Collisionless and Collisional Tearing Mode in Gyrokinetics

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Introduction

We study magnetic reconnection in the presence of a guide field using gyrokinetic code AstroGK.

Guide field reconnections (or component reconnections) are often observed in astrophysical situations, not to mention in fusion experiments.

Gyrokinetics includes various kinetic effects, such as FLR, electron inertia, tensorial pressures, important to understand collisionless magnetic reconnection.

Gyrokinetics assumes strong guide field. Reconnection process may differ in gyrokinetics from anti-parallel reconnection (no guide field). Understanding of gyrokinetic reconnection complements that by weak guide field cases, and contributes to gain insights for how kinetic processes play roles in magnetic reconnection.

Even though reconnection process occurs in collisionless situation, collisions are still important to smooth out velocity space structures.

Relation between microscopic collisions and macroscopic resistivity is not trivial. We also intensively investigate the relation of them.
AstroGK: Basic equations

The distribution function of particles is given by

\[ f = \left(1 - \frac{q\phi}{T_0}\right) f_0 + h, \]

where

\[ f_0 = \frac{n_0/\sqrt{\pi}v_{th}^3}{\exp(-v^2/v_{th}^2)} \]

is the Maxwellian, and the thermal velocity is given by

\[ v_{th} = \sqrt{2T_0/m}. \]

The equations to solve are the gyrokinetic equation for

\[ h = h(R, V_\perp, V_\parallel), \]

\[
\frac{\partial h}{\partial t} + V_\parallel \frac{\partial h}{\partial Z} + \frac{1}{B_0} \{\langle \chi \rangle_R, h\} - \langle C(h)\rangle_R = q \frac{f_0}{T_0} \frac{\partial \langle \chi \rangle_R}{\partial t},
\]

(1)

\[ \chi = \phi - \mathbf{v} \cdot \mathbf{A} \]

and the field equations for \( \phi(r) \), \( A_\parallel(r) \), and \( \delta B_\parallel(r) \),

\[
\sum_s \left[ -\frac{q_s^2 n_0 s \phi}{T_0 s} + q_s \int \langle h_s \rangle_r \mathbf{v} d\mathbf{v} \right] = 0,
\]

(2)

\[
\nabla^2_\perp A_\parallel = -\mu_0 \sum_s q_s \int \langle h_s \rangle_r v_\parallel d\mathbf{v}
\]

(3)

\[
B_0 \nabla_\perp \delta B_\parallel = -\mu_0 \nabla_\perp \cdot \sum_s \int m \mathbf{v}_\perp \mathbf{v}_\perp h_s \rangle_r d\mathbf{v}.
\]

(4)
AstroGK: Normalization

Time and Space

\[ t = \frac{a_0}{v_{\text{th}0}} \hat{t} \quad (v_{\text{th}0} = \sqrt{2T_{00}/m_0}), \quad z = a_0 \hat{z}, \quad x = \rho_0 \hat{x}. \]  

(5)

Species temperature, mass, charge

\[ m_s = m_0 \hat{m}_s, \quad T_{0s} = T_{00} \hat{T}_{0s}, \quad q_s = q_0 \hat{q}_s. \]  

(6)

Fields

\[ \frac{a_0}{\rho_0} \frac{q_0 \phi}{T_{00}} = \hat{\phi}, \quad \frac{a_0}{\rho_0} v_{\text{th}0} \frac{q_0 A_{||}}{T_{00}} = \hat{A}_{||}, \quad \frac{a_0}{\rho_0} \delta B_{||} = B_0 \delta \hat{B}_{||}. \]  

(7)

Distribution function

\[ h_s = \frac{\rho_0}{a_0} f_{0s} \hat{h}_s, \quad (f_{0s} = \frac{1}{\pi^{3/2}} \frac{n_{0s}}{v_{\text{th},s}^3} e^{-v^2/v_{\text{th},s}^2}). \]  

(8)
Collision Operator

Recently, linearized collision operators for gyrokinetic simulations, which satisfies physical requirements are established and implemented in AstroGK. [Abel et al, Phys. Plasmas 15, 122509 (2008), Barnes et al, submitted to Phys. Plasmas (2008).]

The operators are the pitch-angle scattering (Lorentz), the energy diffusion, and moments conserving corrections to those operators for like-particle collisions. Electron-ion collisions consists of pitch angle scattering by background ions and ion drag are also included.

