The XXV International Symposium on Lattice Field Theory

The spectrum of closed loops of fundamental flux in D=2+1 $SU(N)$ gauge theories

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I. Introduction.

General question:

→ What effective string theory describes flux tubes in $SU(N)$ gauge theories?

Two cases:

→ Open string

→ **Closed string** ← in $D = 2 + 1$

During the last decade:

→ 3D, 4D with Z_2, Z_4,$U(1)$, $SU(N \leq 6)$ (Caselle and collaborators, Gliozzi and collaborators, Kuti and collaborators, Lüscher&Weisz, Majumdar and collaborators, Teper and collaborators, Meyer)

Questions to be studied in $D = 2 + 1$ dimensional $SU(N)$ theories:

→ Calculation of excited states, and states with $p_{||} \neq 0$ and $P = \pm$

→ What is the degeneracy pattern of these states?

→ What is the leading correction in E_n^2?

II. Theoretical expectations A.

The Spectrum of the Nambu-Goto (NG) String Model

- Action of Nambu-Goto free bosonic string leads to:
  - → Spectrum given by:

$$E^2_{N_L,N_R,q,w} = (\sigma l w)^2 + 8\pi\sigma\left(\frac{N_L+N_R}{2} - \frac{D-2}{24}\right) + \left(\frac{2\pi q}{l}\right)^2 + p_\perp^2.$$

  - → Described by:
    1. The winding number w (w=1),
    2. The winding momentum $p_{||} = 2\pi q/l$ with $q = 0, \pm 1, \pm 2,$
    3. The transverse momentum $p_\perp$ ($p_\perp = 0$),
    4. N_L and N_R connected through the relation: $N_R - N_L = qw$.

$$N_L = \sum_{k>0} \sum_{n_L(k)>0} n_L(k)k \quad \text{and} \quad N_R = \sum_{k'>0} \sum_{n_R(k')>0} n_R(k')k'$$

  - → String states are eigenvectors of P (In $D = 2 + 1$) with eigenvalues:

$$P = (-1)^{\sum_{i=1}^{m} n_L(k_i) + \sum_{j=1}^{m'} n_R(k'_j)}$$

II. Theoretical expectations B.

Effective string theory

- First prediction for $w = 1$ and $q = 0$ (Lüscher, Symanzik&Weisz. 80):

$$E_n = \sigma l + \frac{4\pi}{l}\left(n - \frac{D-2}{24}\right) + \mathcal{O}\left(1/l^2\right).$$

- Lüscher&Weisz effective string action (Lüscher&Weisz. 04):

$\rightarrow$ Using open-closed string duality:

$*$ For any D the $\mathcal{O}\left(1/l^2\ (1/l)\right)$ (Boundary term) is absent from $E_n(E_n^2)$

$\rightarrow$ Spectrum in $D = 2 + 1$ (Drummond '04, Dass and Matlock '06 for any D.):

$$E_n = \sigma l + \frac{4\pi}{l}\left(n - \frac{1}{24}\right) - \frac{8\pi^2}{\sigma l^3}\left(n - \frac{1}{24}\right)^2 + \mathcal{O}\left(1/l^4\right)$$

$\rightarrow$ Equivalently:

$$E_n^2 = (\sigma l)^2 + 8\pi\sigma\left(n - \frac{1}{24}\right) + \mathcal{O}\left(1/l^3\right),$$

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→ Equivalently:

$$E_n^2 = E_{NG}^2 + \mathcal{O}\left(1/l^3\right) \longrightarrow \text{Fit}: E_{\text{fit}}^2 = E_{NG}^2 - \sigma \frac{C_p}{\left(l\sqrt{\sigma}\right)^p} \quad (p \geq 3)$$

III. Lattice Calculation

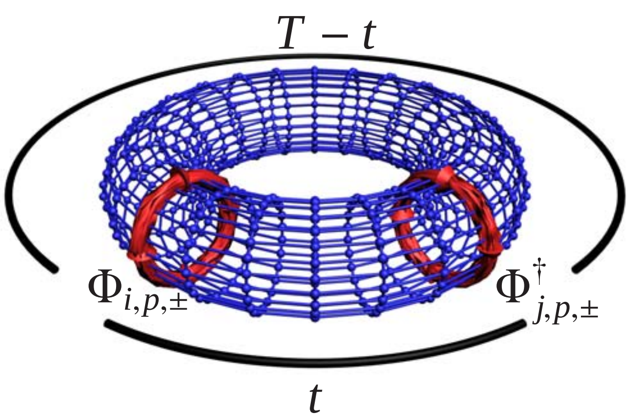
Our approach:

- Construct a large basis of operators (80 – 200) with transverse deformations.
- Calculate the correlation matrix $C_{ij,p,\pm}(t) = \langle \Phi^{\dagger}_{i,p,\pm}(t)\, \Phi_{j,p,\pm}(0) \rangle$.
- Use the variational technique to extract correlators of different states.
- Fit our results using single cosh fits, and look at large t mass plateaus.

Example:

$$\phi_L = \mathrm{Tr}\left\{ \cdots \right\} \text{ and } \phi_R = \mathrm{Tr}\left\{ \cdots \right\}$$

$$\Phi_{p,\pm} = \frac{1}{L_{\parallel} L_{\perp}} \sum_{x_{\parallel}, x_{\perp}} \{\phi_L \pm \phi_R\}\, e^{ipx_{\parallel}}$$



III. Lattice Calculation

Monte-Carlo:

- We define our gauge theory on a 3D discretized periodic Euclidean space-time with $L \times L_\perp \times L_T$ sites.

