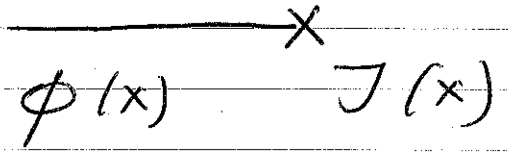
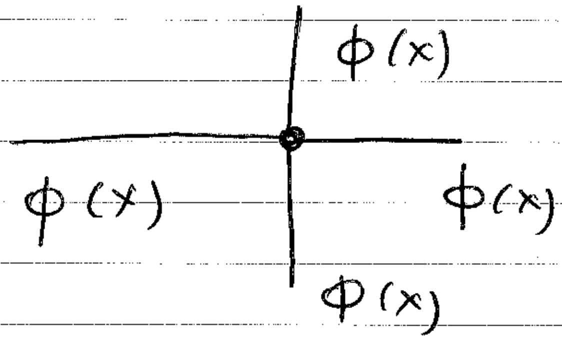
Examples of $\mathcal{H}_I(x)$

- $\mathcal{H}_I(x) = J(x)\,\phi(x)$ (external source)

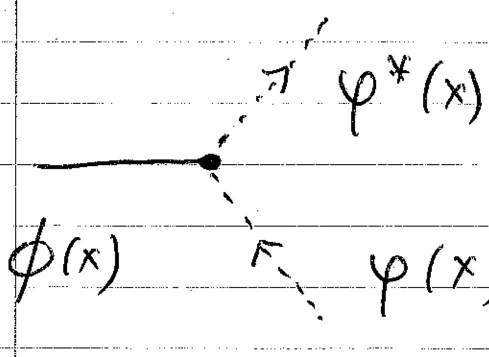


- $\mathcal{H}_I(x) = \frac{\lambda}{4!}\,\phi^4(x)$ $\left(\sim \phi J,\ J \sim \phi^3\right)$



- $\mathcal{H}_I(x) = g\,\varphi^*(x)\,\varphi(x)\,\phi(x)$

$\left(\sim J\phi,\ J \sim \varphi^*\varphi\right)$



Example: $\mathcal{H}_I^{ip} = \frac{\lambda}{4!}\phi_{ip}^4$

Consider $S_{fi} = \langle f | \hat{S} | i \rangle$, where

$$|i\rangle = |p\rangle = a_{\bar{p}}^{+}|0\rangle$$

$$|f\rangle = |q\rangle = a_{\bar{q}}^{+}|0\rangle$$

$$S_{fi} \equiv \mathcal{A} = \langle q | T \exp\left[\frac{-i}{\hbar}\int d^4x\, \mathcal{H}_I^{ip}(x)\right] | p \rangle$$

$$S = T\Big(\mathbb{1} - \frac{i\lambda}{4!}\int d^4x\, \phi_{ip}^4(x) -$$

$$- \frac{1}{2!}\left(\frac{\lambda}{4!}\right)^2 \int d^4x_1\, d^4x_2\ \phi_{ip}^4(x_1)\,\phi_{ip}^4(x_2) + \dots\Big)$$

$$\mathcal{A} = \langle 0 | a_{\bar{q}}\, a_{\bar{p}}^{+} | 0 \rangle - \frac{i\lambda}{4!}\int d^4x \langle 0 | T a_{\bar{q}}\, \phi_{ip}^4(x)\, a_{\bar{p}}^{+} | 0 \rangle$$

$$+ O(\lambda^2)$$

- $\langle 0 | a_{\bar{q}}\, a_{\bar{p}}^{+} | 0 \rangle = \langle q | p \rangle = (2\pi)^3 2\omega_{\bar{p}}\, \delta^{(3)}(\bar{p}-\bar{q})$
- $\langle 0 | T a_{\bar{q}}\, \phi_{ip}(x)\,\phi_{ip}(x)\,\phi_{ip}(x)\,\phi_{ip}(x)\, a_{\bar{p}}^{+} | 0 \rangle$