We, here, mainly discuss the electron-ion collisions since it contributes to resistivity. The operator is given by (in Fourier space)

\[
C_{ei}(h_{e,k}) = \nu_{ei} \left( \frac{v_{th,e}}{V} \right)^{3} \left( \frac{1}{2} \frac{\partial}{\partial \xi} (1 - \xi^2) \frac{\partial h_{e,k}}{\partial \xi} - \frac{1}{4} (1 + \xi^2) \frac{V^2}{v_{th,e}^2} k_\perp^2 \rho_e^2 h_{e,k} + \frac{2V_{\parallel} J_0(\alpha_e) u_{\parallel,i,k}}{v_{th,e}^2} f_{0e} \right)
\]

We examine how this collision operator relates with resistivity which decays the current.
Spitzer Resistivity

From the fluid picture current decays due to collisional resistivity as

$$\frac{\partial J}{\partial t} = -\frac{\eta}{\mu_0} k^2 J,$$

(10)

and the decay rate is $\tau^{-1}_{\text{decay}} = (\eta/\mu_0) k^2$. Using the Spitzer resistivity given by

$$\eta = \frac{m_e}{1.98 \tau_e n_e e^2},$$

(11)

where $\tau_e = 3\sqrt{\pi}/(4\nu_{ei})$, the decay rate is casted into the following form,

$$\tau^{-1}_{\text{decay}} = C\nu_{ei} (d_e k)^2$$

(12)

where the constant $C = 4/(1.98 \times 3\sqrt{\pi}) \sim 0.380$. We will determine $C$ from numerical simulations.
Resistivity Estimate

We start the test with the following parameters $\nu_{ei} = 10$, $\beta = 10^{-4}$, $k_{\perp} = 1$, $m_i = 1$, $m_e = 10^{-4}$, $n_{0i} = 1$, $n_{0e} = 1$, $q_i = 1$, $q_e = -1$ $T_{0i} = 1$, $T_{0e} = 1$, and $u_{\parallel,i}(t = 0) = -1$, $u_{\parallel,e}(t = 0) = 0$ (ion drag is off). For such a small $\beta$ value, the magnetic fluctuation and its temporal change is very small, and we may approximate

$$\frac{\partial h_e}{\partial t} = C_{ei}, \quad \frac{\partial h_i}{\partial t} = 0.$$ (13)

![Graph showing Resistivity Estimate](image)
Effects of e-e collisions and ion drag

We include ee collisions (Lorentz and energy diffusion) in addition to ei collision. Estimated $C'$ is about $C' \sim 1.15 \, (w/ \, L)$, $C' \sim 2 \, (w/ \, L+E)$ ($C' \sim 0.84 \, w/o \, ee \, collisions$)

Even if we include the ion drag effect, current decay rate does not change as long as $J \gg J_{\infty}$.
Parameter Dependence of Resistivity

Resistivity is proportional to $\nu_{ei}$ and inversely proportional to $\beta$ as expected.

- Linearity to $\nu_{ei}$ is exactly held
- By fitting $1/\beta$ function, we obtain $C \sim 0.426$, which is fairly close to Spitzer ($C \sim 0.380$).
- Conservation of momentum was relatively bad in AstroGK, which causes overestimate of resistivity. This is fixed very recently.
- $1/\beta$ does not fit for small $\beta \lesssim 0.001$. 
Collisional Tearing Mode Theory

Time Scales

- Hydromagnetic time scale: \( \tau_H \equiv \tau_A/(kLB_0 y/B_0) \)
- Resistive time scale: \( \tau_R \equiv \mu_0 L^2/\eta \)

Assumptions

- Time scale separations
  \[
  1/\tau_R \ll \omega \ll 1/\tau_H \tag{14}
  \]
- Scale separations
  \[
  \ell_\eta \ll a \tag{15}
  \]
  where \( \ell_\eta \) is the resistive layer width

Dispersion relation

\[
\Delta' a = -\frac{\pi}{8} \gamma^{5/4} \tau_H^{1/2} \tau_R^{3/4} \frac{\Gamma((\lambda^{3/2} - 1)/4)}{\Gamma((\lambda^{3/2} + 5)/4)}
\]

\[
\lambda = \gamma \tau_H^{2/3} \tau_R^{1/3}
\]
Collisionless Tearing Mode Theory


\[
\frac{\Gamma^2 \rho_s}{G(\Gamma/\sqrt{\beta})} + \frac{2}{\Delta'} = \frac{2G(\Gamma/\sqrt{\beta})}{\pi \Gamma} \delta
\]  