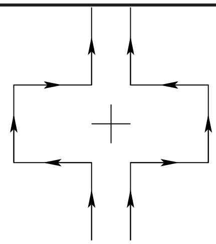
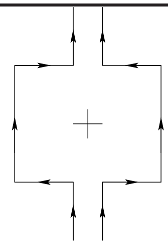
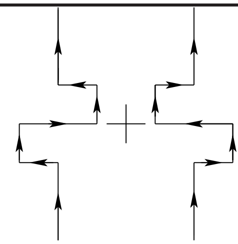
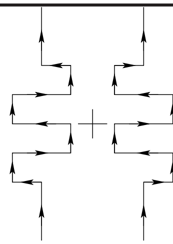
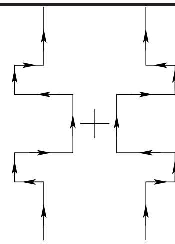
- We choose to use the standard Wilson action:

$$S_{\mathrm{W}} = \beta \sum_P \left[1 - \frac{1}{N} \mathrm{ReTr} U_P \right]$$

with: $\beta = \frac{2N}{ag^2}$.

- Simulation:
  - We use a mixture of Kennedy-Pendelton heat bath and over-relaxation steps for updating $SU(2)$ subgroups of $SU(N)$.
  - We use Cabibbo-Marinary for updating $SU(N)$.

III. Lattice Calculation: Operators for $P = +$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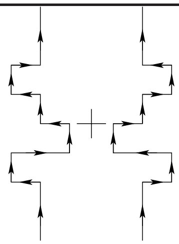	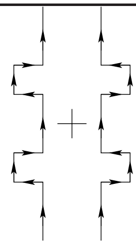	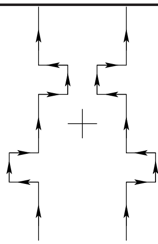	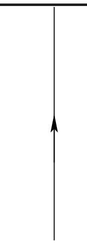	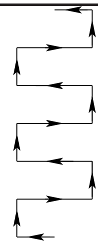
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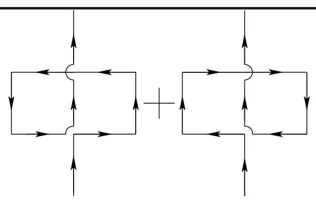	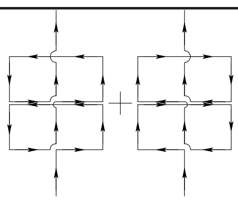	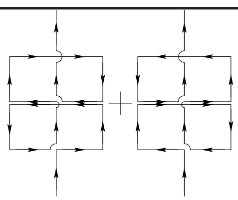	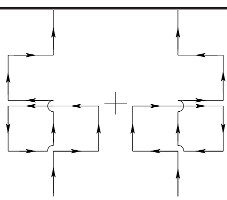	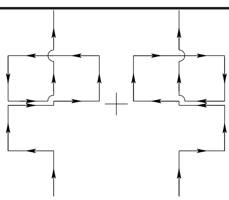