Wick's theorem $\Rightarrow$ 2 types of contractions

1) $a_q \, \phi \, \phi \, \phi \, \phi \, a_p^+$

$a_q \, \phi \, \phi \, \phi \, \phi \, a_p^+$

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They contribute $3 \langle 0 | a_q a_p^+ | 0 \rangle \langle 0 | T \phi \phi | 0 \rangle$

$\times \langle 0 | T \phi \phi | 0 \rangle$

2) $a_q \, \phi \, \phi \, \phi \, \phi \, a_p^+$ and similar ones

$\Rightarrow 12 \langle 0 | a_q \phi_{in}(x) | 0 \rangle \langle 0 | T \phi_{in}(x) \phi_{in}(x) | 0 \rangle$

$\times \langle 0 | \phi_{in}(x) a_p^+ | 0 \rangle$

• $\langle 0 | \phi_{in}(x) a_p^+ | 0 \rangle =$

$= \langle 0 | \int d^3\tilde{q} \left(a_q e^{-iqx} + a_q^+ e^{iqx} \right) a_p^+ | 0 \rangle =$

$= \int d^3\tilde{q} \, \langle 0 | a_q a_p^+ | 0 \rangle e^{-iqx} = e^{-ipx}.$

Similarly, $\langle 0 | a_q \phi_{in}(x) | 0 \rangle = e^{iqx}$.

Recall also:

$$\mathcal{D}_F(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\varepsilon} e^{-ik(x-y)}$$

$$\delta^{(4)}(x-y) = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)}$$

We now have all ingredients to compute part 2) :

$$\mathcal{A}_1^{(2)} = -\frac{i\lambda}{4!} 12 \int d^4x \, e^{-i(p-q)x} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\varepsilon}$$

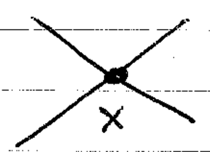
$$= \frac{\lambda}{2} \delta^{(4)}(p-q) \int \frac{d^4k}{k^2 - m^2 + i\varepsilon}$$

What about part 1) ?

But first, we draw Feynman diagrams for $\mathcal{A}_1^{(2)}$!

- draw interaction vertices at $x_1 \dots x_n$

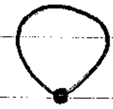
Here we have only one :

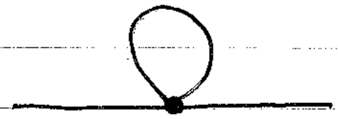


- incoming lines: $\phi(x)\, a_p^+$

outgoing lines: $a_q\, \phi(x)$

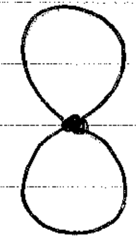
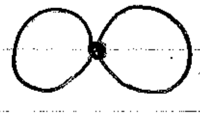
- propagators $D_F(x-y)$ connect vertices

Here $y = x$, so we have 

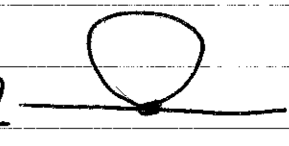
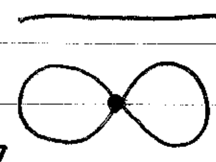
Together: 

Now, part 1) has a piece not related to x: $\langle 0 | a_q\, a_p^+ | 0 \rangle$ represented by

and also x-related piece:

 or 

In total:

$\mathcal{A} =$ $+ 12$  $+ 3$  $+ O(\lambda^2)$

connected / disconnected

vacuum diagram $\langle 0 | S | 0 \rangle$

$$\mathcal{A}_2 = -\frac{\lambda^2}{2!(4!)^2} \langle 0 | \int\!\!\int d^4x \, d^4y \, T\left[a_q \, \phi_{ip}(x) \phi_{ip}(x) \phi_{ip}(x) \phi_{ip}(x) \right.$$

$$\left. \phi_{ip}(y) \phi_{ip}(y) \phi_{ip}(y) \phi_{ip}(y) \, a_p^\dagger \right] | 0 \rangle$$

This gives 6 diagrams: