MHD turbulence in a box

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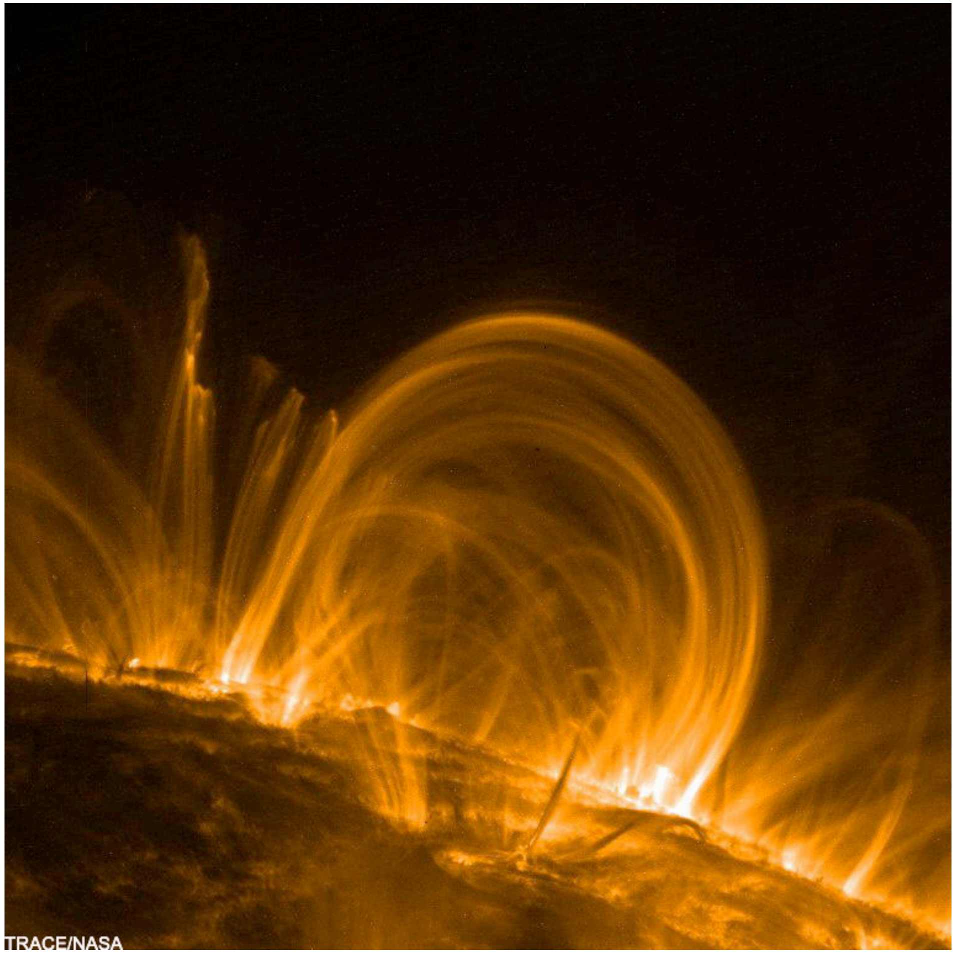
MHD box



Reduced MHD

- Equation for the Fourier transform of the Elsasser variable:

$$\dot{a}_k^{\pm} = \epsilon \sum_{1,2} V_{k12} e^{\pm 2 i k_{\parallel 1} t} a_1^{\mp} a_2^{\pm} \delta_{12}^k,$$

$$\boxed{k_\parallel \ll k_\perp}$$

where

$$a_{1,2}^{\pm} = a^{\pm}(\mathbf{k}_{1,2}),$$

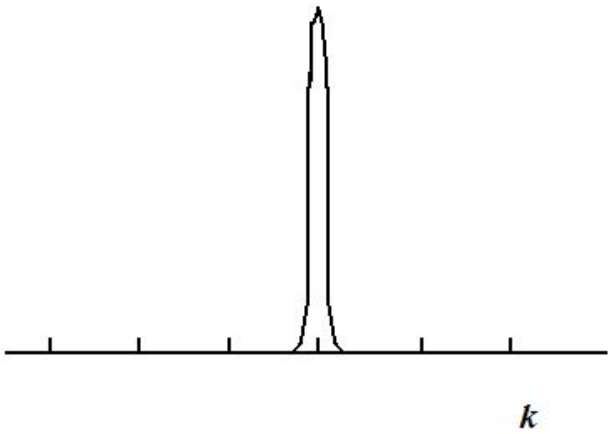
$$V_{k12} = V(\mathbf{k}_\perp, \mathbf{k}_{\perp 1}, \mathbf{k}_{\perp 2}) = \frac{(\mathbf{k}_\perp \times \mathbf{k}_{\perp 2})(\mathbf{k}_{\perp 1} \times \mathbf{k}_{\perp 2})_\parallel}{\mathbf{k}_\perp \mathbf{k}_{\perp 1} \mathbf{k}_{\perp 2}}$$

