Large Helical Device (LHD)

Heliotron configuration

- No net plasma current required
  - Suitable for steady-state operation

Max. parameters

- \( R = 3.9 \text{ m} \)
- \( a = 0.6 \text{ m} \)
- \( V = 30 \text{ m}^3 \)
- \( B = 3 \sim 4 \text{ T} \)
- \( n = 1.1 \times 10^{21} \text{ m}^{-3} \)
- \( T_e = 10 \text{ keV} \)
- \( T_H = 6.9 \text{ keV} \)
- \( <\beta> = 5 \% \)

1-hour discharge
Classification of particle orbits

Tokamak

\[ B = B_0 \left( 1 - \varepsilon_t \cos \theta \right) \]

Helical System

\[ B = B_0 \left[ 1 - \varepsilon_t \cos \theta - \varepsilon_h \cos (L \theta - M \zeta) \right] \]
Collisionless Time Evolution of Zonal Flows in Helical Systems

[Sugama & Watanabe, PRL (2005), Phys. Plasmas (2006)]

Zonal-flow potential

\[
\frac{e\phi_{k_\perp}(t)}{T_i} = \mathcal{K}(t) \frac{e\phi_{k_\perp}(0)}{T_i} + \frac{1}{n_0 \langle k_\perp^2 a_i^2 \rangle} \int_0^t dt' \mathcal{K}(t-t') \left\{ 1 - \frac{2}{\pi} \langle (2\epsilon_H)^{1/2} \{1 - g_{i1}(t-t', \theta)\} \rangle \right\}^{-1} \\
\times \left\langle \int_{k^2 < 1} d^3 v \, e^{-ik_r \vec{v} \cdot \vec{r}_{d,r}(t-t')} \, F_{i0} S_{ik_\perp}(t') + \int_{k^2 > 1} d^3 v \, F_{i0} S_{ik_\perp}(t') \{1 + ik_r (\Delta_r - \langle \Delta_r \rangle_{po})\} \right\rangle
\]

Response function \( = \) GAM component + Residual component

\[
\mathcal{K}(t) = \mathcal{K}_{GAM}(t)[1 - \mathcal{K}_L(t)] + \mathcal{K}_L(t)
\]

\[
\mathcal{K}(t = 0) = 1 \quad \mathcal{K}(t) \rightarrow \mathcal{K}_L(t) \text{ as } \mathcal{K}_{GAM}(t) \rightarrow 0
\]

GAM response function

\[
\mathcal{K}_{GAM}(t) = \cos(\omega_G t) \exp(\gamma t)
\]

Long-time response function

\[
\mathcal{K}_L(t) \equiv \frac{1 - (2/\pi) \langle (2\epsilon_H)^{1/2} \{1 - g_{i1}(t, \theta)\} \rangle}{1 + G + \mathcal{E}(t)' \langle n_0 \langle k_\perp^2 a_i^2 \rangle \rangle}
\]

\[
\mathcal{E}(t) = \frac{2}{\pi} n_0 \left[ \langle (2\epsilon_H)^{1/2} \{1 - g_{i1}(t, \theta)\} \rangle - \frac{3}{2} \langle k_\perp^2 a_i^2 \rangle \right] \\
\times \langle (2\epsilon_H)^{1/2} \{1 - g_{i2}(t, \theta)\} \rangle + \frac{T_i}{T_e} \langle (2\epsilon_H)^{1/2} \{1 - g_{e1}(t, \theta)\} \rangle
\]

\(\mathcal{E}(t)\) represents effects of shielding of potential due to helical-ripple-trapped particles.
Comparison between Zonal-Flow Responses in Tokamak and Helical Plasmas

\[ q = 1.5, \quad \varepsilon_t = 0.1, \quad k_r a_i = 0.131 \]

Tokamak

Helical plasma \( (L=2, M=10) \)

Helical ripples enhance GAM damping rate.

