Particle Theory Journal Club

k-strings in $SU(N)$ gauge theories

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Mostly based on:

AA, Barak Brigoltz and Mike Teper: arXiv:0709.0693 and arXiv:0709.2981 ($k = 1$ in 2+1 dimensions)

Barak Bringoltz and Mike Teper: arXiv:0708.3447 and arXiv:0802.1490 ($k > 1$ in 2+1 dimensions)

AA, Barak Bringoltz and Mike Teper: Work in progress (excited k-strings, 3+1 dimensions)

I. Introduction: General

General question:

→ What effective string theory describes k-strings in $SU(N)$ gauge theories?

Two cases:

→ Open k-strings —

→ Closed k-strings ◯

During the last decade:

→ 3D, 4D with Z_2, Z_4,$U(1)$, $SU(N \leq 6)$ (Caselle and collaborators, Gliozzi and collaborators, Kuti and collaborators, Lüscher&Weisz, Majumdar and collaborators, Teper and collaborators, Meyer)

Questions to be studied in $D = 2 + 1$ dimensional $SU(N)$ theories:

→ Calculation of excited states, and states with $p_{||} \neq 0$ and $P = \pm$ with $k = 1, 2$

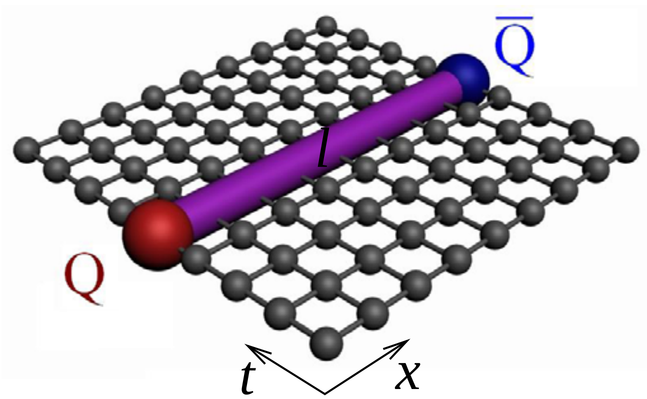
→ What is the degeneracy pattern of these states?

→ Do k-strings fall into specific irreducible representations?

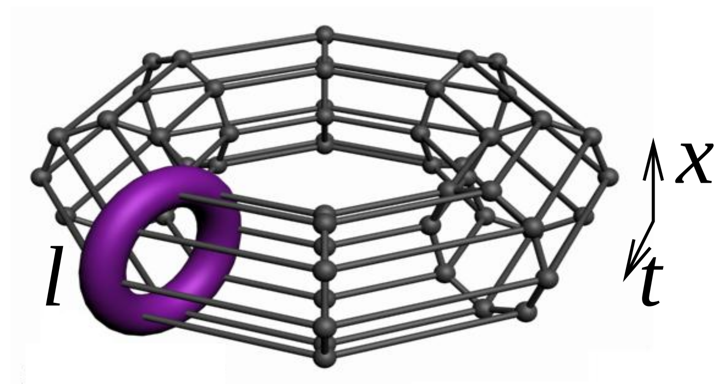
I. Introduction: General

Open flux tube ($k = 1$ string)

Closed flux tube ($k = 1$ string)



periodic b.c $\longrightarrow$



$$\Phi(l,t) = \phi^{\dagger}(0,t)U(0,l;t)\phi(l,t)$$

$$\Phi(l,t) = \mathrm{Tr}U(l;t)$$

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→ **Closed k-strings** ○

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→ What is the degeneracy pattern of these states?

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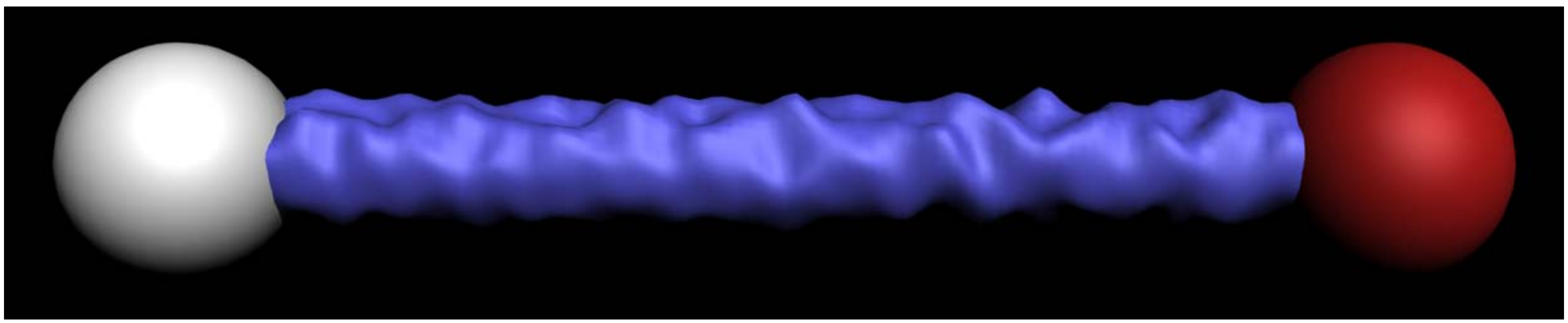
I. Introduction: k- strings

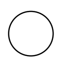
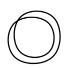
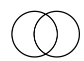
What is a k-string?

- Confinement in 3-d $SU(N)$ leads to a linear potential between colour charges in the fundamental representation.



- For $SU(N \geq 4)$ there is a possibility of new stable strings which join test charges in representations higher than the fundamental!
- We can label these by the way the test charge transforms under the center of the group: $\psi(x) \longrightarrow z^k \psi(x)$, $z \in Z_N$.
- The string has N-ality k, ($k = 2 : \square \bigotimes \square = \square\square \bigoplus \begin{smallmatrix}\square\\\square\end{smallmatrix}$) (fund. $\bigotimes$ fund. = symm. $\bigoplus$ anti.)

I. Introduction: k- strings

- The string tension does not depend on the representation $\mathcal{R}$ but rather on its N-ality k.
- The fundamental string has: N-ality $k = 1$, with string tension σ_f.
- A k-string can be thought of as a bound state of k fundamental strings.
- Operators: $\phi = \mathrm{Tr}\{U^{k-j}\}\mathrm{Tr}\{U\}^j$ where U is a Polyakov loop and $j = 0, .., k-1$.
  - For $k = 1$, $\phi \equiv$  $\equiv \mathrm{Tr}\{U\}$
  - For $k = 2$, $\phi_1 \equiv$  $\equiv \mathrm{Tr}\{U^2\}, \phi_2 \equiv$  $\equiv \mathrm{Tr}\{U\}\mathrm{Tr}\{U\}$
- Predictions:
  - Casimir Scaling: $\sigma_{\mathcal{R}} = \sigma_f \cdot \frac{C_{\mathcal{R}}}{C_f} => \frac{\sigma_k}{\sigma_f} = \frac{k(N-k)}{(N-1)}$
  - MQCD: $\frac{\sigma_k}{\sigma_f} = \frac{\sin\frac{k\pi}{N}}{\sin\frac{\pi}{N}}$
- We expect: $\sigma_k \stackrel{N\longrightarrow\infty}{\longrightarrow} k\sigma_f$

II. Theoretical expectations A.

The Spectrum of the Nambu-Goto (NG) String Model

- Action of Nambu-Goto free bosonic string leads to:

  $\rightarrow$ Spectrum given by:

$$E^2_{N_L,N_R,q,w} = (\sigma l w)^2 + 8\pi\sigma\left(\frac{N_L+N_R}{2} - \frac{D-2}{24}\right) + \left(\frac{2\pi q}{l}\right)^2 + p_\perp^2.$$

  $\rightarrow$ Described by:

  1. The winding number w (w=1),
  2. The winding momentum $p_{||} = 2\pi q/l$ with $q = 0, \pm 1, \pm 2,$
  3. The transverse momentum $p_\perp$ ($p_\perp = 0$),
  4. N_L and N_R connected through the relation: $N_R - N_L = qw$.

$$N_L = \sum_{k>0}\sum_{n_L(k)>0} n_L(k)k \quad \text{and} \quad N_R = \sum_{k'>0}\sum_{n_R(k')>0} n_R(k')k'$$

  $\rightarrow$ String states are eigenvectors of P (In $D = 2+1$) with eigenvalues:

$$P = (-1)^{\sum_{i=1}^{m} n_L(k_i) + \sum_{j=1}^{m'} n_R(k'_j)}$$

II. Theoretical expectations A.

The seven lowest NG energy levels for the $w = 1$ closed string

level	N_R	N_L	q	$P = +$	$P = -$
0	0	0	0	$\vert 0\rangle$	
1	1	0	1		$\alpha_{-1}\vert 0\rangle$
2	1	1	0	$\alpha_{-1}\bar{\alpha}_{-1}\vert 0\rangle$	
3	2	0	2	$\alpha_{-1}\alpha_{-1}\vert 0\rangle$	$\alpha_{-2}\vert 0\rangle$
4	2	1	1	$\alpha_{-2}\bar{\alpha}_{-1}\vert 0\rangle$	$\alpha_{-1}\alpha_{-1}\bar{\alpha}_{-1}\vert 0\rangle$
5	2	2	0	$\alpha_{-2}\bar{\alpha}_{-2}\vert 0\rangle,\ \alpha_{-1}\alpha_{-1}\bar{\alpha}_{-1}\bar{\alpha}_{-1}\vert 0\rangle$	$\alpha_{-2}\bar{\alpha}_{-1}\bar{\alpha}_{-1}\vert 0\rangle,\ \alpha_{-1}\alpha_{-1}\bar{\alpha}_{-2}\vert 0\rangle$
6	3	1	2	$\alpha_{-3}\bar{\alpha}_{-1}\vert 0\rangle,\ \alpha_{-1}\alpha_{-1}\alpha_{-1}\bar{\alpha}_{-1}\vert 0\rangle$	$\alpha_{-2}\alpha_{-1}\bar{\alpha}_{-1}\vert 0\rangle$

II. Theoretical expectations B.

Effective string theory

- First prediction for $w = 1$ and $q = 0$ (Lüscher, Symanzik&Weisz. 80):

$$E_n = \sigma l + \frac{4\pi}{l}\left(n - \frac{D-2}{24}\right) + \mathcal{O}\left(1/l^2\right).$$

- Lüscher&Weisz effective string action (Lüscher&Weisz. 04):

→ For any D the $\mathcal{O}\left(1/l^2\ (1/l)\right)$ (Boundary term) is absent from $E_n(E_n^2)$

→ Spectrum in $D = 2 + 1$ (Drummond '04, Dass and Matlock '06 for any D.):

$$E_n = \sigma l + \frac{4\pi}{l}\left(n - \frac{1}{24}\right) - \frac{8\pi^2}{\sigma l^3}\left(n - \frac{1}{24}\right)^2 + \mathcal{O}\left(1/l^4\right)$$

→ Equivalently:

$$E_n^2 = (\sigma l)^2 + 8\pi\sigma\left(n - \frac{1}{24}\right) + \mathcal{O}\left(1/l^3\right),$$

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→ Equivalently:

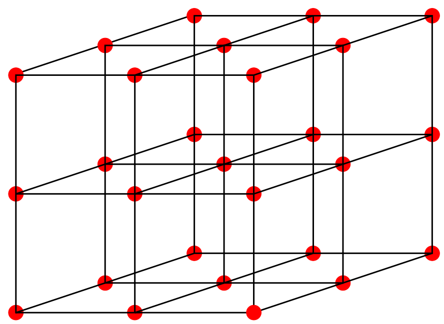
$$E_n^2 = E_{NG}^2 + \mathcal{O}\left(1/l^3\right) \longrightarrow \text{Fit} : E_{\text{fit}}^2 = E_{NG}^2 - \sigma\frac{C_p}{\left(l\sqrt{\sigma}\right)^p} \qquad (p \geq 3)$$

III. Lattice Calculation: Lattice setup

- Usually we are interested in calculating quantities like:

$$\langle \Psi(A) \rangle = \frac{1}{Z} \int \prod_{\vec{x},\mu} dA_\mu(\vec{x}) \Psi(A) e^{-S[A]}$$

- The lattice represents a mathematical trick: It provides a regularisation scheme.