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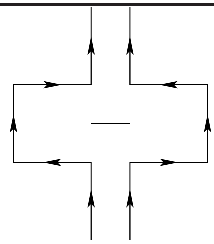
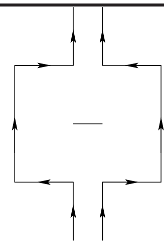
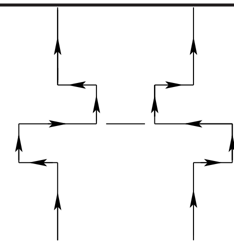
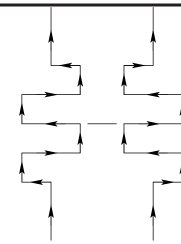
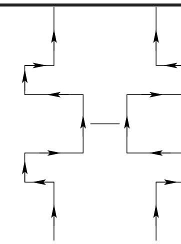
III. Lattice Calculation: Operators for $P = -$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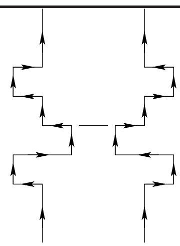	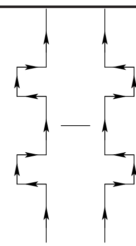	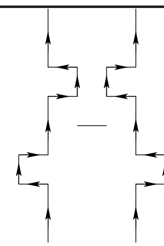		
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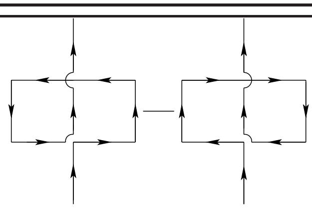	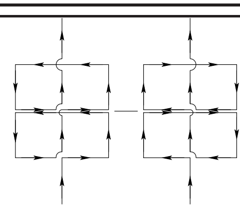	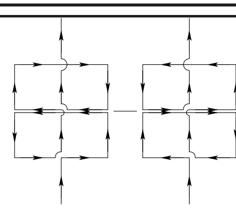	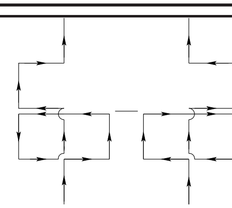	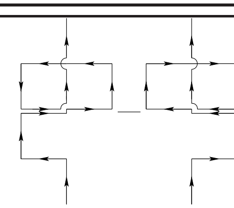
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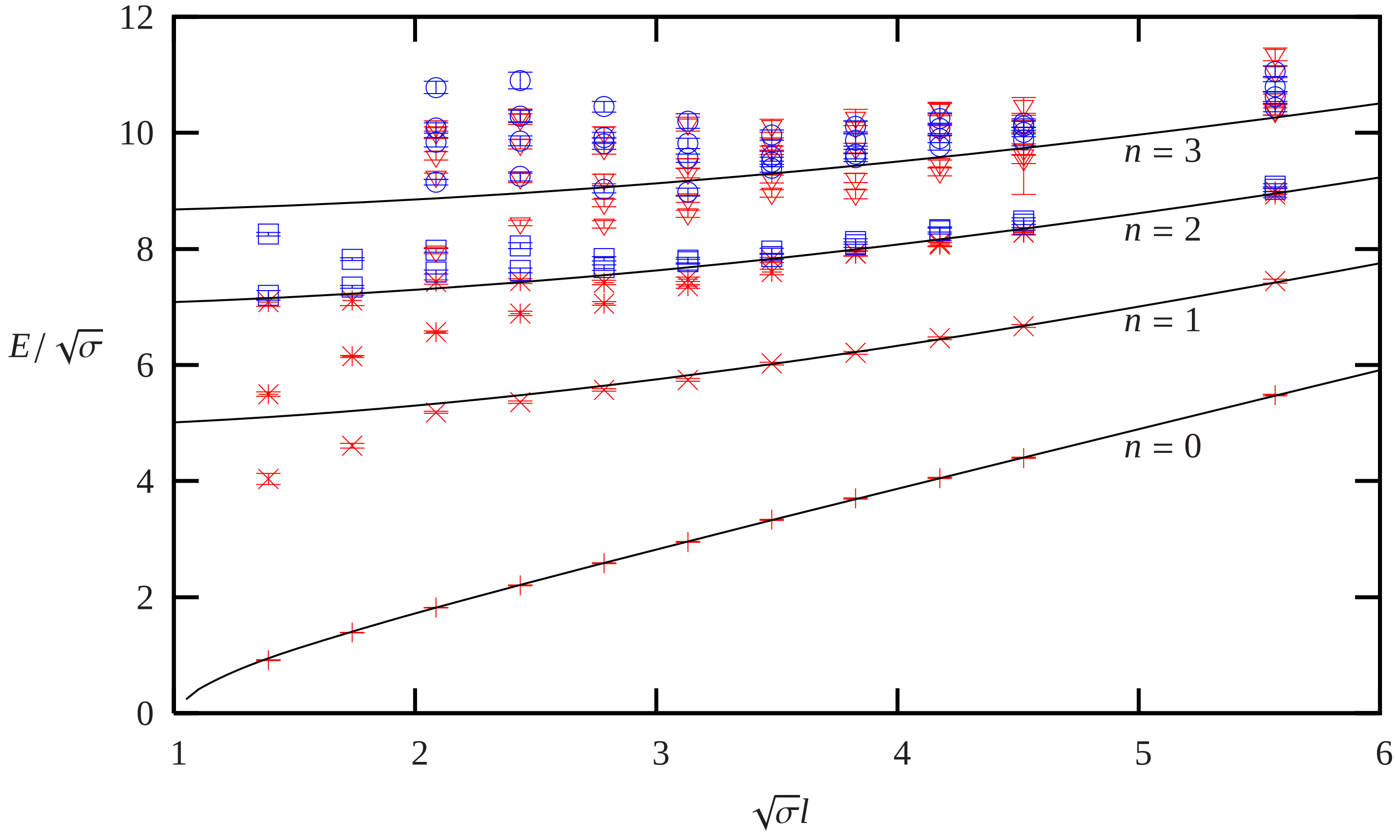
III. Lattice Calculation: Large Basis of Operators

