)}(0) = V$.

$$\langle f|f\rangle = \prod_{j=1}^{n} 2\omega_{q_j} V$$

$$\langle f|\hat{S} - \mathbb{1}|i\rangle = \langle f|i\hat{T}|i\rangle =$$

$$= i\,(2\pi)^4 \delta^{(4)}(p_f - p_i)\, \mathcal{M}(i \to f)$$

$$P_n = \frac{(2\pi)^4 \delta^{(4)}(p_f - p_i)\, V T\, |\mathcal{M}_{fi}|^2}{2\omega_{p_i} V \prod_{j=1}^{n} 2\omega_{q_j} V}$$