- We define our gauge theory on a 3D discretized periodic Euclidean space-time with $L_{\|} \times L_{\perp} \times L_T$ sites.

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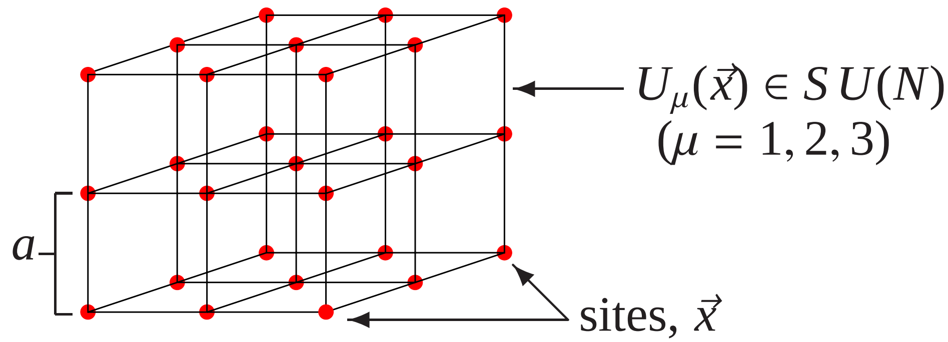


- We define our gauge theory on a 3D discretized periodic Euclidean space-time with $L_{\|} \times L_{\perp} \times L_T$ sites.

III. Lattice Calculation: On the Lattice

- Using link variables, the EFPI is written as:

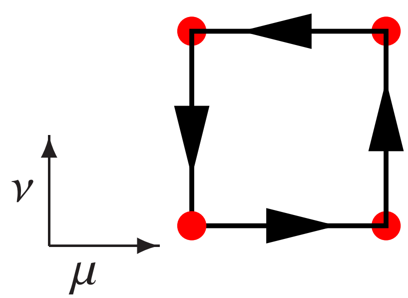
$$\langle \Psi_L(U) \rangle = \frac{1}{Z} \int \prod_{n,\mu} dU_\mu(n) \Psi_L(U) e^{-S_L[U]} \quad (1)$$

Where:

$$S_L = \beta \sum_p \{1 - \frac{1}{N_c} ReTrU_p\} \quad (2)$$

$$\beta = \frac{2N_c}{ag^2} \qquad (D = 2+1) \quad (3)$$

$$U_p = U_\mu(\vec{x}) U_\nu(\vec{x} + a\hat{\mu}) U_\mu^\dagger(\vec{x} + a\hat{\nu}) U_\mu^\dagger(\vec{x}) \quad (4)$$



III. Lattice Calculation: Monte - Carlo Simulations

Why Monte - Carlo?:

- We want to calculate the expectation value of colour singlet operators.
- High multidimensionality of these integrals makes traditional mesh techniques impractical.

  $\longrightarrow$ Monte Carlo Methods.
- We need to generate n_c different field configurations with probability distribution:

$$\prod_l dU_l e^{-\beta \sum_p \{1 - \frac{1}{N_c} ReTrU_p\}} \tag{5}$$

- Then the expectation value of $\Psi_L(U)$ will be just the average over these fields.

$$\langle \Psi_L(U) \rangle = \frac{1}{n_c} \sum_{I=1}^{n_c} \Psi_L(U^I) \pm \mathcal{O}(\frac{1}{\sqrt{n_c}}) \tag{6}$$

III. Lattice Calculation: Energy Calculation

- Masses of certain states can be calculated using the correlation functions of specific operators:

$$\begin{aligned} \langle \Phi^\dagger(t)\Phi(0)\rangle &= |\langle \Omega|\Phi^\dagger|0\rangle|^2 e^{-tm_0} + \sum_{m\geq 1} |\langle \Omega|\Phi^\dagger|m\rangle|^2 e^{-tm_m} \\ &\xrightarrow{t\to\infty} |\langle \Omega|\Phi^\dagger|0\rangle|^2 e^{-tm_0} \end{aligned} \tag{7}$$

- Let us define the effective mass:

$$am_{eff}(t) = -\ln \frac{\langle \Phi^\dagger(t)\Phi(0)\rangle}{\langle \Phi^\dagger(t-a)\Phi(0)\rangle} \tag{8}$$

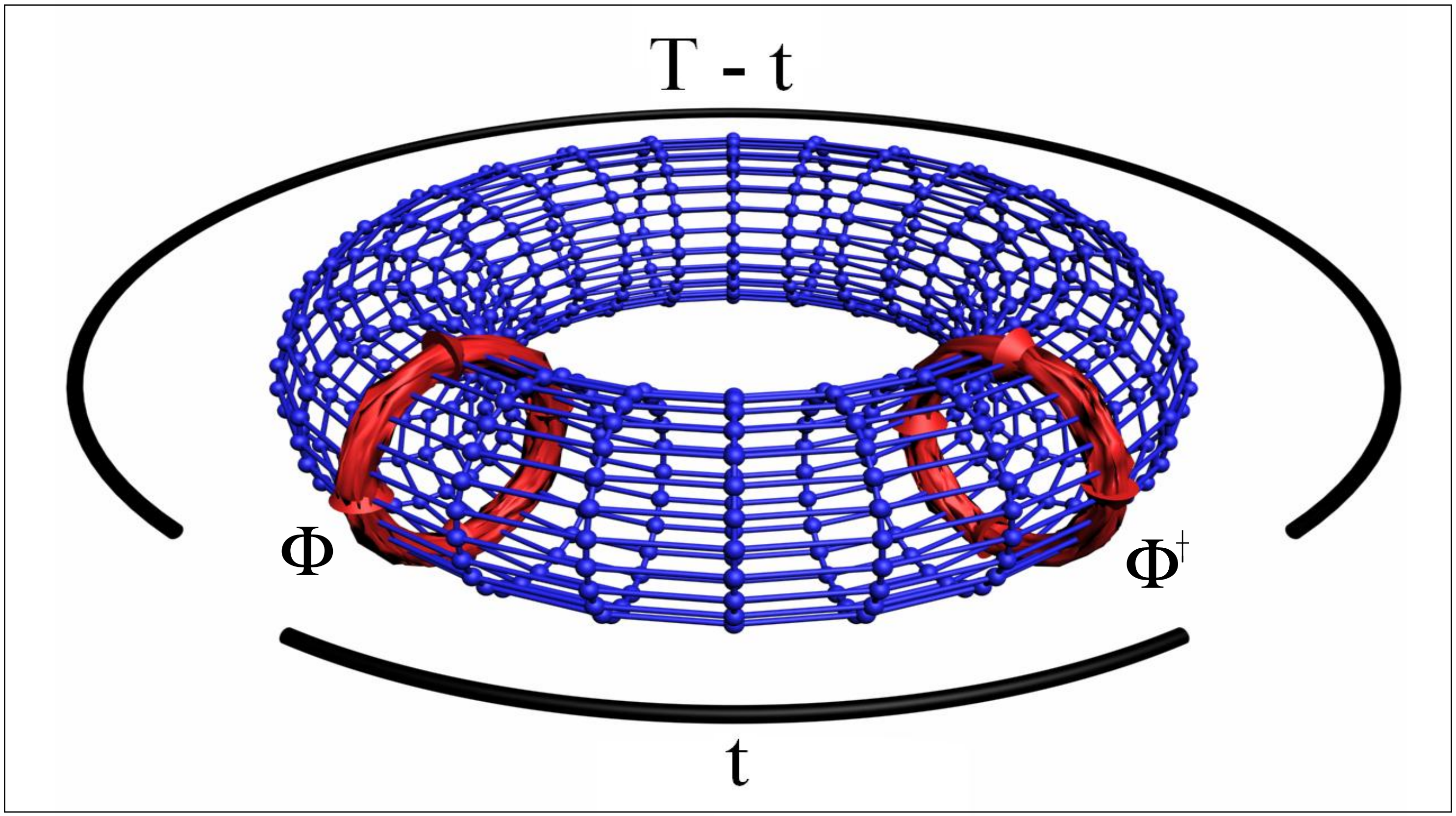
- The mass will be equal to:

$$am_k \simeq am_{eff}(t_0) \tag{9}$$

where t_0 is the lowest value of t for which $m_{eff}(t_0) = am_{eff}(t > t_0)$

III. Lattice Calculation: Energy Calculation

Example: Closed $k = 1$ string:



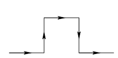
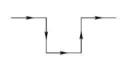
III. Lattice Calculation: Energy Calculation

- We expect to calculate masses for some excited states, so we need to use operators which merely project onto the excited states.
- We construct a basis of operators, $\Phi_i : i = 1, ..., N_O$, with transverse deformations described by the quantum numbers of parity P, winding number w, longitudinal momentum $p_{\|}$ and transverse momentum $p_{\perp} = 0$.

  For example:

$$\Phi_{\pm}^{p_{\|},p_{\perp}} = \frac{1}{L_{\|}L_{\perp}} \sum_{x_{\|},x_{\perp}} \{\phi_u \pm \phi_d\}\, e^{ip_{\|}x_{\|}+ip_{\perp}x_{\perp}}$$

$k = 1$:

$\phi_u = \mathrm{Tr}\ \{$  $\}$ and $\phi_{\mathrm{d}} = \mathrm{Tr}\ \{$  $\}$

III. Lattice Calculation: Energy Calculation

$k = 2$:

1st set of operators: $\phi_u = \mathrm{Tr}\,\{ \_\sqcap\_ \}^2, \;\; \phi_{\mathrm{d}} = \mathrm{Tr}\{ \overline{\;}\sqcup\overline{\;} \}^2$

2nd set of operators: $\phi_u = \mathrm{Tr}\,\{ \_\sqcap\_ \cdot \_\sqcap\_ \}, \;\; \phi_{\mathrm{d}} = \mathrm{Tr}\{ \overline{\;}\sqcup\overline{\;} \cdot \overline{\;}\sqcup\overline{\;} \}$

3rd set of operators: $\phi_u = \mathrm{Tr}\,\{ \_\_\sqcap\_\_ \}, \;\; \phi_{\mathrm{d}} = \mathrm{Tr}\{ \overline{\;\;}\sqcup\overline{\;\;} \}$

Projection onto the Antisymmetric representation:

$\phi_u = [\mathrm{Tr}\,\{ \_\sqcap\_ \}^2 - \mathrm{Tr}\,\{ \_\sqcap\_ \cdot \_\sqcap\_ \}], \;\; \phi_{\mathrm{d}} = [\mathrm{Tr}\{ \overline{\;}\sqcup\overline{\;} \}^2 - \mathrm{Tr}\{ \overline{\;}\sqcup\overline{\;} \cdot \overline{\;}\sqcup\overline{\;} \}]$

Projection onto the Symmetric representation:

$\phi_u = [\mathrm{Tr}\,\{ \_\sqcap\_ \}^2 + \mathrm{Tr}\,\{ \_\sqcap\_ \cdot \_\sqcap\_ \}], \;\; \phi_{\mathrm{d}} = [\mathrm{Tr}\{ \overline{\;}\sqcup\overline{\;} \}^2 + \mathrm{Tr}\{ \overline{\;}\sqcup\overline{\;} \cdot \overline{\;}\sqcup\overline{\;} \}]$

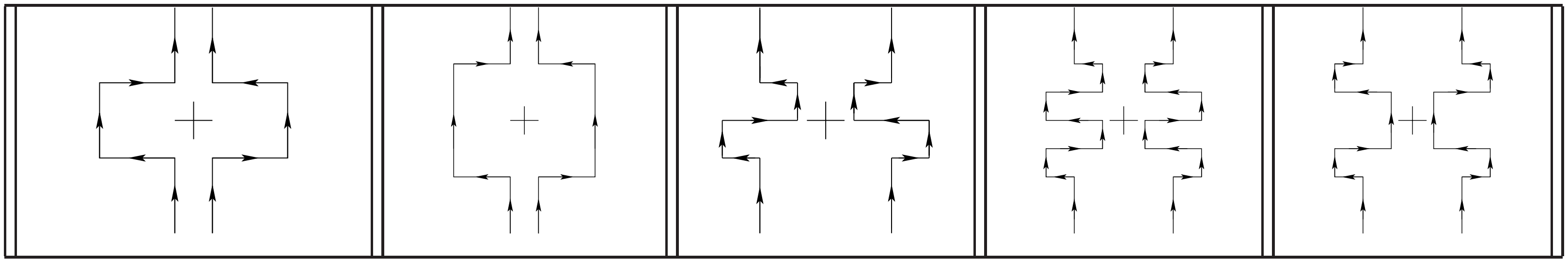
- We calculate the correlation function (Matrix): $C_{ij}(t) = \langle \Phi_i^\dagger(t)\Phi_j(0)\rangle$
- We diagonalize the matrix: $C^{-1}(0)C(a)$.
- We extract the correlator for each state.
- By fitting the results, we extract the mass (energy) for each state.

