**Workshop and Minicourse**
**"Kinetic Equations, Numerical Approaches and Fluid Models for Plasma Turbulence"**
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# Gyrokinetic Theory and Simulation of Zonal Flows and ITG Turbulence

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# OUTLINE

- **Introduction**
  **Zonal Flows and Turbulence**

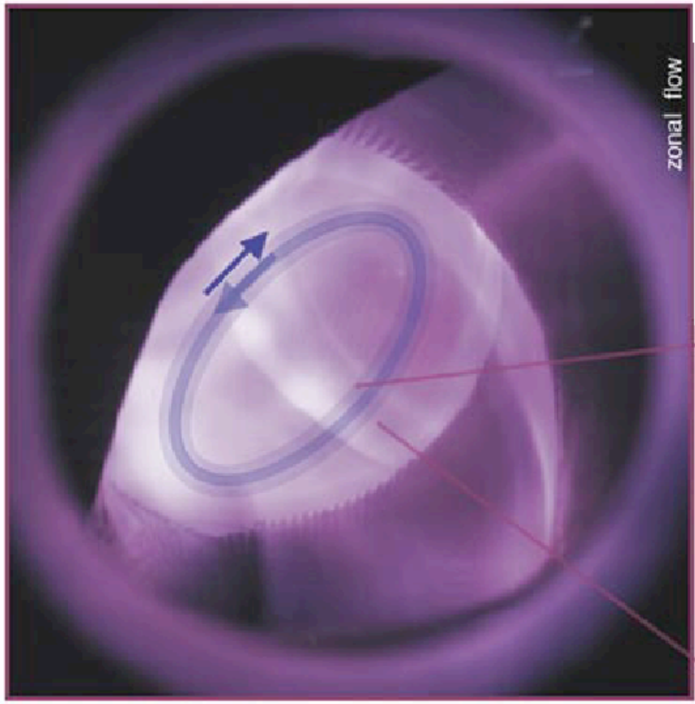
- **Zonal Flows driven by ITG Turbulence**
  **Geodesic Acoustic Mode (GAM), Residual Zonal Flow**

- **Collisionless Kinetic-Fluid Model for describing Zonal Flows**

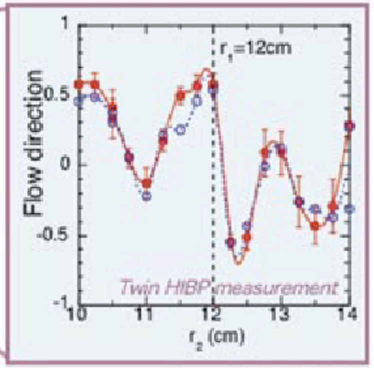
- **Zonal Flows and ITG Turbulence in Helical Plasmas**

- **Summary**

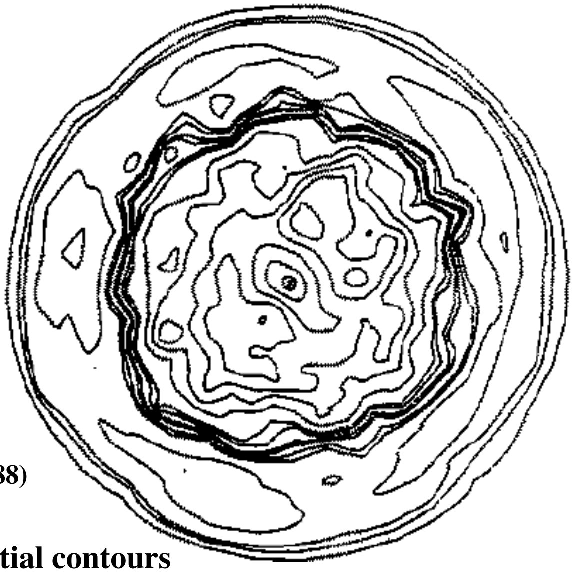
# Zonal Flows observed in Nature, Experiments, and Simulations.




Jupiter




CHS Plasma in NIFS
Fujisawa *et al.*, PRL (2004)

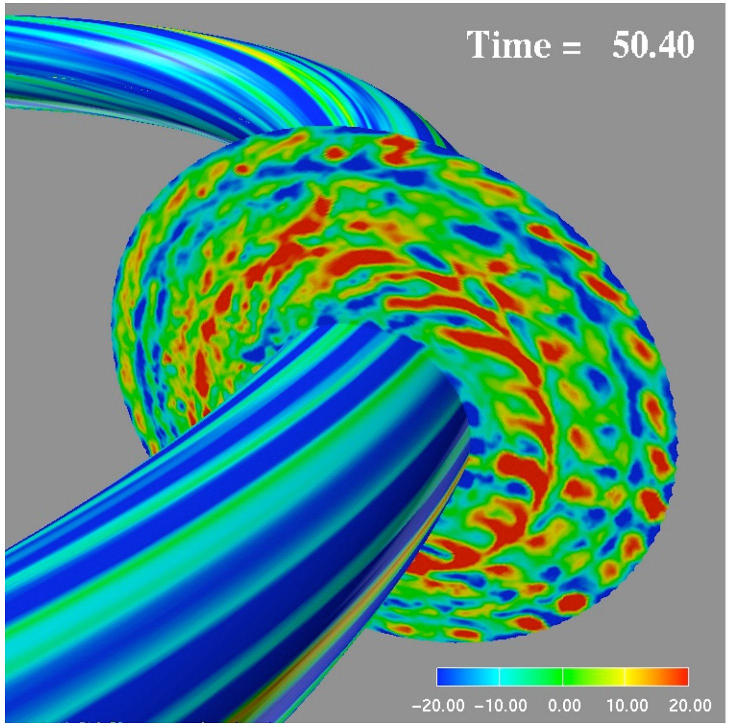


Potential contours

Zonal flows generated by resistive drift-interchange turbulence in a cylindrical plasma

Hasegawa & Wakatani, PRL (1987)
Sugama, Wakatani & Hasegawa, PoF (1988)

Gyrokinetic ITG Turbulence Simulation



# Zonal Flow Dynamics in Toroidal Systems (Toroidal ITG Turbulence Case)

**EXB zonal flow** → suppression → **plasma turbulence anomalous transport**

**plasma turbulence anomalous transport** → generation → **EXB zonal flow**

Reviewed by Diamond *et al*. PPCF (2005)
Itoh *et al*. PoP (2006)

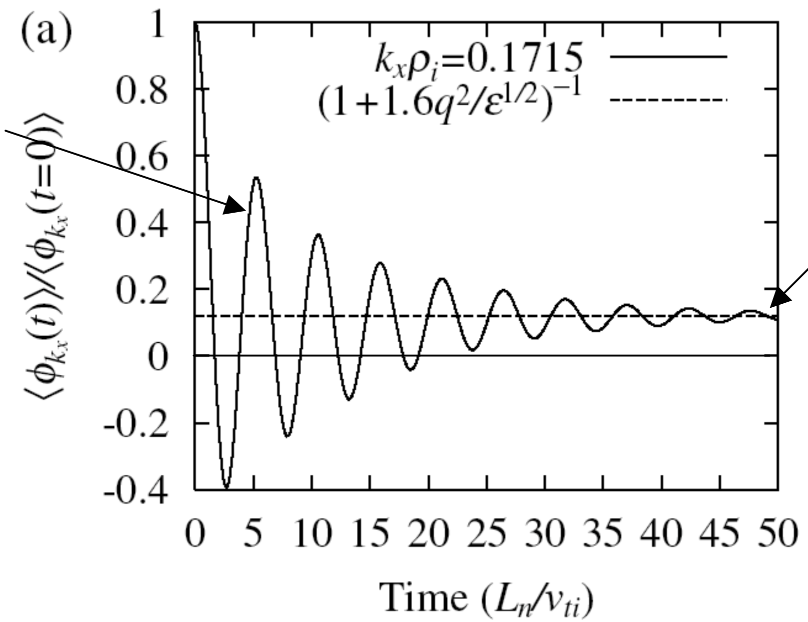
**Response of zonal flows to initial perturbation (or source term)**

