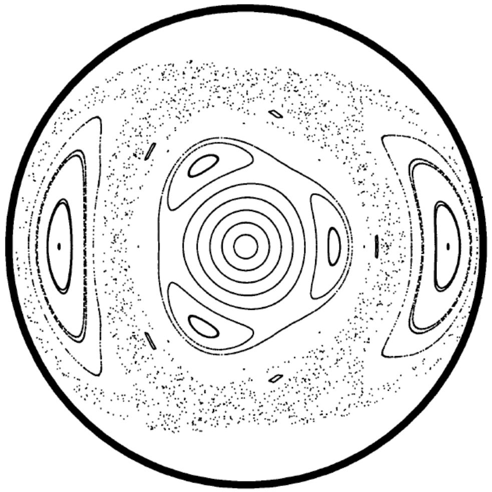
Are the field lines in Tokamaks stochastic?

Steve Cowley, UKAEA Culham and Imperial College.



Introduction.

•Is transport in tokamaks due to wandering field lines?

•Are field lines in tokamak drift-wave turbulence stochastic?

•I will extend the frozen - in - theorem for ion scale turbulence *(ITG and TEM).*

•I will suggest that the field is only stochastic on electron scales.

Transport Stochasticity and Turbulence -- τ_E

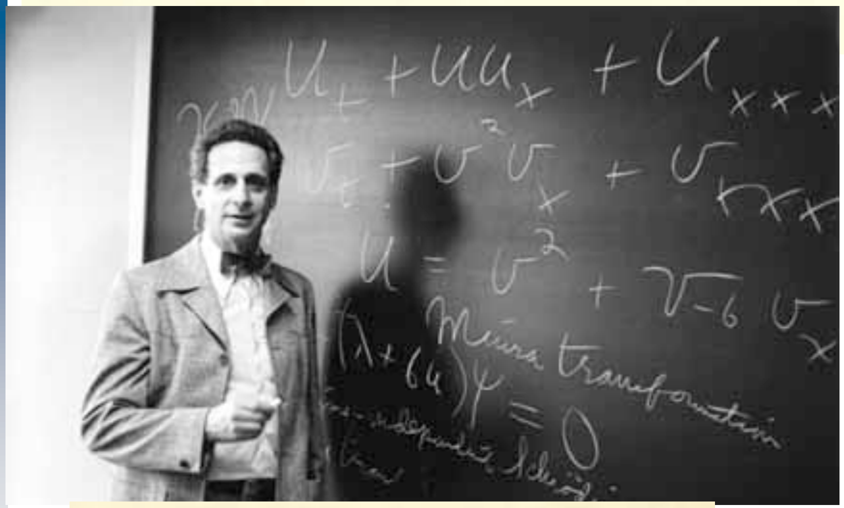
Mapping of field lines -- the need for magnetic surfaces