III. Lattice Calculation: Large Basis of Operators

Using this large basis of operators:

- We extract masses of excited states. (up to 15 states for $k = 1$)
- It increases the Overlaps (using single exponential fits):
  - Ground state $\sim 99 - 100\%$,
  - First excited state $\sim 98 - 100\%$ ($\sim 90 - 95$ with just $\times 5bl$),
  - Second excited state $\sim 95 - 99\%$ ($\sim 85 - 90$ with just $\times 5bl$),
- We can extract energies of non-zero winding momentum states.
- We can extract energies of $P = -$ states.
- It increases computational time moderately.(ex. $\times 6$ for $L = 16a$)

III. Lattice Calculation: Operators for $P = +$ and $k = 1$



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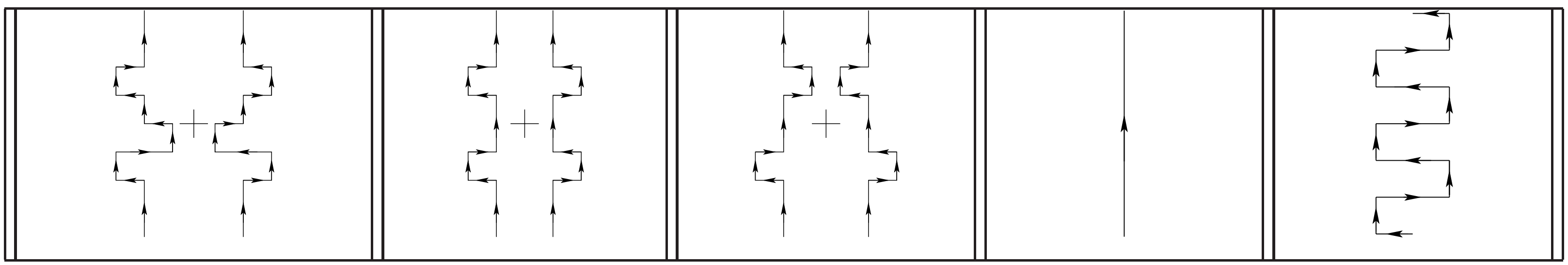
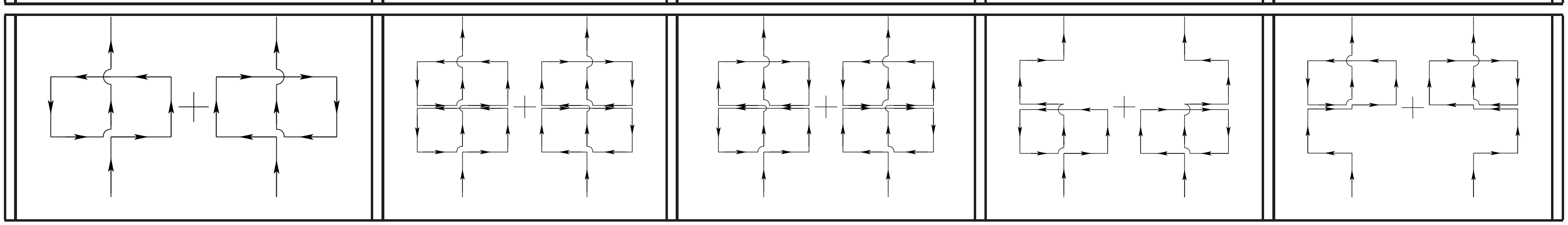


$\times 5\ bl$	$\times 5\ bl$	$\times 5\ bl$	$\times 5\ bl$	$f(L_{\|}, L_{\perp}, bl)$



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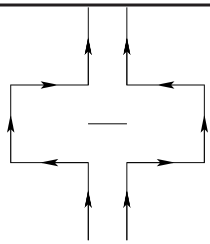
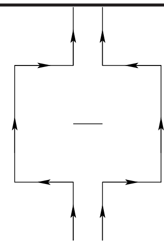
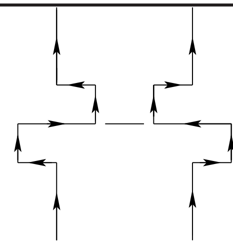
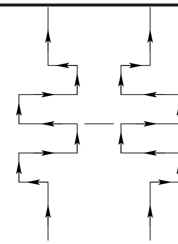
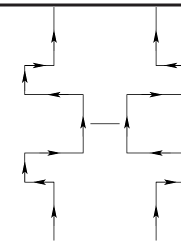
III. Lattice Calculation: Operators for $P = -$ and $k = 1$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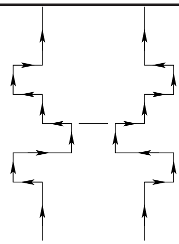	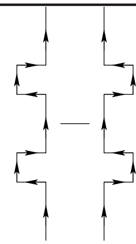	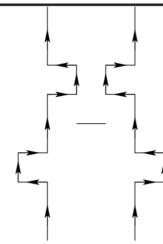		
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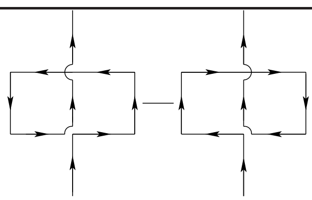	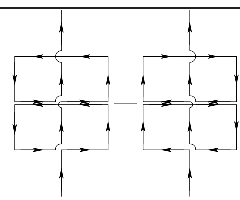	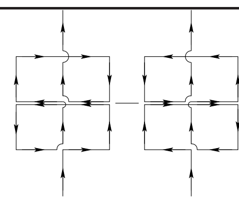	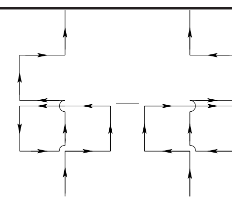	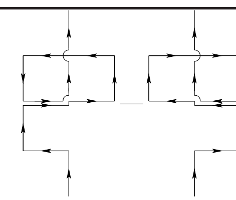
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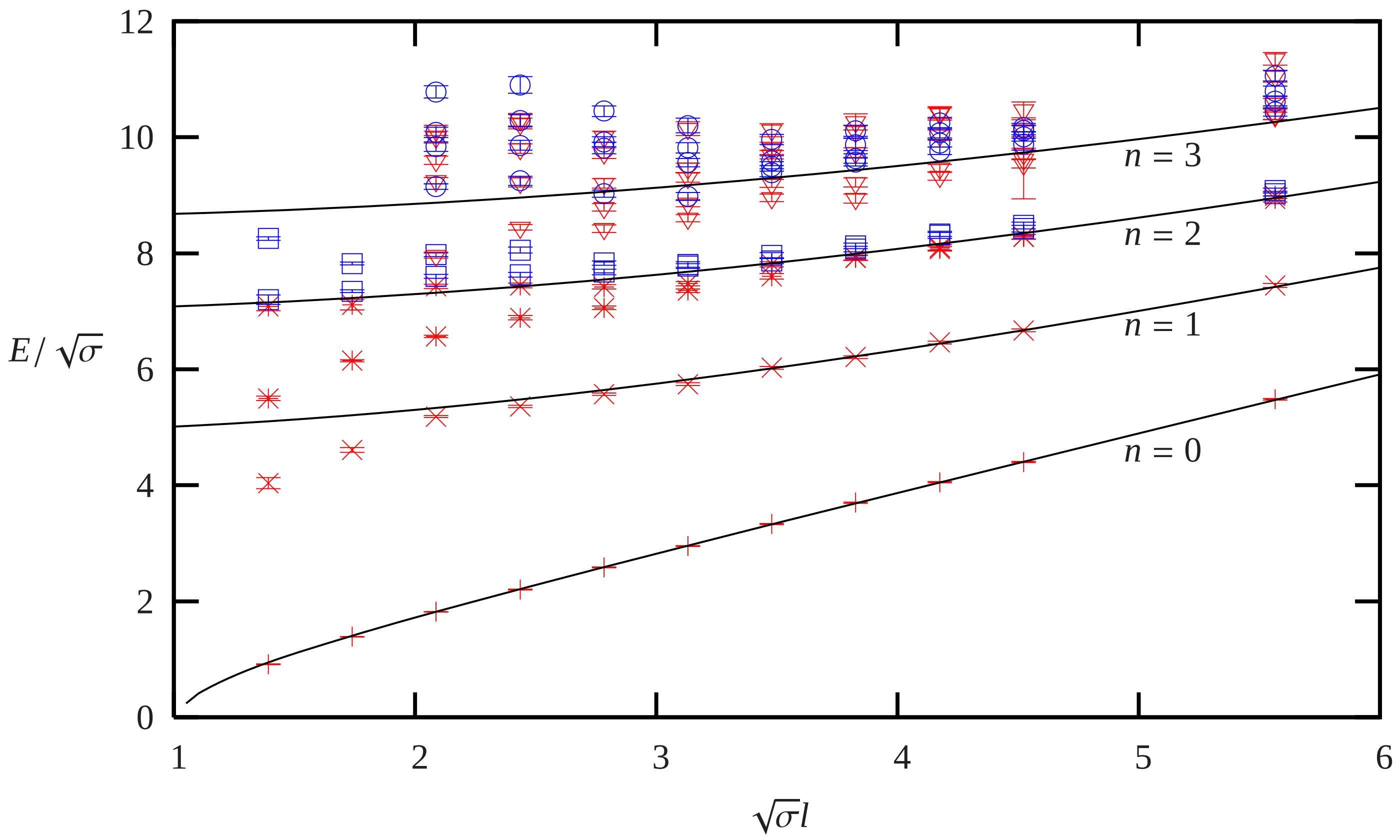
IV. Results: Spectrum of $SU(3)$ and $\beta = 21.0$ for $k = 1$

