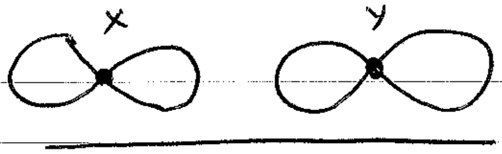
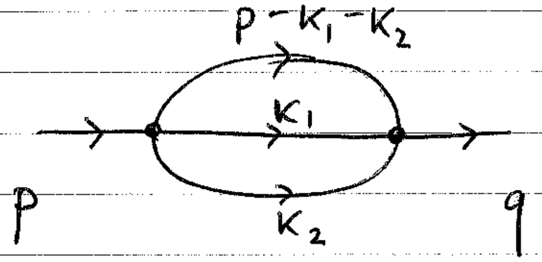


– with appropriate symmetry factors.

Explicitly:



$$\Rightarrow (2\pi)^4 \delta^{(4)}(p-q) \frac{(-i\lambda)^2}{6} \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \times$$

$$\times \frac{i}{k_1^2 - m^2 + i\varepsilon} \frac{i}{k_2^2 - m^2 + i\varepsilon} \frac{i}{(p-k_1-k_2)^2 - m^2 + i\varepsilon}.$$

Feynman rules for $\lambda\phi^4$ theory
(in momentum space)

- Each <u>vertex</u> gives a factor $-i\lambda$
- Each <u>internal line</u> gives a propagator

$$\frac{i}{k^2 - m^2 + i\varepsilon}$$

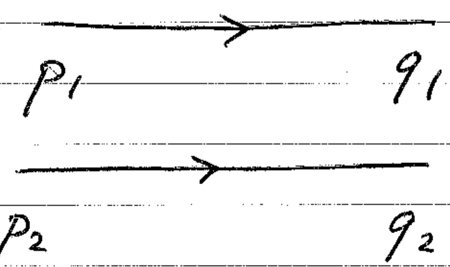
- At each vertex, total momentum is conserved
- Integrate over internal momenta $\int \frac{d^4 k}{(2\pi)^4}$
- Include symmetry factors
- Include overall $(2\pi)^4 \delta^{(4)}(p_f - p_i)$ factor

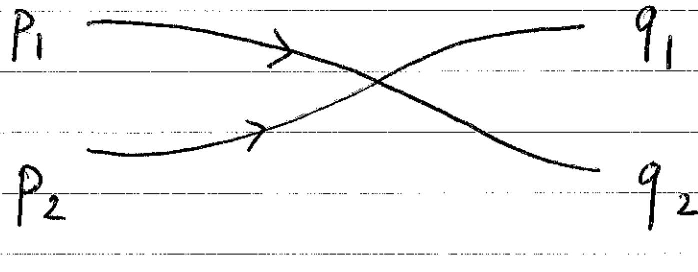
Scattering: 2 → 2

Consider now $S_{fi} = \langle q_1 q_2 | \hat{S} | p_2 p_1 \rangle$

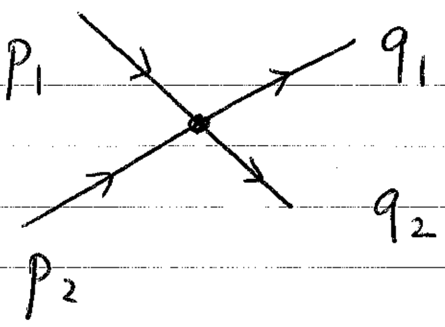
$= \langle 0 | a_{q_1} a_{q_2} \hat{S} a^\dagger_{p_2} a^\dagger_{p_1} | 0 \rangle$

λ^0:

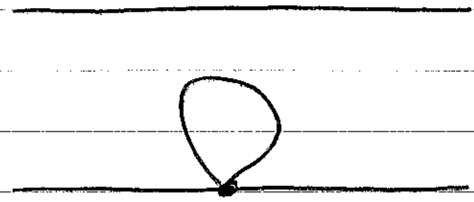


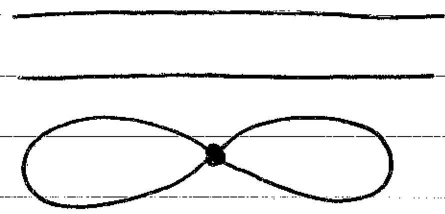


λ :



(*)





$+ \; O(\lambda^2)$.

Connected diagram (*)

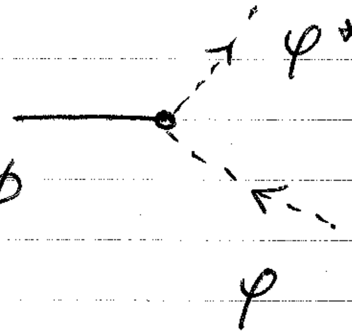
$\Rightarrow (2\pi)^4 \delta^{(4)}(q_1 + q_2 - p_1 - p_2)(-i\lambda)$

$\Rightarrow$ lowest order trans. prob. $\sim |S_{fi}|^2 \sim \lambda^2$.

In $\lambda\phi^4$ theory, expect scattering cross-section $\sim \lambda^2$, $\lambda \ll 1$.

Example: scalar Yukawa theory

$$\mathcal{H}_I(x) = g\, \varphi^*(x)\, \varphi(x)\, \phi(x)$$



Scattering $2 \to 2$: $NN \to NN$

$$|i\rangle = a_+^\dagger(p_1)\, a_+^\dagger(p_2)\, |0\rangle = |p_1, p_2\rangle$$

$$|f\rangle = a_+^\dagger(q_1)\, a_+^\dagger(q_2)\, |0\rangle = |q_1, q_2\rangle$$

Note: can also consider $\bar{N}N \to \bar{N}N$ or $\bar{N}\bar{N} \to \bar{N}\bar{N}$ (charge is conserved)

Amplitudes with $NN \to \bar{N}\bar{N}$ etc do not appear.

$O(g)$ contribution vanishes:

$$\langle 0 |\, T\left(a_{q_1}\, a_{q_2}\, \varphi^* \varphi\, \phi\, a_{p_1}^\dagger a_{p_2}^\dagger \right) |0\rangle$$

unpaired $\Rightarrow 0$.

Convenient to consider $S = 1 + iT$

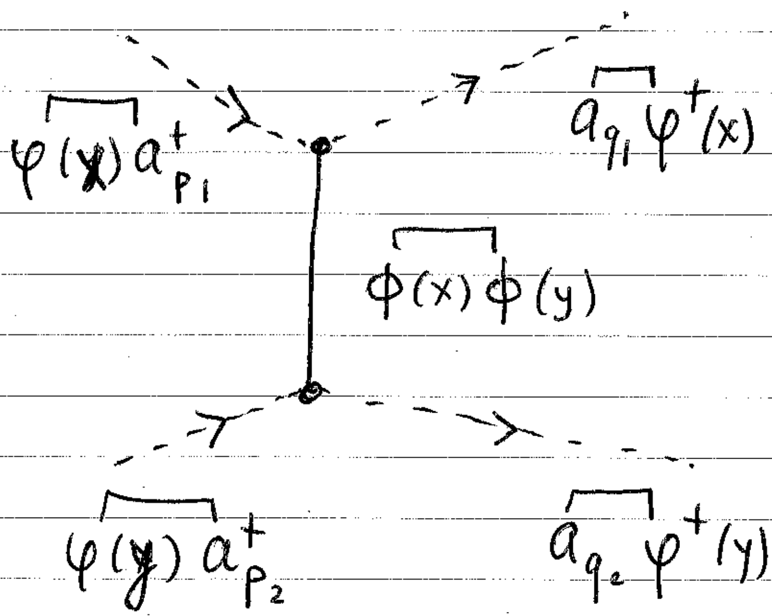
$O(g^2)$ contribution to iT:

$$T_2 = \frac{(-ig)^2}{2}\int d^4x\, d^4y\; T\left(\varphi^\dagger(x)\varphi(x)\phi(x)\varphi^\dagger(y)\varphi(y)\phi(y)\right)$$