Using this large basis of operators:

- We extract masses of excited states.(up to 15 states)
- It increases the Overlaps (using single exponential fits):
  - Ground state $\sim 99 - 100\%$,
  - First excited state $\sim 98 - 100\%$ ($\sim 90 - 95$ with just $\times 5bl$),
  - Second excited state $\sim 95 - 99\%$ ($\sim 85 - 90$ with just $\times 5bl$),
- We can extract energies of non-zero winding momentum states.
- We can extract energies of $P = -$ states.
- It increases computational time moderately.(ex. $\times 6$ for $L = 16a$)

IV. Results: Spectrum of $SU(3)$ and $\beta = 21.0$

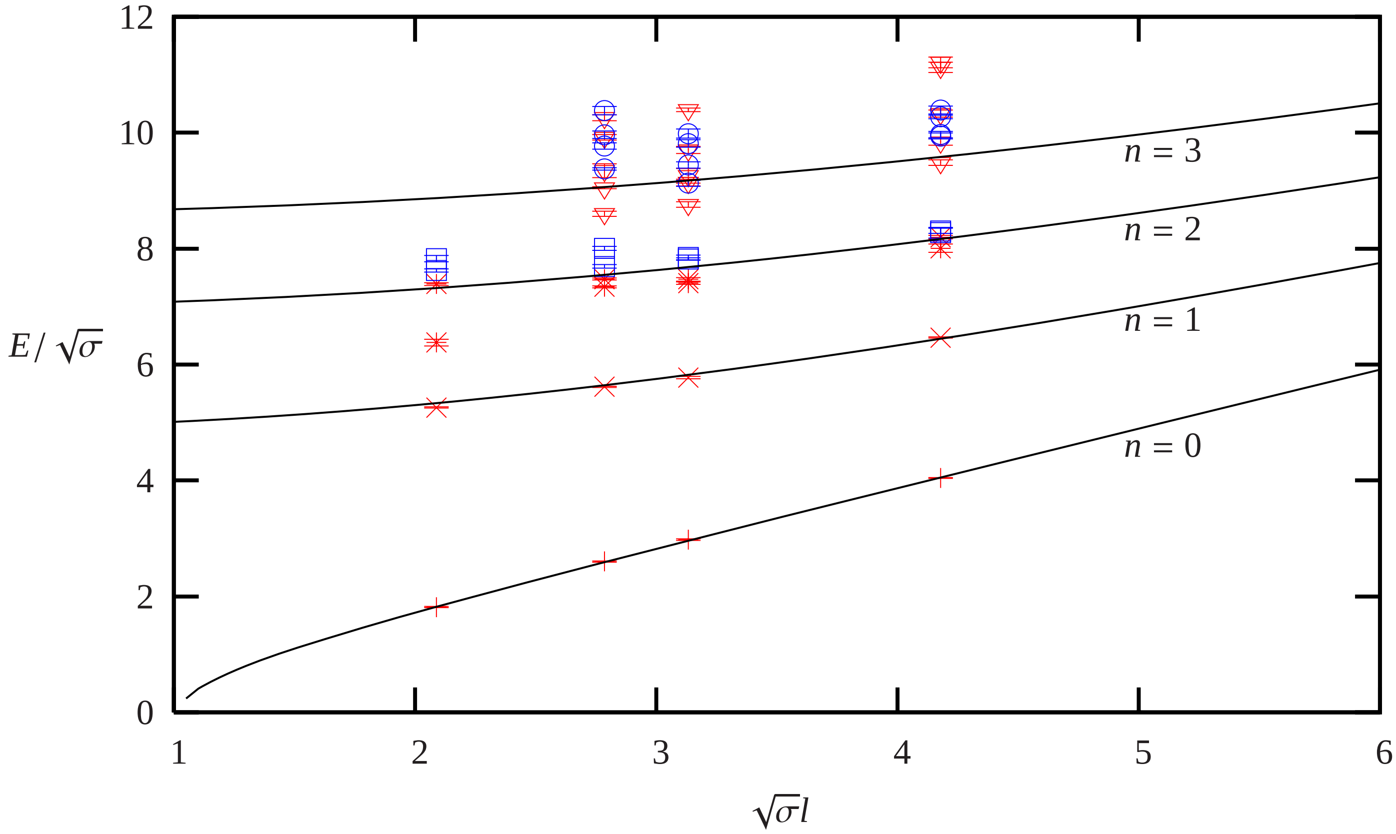
Group: $SU(3)$, $\underline{a} \simeq 0.08\,fm$, **Quantum Numbers**: $P = +, -$ and $q = 0$



NG Prediction: $E_n^2 = (\sigma l)^2 + 8\pi\sigma\left(n - \frac{1}{24}\right)$, where $n = N_L = N_R$ since $q = 0$.

IV. Results: Spectrum of $SU(3)$ and $\beta = 40.0$

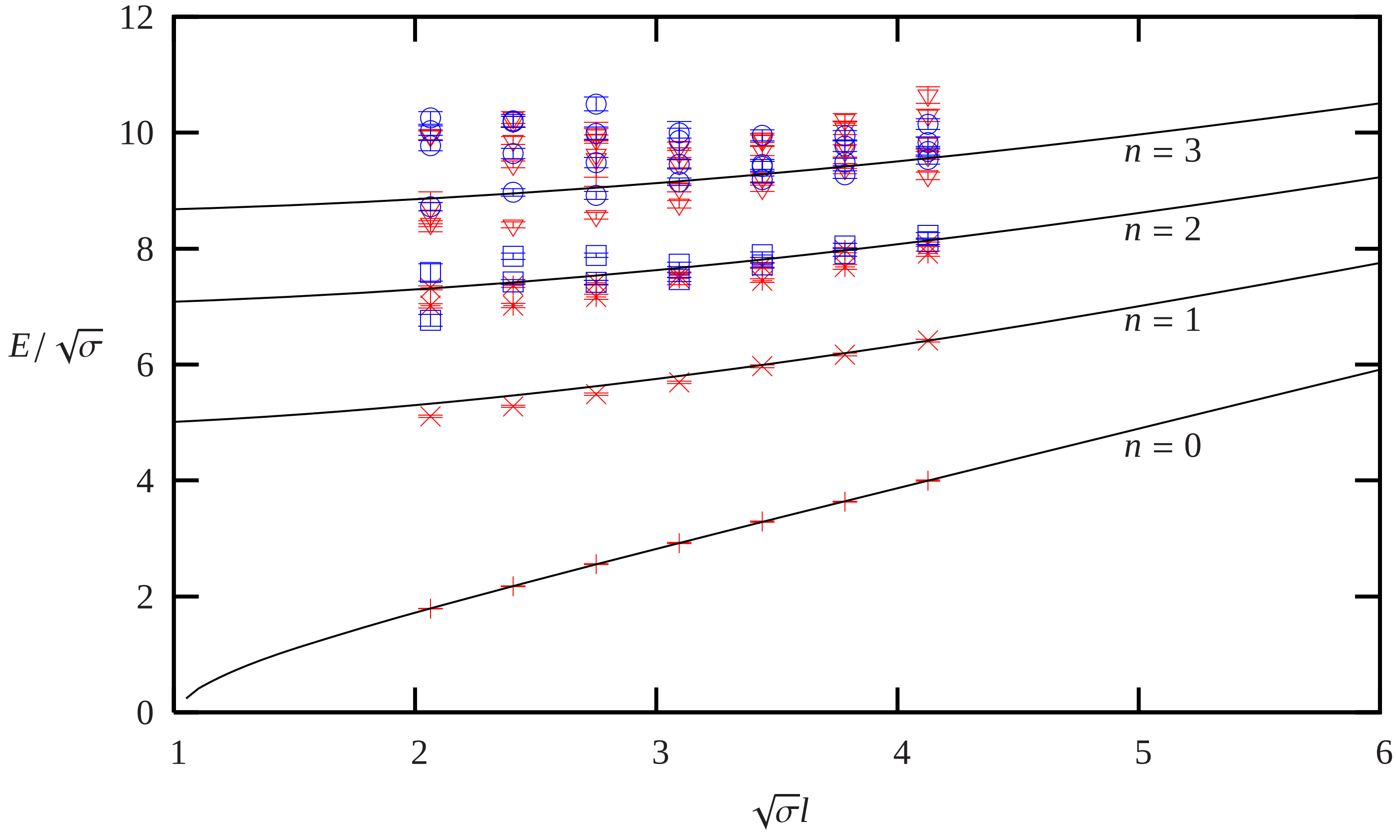
Group: $SU(3)$, $\underline{a} \simeq 0.04\,fm$, **Quantum Numbers**: $P = +, -$ and $q = 0$