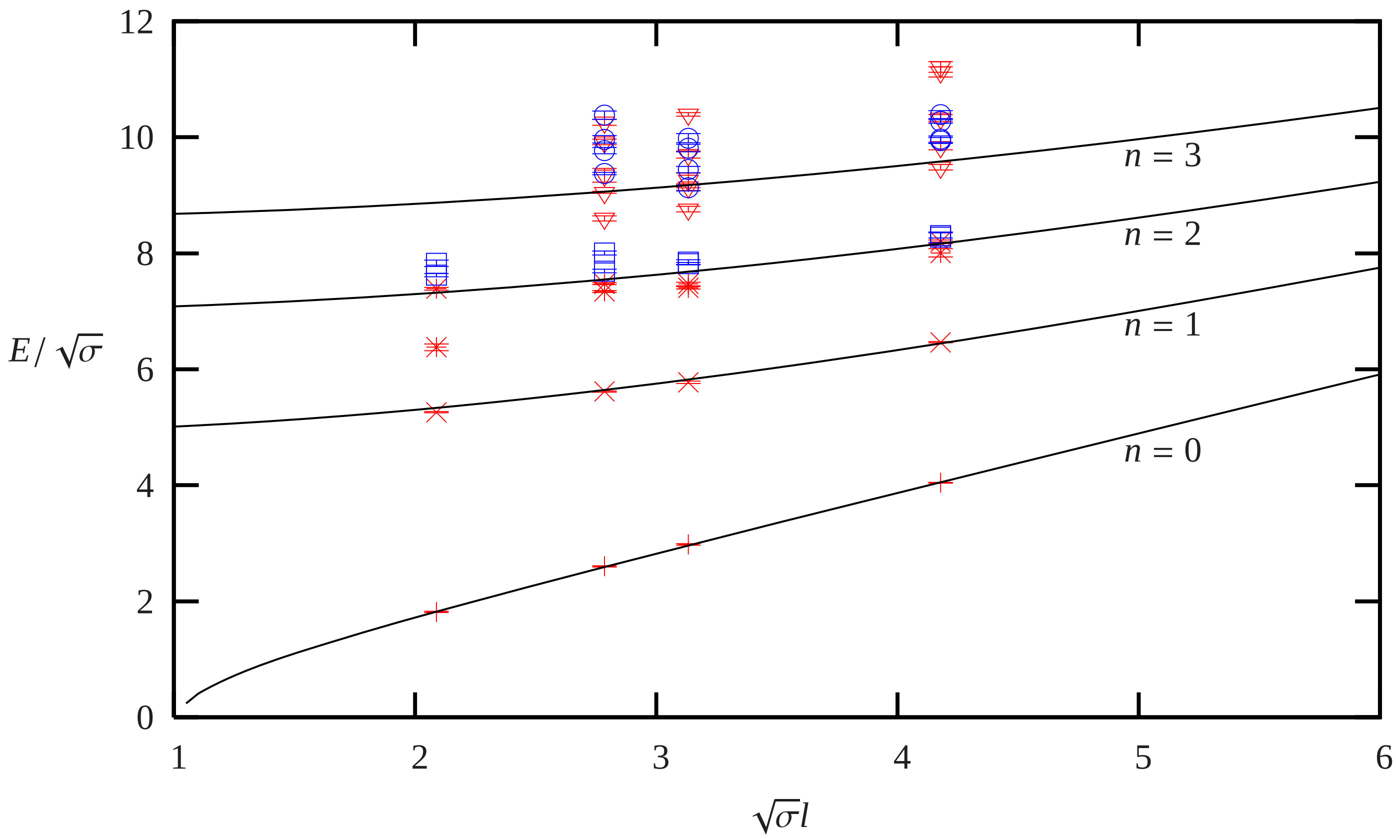
Group: $SU(3)$, $\underline{a} \simeq 0.08\,fm$, **Quantum Numbers**: $P = +, -$ and $q = 0$



NG Prediction: $E_n^2 = (\sigma l)^2 + 8\pi\sigma\left(n - \frac{1}{24}\right)$, where $n = N_L = N_R$ since $q = 0$.

IV. Results: Spectrum of $SU(3)$ and $\beta = 40.0$ for $k = 1$

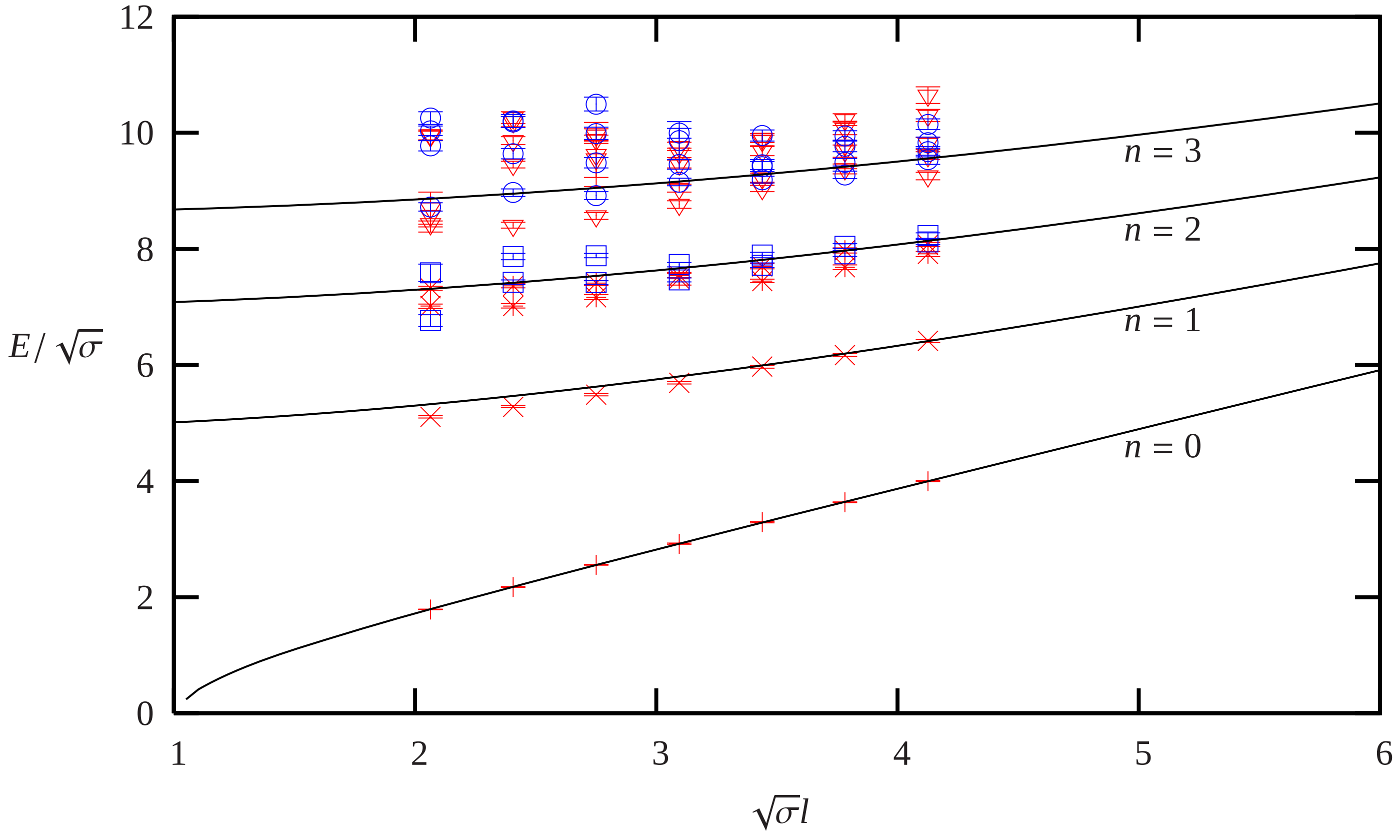
Group: $SU(3)$, $a \simeq 0.04\, fm$, **Quantum Numbers**: $P = +, -$ and $q = 0$



NG Prediction: $E_n^2 = (\sigma l)^2 + 8\pi\sigma\left(n - \frac{1}{24}\right)$, where $n = N_L = N_R$ since $q = 0$.

IV. Results: Spectrum of $SU(6)$ and $\beta = 90.0$ for $k = 1$

Group: $SU(6)$, $\underline{a} \simeq 0.08\,fm$, **Quantum Numbers**: $P = +, -$ and $q = 0$

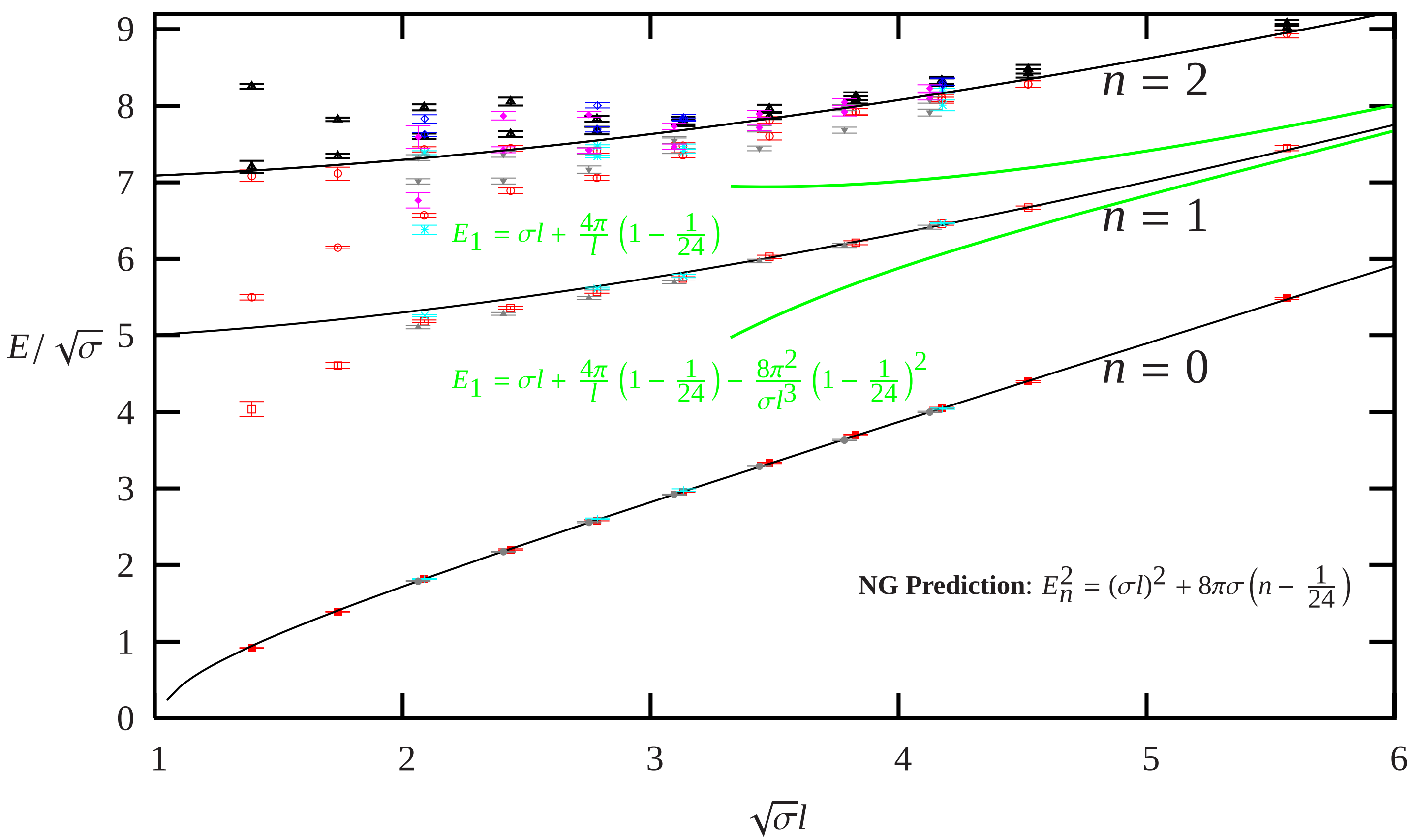


NG Prediction: $E_n^2 = (\sigma l)^2 + 8\pi\sigma\left(n - \frac{1}{24}\right)$, where $n = N_L = N_R$ since $q = 0$.