**GAM oscillations** → **Collisionless damping** → **Residual zonal flow** ⇢ **Collisional damping**

Hinton & Rosenbluth PPCF (1999)

(Geodesic Acoustic Mode)
[Winsor *et al*., PoF (1968)]

**GAM**




**Residual zonal flow**
[Rosenbluth & Hinton, PRL (1998)]

**An initially given zonal-flow perturbation is not completely damped in collisionless processes.**

# Geodesic Accoustic Mode (GAM)

$$i\,\omega\,\delta n/n = \mathrm{div}\,\boldsymbol{V}_{\mathrm{EXB}} = (2cE_r/RB)\sin\theta \qquad \boldsymbol{V}_{\mathbf{EXB}} = c\boldsymbol{E}\mathrm{X}\boldsymbol{B}/B^2$$

$$J_r^{dia} = -(c/B)\mathrm{grad}_\theta(\delta p) \qquad B = B_0R_0/R = B_0/(R_0 + r\cos\theta)$$

$$=(i/\omega)\,(2\gamma\, pc^2E_r/r\,RB^2)\cos\theta \qquad (\,\delta p/p = \gamma\,\delta n/n\,)$$

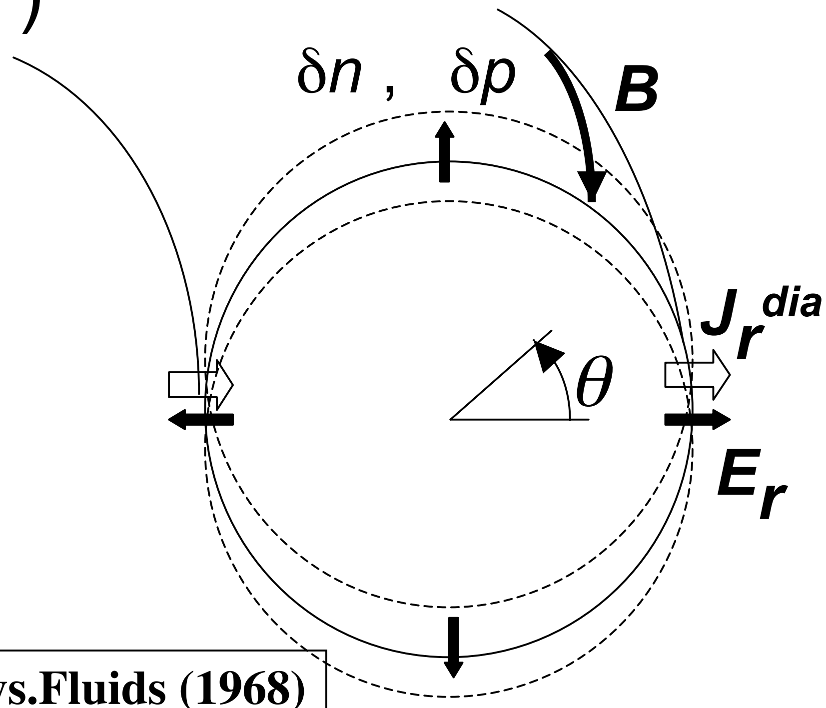
$$<J_r^{dia}> = (i/\omega)\,(2\gamma\, pc^2E_r/R_0^{\,2}B_0^{\,2})$$

$$<J_r^{pol}> = (-\,i\,\omega)\,(nmc^2E_r/B_0^{\,2})$$

$$<J_r^{pol} + J_r^{dia}> = 0$$

$$\Longrightarrow \qquad \omega^2 = 2(c_s/R_0)^2$$

$$c_s = (\gamma p/nm)^{1/2}$$




Winsor *et al.,* **Phys.Fluids (1968)**

# Gyrokinetic Equations (for ITG Turbulence)

$$k_\perp \rho_i \approx 1, \quad k_\perp \rho_e << 1$$

## Ion gyrokinetic equation for $\delta f(x, v_{||}, \mu, t)$

$$\boxed{\left[\frac{\partial}{\partial t} + v_{||}\hat{\mathbf{b}}\cdot\nabla + \mathbf{v}_d\cdot\nabla - \mu\left(\hat{\mathbf{b}}\cdot\nabla\Omega\right)\frac{\partial}{\partial v_{||}}\right]\delta f + \frac{c}{B_0}\left\{\psi,\delta f\right\} = \left(\mathbf{v}_* - \mathbf{v}_d - v_{||}\hat{\mathbf{b}}\right)\cdot\frac{e\nabla\psi}{T_i}F_M + C\left(\delta f\right)}$$

**Gyrocenter drift**
**&**
**Diamagnetic drift**

$$\mathbf{v}_d\cdot\nabla = -\frac{v_{||}^2 + \Omega\mu}{\Omega R_0}\left[\left(\cos z + \hat{s}z\sin z\right)\frac{\partial}{\partial y} + \sin z\frac{\partial}{\partial x}\right],$$

