

Vectors and Matrices

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Outline

- 1) Vector spaces and vectors
 - 1.1) Vectors in \mathbb{R}^n
 - 1.2) Vector spaces
 - 1.3) Linear combinations, linear independence
 - 1.4) Basis and dimension
- 2) Vector in \mathbb{R}^n , geometrical applications
 - 2.1) Scalar product in \mathbb{R}^n
 - 2.2) Vector product in \mathbb{R}^3
 - 2.3) Some geometry, lines and planes
- 3) Linear maps and matrices
 - 3.1) Linear maps
 - 3.2) Matrices and their properties
 - 3.3) Row/column operations, Gaussian elimination
 - 3.4) Change of basis
 - 3.5) Dual vector space
- 4) Systems of linear equations
 - 4.1) General structure of solutions
 - 4.2) Solution by “explicit calculation”
 - 4.3) Solution by row reduction
- 5) Determinants
 - 5.1) Definition of determinant
 - 5.2) Properties and calculation
 - 5.3) Applications
- 6) Scalar products
 - 6.1) Real and hermitian scalar products
 - 6.2) Orthonormal basis, Gram-Schmidt procedure
 - 6.3) Adjoint linear map

- 6.4) Orthogonal and unitary maps
- 7) Eigenvectors and eigenvalues
 - 7.1) Basic concepts
 - 7.2) Characteristic polynomial
 - 7.3) Diagonalization of matrices
 - 7.4) Applications

Literature

A large number of textbooks on the subject can be found, varying in style from “Vectors and Matrices for Dummies” to hugely abstract treatises. I suggest a trip to the library in order to pick one or two books in the middle ground that you feel comfortable with. Below is a small selection which have proved useful in preparing the course.

- *Mathematical Methods for Physics and Engineering*, K. F. Riley, M. P. Hobson and S. J. Bence, CUP 2002.
This is the recommended book for the first year physics course which covers vectors and matrices and much of the other basic mathematics required. As the title suggests it is a “hands-on” book, strong on explaining methods and concrete applications, rather weaker on presenting a coherent mathematical exposition.
- *Linear Algebra*, S. Lang, Springer, 3rd edition.
A nice mathematics books, written by a famous Mathematician and at a fairly informal level, but following the mathematical logic of the subject.
- *Linear Algebra. An Introductory Approach*, C. W. Curtis, Springer 1996.
A useful mathematics book but, despite the understating title, more formal than Lang.
- *Linear Algebra*, K. Jänich, Springer 1994.
A mathematics book but with an attempt at intuitive presentation (many figures) and some connections to physics.

Problem sheets

- Problem set 1 (Vectors, vector spaces and geometry)
- Problem set 2 (Matrices, linear equations and linear maps)
- Problem set 3 (Determinants and scalar products)
- Problem set 4 (Eigenvectors, eigenvalues and diagonalization)