NG Prediction: $E_n^2 = (\sigma l)^2 + 8\pi\sigma\left(n - \frac{1}{24}\right)$, where $n = N_L = N_R$ since $q = 0$.

IV. Results: Spectrum of $SU(6)$ and $\beta = 90.0$

Group: $SU(6)$, $a \simeq 0.08\,fm$, **Quantum Numbers**: $P = +, -$ and $q = 0$

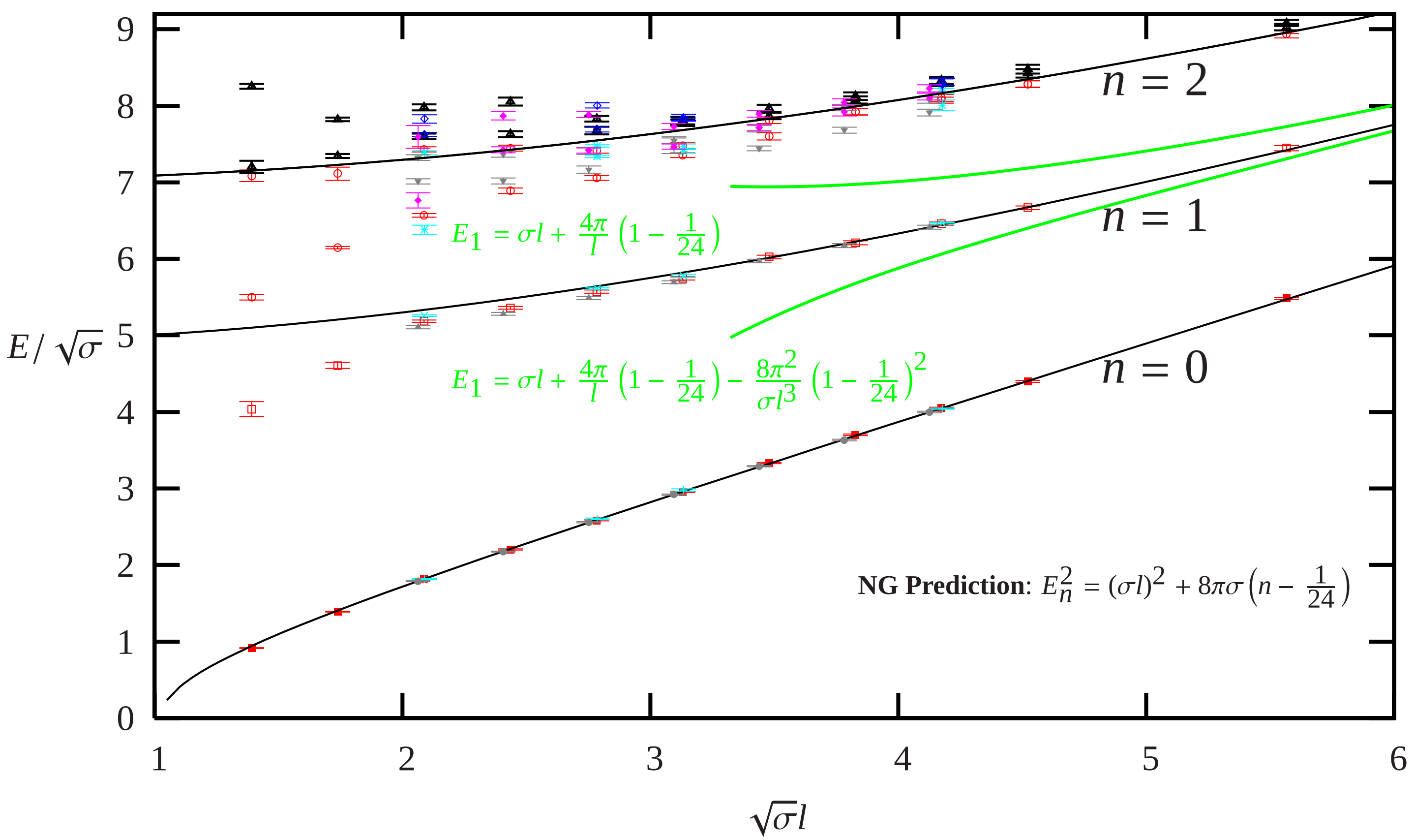


NG Prediction: $E_n^2 = (\sigma l)^2 + 8\pi\sigma\left(n - \frac{1}{24}\right)$, where $n = N_L = N_R$ since $q = 0$.