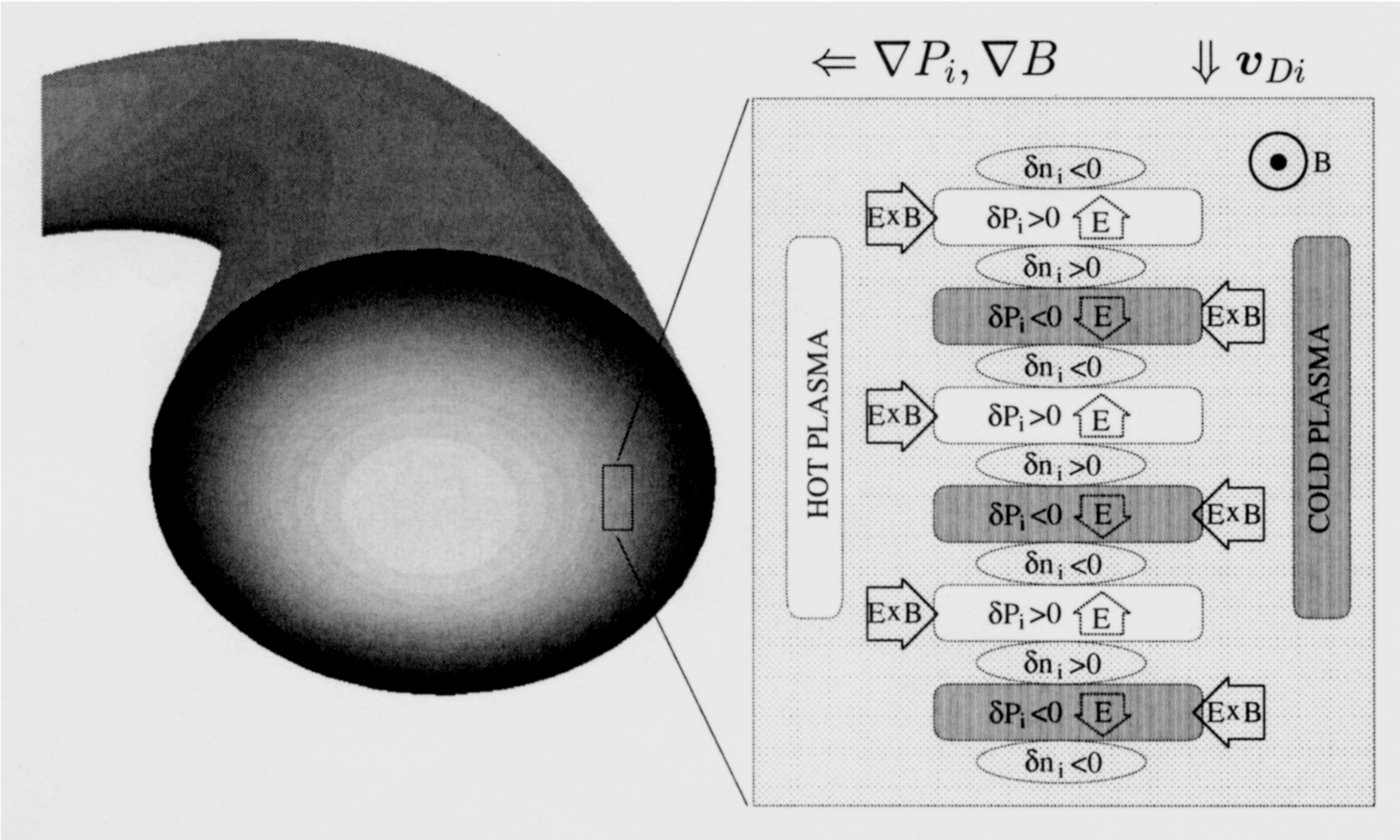
$$\mathbf{v}_* = -\frac{cT_i}{eL_nB_0}\left[1 + \eta_i\left(\frac{mv^2}{2T_i} - \frac{3}{2}\right)\right]\hat{\mathbf{y}}, \quad \mu = \frac{v_\perp^2}{2\Omega}$$

## Quasineutrality condition & Adiabatic electron assumption

$$\boxed{\int J_0\left(k_\perp v_\perp/\Omega\right)\delta f\,\mathrm{d}^3v - \left[1 - \Gamma_0\left(k_\perp^2\right)\right]\frac{e\phi}{T_i} = \frac{e}{T_e}\left(\phi - \langle\phi\rangle\right), \quad k_\perp^2 = \left(k_x + \hat{s}zk_y\right)^2 + k_y^2}$$

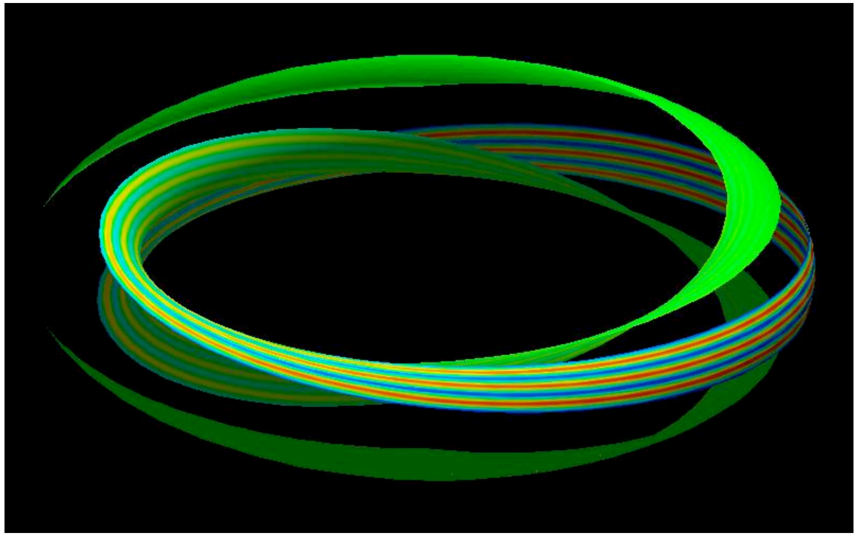
**Ion polarization**

# Physical mechanism of the toroidal ITG mode

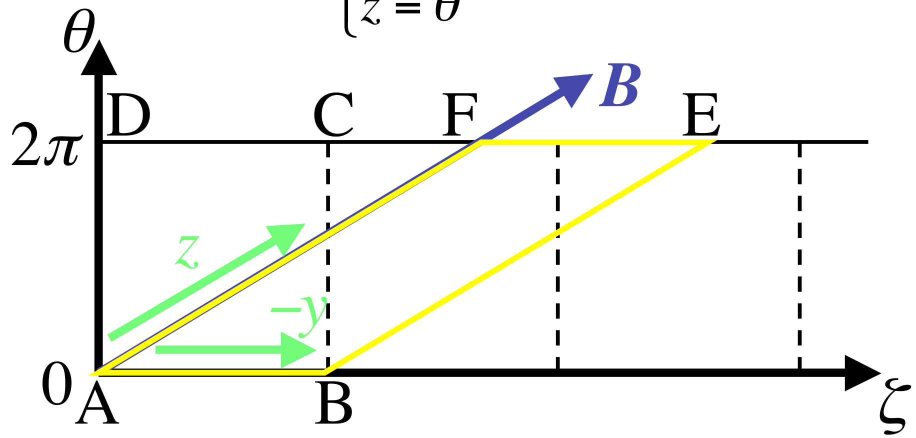


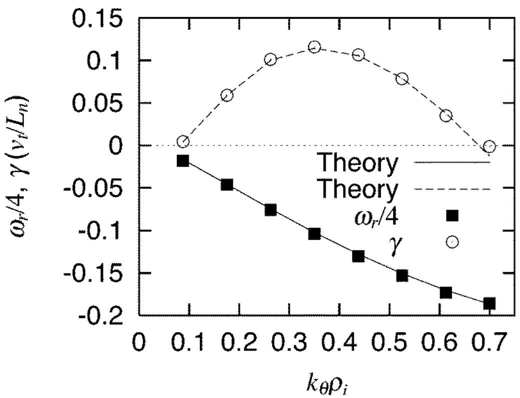
# Simulation Model for Tokamak Configuration

- **Toroidal Flux Tube Model**



$$\begin{cases} x = r - r_0 \\ y = \dfrac{r_0}{q_0}\left[q(r)\theta - \zeta\right] \\ z = \theta \end{cases}$$




Linear Benchmark Test of ITG Instablities

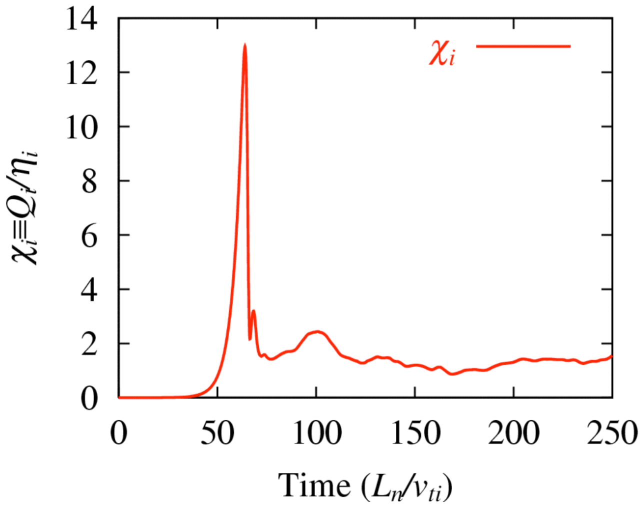
**GKV code** [Watanabe & Sugama, Nucl.Fusion 46, 24(2006)]