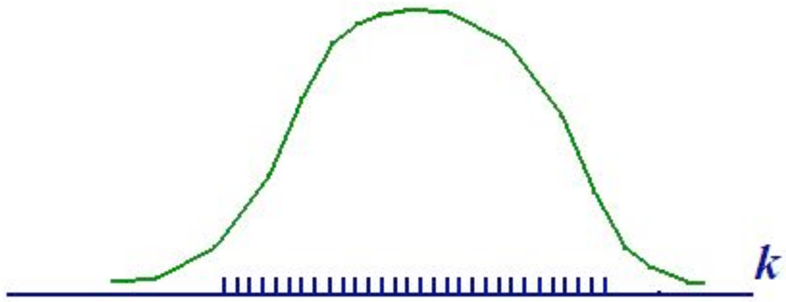
is the interaction coefficient and δ_{12}^k is the Kronecker delta

$$\delta_{12}^k = 1 \;\text{ if }\; \mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2 \qquad \text{and} \qquad \delta_{12}^k = 0 \;\text{ if }\; \mathbf{k} \neq \mathbf{k}_1 + \mathbf{k}_2.$$

Alfven waves. Exact and quasi resonances

$$\boxed{|\omega_{k_1} \pm \omega_{k_2} - \omega_{k_3}| < \delta, \quad \mathbf{k}_1 \pm \mathbf{k}_2 - \mathbf{k}_3 = 0.}$$





- Weak excitations: exact
- Stronger: quasi

$$\omega_{\rm nl} \sim 1/\tau_{\rm nl} \ll \Delta\omega = \Delta k_\parallel = 2\pi/L_\parallel,$$

$$\omega_{\rm nl} \sim 1/\tau_{\rm nl} \gg \Delta\omega = \Delta k_\parallel = 2\pi/L_\parallel,$$

Very weak turbulence

$$\omega_{\rm nl} \sim 1/\tau_{\rm nl} \ll \Delta\omega = \Delta k_{\parallel} = 2\pi/L_{\parallel},$$

- Ignore oscillating contributions

$$\dot{a}^{\pm}(\mathbf{k}_{\perp}, \mathbf{k}_{\parallel}) = \epsilon \sum_{\mathbf{k}_{\perp 1}, \mathbf{k}_{\perp 2}} V_{k12} a^{\mp}(\mathbf{k}_{\perp 1}, 0) a^{\pm}(\mathbf{k}_{\perp 2}, \mathbf{k}_{\parallel}) \delta(\mathbf{k} - \mathbf{k}_{\perp 1} - \mathbf{k}_{\perp 2}).$$

•3D waves are slaved to the purely 2D mode.
•2D component is detached from the 3D waves

Very weak turbulence

$$\dot{a}^{\pm}(\mathbf{k}_\perp, \mathbf{k}_\parallel) = \epsilon \sum_{\mathbf{k}_{\perp 1}, \mathbf{k}_{\perp 2}} V_{k12} a^{\mp}(\mathbf{k}_{\perp 1}, 0) a^{\pm}(\mathbf{k}_{\perp 2}, \mathbf{k}_\parallel) \delta(\mathbf{k} - \mathbf{k}_{\perp 1} - \mathbf{k}_{\perp 2}).$$

$$a^{\pm}(\mathbf{k}_\perp, \mathbf{k}_\parallel) = a_\perp^{\pm}(\mathbf{k}_\perp) a_\parallel^{\pm}(\mathbf{k}_\parallel),$$

$$\dot{a}_\perp^{\pm}(\mathbf{k}_\perp) = \epsilon \sum_{\mathbf{k}_{\perp 1}, \mathbf{k}_{\perp 2}} V_{k12} a_\perp^{\mp}(\mathbf{k}_{\perp 1}) a_\perp^{\pm}(\mathbf{k}_{\perp 2}) \delta(\mathbf{k} - \mathbf{k}_{\perp 1} - \mathbf{k}_{\perp 2}).$$

- This is the 2D RMHD equation

•Now find the characteristic timescale: $\omega_{\rm nl} \sim k_\perp \tilde{b}$

Condition or realizability of the slaved regime

$$\omega_{\mathrm{nl}} \sim 1/\tau_{\mathrm{nl}} \ll \Delta\omega = \Delta k_{\|} = 2\pi/L_{\|},$$

$$\omega_{\mathrm{nl}} \sim k_{\perp}\tilde{b}$$

$$\frac{\tilde{b}}{B_0} \ll \frac{1}{k_{\|} L_{\|}}$$

Wave turbulence description

$$\omega_{\rm nl} \sim 1/\tau_{\rm nl} \gg \Delta\omega = \Delta k_{\|} = 2\pi/L_{\|},$$

- Wave kinetic equation of the energy spectrum

$$\dot{n}^{\pm}_{\perp k} = \pi\epsilon^2 \int V^2_{k12}\, n^{\mp}_{\perp 1}[n^{\pm}_{\perp 2} - n^{\pm}_{\perp k}]\delta(\mathbf{k}_\perp - \mathbf{k}_{\perp 1} - \mathbf{k}_{\perp 2})\, \mathrm{d}\mathbf{k}_{\perp 1}\, \mathrm{d}\mathbf{k}_{\perp 2}.$$

now estimate the characteristic nonlinear frequency broadening

$$\omega_{\rm nl} \sim \frac{k_\perp^2 \tilde{b}^2}{k_\|}.$$

Applicability of WT:

$$\frac{k_\|}{k_\perp} \gg \frac{\tilde{b}}{B_0} \gg \frac{2\pi k_\|^{1/2}}{k_\perp L_\|^{1/2}}.$$