IV. Results: Spectrum of $SU(N)$ for $k = 1$

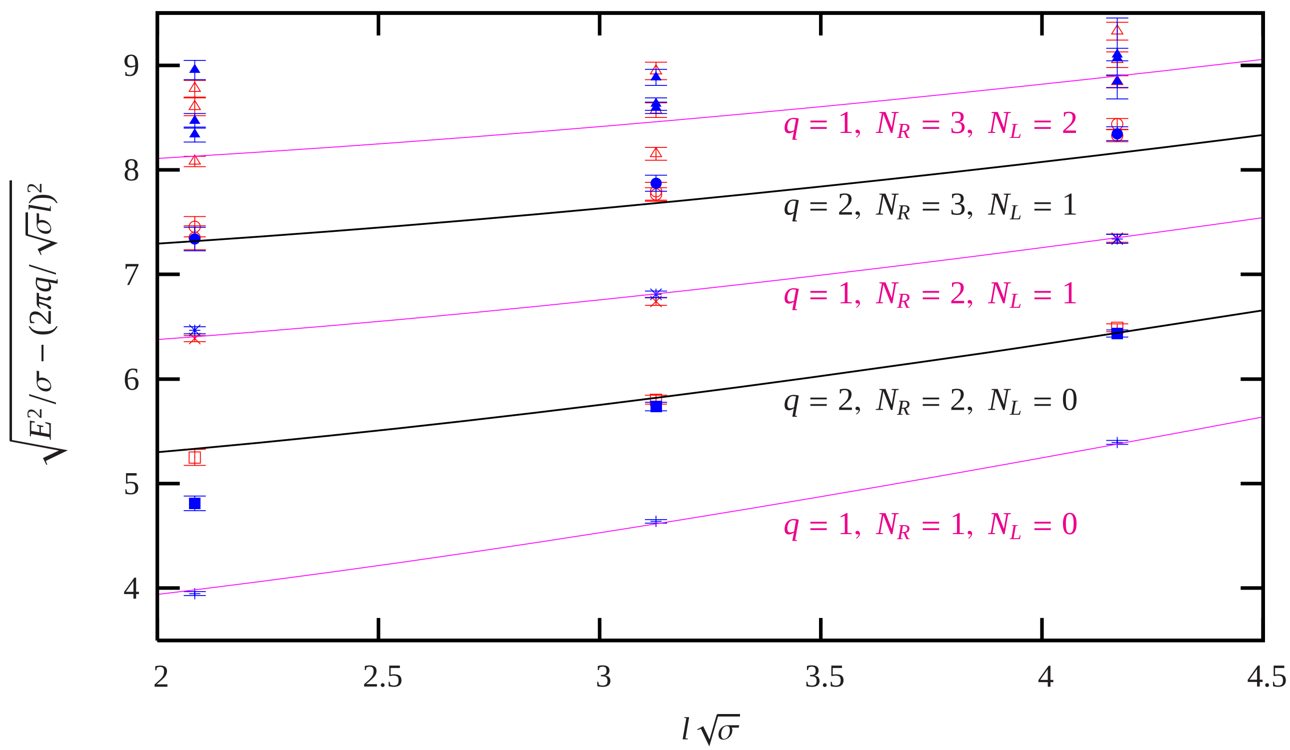
Groups: $SU(3)$ and $SU(6)$, $\underline{a} \simeq 0.04\,fm$ and $0.08\,fm$,

Quantum Numbers: $P = \pm$ and $q = 0$



IV. Results: Non-zero winding momentum for $k = 1$.

Group: $SU(3)$, $\underline{a} \simeq 0.08\,fm$, **Quantum Numbers**: $P = +, -$, $q = 1, 2$ and $w = 1$

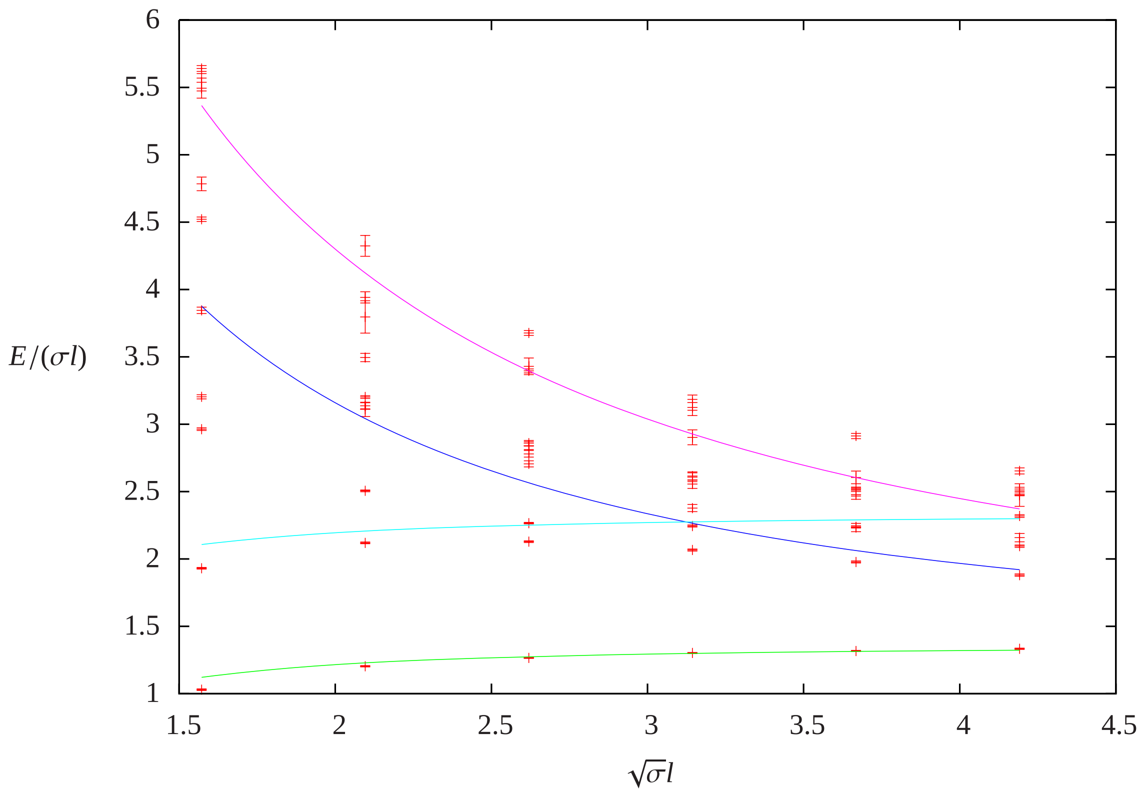


NG Prediction: $E^2 - (2\pi q/l)^2 = (\sigma l w)^2 + 8\pi\sigma\left(\frac{N_R+N_L}{2} - \frac{D-2}{24}\right)$.

Constraint: $N_R - N_L = qw$

IV. Results: Spectrum of $SU(4)$ for $P = +$ and $k = 2$.

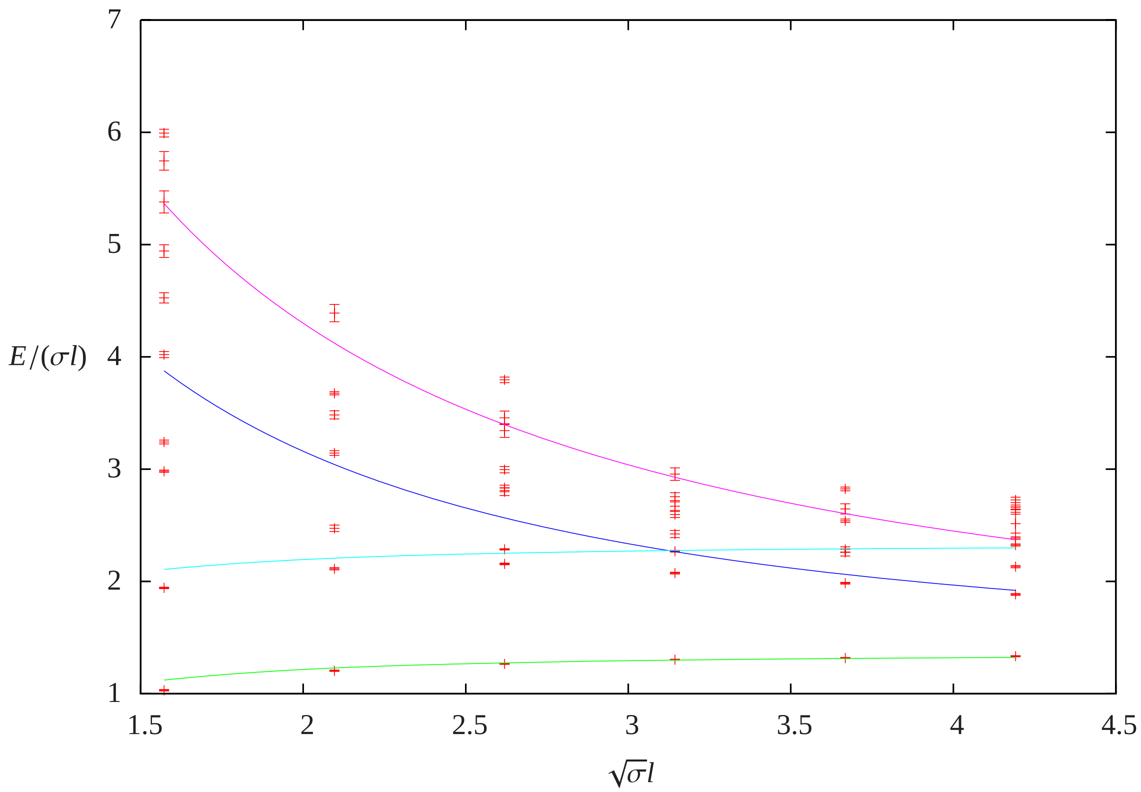
Group: $SU(4)$, $\underline{a} \simeq 0.06\,fm$, **Quantum Numbers**: $P = +$, $q = 0$ and $k = 2$



Basis: $\mathrm{Tr}U^2_{(w=1)+} + \mathrm{Tr}U^2_{(w=1)-}$, $\mathrm{Tr}(U_{(w=1)+})^2 + \mathrm{Tr}(U_{(w=1)-}{}^2)$, $\mathrm{Tr}U_{(w=2)+} + \mathrm{Tr}U_{(w=2)-}$

IV. Results: Spectrum of $SU(4)$ for $P = +$ and $k = 2$.

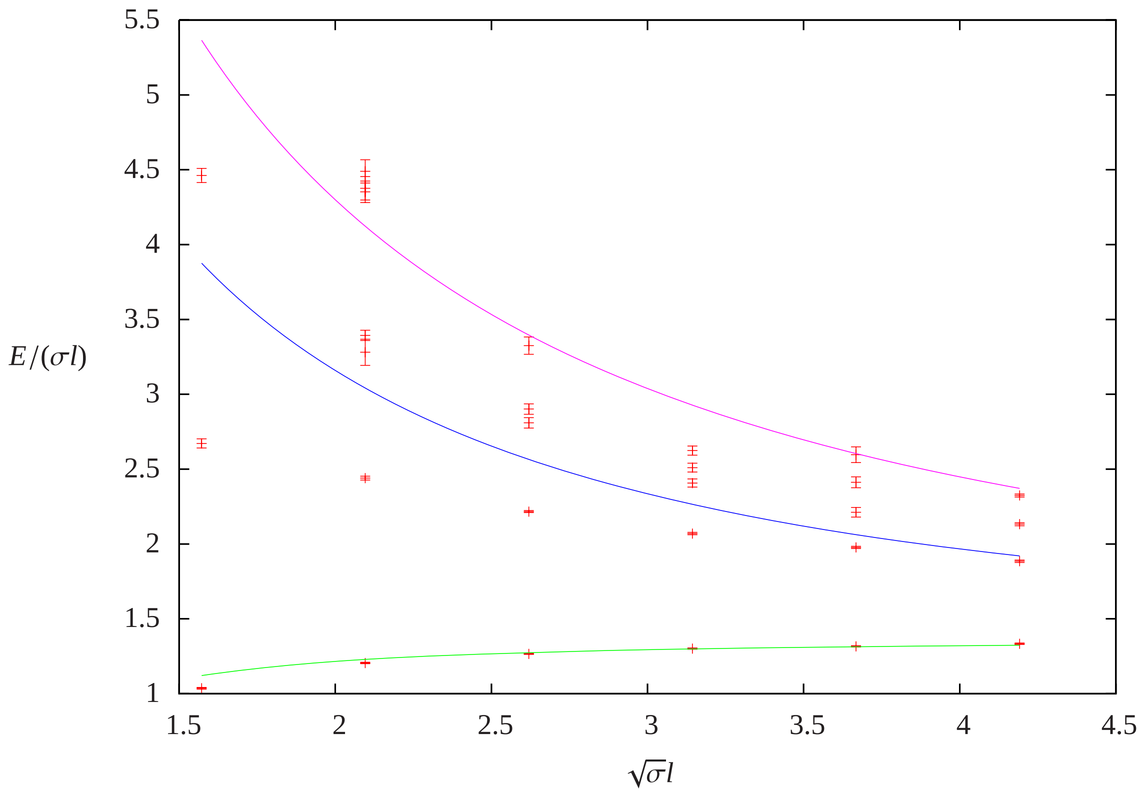
Group: $SU(4)$, $\underline{a} \simeq 0.06 fm$, **Quantum Numbers**: $P = +$, $q = 0$ and $k = 2$