**The gyrokinetic equation is directly solved in the 5-D phase space.**

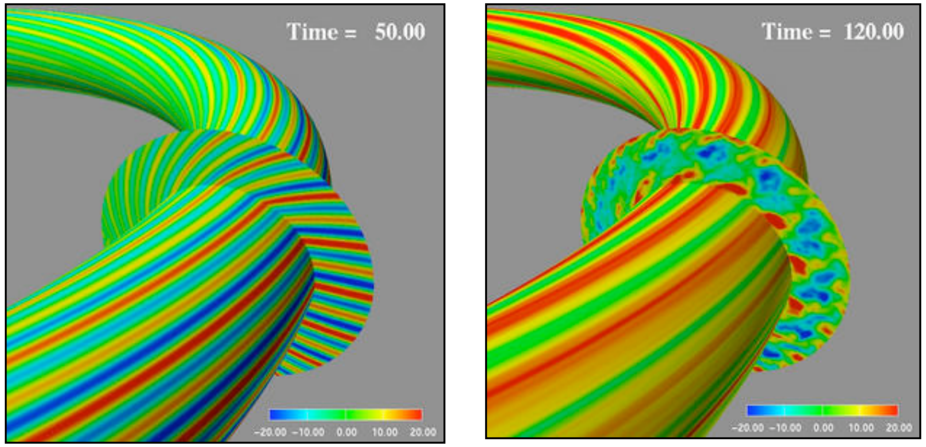
# Gyrokinetic Simulation of Toroidal ITG Turbulence

[Watanabe & Sugama, NF (2006)]

**Time evolution of anomalous ion heat diffusivity**




**Structures of electrostatic potential**




**Ion energy flux**

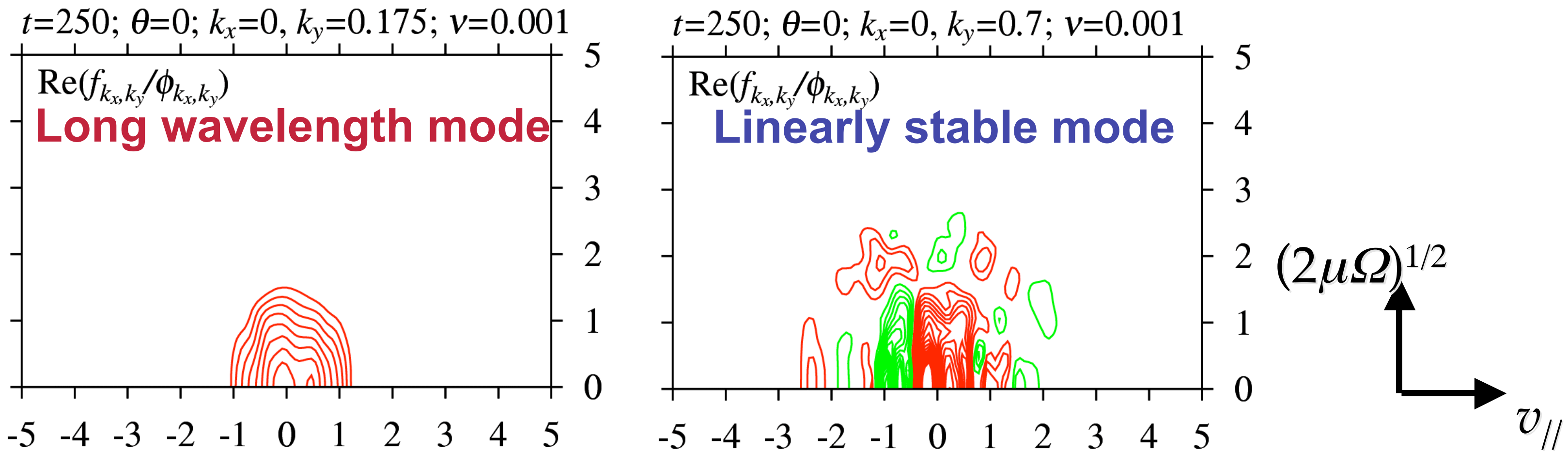
$$Q_i = \left\langle \frac{c}{B} \int d^3v \frac{1}{2} m_i v^2 \sum_{\mathbf{k}_\perp} J_0(k_\perp \rho_i) \mathrm{Im}\left[ f_{i\mathbf{k}_\perp}^{(g)*} \phi_{\mathbf{k}_\perp} \right] (\mathbf{k}_\perp \times \mathbf{b}) \cdot \nabla r \right\rangle$$

**Cyclone DIII-D base case**

$\varepsilon = r/R = 0.18,\ \ q = 1.4,\ \ s = (r/q)(dq/dr) = 0.78$

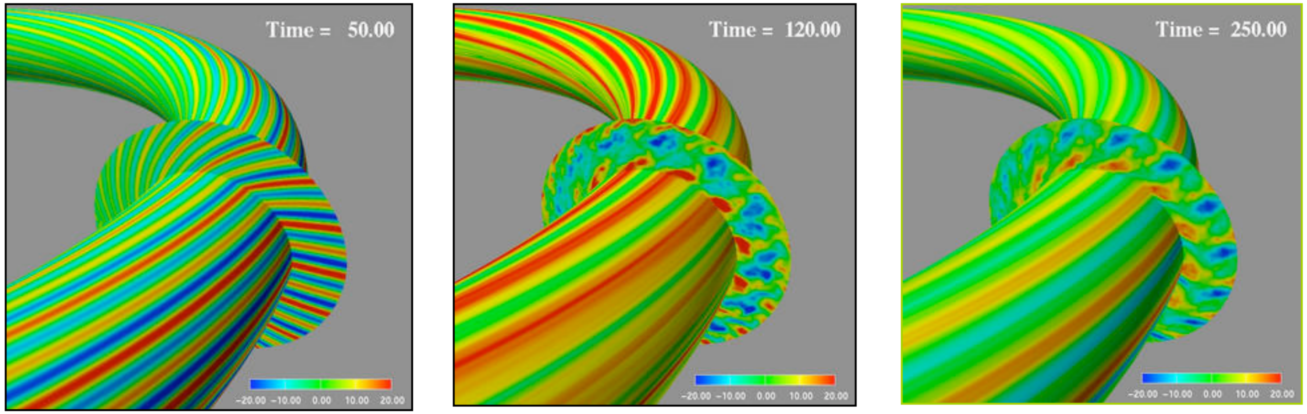
$T_e/T_i = 1,\ \ \eta_i = L_n/L_T = 3.114,\ \ R/L_T = 6.92$

# Velocity-Space Structures of *f* in Toroidal ITG Turbulence




- **Fine-scale structures generated by the ballistic motions appear in the stable mode, and are dissipated by collisions, while the transport is driven by macro-scale vortices.**
- **High-velocity space resolution is necessary for weakly collisional plasmas.**