IV. Results: Spectrum of $SU(N)$

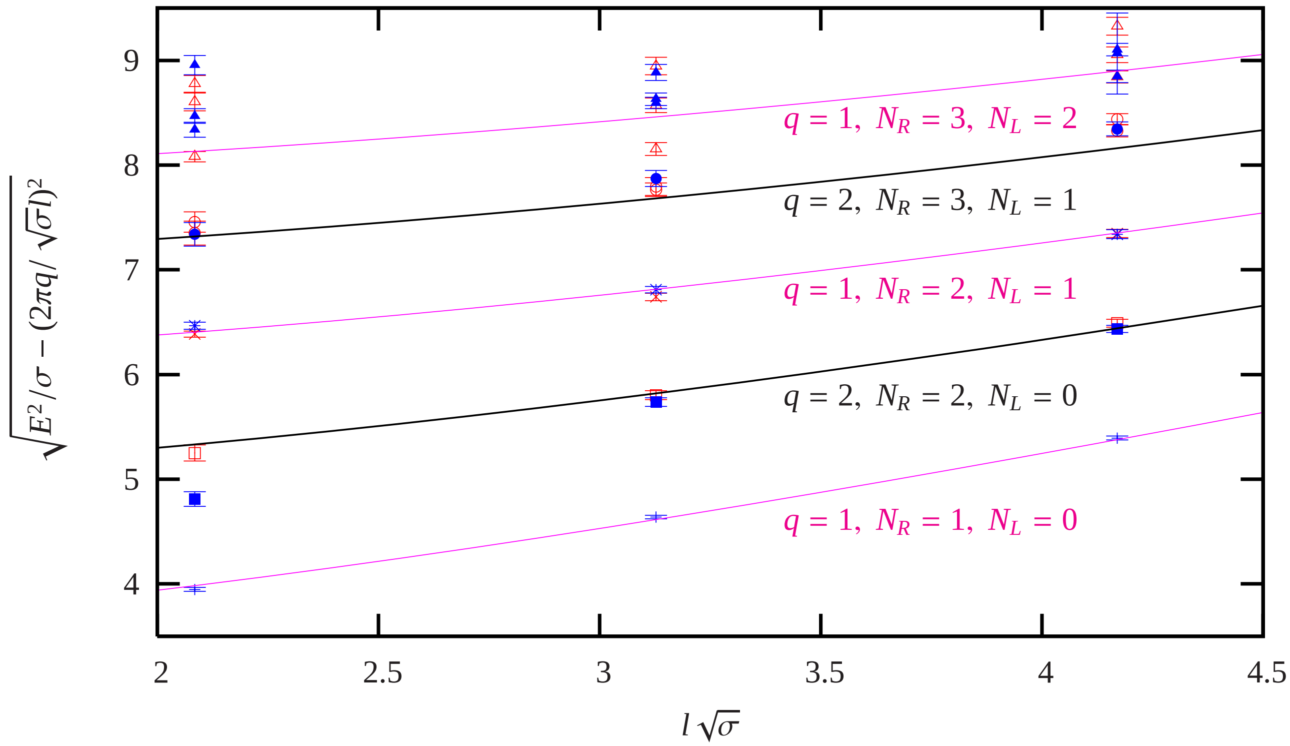
Groups: $SU(3)$ and $SU(6)$, $\underline{a} \simeq 0.04\,fm$ and $0.08\,fm$,

Quantum Numbers: $P = \pm$ and $q = 0$



IV. Results: Non-zero winding momentum.

Group: $SU(3)$, $\underline{a} \simeq 0.08\,fm$, **Quantum Numbers**: $P = +, -$, $q = 1, 2$ and $w = 1$



NG Prediction: $E^2 - (2\pi q/l)^2 = (\sigma l w)^2 + 8\pi\sigma\left(\frac{N_R+N_L}{2} - \frac{D-2}{24}\right)$.