SOME PROPERTIES OF ROTATIONAL TRANSFORMS

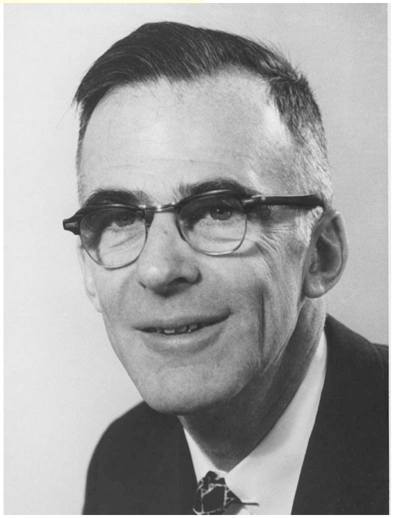
February 18, 1952 NYO-998
PM-S-5

$$\frac{1}{g}(X_{n+1} - X_n) = \sum_i g^i U^{(i)}(X_n);$$

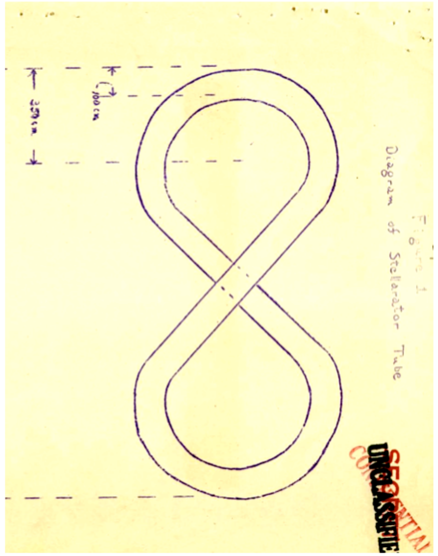
Martin D. Kruskal



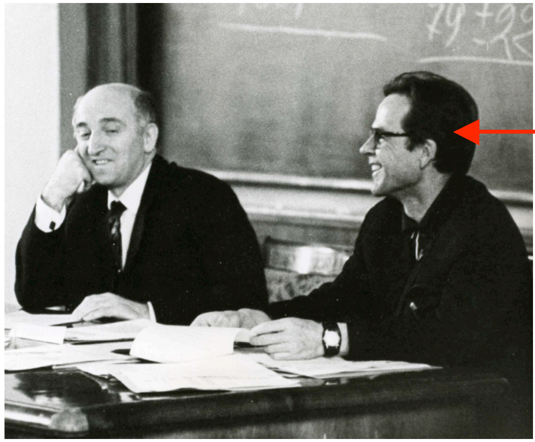
Martin D. Kruskal



Lyman Spitzer



Transport Stochasticity and Turbulence -- τ_E

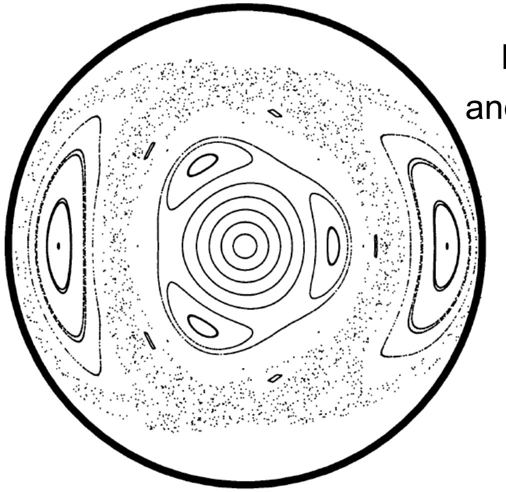


Chrikov-Taylor map

$$p_{n+1} = p_n + K\sin(\theta_n)$$
$$\theta_{n+1} = \theta_n + p_{n+1}$$

Field Line Diffusion

Island overlap.



N. **Rosenbluth**, R. Z. **Sagdeev**, J. B. **Taylor**, and. G. M. **Zaslavsky**, Nucl. Fusion 6 , 297 (1966)

$$D_B = L_c\left(\frac{\delta B_r}{B}\right)^2$$

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Zaslavsky, Rechester, White, Rosenbluth, Taylor, Chrikov, Itoh….

Test particle transport -- τ_E

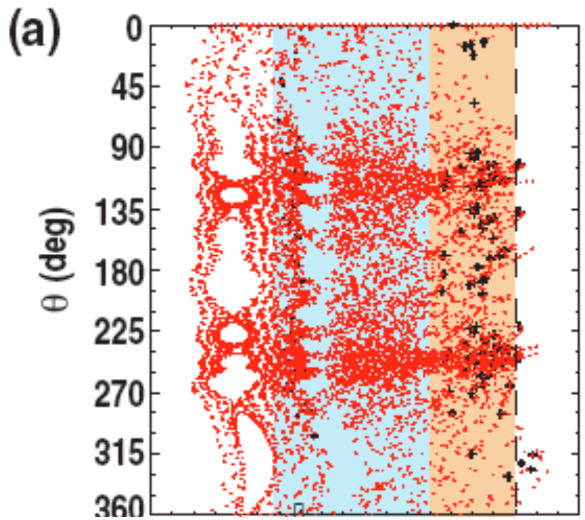
Test particles moving in prescribed stochastic fields.