# Time Evolution of Tokamak ITG Turbulence



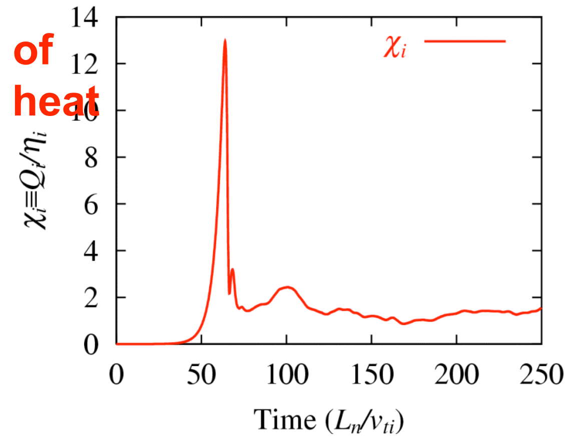

Electrostatic potential patterns

**Watanabe & Sugama, Nucl.Fusion 46, 24(2006)**

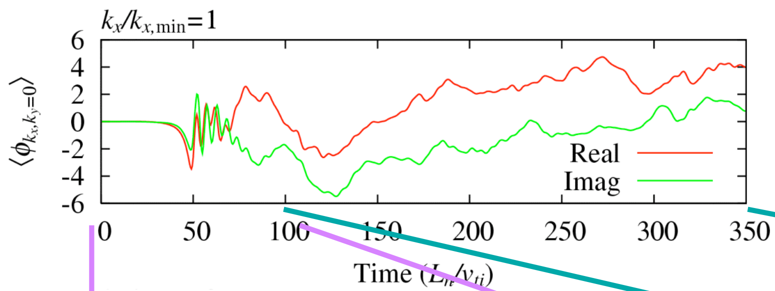
**Cyclone DIII-D base case**

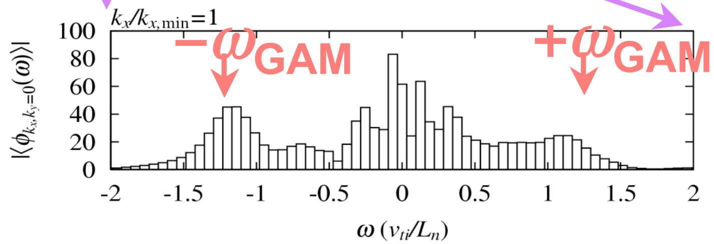
**$k_{x,min}$= 0.1715, $k_{y,min}$= 0.175, $\nu$ = 0.001 $v_{ti}/L_n$ (Lenard-Berstein collision model)**

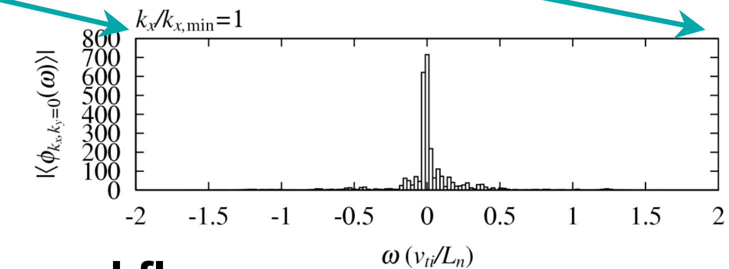
Time evolution of anomalous ion heat diffusivity




**Time evolution of a zonal flow comp.**




**Power spectrum of a zonal flow comp.**

# Responses to initial condition and to nonlinear sources

**Perturbed gyrocenter distribution function :**

$$\delta f^{(\mathrm{g})}_{a\mathbf{k}_\perp}(t) = \sum_{a'=i,e} \left[ \boxed{U_{aa'}(t)} \delta f^{(\mathrm{g})}_{a'\mathbf{k}_\perp}(0) + \int_0^t dt' \boxed{U_{aa'}(t-t')} F_{a'0} S_{a'\mathbf{k}_\perp}(t') \right]$$

**Propagator :** $\boxed{U_{aa'}(t) \;\; (a, a' = i, e)}$ $\boxed{N_a(t) \equiv \int d^3v \, U_{ia}(t) - \int d^3v \, U_{ea}(t)}$

**Zonal-flow potential :**

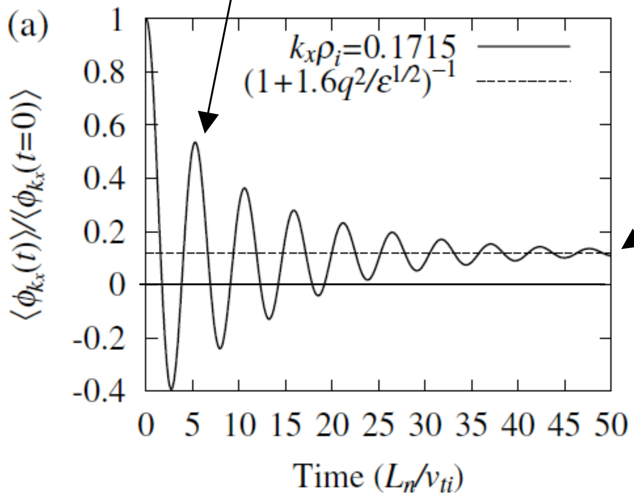
$$\frac{e\phi_{\mathbf{k}_\perp}(t)}{T_i} = \frac{1}{n_0 \left[1 - \Gamma_0(b)\right]} \sum_{a=i,e} \left[ \boxed{N_a(t)} \delta f^{(\mathrm{g})}_{a\mathbf{k}_\perp}(0) + \int_0^t dt' \boxed{N_a(t-t')} F_{a0} S_{a\mathbf{k}_\perp}(t') \right]$$

**Response to a given turbulence source can be evaluated by solving a linear initial value problem.**

# Gyrokinetic Simulation of EXB Zonal Flow Damping in Tokamaks

Watanabe & Sugama,
Nucl.Fusion 46, 24(2006)

GAM

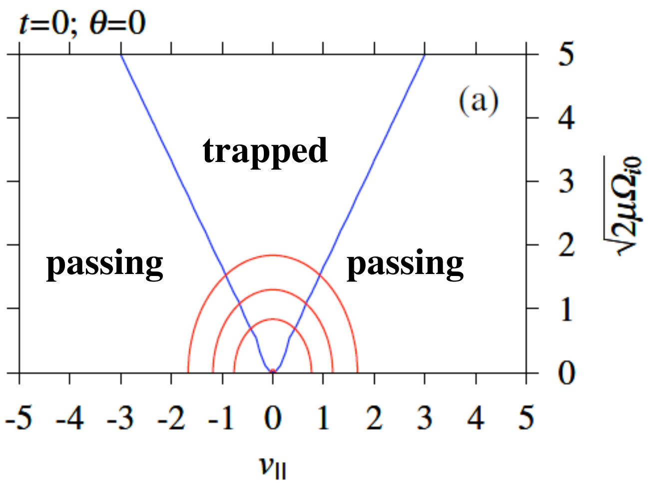



**Undamped residual flow**
**[Rosenbluth & Hinton, PRL(1998)]**

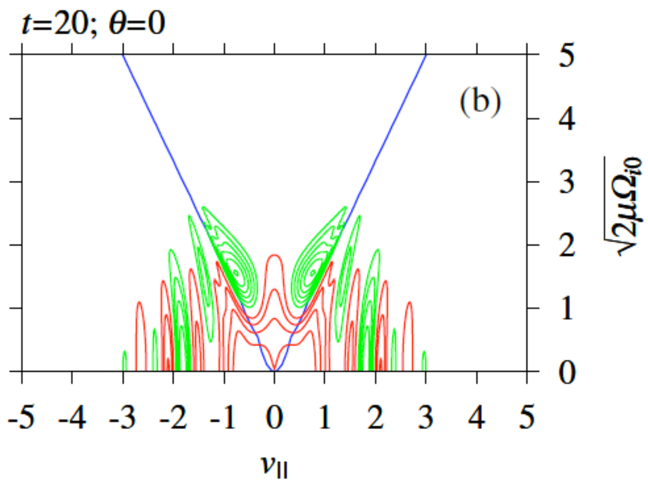
$$\phi_{k_r0}(\infty) = \phi_{k_r0}(0)/(1+1.6q^2/\epsilon^{1/2})$$