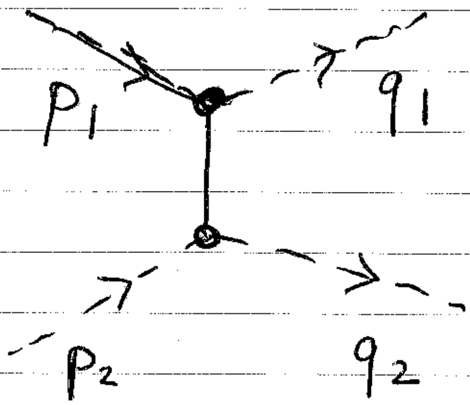
We have:

$$\langle 0|\, T\, a_{q_1} a_{q_2}\, \varphi^\dagger(x)\varphi(x)\phi(x)\varphi^\dagger(y)\varphi(y)\phi(y)\, a^\dagger_{p_2} a^\dagger_{p_1} |0\rangle$$

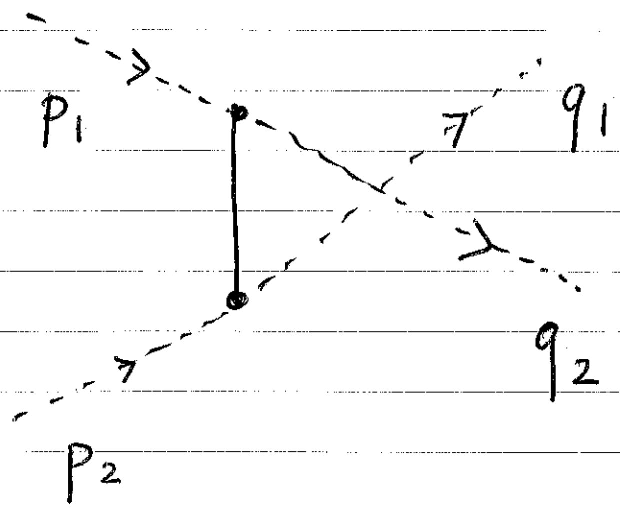
Diagram:



Simpler notation:



Another diagram:



Algebraically:

$$D_F(x-y) :\varphi^+(x)\varphi(x)\varphi^+(y)\varphi(y):$$

$$\langle q_1 q_2 | :\varphi^+(x)\varphi(x)\varphi^+(y)\varphi(y): | p_1 p_2 \rangle$$

$$= \langle q_1 q_2 | \varphi^+(x)\varphi^+(y) | 0 \rangle \langle 0 | \varphi(x)\varphi(y) | p_1 p_2 \rangle$$

$$= e^{i[(q_1-p_1)x + (q_2-p_2)y]} +$$

$$+ e^{i[(q_2-p_1)x + (q_1-p_2)y]} +$$

$$+ e^{i[(q_1-p_1)y + (q_2-p_2)x]} +$$

$$+ e^{i[(q_2-p_1)y + (q_1-p_2)x]}$$

Indeed, recall that

$$\langle 0|\varphi(x)|p\rangle =$$

$$= \langle 0|\int d^3\tilde{q}\left(a_+(q)e^{-iqx} + a_-^+(q)e^{iqx}\right)a_+^+(p)|0\rangle$$

$$= \langle 0|\int d^3\tilde{q}\,[a_+(q)\,a_+^+(p)]\,|0\rangle e^{-iqx} =$$

$$= \langle 0|\int \frac{d^3q}{(2\pi)^3}\frac{1}{2\omega_q}(2\pi)^3 2\omega_p\,\delta^{(3)}(\bar{p}-\bar{q})\,e^{-iqx}|0\rangle$$

$$= e^{-ipx}.$$

$$T_2 = \frac{(-ig)^2}{2}\int d^4x\,d^4y\int\frac{d^4k}{(2\pi)^4}\frac{i\,e^{ik(x-y)}}{k^2-m^2+i\varepsilon}\times$$

$$\times\Big(e^{i[(q_1-p_1)x+(q_2-p_2)y]} + e^{i[(q_2-p_1)x+(q_1-p_2)y]}$$

$$+\{x\leftrightarrow y\}\Big)$$