T. H. Stix, Nucl. Fusion 183,353 (1978)

A. B. Rechester and M. N. Rosenbluth, Phys. Rev. Lett. ***3,*** 38 (1978)

$$\chi_r \sim v_{\|} D_B$$ *Collisionless*

$$\sim \frac{\chi_{\|}}{L_c} D_B$$ *Collisional*



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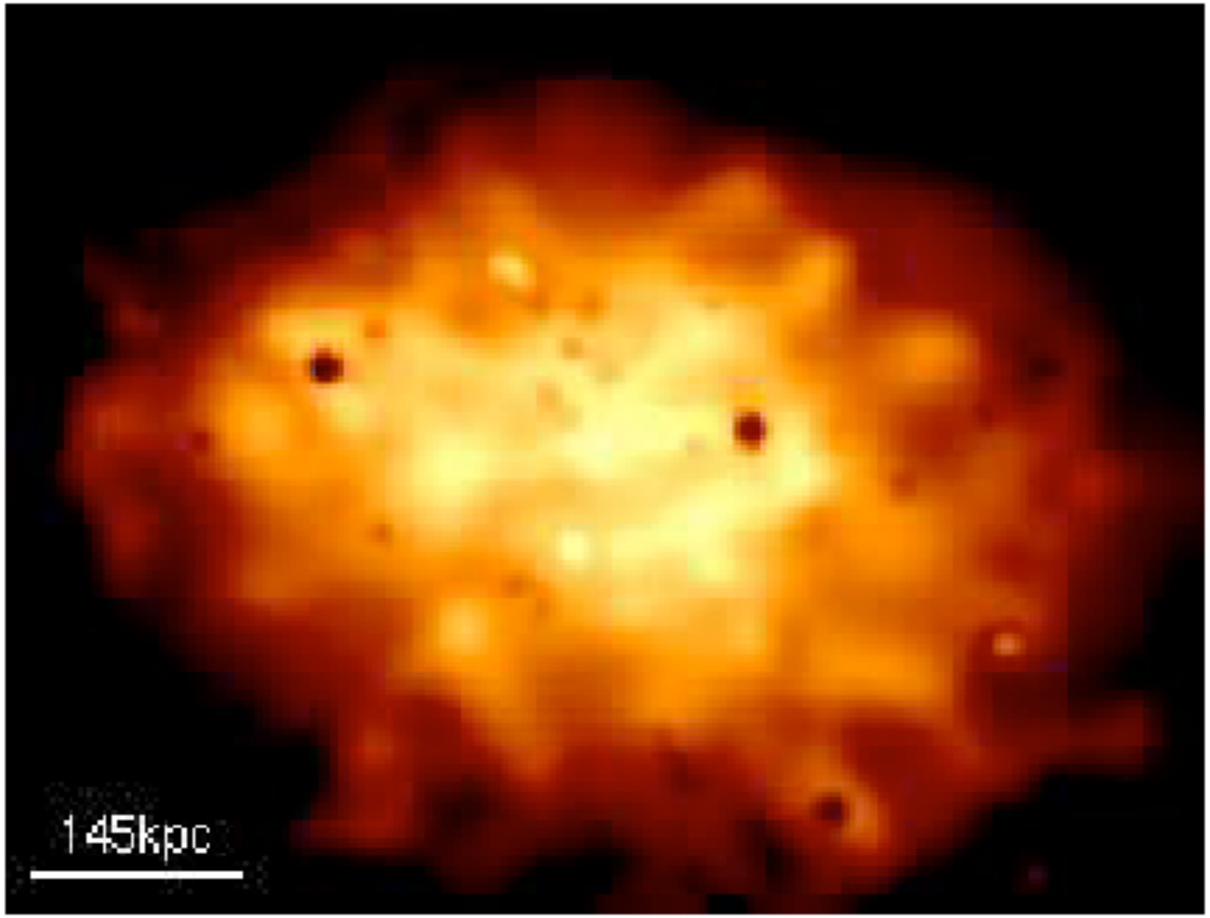
Zaslavsky, Rechester, White, Rosenbluth, Taylor, Chrikov….

