Similarly, one defines

$$\Delta_A(x-y) = \theta(y^0 - x^0)\,[\phi(x), \phi(y)]$$

and $\bar{\Delta}(x-y) = \frac{1}{2}(\Delta_R + \Delta_A)$.

- D_F, Δ_R, Δ_A and $\bar{\Delta}$ obey the inhomogeneous KG eq. (propagation functions)

$$(\Box_x + m^2)\, G = \delta^{(4)}(x-y).$$

- We can also use $\Delta(x-y)$ Pauli-Jordan

$$\Delta(x-y) = \langle 0 | [\phi(x), \phi(y)] | 0 \rangle$$

$$\begin{cases} \Delta_+(x-y) = \langle 0 | \phi(x)\phi(y) | 0 \rangle \\ \Delta_-(x-y) = \langle 0 | \phi(y)\phi(x) | 0 \rangle \end{cases}$$

Whightman functions

$$\Delta^{(1)}(x-y) = \langle 0 | \{\phi(x)\, \phi(y)\} | 0 \rangle$$

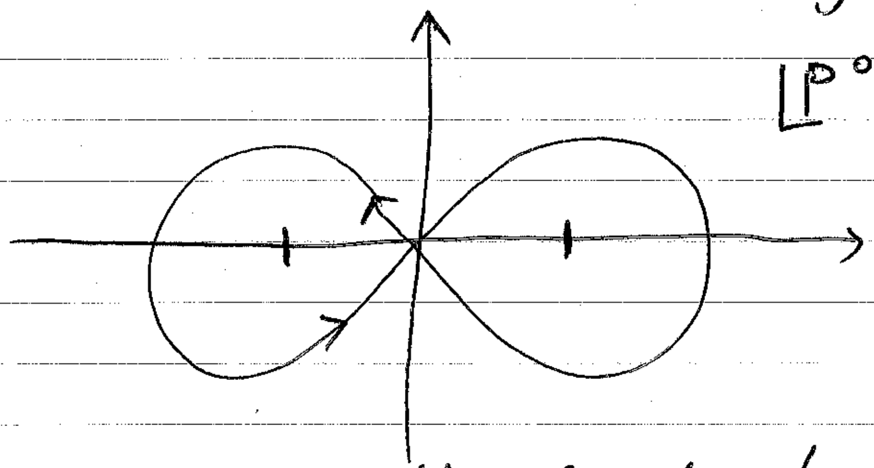
anticommutator

Hadamar function

They can all be obtained from the same integral representation

$$\Delta(x) = -\int_C \frac{d^4p}{(2\pi)^4} \frac{e^{-ipx}}{p^2 - m^2}$$

with various contours, e.g. for $\Delta^{(1)}$



- Δ, $\Delta_\pm$, $\Delta^{(1)}$ obey the homogeneous KG eq. (commutation functions)

$$(\Box_x + m^2)\Delta = 0$$

- All these functions are Lor.-invar.
- There are numerous relations among them (in position or momentum space) $\Rightarrow$ usually enough to know one.
- Can define similar functions for composite operators, e.g.

$$\Delta_R^{\mu\nu}(x-y) = -\theta(x^0-y^0)\left[J^\mu(x), J^\nu(y)\right]$$

Quantum complex scalar field

$$\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi - m^2 \varphi^* \varphi$$

$$(\Box + m^2)\varphi = 0 \qquad (\Box + m^2)\varphi^* = 0$$

$$\pi(x) = \dot{\varphi}^*$$

$$\mathcal{H} = \pi^* \pi + \nabla\varphi^* \nabla\varphi + m^2 \varphi^* \varphi$$

$$\varphi = (\phi_1 + i\phi_2)/\sqrt{2} \qquad 2 \text{ real d.o.f.}$$

$$\varphi \to \hat{\varphi}, \quad \varphi^* \to \hat{\varphi}^+$$

$$[\varphi(t,\bar{x}), \varphi(t,\bar{y})] = 0$$

$$[\varphi^+(t,\bar{x}), \varphi^+(t,\bar{y})] = 0$$

but

$$[\varphi(t,\bar{x}), \pi(t,\bar{y})] = i\delta^{(3)}(\bar{x}-\bar{y})$$

$$[\varphi^+(t,\bar{x}), \pi^+(t,\bar{y})] = i\delta^{(3)}(\bar{x}-\bar{y})$$

Solutions to KG eq.:

$$\varphi(x) = \int d^3\tilde{p} \left(a_+(p) e^{-ipx} + a_-^+(p) e^{ipx} \right)$$

$$\varphi^+(x) = \int d^3\tilde{p} \left(a_- e^{-ipx} + a_+^+ e^{ipx} \right)$$

$$[a_\pm(p), a_\pm^\dagger(q)] = (2\pi)^3 2\omega_p \delta^{(3)}(\bar{p}-\bar{q})$$

So we have 2 types of states:

$$|p,+\rangle = a_+^\dagger |0\rangle$$

$$|p,-\rangle = a_-^\dagger |0\rangle$$

Introduce:

$$N_\pm = \int d^3\tilde{p}\; a_\pm^\dagger a_\pm$$

For a <u>free</u> theory, $[N_\pm, H] = 0$.

We have $U(1)$ symmetry in this theory and a Noether charge ($\Rightarrow$ operator)

$$Q = i\int d^3x \left(\varphi^\dagger \pi^\dagger - \pi\varphi\right)$$

<u>exercise</u>: show that (after normal ord.)

$$Q = \int d^3\tilde{p} \left(a_+^\dagger a_+ - a_-^\dagger a_-\right) = N_+ - N_-$$

$|p,+\rangle$: particles $\qquad [Q,H] = 0$

$|p,-\rangle$: antiparticles $\qquad$ in inter. theory as well.