(a) \( \varepsilon_h = 0 \)

\[ <\phi_k(t)>/<\phi_k(0)> \]

(b) \( \varepsilon_h = 0.1 \)

\[ <\phi_k(t)>/<\phi_k(0)> \]
The long-time limit of the response kernel

\[ \mathcal{K}_e \equiv \lim_{t/\tau_e \to +\infty} \mathcal{K}_L(t) \]

\[ = \langle k_\perp^2 a_i^2 \rangle \left[ 1 - \frac{2}{\pi} \langle (2\epsilon_H)^{1/2} \rangle \right] \]
\[ \times \left\{ \langle k_\perp^2 a_i^2 \rangle \left[ 1 - \frac{3}{\pi} \langle (2\epsilon_H)^{1/2} \rangle + C \right] \right. \]
\[ \left. + \frac{2}{\pi} \left( 1 + T_i/T_e \right) \langle (2\epsilon_H)^{1/2} \rangle \right\}^{-1} \]

depends on the depth of helical ripples \( \mathcal{E}_H \) as well as on the radial wave number \( k_r \).
The perturbed ion gyrocenter distribution is given by the analytical solution:

\[
\delta f_{ik_\perp}^{(g)} (t) = \frac{e\phi_{k_\perp} (0)}{T_i} F_{i0} \left[ k_r^2 a_i^2 e^{-i k_r \bar{v}_d r t} - K_L (t) \right] \\
\times \left( 1 - \frac{1}{4} k_r^2 \rho^2 \right) \left( 1 - e^{-i k_r \bar{v}_d r t} \right) \text{ for } \kappa^2 < 1
\]

\[
\delta f_{ik_\perp}^{(g)} (t) = \frac{e\phi_{k_\perp} (0)}{T_i} F_{i0} \left[ k_r^2 a_i^2 - K_L (t) \left\{ i k_r (\Delta_r - \langle \Delta_r \rangle_{po}) \right. \right] \\
+ \left. \frac{1}{2} k_r^2 (\Delta_r - \langle \Delta_r \rangle_{po})^2 \right\} \text{ for } \kappa^2 > 1.
\]

These are derived from taking the average along the rapid particle motion along the field line.
Perturbed gyrocenter distribution

\[ \delta f(v_\parallel, v_\perp) \]

Simulation

(a) \( \theta = 0 \)

(b) \( \theta = 8\pi/13 \)

Helical plasma

\[ q = 1.5, \quad \epsilon_h = 0.1, \quad L = 2, \quad M = 10 \]

\[ t = 12.5 \left( \frac{R_\phi}{v_\phi} \right) \]

Theory (rapid oscillations dropped)

(a) \( \theta = 0 \)

(b) \( \theta = 8\pi/13 \)
In the LHD experiments, better confinement is observed in the inward-shifted magnetic configurations, where lower neoclassical ripple transport but more unfavorable magnetic curvature driving pressure-gradient instabilities are anticipated.

Anomalous transport is also improved in the inward shifted configuration.

In this work, we investigate effects of changes in helical magnetic configuration on anomalous transport and zonal flows based on ITG turbulence simulation and zonal-flow response theory to show that neoclassical optimization contributes to reduction of anomalous transport by enhancing the zonal-flow level.
Safety factor
$q = 1.9$ (standard)  $1.7$ (inward)

Magnetic shear parameter
$s = -0.85$ (standard)  $-0.96$ (inward)

Magnetic surface at $r = 0.6a$

<table>
<thead>
<tr>
<th>$r/R_0$</th>
<th>$\varepsilon_t$</th>
<th>$\varepsilon_h/\varepsilon_t$</th>
<th>$\varepsilon/\varepsilon_t$</th>
<th>$\varepsilon_{\pm}/\varepsilon_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.099</td>
<td>0.087</td>
<td>0.91</td>
<td>-0.28</td>
<td>0.0</td>
</tr>
<tr>
<td>0.114</td>
<td>0.082</td>
<td>1.20</td>
<td>-0.74</td>
<td>-0.24</td>
</tr>
</tbody>
</table>

$|B|$ along the fieldline

Inward slightly more unstable
bad curvature

standard
Magnetic curvature along the fieldline

standard

Inward smaller neoclassical transport
Growth rates $\gamma$ and real frequencies $\omega_r$ of ITG modes

The maximum growth rate is slightly larger for the inward-shifted case.

