Groups and Representations

MMathPhys/MScMTP Lecture Course

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Outline

1) Groups and representations
2) Finite groups
3) Lie groups
    a) Lie groups
    b) Lie algebras
4) Examples
    a) Lorentz and Poincaré group
    b) $SU(n)$ and tensor methods
    c) $SO(n)$, spinor representations
5) Classification of simple Lie algebras
6) Representations and Dynkin formalism

Course information

Modern theories of particle physics are based on symmetry principles and use group theoretical tools extensively. Besides the standard Poincaré/Lorentz invariance of all such theories, one encounters internal (continuous) groups such as $SU(3)$ in QCD, $SU(5)$ and $SO(10)$ in grand unified theories (GUTs), and E_6 and E_8 in string theory. Discrete groups also play an important role in particle physics model building, for example in the context of models for fermion masses. The main purpose of this course is to develop the understanding of groups and their representations, including finite groups and Lie groups. Emphasis is placed on a mathematically satisfactory exposition as well as on applications to physics and practical methods needed for "routine" calculations.

Literature

- W. Fulton and J. Harris, "Representation Theory", Springer, Graduate Texts in Mathematics.
- T. Bröcker and T. tom Dieck, "Representations of Compact Lie Groups", Springer, Graduate Texts in Mathematics.
- H. Boerner, "Representations of groups: with special consideration for the needs of modern physics", North-Holland Pub. Co., 1963.
- R. Slansky, "Group Theory for Unified Model Building", *Phys. Rep.* **79** (1981) 1.
- B. G. Wybourne, "Classical Groups for Physicists", Wiley (1974).
- M. Gourdin, "Unitary Symmetries", North Holland (1967).
- R. N. Cahn, "Semi-Simple Lie Algebras and Their Representations", Benjamin/Cummings (1984).
- H. Georgi, "Lie Algebras in Particle Physics", Benjamin/Cummings (1982).
- P. Ramond, "Group Theory: A Physicist's Survey", CUP, (2010).

Prerequisites

The course assumes knowledge of

- linear algebra

- the groups $SO(3)$ and $SU(2)$, as, for example, encountered in the context of quantum mechanics

It will help to have come across the following subjects

- some basic differential geometry
- the $SU(3)$ quark model
- gauge-field theories