Basis: $\mathrm{Tr}U^2_{(w=1)+} + \mathrm{Tr}U^2_{(w=1)-}$, $\mathrm{Tr}(U_{(w=1)+})^2 + \mathrm{Tr}(U_{(w=1)-}{}^2)$

IV. Results: Spectrum of $SU(4)$ for $P = +$ and $k = 2A$.

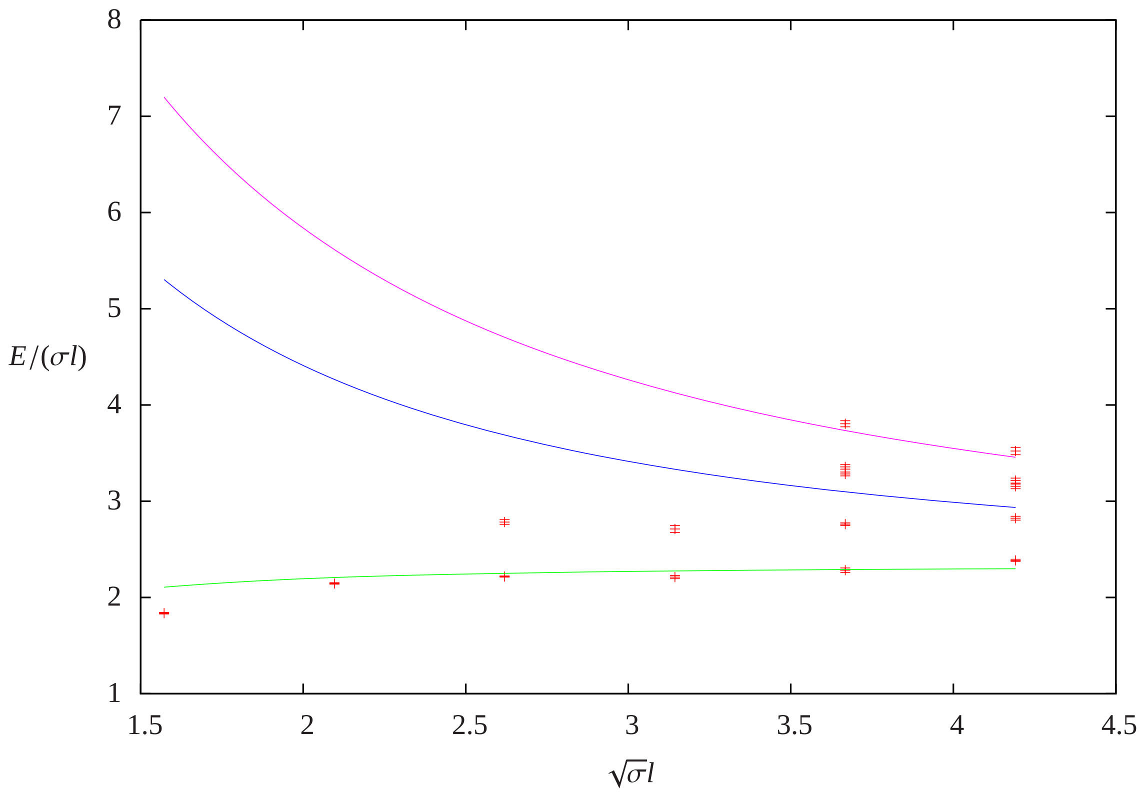
Group: $SU(4)$, $\underline{a} \simeq 0.06 fm$, **Quantum Numbers**: $P = +$, $q = 0$ and $k = 2$



Basis: $[\mathrm{Tr}U^2_{(w=1)+} - \mathrm{Tr}(U_{(w=1)+})^2] + [\mathrm{Tr}U^2_{(w=1)-} - \mathrm{Tr}(U_{(w=1)-}{}^2)]$

IV. Results: Spectrum of $SU(4)$ for $P = +$ and $k = 2S$.

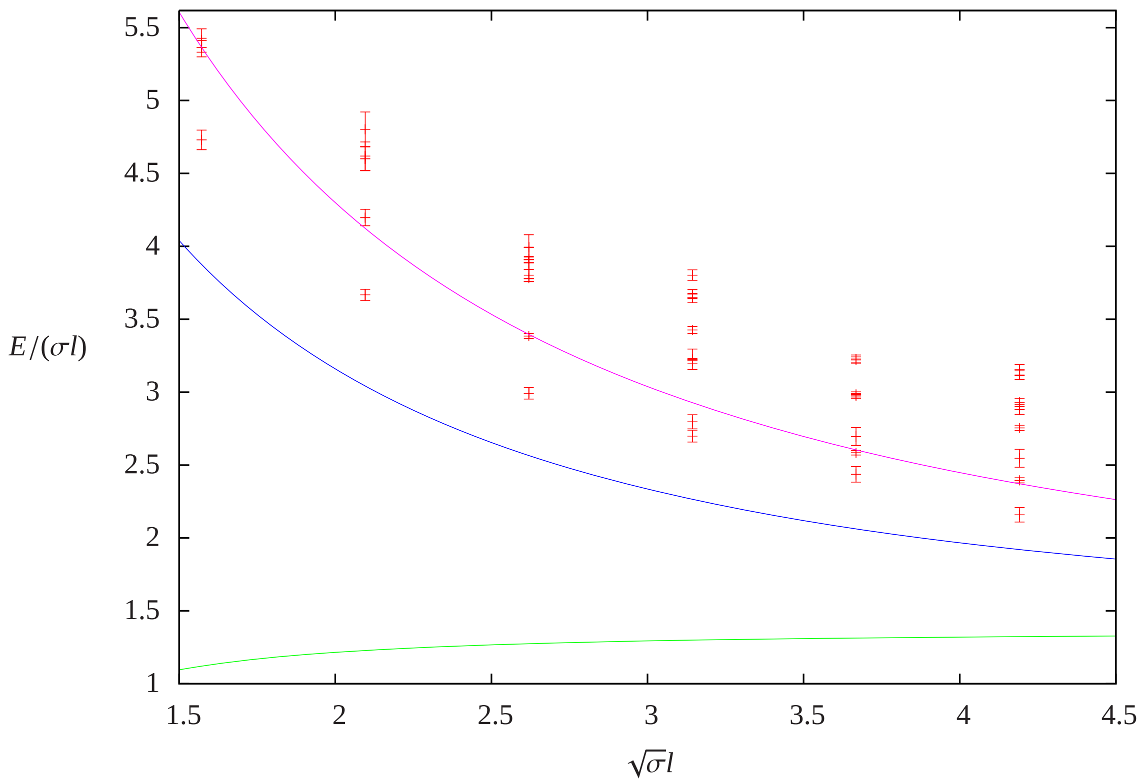
Group: $SU(4)$, $\underline{a} \simeq 0.06 fm$, **Quantum Numbers**: $P = +$, $q = 0$ and $k = 2$



Basis: $[\mathrm{Tr}U^2_{(w=1)+} + \mathrm{Tr}(U_{(w=1)+})^2] + [\mathrm{Tr}U^2_{(w=1)-} + \mathrm{Tr}(U_{(w=1)-}{}^2)]$

IV. Results: Spectrum of $SU(4)$ for $P = -$ and $k = 2$.

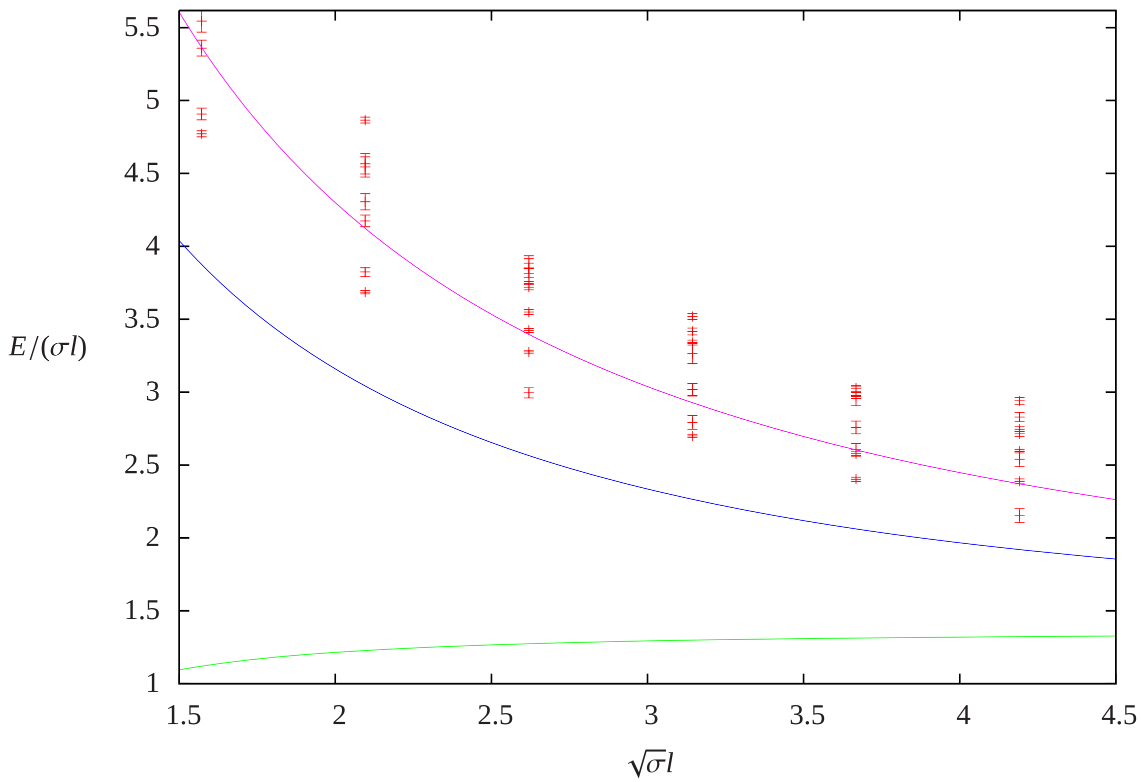
Group: $SU(4)$, $a \simeq 0.06\,fm$, **Quantum Numbers**: $P = -$, $q = 0$ and $k = 2$



Basis: $\mathrm{Tr}U^2_{(w=1)+} + \mathrm{Tr}U^2_{(w=1)-}$, $\mathrm{Tr}(U_{(w=1)+})^2 + \mathrm{Tr}(U_{(w=1)-}{}^2)$