Galaxy Clusters -- Confined by stochastic field?



Outside is 10KeV
Inside is <1KeV
Thermal equilibriation
Shorter than lifetime,
Magnetic confinement
With stochastic fields?

The Coma Cluster: pressure map
[Schuecker *et al.* 2004, *A&A* **426**, 387]

Electrostatic or Electromagnetic Turbulence?

Current tokamaks and ITER will have $\beta \sim \varepsilon/q^2 \sim a/q^2R$ and therefore turbulence will have strong magnetic component. We expect:

$$\frac{\delta B}{B} \sim \frac{\Delta}{qR}, \qquad k_{\|} \sim \frac{1}{qR}$$

Field line crosses a whole Eddy going once around the tokamak.

$$D_B = \frac{\Delta^2}{qR} \quad ??$$

Field line diffusion if field is incoherent each time around.

Electrostatic or Electromagnetic Turbulence?

Estimating the thermal diffusivities we get:

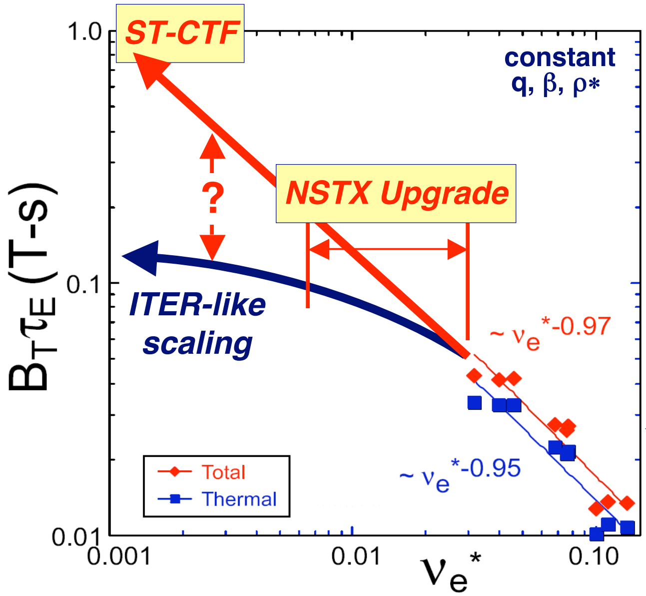
$$\chi_i \sim v_{thi} D_B$$

Ion thermal diffusivity from both cross field and along field line motion.

$$\chi_e \sim v_{the} D_B \sim 60 \chi_i$$

Electron thermal diffusivity from motion along stochastic field line.

Spherical Tokamaks



Normalized electron collisionality $\nu_e^* \propto n_e / T_e^2$

Stan Kaye et. al NF 2007.

- Future ST's are projected to operate at 10-100 times lower normalized collisionality ν^*

ITER $B\tau_E$ (e-static g-Bohm) $\propto \rho_*^{-3}\ \beta^0\ \boxed{\nu_*^{-0.14}}\ q^{-1.7}$

Petty et al., PoP, Vol. 11 (2004)

- Conventional tokamaks observe weak inverse dependence of confinement on ν^*
- NSTX observes much stronger scaling vs. ν^*
  - Does favorable scaling extend to lower ν^* ?
  - What modes dominate e-transport in ST ?

Self Consistent Transport.

The motion of the particles affects the field -- test particle transport typically ignores the correlation of motion and field fluctuations.

Electrons provide constraints on the fluctuating field.

This area is much less researched although there is some -- *e.g. Itoh and Itoh.*

Frozen in Magnetic Field.