**After GAM oscillations are damped in the collisionless process (Landau damping), the zonal-flow potential approaches the theoretical value predicted by the Rosenbluth-Hinton theory.**

**initial ($t$=0) distribution**




***t*** **increases**




**Results from**
**Gyrokinetic Vlasov (GKV) code**

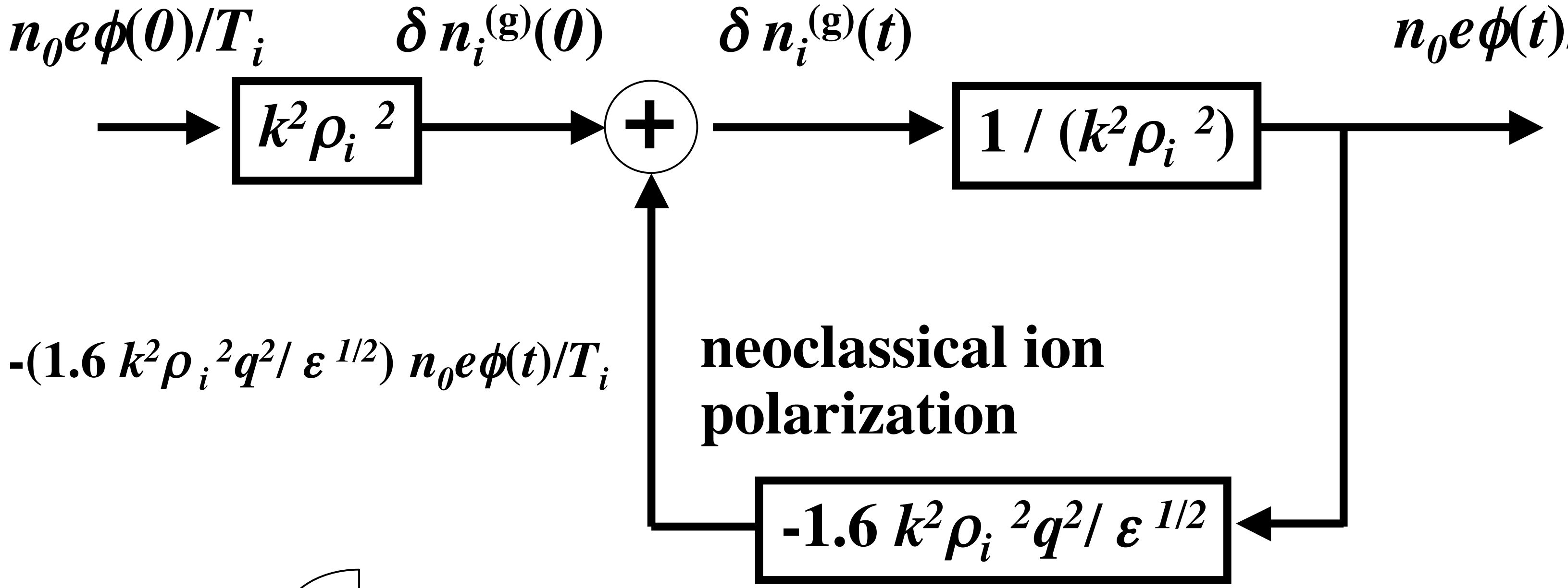
**Real part of the ion gyrocenter distribution function** $\delta f(v_{||},\mu)$

## Residual zonal-flow potential for $k\rho_i < 1$ (ITG)

**quasineutrality condition** $\delta n_i^{(g)} = k^2\rho_i^{\,2}\, n_0 e\phi / T_i$ **classical ion polarization**

$\delta n_i^{(g)}(0) = k^2\rho_i^{\,2}\, n_0 e\phi(0)/T_i$

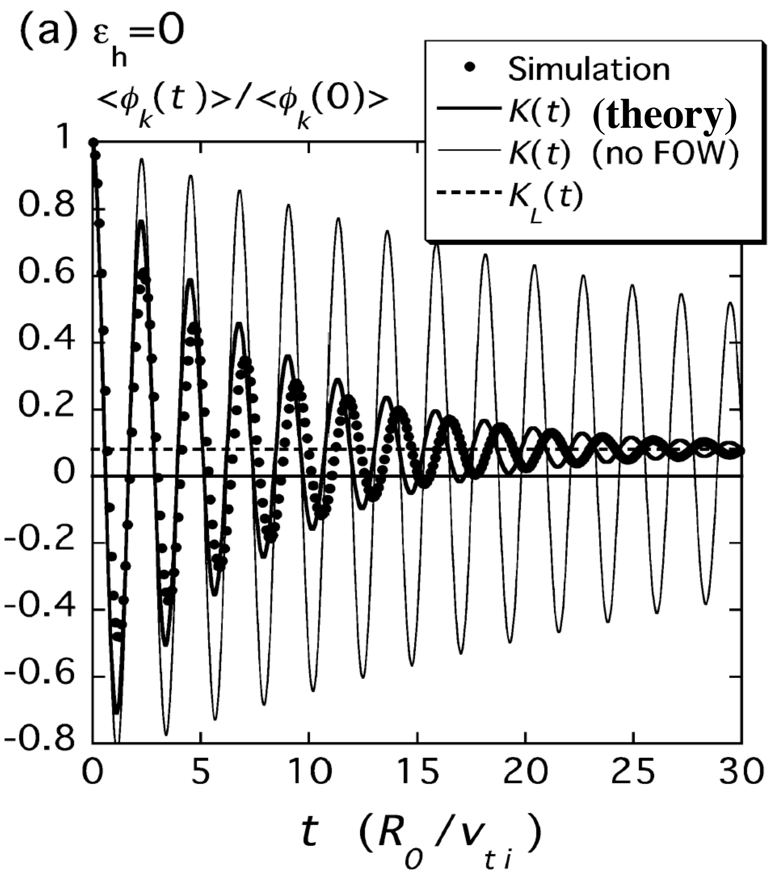
$n_0 e\phi(t)/T_i = \delta n_i^{(g)}(t)/(k^2\rho_i^{\,2})$