IV. Results: Spectrum of $SU(4)$ for $P = -$ and $k = 2$.

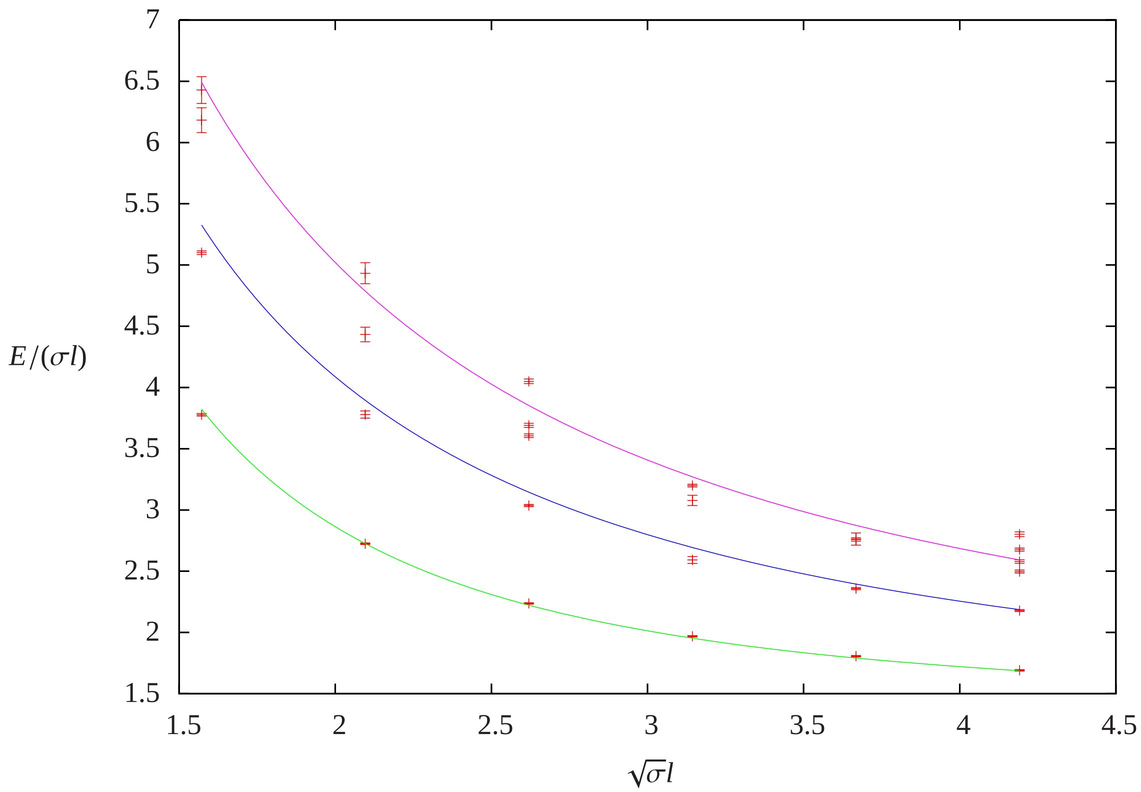
Group: $SU(4)$, $\underline{a} \simeq 0.06 fm$, **Quantum Numbers**: $P = -$, $q = 0$ and $k = 2$



Basis: $\mathrm{Tr}U^2_{(w=1)+} + \mathrm{Tr}U^2_{(w=1)-}$, $\mathrm{Tr}(U_{(w=1)+})^2 + \mathrm{Tr}(U_{(w=1)-}{}^2)$, $\mathrm{Tr}U_{(w=2)+} + \mathrm{Tr}U_{(w=2)-}$

IV. Results: Spectrum of $SU(4)$ for $P = -$, $q = 1$, $k = 2A$.

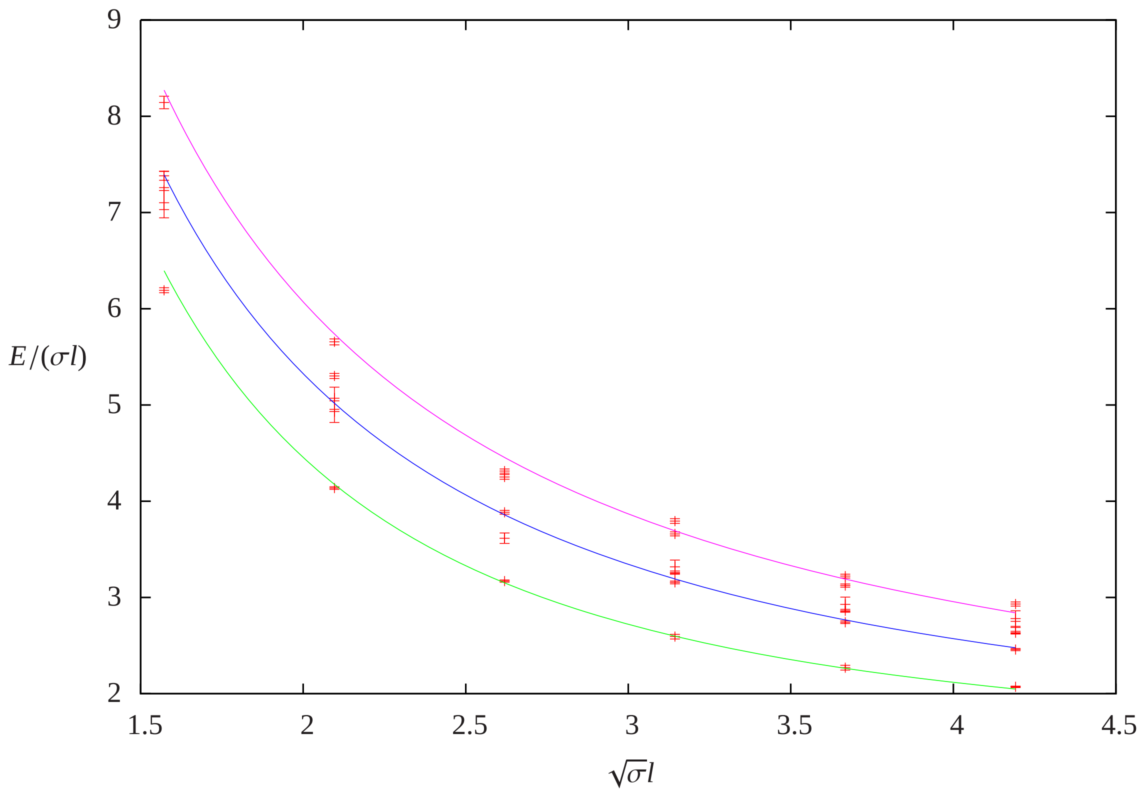
Group: $SU(4)$, $\underline{a} \simeq 0.06 fm$, **Quantum Numbers**: $P = -$, $q = 1$ and $k = 2A$



Basis: $[\mathrm{Tr}U^2_{(w=1)+} - \mathrm{Tr}(U_{(w=1)+})^2] + [\mathrm{Tr}U^2_{(w=1)-} - \mathrm{Tr}(U_{(w=1)-}{}^2)]$

IV. Results: Spectrum of $SU(4)$ for $P = -$, $q = 2$, $k = 2A$.

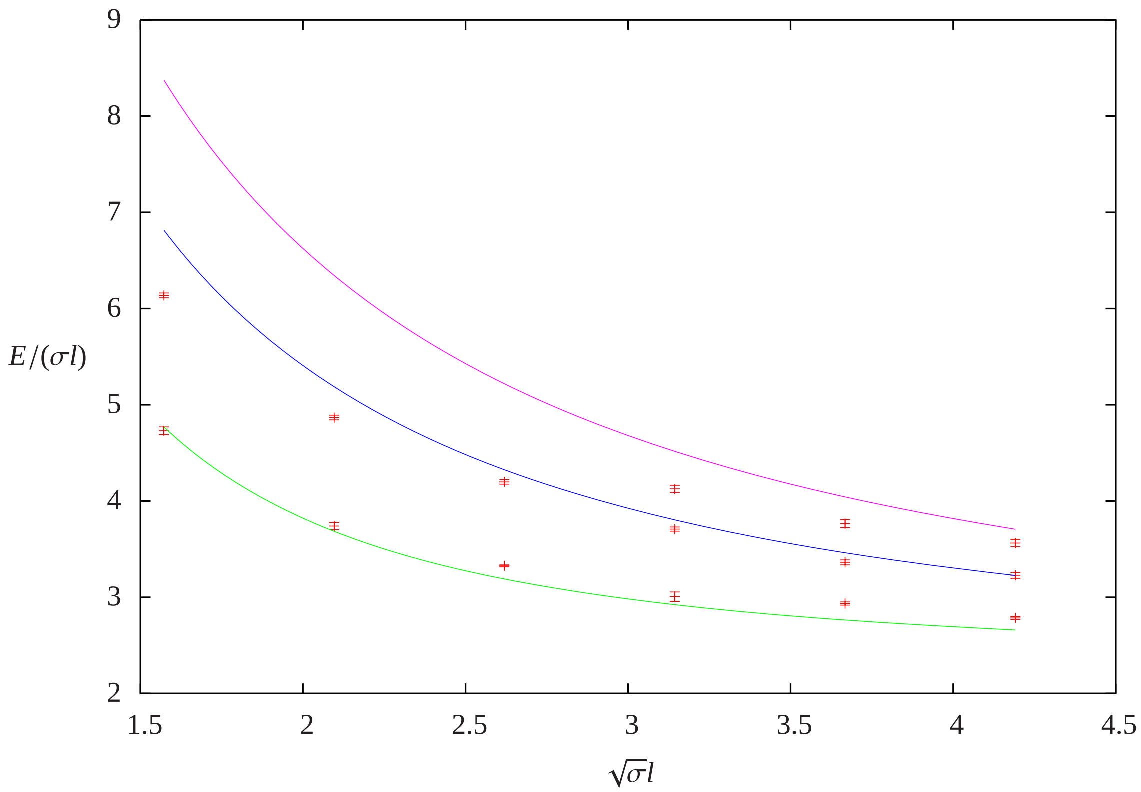
Group: $SU(4)$, $\underline{a} \simeq 0.06\,fm$, **Quantum Numbers**: $P = -$, $q = 2$ and $k = 2A$



Basis: $[\mathrm{Tr}U^2_{(w=1)+} - \mathrm{Tr}(U_{(w=1)+})^2] + [\mathrm{Tr}U^2_{(w=1)-} - \mathrm{Tr}(U_{(w=1)-}{}^2)]$

IV. Results: Spectrum of $SU(4)$ for $P = -$, $q = 1$, $k = 2S$.

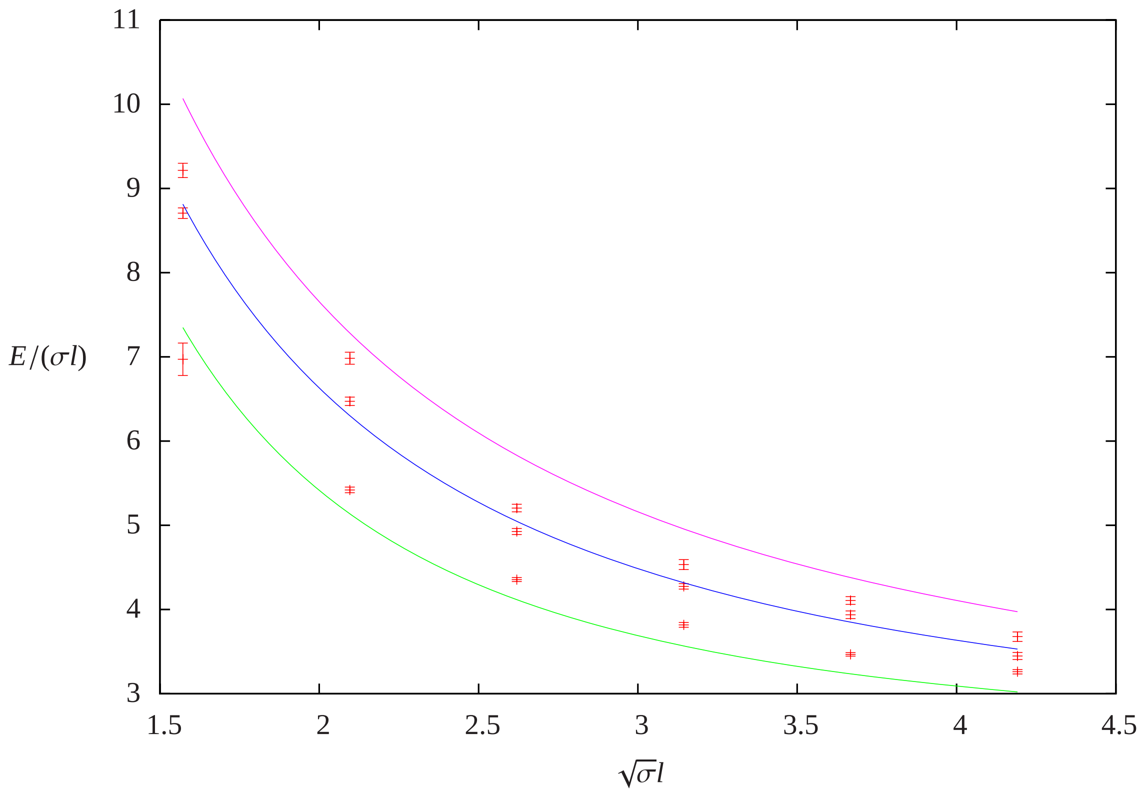
Group: $SU(4)$, $\underline{a} \simeq 0.06 fm$, **Quantum Numbers**: $P = -$, $q = 1$ and $k = 2S$