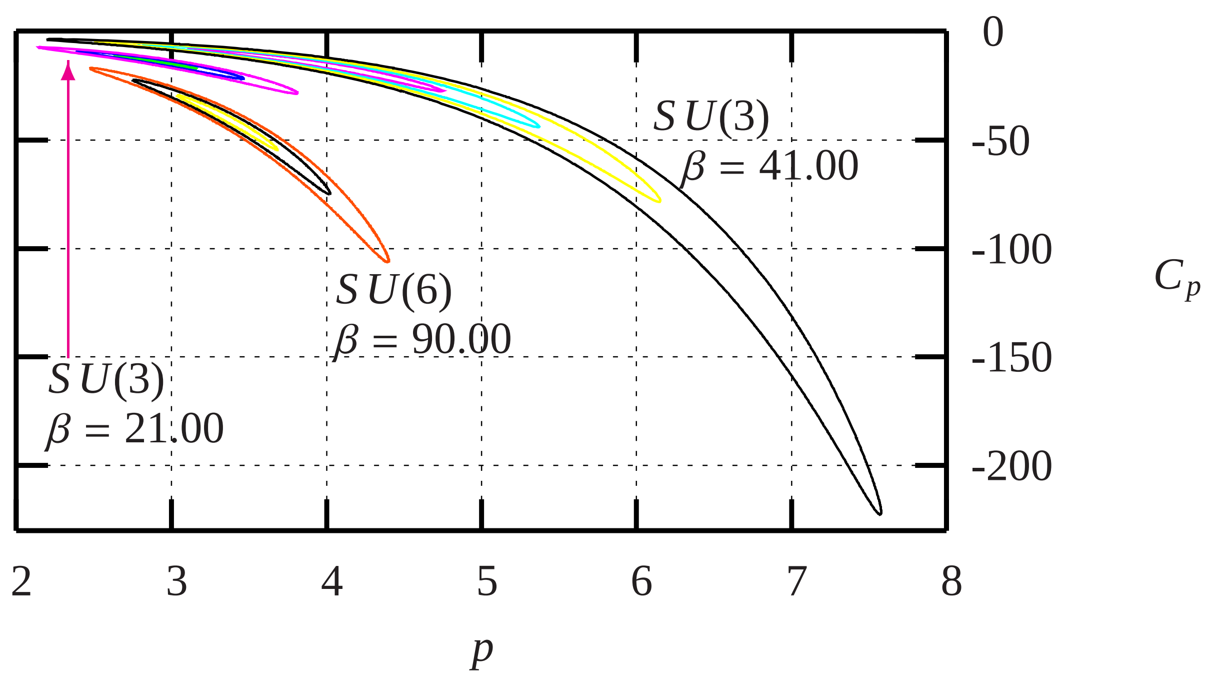
Constraint: $N_R - N_L = qw$

IV. Results: Fits.

Ansatz: $E_{\text{fit}}^2 = E_{NG}^2 - \sigma \frac{C_p}{(l\sqrt{\sigma})^p}$,

String tension: Calculated using the ground state, fixing $p = 3$

$e^{-\frac{1}{2}\chi^2} = f(p, C_p)$



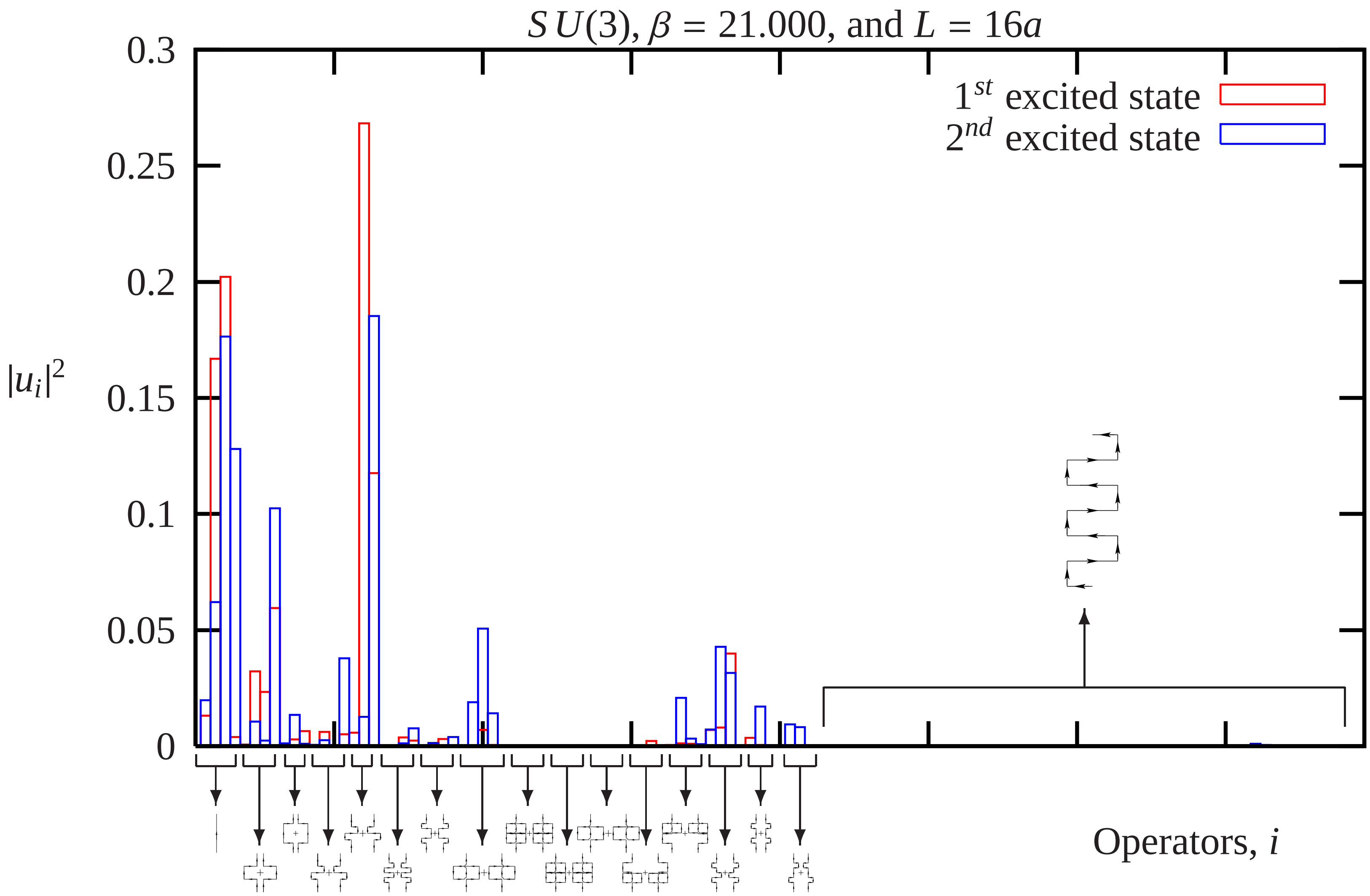
First excited states exclude $p = 1$ (Boundary term)!