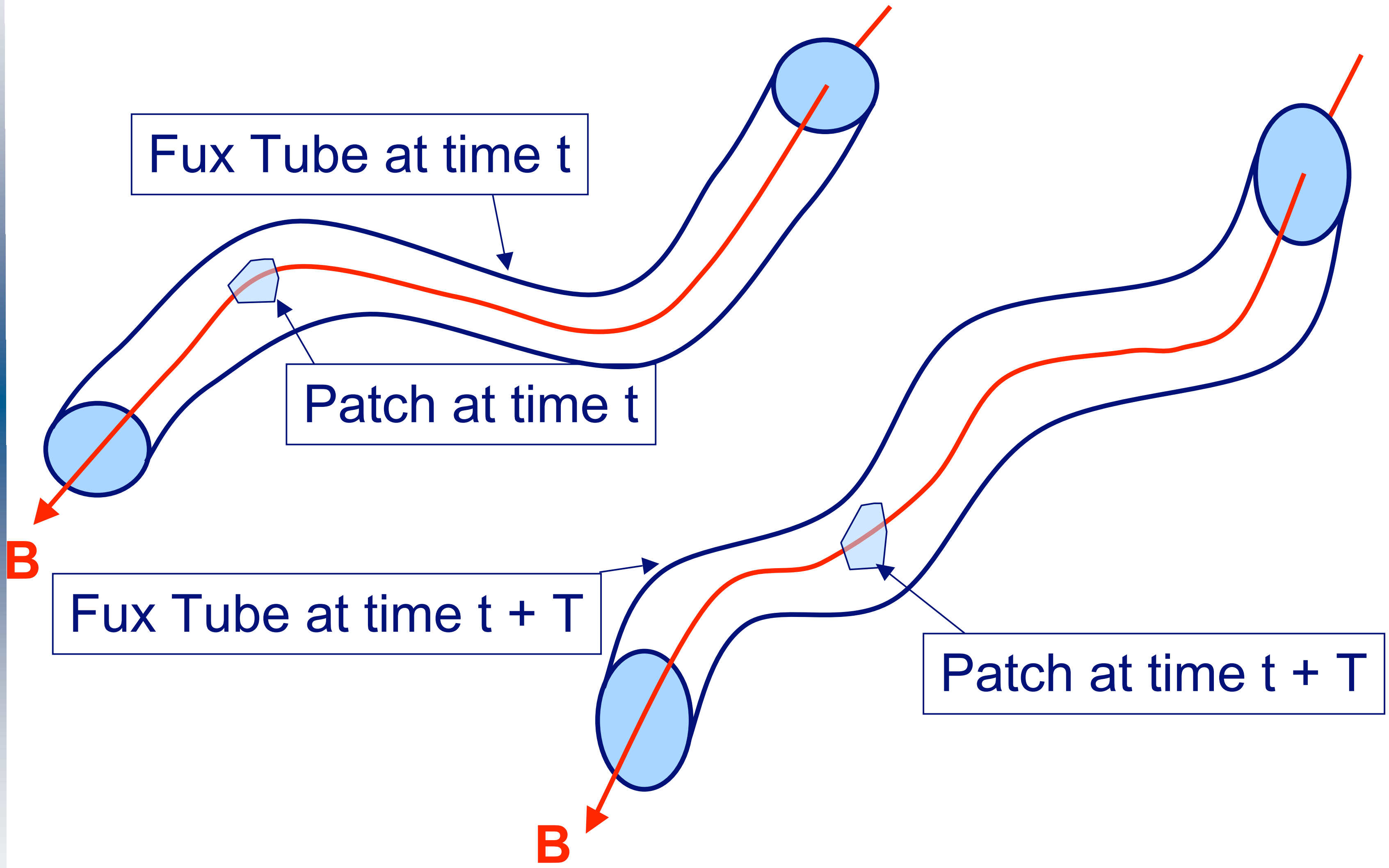
Alfven 1945, Kelvin, Cauchy Ideal Ohm's law: $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0.$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

Flux Freezing. If **E + v×B** = 0 the magnetic flux through a loop that moves with the plasma flow, **v**, is constant in time.

Frozen Field Lines. If **E + v×B** = 0 the magnetic field lines change as though they are simply convected with velocity, **v**. Thus we say that the field lines are frozen to the plasma

Velocity of Magnetic Field lines = Velocity of Plasma = v.



When are Field Lines Frozen?