Basis: $[\mathrm{Tr}U^2_{(w=1)+} + \mathrm{Tr}(U_{(w=1)+})^2] + [\mathrm{Tr}U^2_{(w=1)-} + \mathrm{Tr}(U_{(w=1)-}{}^2)]$

IV. Results: Spectrum of $SU(4)$ for $P = -$, $q = 2$, $k = 2S$.

Group: $SU(4)$, $a \simeq 0.06\,fm$, **Quantum Numbers**: $P = -$, $q = 2$ and $k = 2S$



Basis: $[\mathrm{Tr}U^2_{(w=1)+} + \mathrm{Tr}(U_{(w=1)+})^2] + [\mathrm{Tr}U^2_{(w=1)-} + \mathrm{Tr}(U_{(w=1)-}{}^2)]$

V. Summary

- We constructed a large basis of operators characterized by the quantum numbers of **parity** P, and **winding momentum** $2\pi q/l$,
- We calculated the energies of closed k-strings in D=2+1 described by $P = \pm$ for:

  $\rightarrow$ $SU(3)$ with $k = 1, \beta = 21.0$ ($a \simeq 0.08$fm) and $q = 0, \pm1, \pm2$,

  $\rightarrow$ $SU(3)$ with $k = 1, \beta = 40.0$ ($a \simeq 0.04$fm) and $q = 0$,

  $\rightarrow$ $SU(4)$ with $k = 1, 2, \beta = 50.0$ ($a \simeq 0.06$fm) and $q = 0, \pm1, \pm2$,

  $\rightarrow$ $SU(5)$ with $k = 1, 2, \beta = 80.0$ ($a \simeq 0.06$fm) and $q = 0, \pm1, \pm2$,

  $\rightarrow$ $SU(6)$ with $k = 1, \beta = 90.0$ ($a \simeq 0.08$fm) and $q = 0$.
- We fit our data for the ground state using $E^2_{\text{fit}} = E^2_{NG} - \sigma C_p / \left(l\sqrt{\sigma}\right)^p$ and $p = 3$, and extract σ.
- Using σ we compare our results to Nambu-Goto:

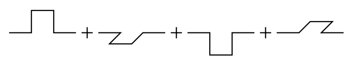
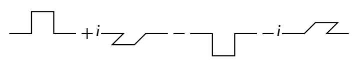
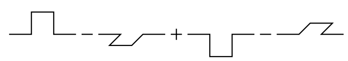
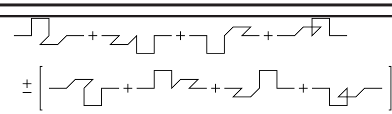
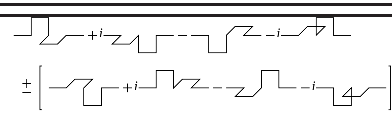
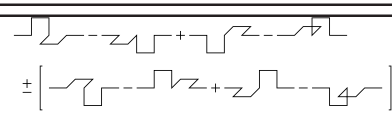
  $\rightarrow$ **Nambu-Goto is <u>VERY</u> good**

V. Summary

- k-strings know about the full $SU(N)$ gauge group.
- We observe additional states...

VI. Future project: $3 + 1$ Dimensions.

Example of Operators:

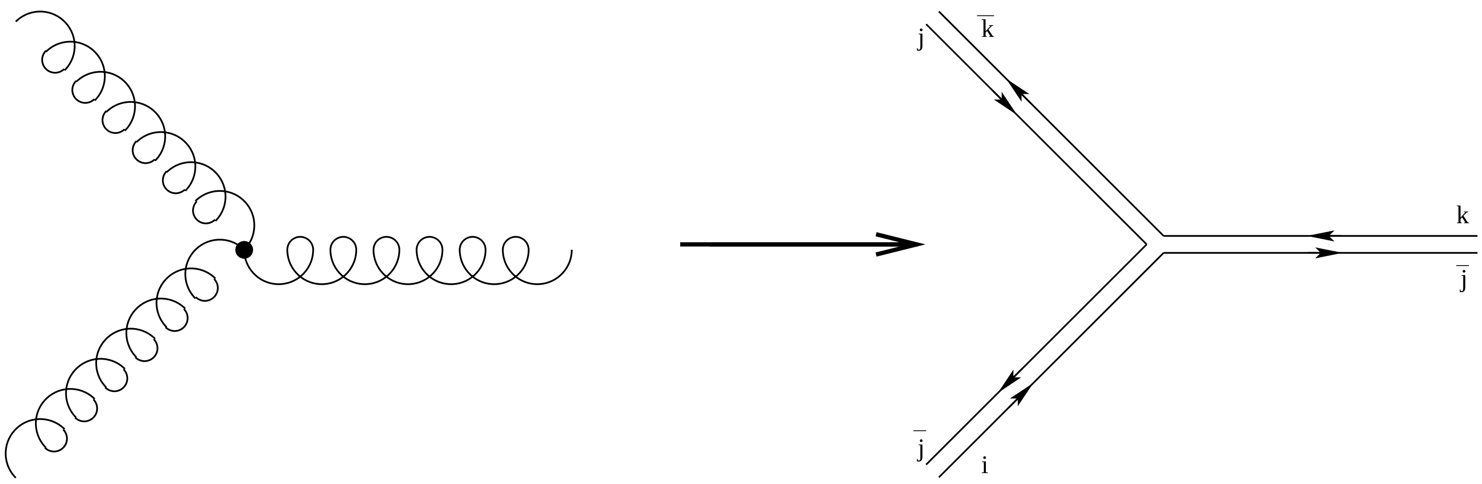
Operators	CP	J
	+	0
	+	0
	-	1
	+	2
	±	0
	±	1
	±	2

VII. Appendix 1: Large-N_c

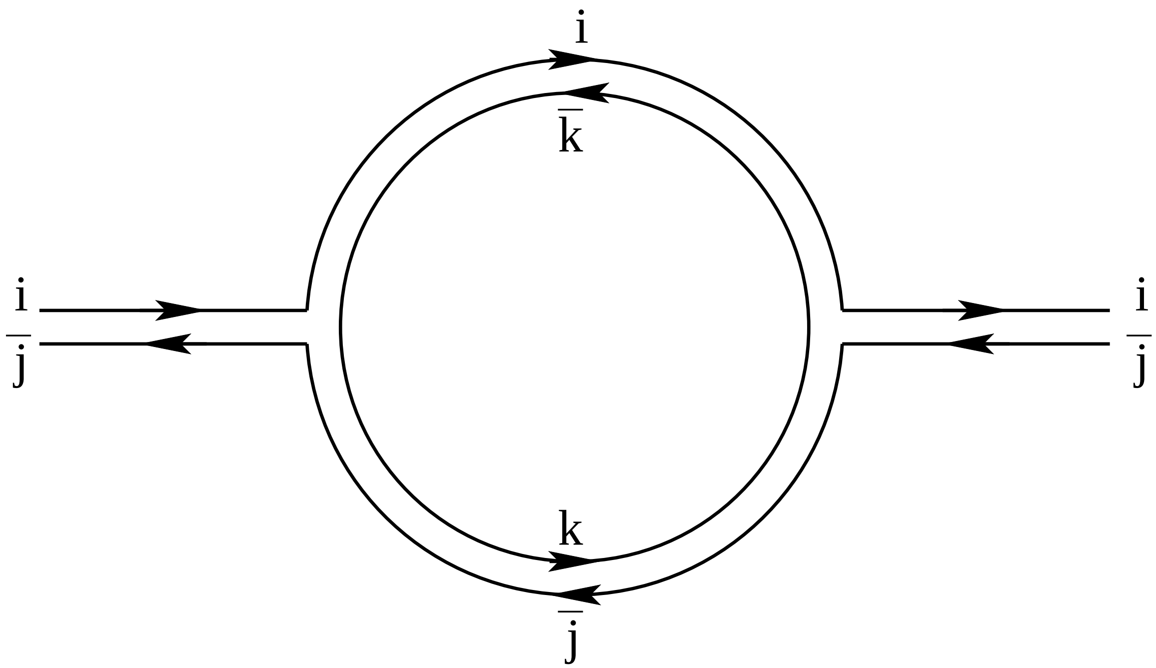
- Quantum chromodynamics is the theory of strong interactions based on the gauge group $SU(N_c = 3)$.
- Yet some of its essential properties including confinement, are poorly understood.
- It is useful to find a 'Neighbouring' field theory that one can analyze more simply.
- 'Neighbouring Theory' $\longrightarrow SU(N_c \longrightarrow \infty)$.
- Expansion parameter: $\longrightarrow \frac{1}{N_c}$.

VII. Appendix 2: ’T Hooft’s coupling

- ’T Hooft’s coupling: $\lambda = g^2 N_c$.
- ’T Hooft’s double line diagrammatic representation:

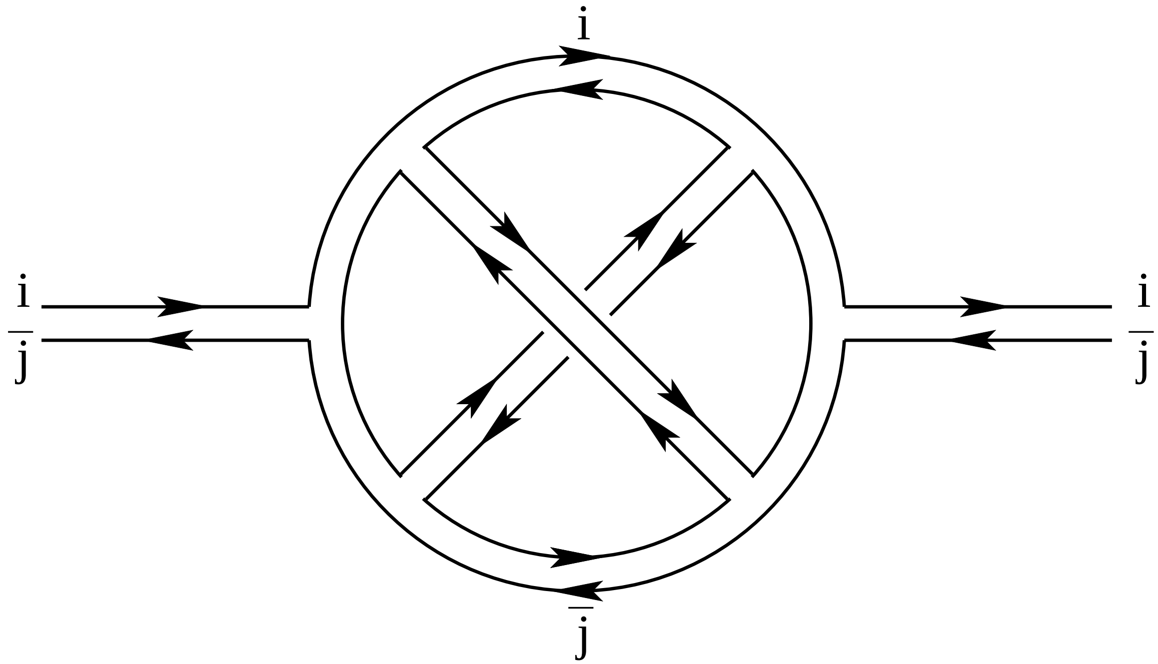


VII. Appendix 3. Planar Diagram



$$\sim g^2 \times N_c (\text{1 closed loop}) = \frac{\lambda}{N_c} \times N_c = \lambda$$

VII. Appendix 4. Non-Planar Diagram



$$\sim g^6 \times N_c(1 \text{ closed loop}) = \frac{\lambda^3}{N_c^3} \times N_c = \frac{\lambda^3}{N_c^2} \xrightarrow{N_c \to \infty} 0$$

VII. Appendix 2: Contribution of Operators