$$\phi(t) = \phi(0)\,/(1+1.6\,{}^2q^2/\varepsilon^{1/2})$$

# Comparison between Gyrokinetic Theory and Simulation Results of Zonal-Flow Dynamics

**Time evolution of zonal-flow electrostatic potential in a tokamak**

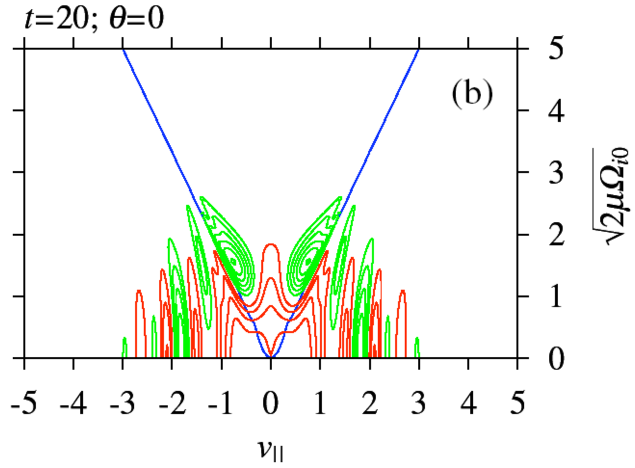



**Finite orbit width (FOW) enhances GAM damping.**

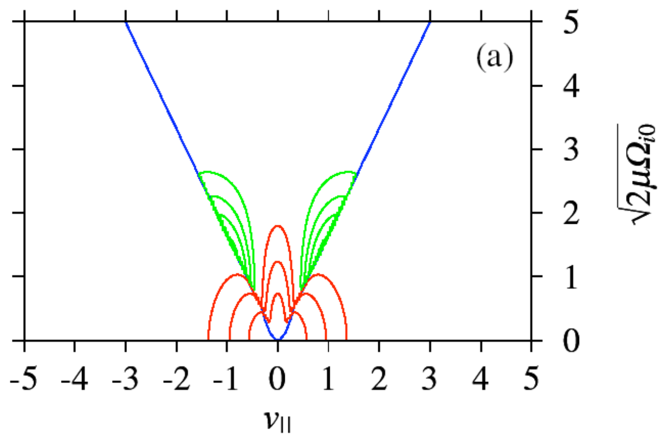
**Sugama & Watanabe,**
**J. Plasma Phys. (2006) Phys. Plasmas (2006)**

**Gyrocenter distribution function** $\delta f(v_{//},\mu)$

**Simulation results**




Analytical solution **(rapid oscillations dropped)**



# Collisionless Kinetic-Fluid Model of Zonal Flows in Toroidal Plasmas

**Sugama, Watanabe & Horton, Phys. Plasmas (2007)**

# Gyrofluid Equations for zonal flows $\mathbf{k}_\perp = k_r \nabla r$

$$\left[\delta n,\ n_0 u_\parallel,\ \delta p_\parallel,\ \delta p_\perp\right] \equiv \int d^3v\, \delta f \left[1,\ v_\parallel,\ m v_\parallel^2,\ \frac{1}{2} m v_\perp^2\right]$$

**Continuity eq.**

$$\frac{\partial}{\partial t}\delta n + \mathbf{B}\cdot\nabla\left(n_0 u_\parallel / B\right) - i\left(\frac{kq}{\varepsilon m \Omega}\right)(\mathbf{b}\cdot\nabla \ln B)\left(\delta p_\parallel + \delta p_\perp\right)$$

$$= i\left(\frac{kq}{\varepsilon m \Omega}\right)(\mathbf{b}\cdot\nabla \ln B)\, n_0 e\, \phi\, e^{-(ka)^2/2}\left(2 - \frac{(ka)^2}{2}\right) + \int S\, F_0\, d^3v$$

**need for closure**

**Parallel momentum balance**

$$\frac{\partial}{\partial t}\left(m n_0 u_\parallel\right) + \mathbf{B}\cdot\nabla\left(\delta p_\parallel / B\right) + \delta p_\perp (\mathbf{b}\cdot\nabla \ln B) - i\left(\frac{kq}{\varepsilon \Omega}\right)(\mathbf{b}\cdot\nabla \ln B)\left(q_\parallel + q_\perp + 4 p_0 u_\parallel\right)$$

$$= n_0 e\left[-\mathbf{b}\cdot\nabla\left(e^{-(ka)^2/2}\phi\right) + \frac{(ka)^2}{2} e^{-(ka)^2/2}\phi\, (\mathbf{b}\cdot\nabla \ln B)\right] + \int S\, F_0\, m v_\parallel\, d^3v$$

**Parallel pressure**

$$\frac{\partial}{\partial t}\delta p_\parallel + \mathbf{B}\cdot\nabla\left[\left(q_\parallel + 3 p_0 u_\parallel\right)/B\right] + 2(\mathbf{b}\cdot\nabla \ln B)\left(q_\perp + p_0 u_\parallel\right) - i\left(\frac{kq}{\varepsilon \Omega}\right)(\mathbf{b}\cdot\nabla \ln B)\left(\delta r_{\parallel,\parallel} + \delta r_{\parallel,\perp}\right)$$

$$= i\left(\frac{kq}{\varepsilon m \Omega}\right)(\mathbf{b}\cdot\nabla \ln B)\, p_0 e\, \phi\, e^{-(ka)^2/2}\left(4 - \frac{(ka)^2}{2}\right) + \int S\, F_0\, m v_\parallel^2\, d^3v$$

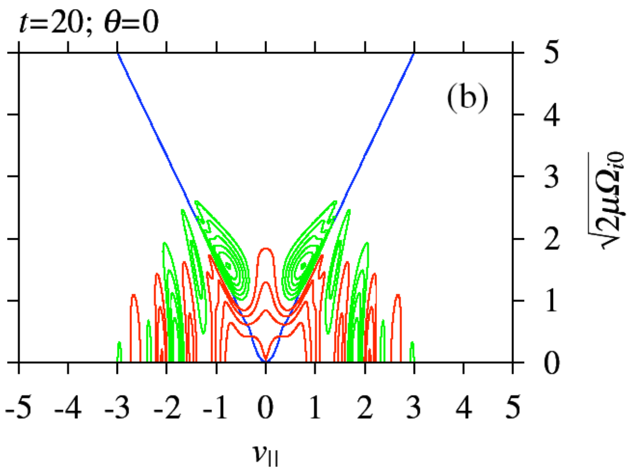
**Perpendiular pressure**

$$\frac{\partial}{\partial t}\delta p_\perp + \mathbf{B}\cdot\nabla\left[\left(q_\perp + p_0 u_\parallel\right)/B\right] - (\mathbf{b}\cdot\nabla \ln B)\left(q_\perp + p_0 u_\parallel\right) - i\left(\frac{kq}{\varepsilon \Omega}\right)(\mathbf{b}\cdot\nabla \ln B)\left(\delta r_{\parallel,\perp} + \delta r_{\perp,\perp}\right)$$

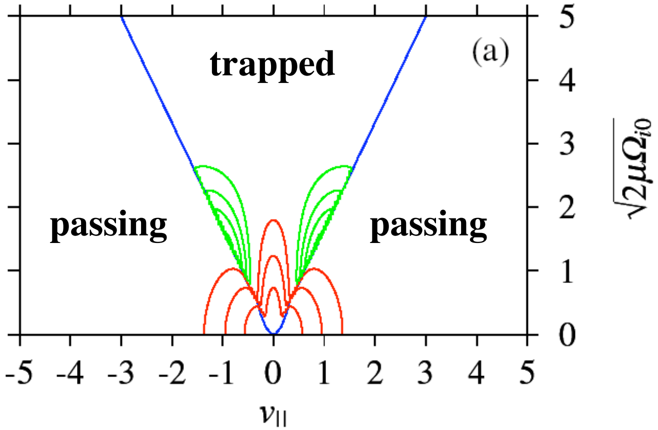
$$= i\left(\frac{kq}{\varepsilon m \Omega}\right)(\mathbf{b}\cdot\nabla \ln B)\, p_0 e\, \phi\, e^{-(ka)^2/2}\left(3 - \frac{3}{2}(ka)^2 + \frac{1}{8}(ka)^4\right) + \int S\, F_0\, \frac{m v_\perp^2}{2}\, d^3v$$