Newcomb 1958

Suppose we can write

$$\mathbf{E} = \mathbf{E}_\perp + E_\| \mathbf{b}, \qquad E_\| = \mathbf{b} \cdot \nabla \zeta$$

$$\tilde{\mathbf{v}} = \frac{1}{B^2}[(\mathbf{E} - \nabla\zeta) \times \mathbf{B}],$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\tilde{\mathbf{v}} \times \mathbf{B}).$$

velocity

This only makes sense if
ζ is finite and single valued.

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Frozen Field Lines with Pressure and Hall Terms

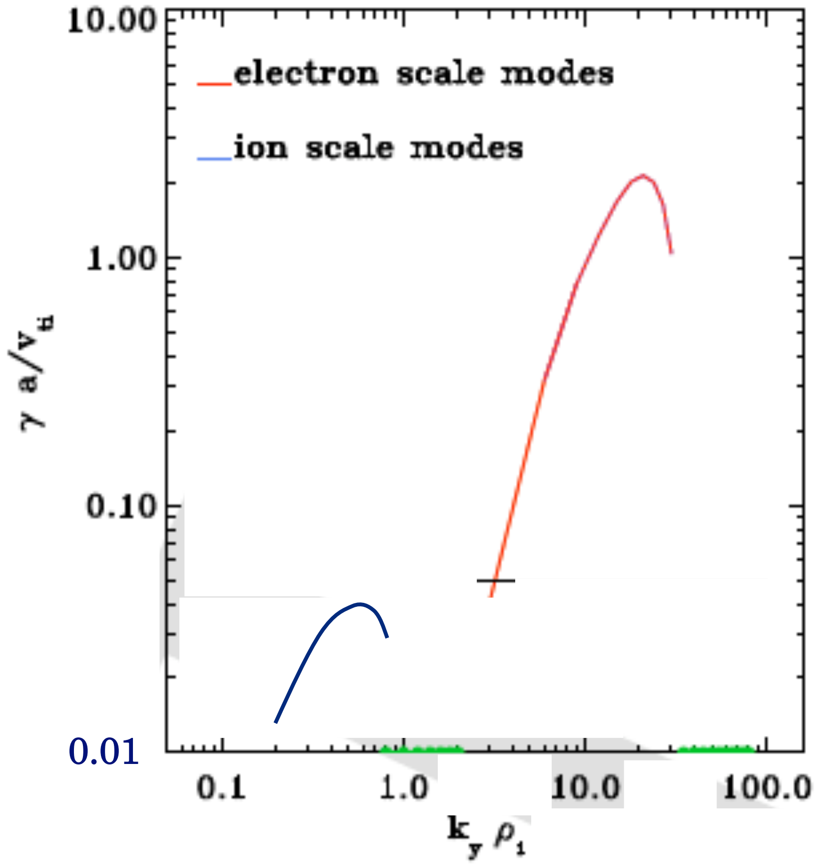
$$\mathbf{E} + \mathbf{v_e} \times \mathbf{B} = -\frac{\nabla p_e}{en_e}. \quad \mathbf{B} \cdot \nabla T_e = 0$$

$$\tilde{\mathbf{v}} = \mathbf{v_e} + \frac{\ln n_e}{eB^2}(\nabla T_e \times \mathbf{B})$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\tilde{\mathbf{v}} \times \mathbf{B}).$$

Field lines are frozen but not to either species.

Ion Scale -- Electron Scale Turbulence?



Short wavelength dominates growth rate.

Long wavelength dominates turbulent transport

$$\chi \sim \frac{\Delta^2}{\tau} \sim \frac{\gamma}{k^2}$$

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Ion Scale Turbulence in Tokamaks

Most of the tokamak turbulence seems to satisfy:

$$\omega \sim \omega^* \sim k_\parallel v_{thi} \sim \frac{v_{thi}}{qR} \ll k_\parallel v_{the} \quad (\sim \frac{v_{the}}{qR})$$

Electron response to highest order is due to rapid motion along the field. The guiding center motion holds.