For the inward-shifted case, more unfavorable curvature but lower $q$ and higher magnetic shear $s$.

$$\eta_i = L_n/L_{Ti} = 3$$
$$L_n/R_0 = 0.3$$
$$T_e/T_i = 1$$
Zonal-flow responses for the **standard** and **inward-shifted** configurations

Theoretical predictions of GAM oscillation damping and **residual zonal flow enhanced in the inward-shifted configuration** are in qualitative agreement with simulation results.
Higher zonal-flow response for the inward-shifted configuration is caused by slower radial drift of helical-ripple-trapped particles.

Oscillatory distribution produced by radial drift of helically-trapped particles gives higher shielding of potential and lower zonal-flow response.
Gyrokinetic Simulation of ITG Turbulence and Zonal Flows in LHD Plasma

Watanabe, Sugama, & Ferrando, PRL(2008)

Sugama Watanabe, & Ferrando, PFR(2008)

Color contours of electrostatic potential

More than 50 billions of grid points used in 5D phase space

\[ N_x \cdot N_y \cdot N_z \cdot N_{\parallel} \cdot N_{\perp} = 128 \times 128 \times 512 \times 128 \times 48 \approx 5.15 \times 10^{10} \]

The gyrokinetic Vlasov simulations of the ITG turbulence were carried out by the Earth Simulator under the support from JAMSTEC.
ITG Turbulence Simulations for the Standard and Inward-Shifted Configurations in LHD

Smaller $\chi_i$ and larger zonal flows are found in the saturated turbulent state for the inward-shifted configuration than for the standard one!

![Graph showing turbulent thermal diffusivity and squared zonal-flow potential](image)

$$\langle \phi \rangle^2 = \int_0^\infty S(k_r) dk_r$$

$k_r$ spectrum of zonal-flow potential (averaged over $60 < t < 250$)
ITG Turbulence and Zonal Flows in LHD Plasma

Color contour of potential perturbations

Inward-Shifted Case

Standard Case

Radial profiles of zonal-flow potential $\langle \phi \rangle$

$\langle \phi \rangle = \frac{(r-r_0)}{\rho_{ti}} \ (r_0 = 0.6 \ a)$

inward-shifted configuration

standard configuration
Profiles of zonal-flow potential

\[
\langle \phi \rangle^2 = \int_0^\infty S(k_r) \, dk_r
\]

\[
\langle \phi \rangle \quad \left( \frac{r-r_0}{\rho_{ti}} \right) \quad (r_0 = 0.6 \, a)
\]

Time-averaged \( \langle \phi \rangle \)

\[
\langle \phi \rangle = \frac{1}{\mathcal{L}_n} \int_{\mathcal{L}_n} S(k_r) \, dk_r
\]

Larger stationary zonal flows are generated in the inward-shifted configuration.
Spatio-Temporal Profiles of Zonal-Flow Potential

**Inward-shifted configurations**

**Standard configurations**

![Graphs showing spatio-temporal profiles](image)

- **Potential**
- **Radial direction**
- **Time**
Summary

- Zonal flows are an attractive mechanism to regulate plasma turbulence and it is important to optimize geometry for enhancing residual zonal flows which reduce anomalous transport.

- A kinetic-fluid closure model to describe zonal flow dynamics (residual zonal flow and GAM damping) in tokamaks is presented.

- Effects of changes in helical magnetic configuration on anomalous transport and zonal flows are investigated based on ITG turbulence simulation and zonal-flow response theory.

- The inward-shifted LHD configuration enhances zonal-flow generation and reduces turbulent thermal diffusivity even though it is linearly more unstable.

- Thus, neoclassical optimization contributes to reduction of anomalous transport by increasing residual zonal flows. This gives a physical mechanism to explain better confinement observed in the LHD experiments with the inward plasma shift.