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Intermediate intensities

$$\frac{1}{(k_\parallel L_\parallel)} \ll \frac{\tilde{b}}{B_0} \ll \frac{1}{(k_\parallel L_\parallel)^{1/2}}$$

- In this range both assumptions lead to contradictions:

$$\omega_{\rm nl} \sim 1/\tau_{\rm nl} \ll \Delta\omega = \Delta k_\parallel = 2\pi/L_\parallel,$$

$$\omega_{\rm nl} \sim 1/\tau_{\rm nl} \ll \Delta\omega = \Delta k_\parallel = 2\pi/L_\parallel,$$

Conclusion: $\omega_{\rm nl} \sim \Delta\omega$

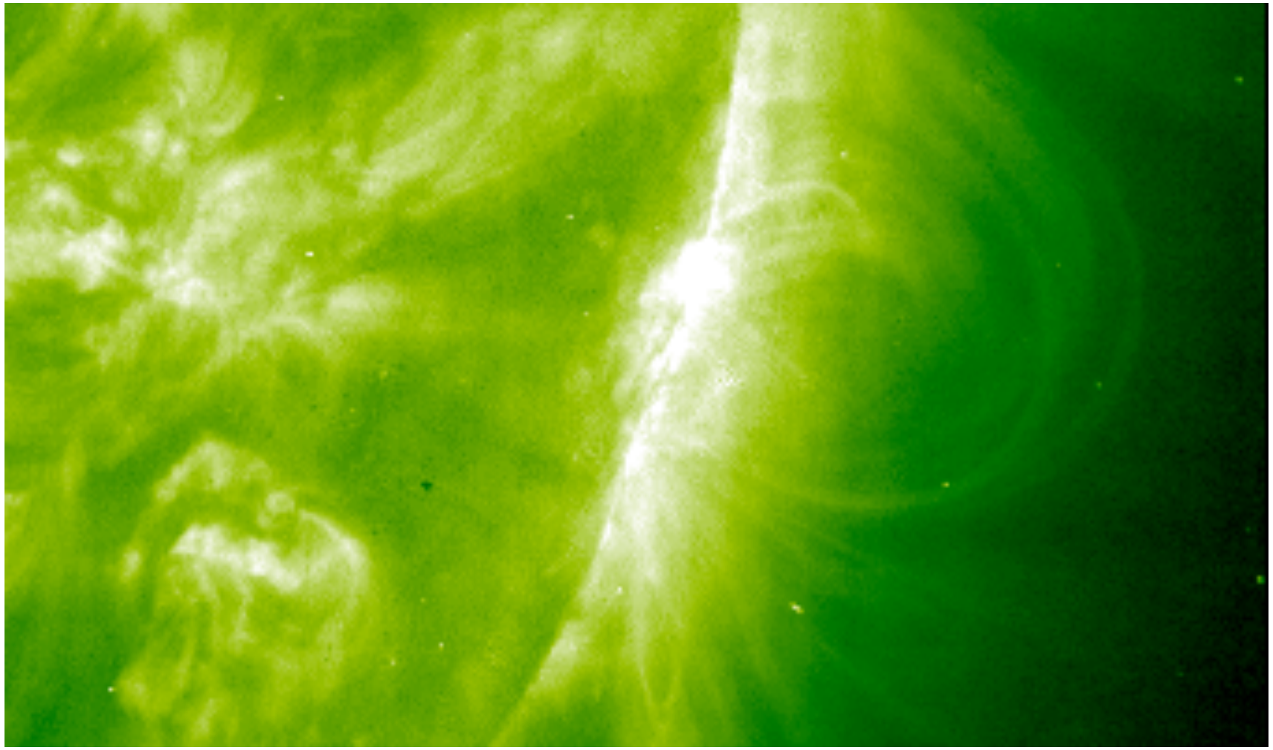
Mesoscopic MHD turbulence

- Put intemediate forcing at low kz's:
  $c_A^3 \Delta_{par}^3 k_{perp}^{-2} < \epsilon < c_A^3 \Delta_{par}^2 k_{perp}^{-2} k_{par}$.
- -> 2D turbulence cascades at a fixed rate. Excess energy is radiated away as 3D waves.
- -> waves are slaved/passive and wave energy does not cascade (energy cascade is inverse for passive scalars).
- -> accumulation of wave energy at low k until δ reaches the critical value
- -> initiation of an "avalanche" spill of the 3D wave energy toward larger k.
- -> value of δ drops below the critical value
- -> repeat the process

Summary

- Weak weak: dynamics of only modes who are in exact resonances -> enslaving to the 2D component.
- Strong weak: quasi-resonances – classical WT (Galtier et al 2000).
- Intermediate weak: two component system. Avalanches of wave energy.

MHD sandpile?



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