$$v_\parallel = \sqrt{\frac{2}{m_e}(\mathcal{E} - \mu B)}$$

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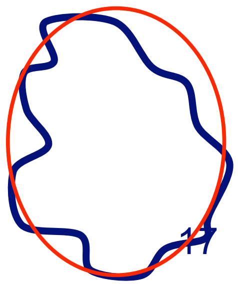
Condition on $E_{||}$

Collisionless electron drift kinetic equation:

perturbed electron distribution function, δf_e, satisfies:

$$v_{\|}\mathbf{b}\cdot\nabla(\delta f_e) = -v_{\|}\frac{eE_{\|}F_M}{T_0} - v_{\|}\mathbf{b}\cdot\{\frac{\nabla n_0}{n_0} + \frac{\nabla T_0}{T_0}(\frac{\mathcal{E}}{T_0} - \frac{3}{2})\}F_M$$

where $\mathbf{b}$ is the unit vector along the full field (perturbed plus unperturbed), $E_{\|} = \mathbf{b}\cdot\mathbf{E}$ and parallel derivative is at fixed $\mathcal{E}$ and μ.



Condition on $E_{||}$

Let

$$\delta f_e = \{\frac{\delta n}{n_0} + \frac{\delta T}{T_0}(\frac{\mathcal{E}}{T_0} - \frac{3}{2})\}F_M + \delta f_H$$

where the homogeneous solution satisfies:

$$v_{\|}\mathbf{b}\cdot\nabla(\delta f_H) = 0$$

Then:

$$v_{\|}\mathbf{b}\cdot\nabla(\{\frac{\delta n}{n_0}+\frac{\delta T}{T_0}(\frac{\mathcal{E}}{T_0}-\frac{3}{2})\}) = -v_{\|}\frac{eE_{\|}}{T_0} - v_{\|}\mathbf{b}\cdot\{\frac{\nabla n_0}{n_0}+\frac{\nabla T_0}{T_0}(\frac{\mathcal{E}}{T_0}-\frac{3}{2})\}$$

Any dependence on ε and μ can be absorbed into δf_H

Condition on $E_{||}$

→ $$\mathbf{b}\cdot\nabla(\delta T + T_0) = 0$$

$$\mathbf{b}\cdot\nabla\{(\delta T + T_0)(\frac{\delta n}{n_0} + \ln n_0)\} = -eE_{||}$$

and clearly

$$E_{||} = \mathbf{b}\cdot\nabla\zeta$$

and

$$\tilde{\mathbf{v}} = \frac{1}{B^2}[(\mathbf{E} - \nabla\zeta)\times\mathbf{B}],$$

Field lines are frozen to the "funny velocity"

Moving Surfaces

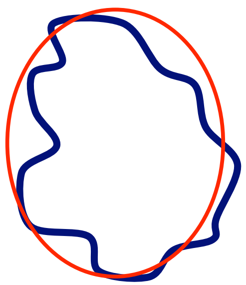
If surfaces aren't broken then they are frozen to $\tilde{\mathbf{v}}$

$$\frac{\partial \tilde{\psi}}{\partial t} + \tilde{\mathbf{v}} \cdot \nabla \tilde{\psi} = 0$$

Equation for moving flux surfaces

δf_H can be found by averaging along moving field line -- flux surface average for passing particles and bounce average for trapped particles. E.g. in the collisionless case the passing particles have:

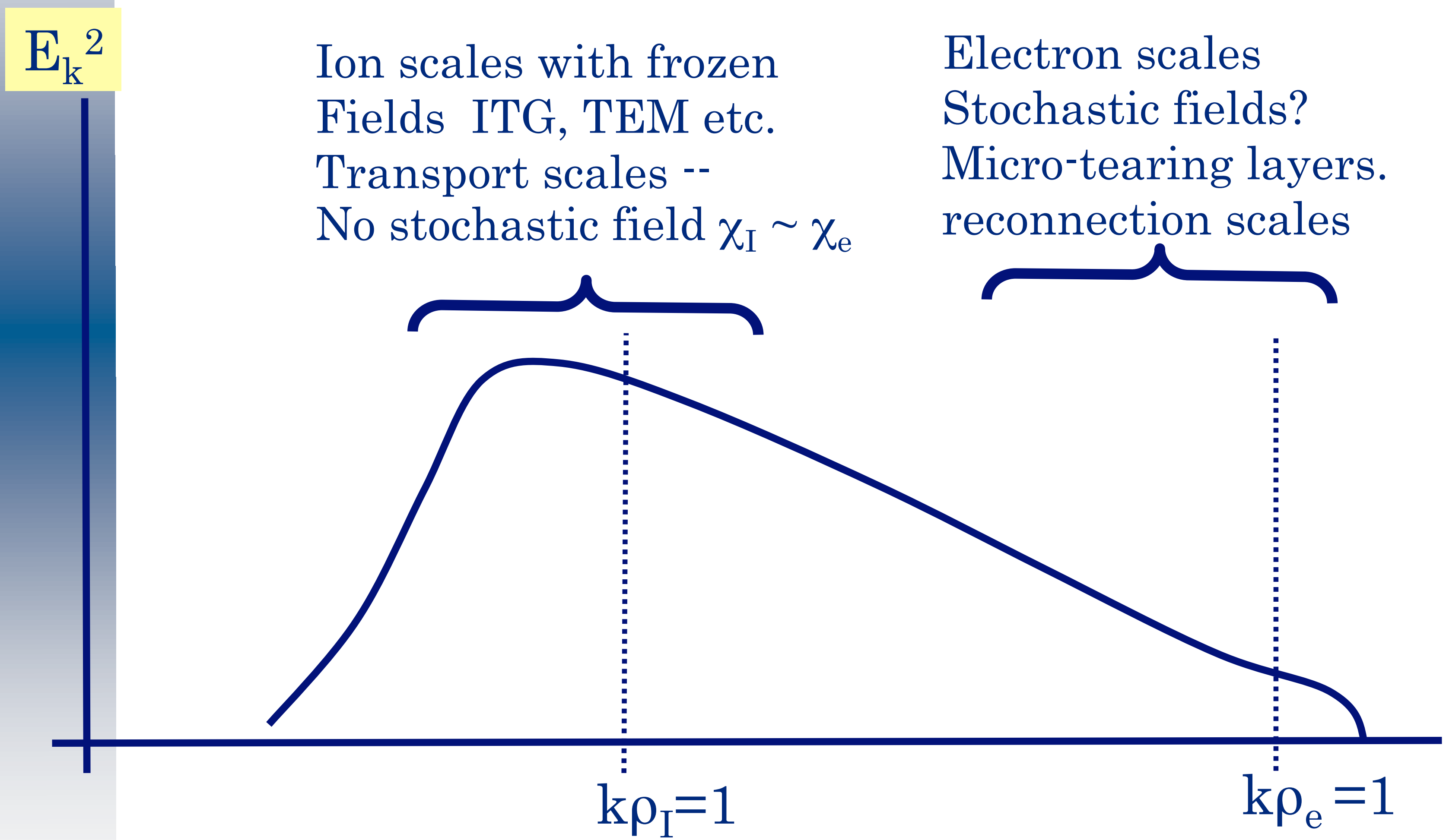
$$\delta f_e = \frac{e(\zeta - \bar{\zeta})}{T_e} F_0$$



Integral over constant $\tilde{\psi}$ surface.

$$with \quad \bar{\zeta} = \frac{\oint ds \frac{B\zeta}{v_{\parallel} |\nabla \tilde{\psi}|}}{\oint ds \frac{B}{v_{\parallel} |\nabla \tilde{\psi}|}}$$

No Stochasticity?



Embedded Micro-tearing

Rational surfaces distorted by electromagnetic ITG/TEM can still tear and make electron scale islands. How would we know they are there?

Electron heat transport from micro-tearing:

$$\chi_e \sim \rho_e^2 \frac{v_{the}}{qR} \sim \chi_i \sqrt{m_e/m_i}$$

Conclusions

•In most Tokamak Turbulence the field lines are frozen to a flow.

•Electrons can still be transported since they don't move with the lines.

•Small scales could be affecting the large scales. We might imagine small scales stochasticity is always present.