(17)

\[\Gamma = \gamma \tau_A / (\rho_s k), \quad \delta^2 = d_e^2 + \eta / (\mu_0 \gamma), \quad \beta = \mu_0 (\gamma_e p_e^{(0)} + \gamma_i p_i^{(0)}) / (B^{(0)})^2, \]

\[G(x) = (\sqrt{x}/2)(\Gamma(1/4 + x/4)/\Gamma(3/4 + x/4)).\]


\[
\frac{Q^2 d_\beta}{G(Q/c_\beta)} + \frac{2}{\Delta'} = \frac{2G(Q/c_\beta)d_e}{\pi Q}
\]  

(18)

\[Q = \gamma \tau_A / (d_\beta k), \quad d_\beta = c_\beta d_i, \quad c_\beta = \sqrt{\beta/(1 + \beta)}, \quad \beta = \mu_0 \gamma_e p_e^{(0)} / (B^{(0)})^2.\]

If \(\beta \ll 1\), \(d_\beta \rightarrow \rho_s\), and \(Q \rightarrow \Gamma\). Thus, for cold ion and collisionless limit, the dispersion relation is same as Mirnov’s.

Later Fitzpatrick and Porcelli removed \(G\) in the RHS by taking into account gyroviscous cancellation. [PoP, 14, 049902 (2007).]
Equilibrium profile

\[ h_e = h_{e,0} \cosh^{-2}\left(\frac{x}{a}\right) 2V_\parallel \]  

(19)

yields the fields \( \phi = \delta B_\parallel = 0 \),

\[ A_\parallel = A_0 \cosh^{-2}\left(\frac{x}{a}\right) \]  

(20)

\( h_{e,0} \) (proportional to \( A_0 \)) is determined such that it gives a desired \( B_{0y} \).

Stability Index \( \Delta' \)

\[ \Delta' a = 2 \left( \frac{6\bar{k}^2 - 9}{\bar{k}(\bar{k}^2 - 4)} - \bar{k} \right) \]  

(21)

\[ \bar{k}^2 = a^2 k^2 + 4 \]
Simulation Setting

Parameters

\[ n_{0e} = n_{0i} = 1, \quad T_{0e} = T_{0i} = 1, \quad -q_e = q_i = 1 \]  
\[ m_e = 10^{-2}, \quad m_i = 1, \]  
\[ \beta_0 = 0.3 \]  

which yield the following spatial scales

\[ \rho_i = 1 \quad \rho_e = 0.1 \]  
\[ d_i = 1.8 \quad d_e = 0.18. \]  

Interpretation of AstroGK time scales

\[ \tau_H / t_0 = \sqrt{\frac{n_{0i} m_i \beta_0}{kB_0y}} \]  
\[ \tau_R / t_0 = 2.63 \nu_{ei}^{-1} a^2 \frac{n_{0e} q_e^2}{m_e \beta_0} \]
Results

Lines are for different kinetic effects ($a/\rho_i = 5$ and $a/\rho_i = 50$)

- Growth rate is independent of collision frequency if it is small
- Growth rate does not scale like resistive scaling due to kinetic effect
- We must further reduce kinetic effect to observe resistive scaling scaling
- Collisionless scaling qualitatively fit to numerical results
- Red line does not change by ion temperature (result not shown)
Summary

- We have confirmed that e-i collisions in addition to full e-e collisions yield expected macroscopic behavior. The resistivity is quantitatively same as Spitzer’s value.
- We have performed collisionless and collisional tearing mode simulations, and have scanned for \( \nu_{ei} \).
- We have observed transition from collisional regime (collision dependent growth rate) to collisionless regime (collision independent growth rate).
- Due to kinetic effects, growth rate does not fit to resistive scaling even in the collisional regime. Further decrease of kinetic effect (larger \( a/\rho_i \)) needed.
- We have also compared the results with collisionless scaling by Fitzpatrick’s and Mirnov’s. Numerical results agree with the theories qualitatively, but are different by a factor.