Also second excited states exclude $p = 1$

V. Summary

- We constructed a large basis of operators characterized by the quantum numbers of **parity** P, and **winding momentum** $2\pi q/l$,
- We calculated the energies of closed flux tubes in D=2+1 described by $P = \pm$ for:
  - $\rightarrow$ $SU(3)$ with $\beta = 21.0$ ($a \simeq 0.08$fm) and $q = 0, \pm 1, \pm 2$,
  - $\rightarrow$ $SU(3)$ with $\beta = 40.0$ ($a \simeq 0.04$fm) and $q = 0$,
  - $\rightarrow$ $SU(6)$ with $\beta = 90.0$ ($a \simeq 0.08$fm) and $q = 0$,
- We fit our data for the ground state using $E^2_{\text{fit}} = E^2_{NG} - \sigma C_p / \left(l\sqrt{\sigma}\right)^p$ and $p = 3$, and extract σ.
- Using σ we compare our results to Nambu-Goto:
  - $\rightarrow$ **Nambu-Goto is VERY good**
- We fit our data for the 1^{st} and 2^{nd} excited states with E^2_{fit}, and extract p
  - $\rightarrow$ 1^{st} **and** 2^{nd} **excited states exclude** $p = 1$ (boundary term Lüscher&Weisz. 04)

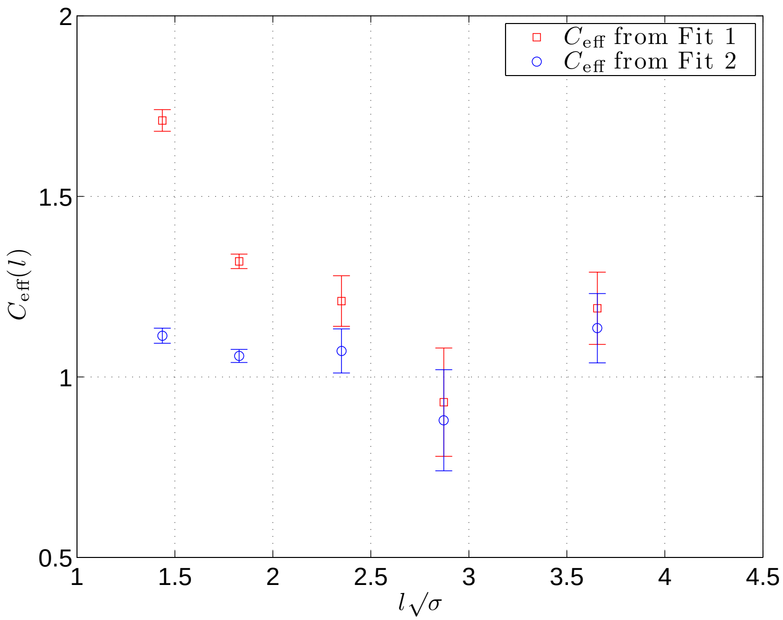
IV. Appendix 1: Contribution of Operators



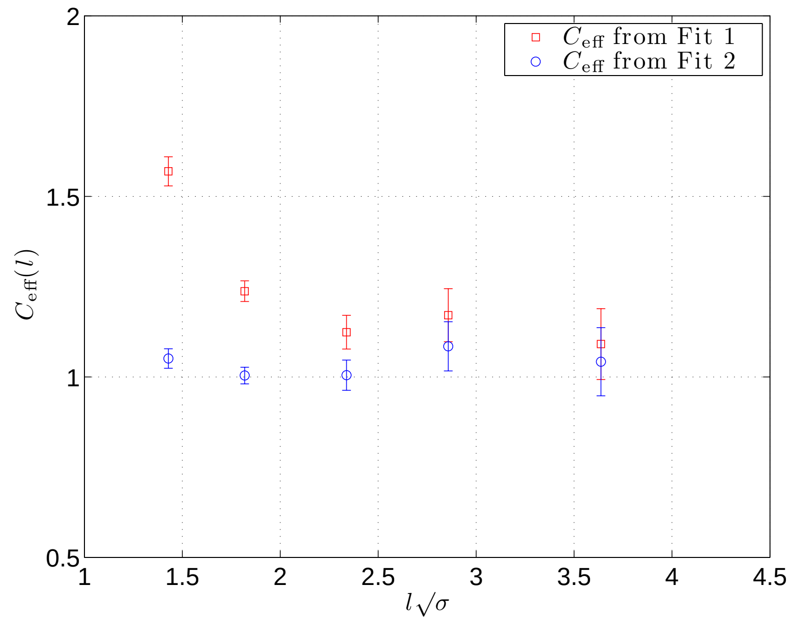
IV. Appendix 2: Why E_n^2?

Ground state:

$SU(3)$ and $\beta = 14.7172$



$SU(5)$ and $\beta = 80.00$



Fit 1: $E_0(l,\sigma) = \sigma l - \frac{\pi}{6l} \times C_{\rm eff}^{(1)}(l)$

Fit 2: $E_0^2(l,\sigma) = (\sigma l)^2 - \frac{\pi\sigma}{3} \times C_{\rm eff}^{(2)}(l).$