## Structures of the perturbed gyrocenter distribution for zonal-flow components (tokamak case)

**Simulation results**




Analytical solution **(rapid oscillations dropped)**




**The gyrocenter distribution for residual zonal flow part can be described by the analytical solution.**

$$f_{k_x,0}(t) = F_{\mathrm{M}} \frac{e\langle \phi_{k_x,0}(0)\rangle}{T_{\mathrm{i}}} [k_x^2 \rho_{\mathrm{i}}^2 + \{\mathrm{i} k_x (\overline{\rho_b} - \rho_b) + k_x^2 (\rho_b \overline{\rho_b} - \tfrac{1}{2}\overline{\rho_b^2} - \tfrac{1}{2}\rho_b^2)/(1 + 1.6 q^2/\epsilon^{1/2})]$$

⇩

**Useful information to derive a kinetic-fluid closure model**

# Closure Model for Zonal Flow Dyanamics (I)

**Parallel heat fluxes**

$$\left[q_\parallel,\, q_\perp\right] \equiv \int d^3v\, \delta f \left[\left(mv_\parallel^2 - 3T\right)v_\parallel,\, \left(\frac{1}{2}mv_\perp^2 - T\right)v_\parallel\right]$$

**Fourth-order moments**

$$\left[\delta r_{\parallel\parallel},\, \delta r_{\parallel\perp},\, \delta r_{\perp\perp}\right] \equiv \int d^3v\, \delta f \left[mv_\parallel^4,\, \frac{1}{2}mv_\parallel^2 v_\perp^4,\, \frac{1}{4}mv_\perp^4\right]$$

$$q = q_\parallel^{(l)} + q_\parallel^{(s)}$$

**($l$) long-time behavior (residual zonal flow)** + **($s$) short-time behavior (GAM damping)**

**using the analytical solution** $\delta f$

$$\begin{aligned}
q_{\parallel\mathbf{k}_\perp}^{(l)} &= -2q_{\perp\mathbf{k}_\perp}^{(l)} = 2p_0 U_{\mathbf{k}_\perp}\left[B - (\beta_2/\beta_1)B^2\right] \\
U_{\mathbf{k}_\perp} &\equiv \beta_1\left(\beta_1 - \langle B^{-2}\rangle\right)^{-1}\left[\langle u_{\parallel\mathbf{k}_\perp}/B\rangle - \langle B^{-2}\rangle\langle Bu_{\parallel\mathbf{k}_\perp}(t=0)\rangle\right. \\
&\quad \left. - (\beta_1 n_0)^{-1}\langle B^{-2}\rangle\left\langle \int d^3v\, F_0 R_{\mathbf{k}_\perp}(t)\overline{(v_\parallel/B)}\right\rangle\right].
\end{aligned}$$

$$\beta_1 = \frac{15}{4}\int_0^{B_M} d\lambda/\langle B/(1-\lambda B)^{1/2}\rangle \qquad R_{\mathbf{k}_\perp}(t) = \int_0^t dt' S_{\mathbf{k}_\perp}(t')$$

$$\beta_2 = \frac{3}{2}\int_0^{B_M} \lambda d\lambda/\langle B/(1-\lambda B)^{1/2}\rangle$$

**different model from Beer & Hammett (1998)**

# Closure Model for Zonal Flow Dyanamics (II)

$$q = q_{\parallel}^{(l)} + q_{\parallel}^{(s)}$$

(*l*) long-time behavior (residual zonal flow) + (*s*) short-time behavior (GAM damping)

**Hammett-Perkins type model**

$$q_{\parallel}^{(s)} = -2\sqrt{\frac{2}{\pi}}\, i n_0 v_t \sum_m \frac{m}{|m|} \delta T_{\parallel m}\, e^{im\theta}$$

$$q_{\perp}^{(s)} = -\sqrt{\frac{2}{\pi}}\, i n_0 v_t \sum_m \frac{m}{|m|} \delta T_{\perp m}\, e^{im\theta}$$

$$n_0 \delta T_{\parallel} = \delta p_{\parallel} - T\delta n$$

$$n_0 \delta T_{\perp} = \delta p_{\perp} - T\delta n$$

## Fourth-order variables

$$\left(\delta r_{\parallel\parallel},\, \delta r_{\parallel\perp},\, \delta r_{\perp\perp}\right) = (3, 1, 2) \times T v_t^2\, \delta n^{(g)}$$

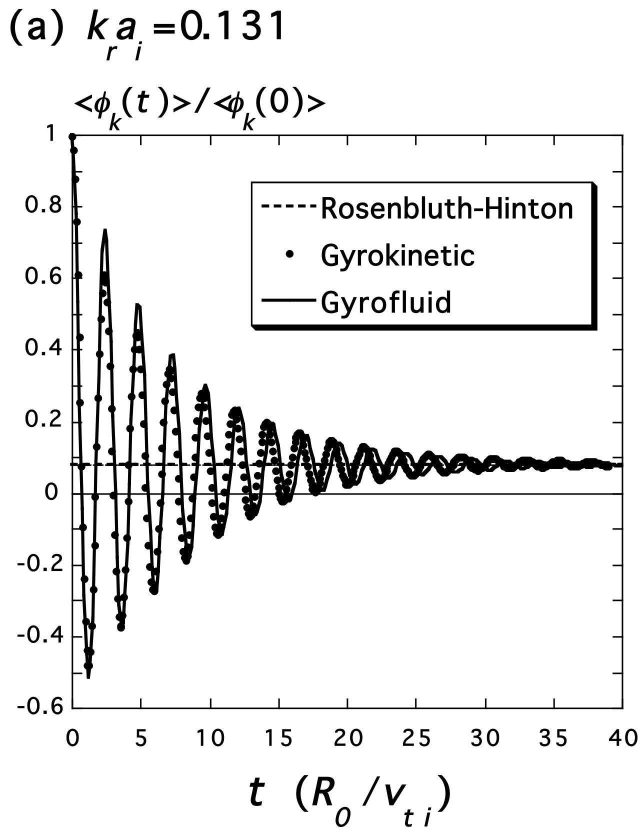
**where the Maxwellian part of the perturbed distribution is taken into account.**

# ITG-Mode-Driven Zonal Flow

**Gyrofluid equations for ions combined with the quasineutrality condition**

$$e^{-b_i/2}\left(\frac{\delta n_{i\mathbf{k}_\perp}^{(\mathrm{g})}}{n_0}-\frac{b_i}{2}\frac{\delta T_{i\perp\mathbf{k}_\perp}}{T_i}\right)-\frac{e\phi_{\mathbf{k}_\perp}}{T_i}\left[1-\Gamma_0(b_i)\right] = \frac{e}{T_e}\left(\phi_{\mathbf{k}_\perp}-\langle\phi_{\mathbf{k}_\perp}\rangle\right)$$

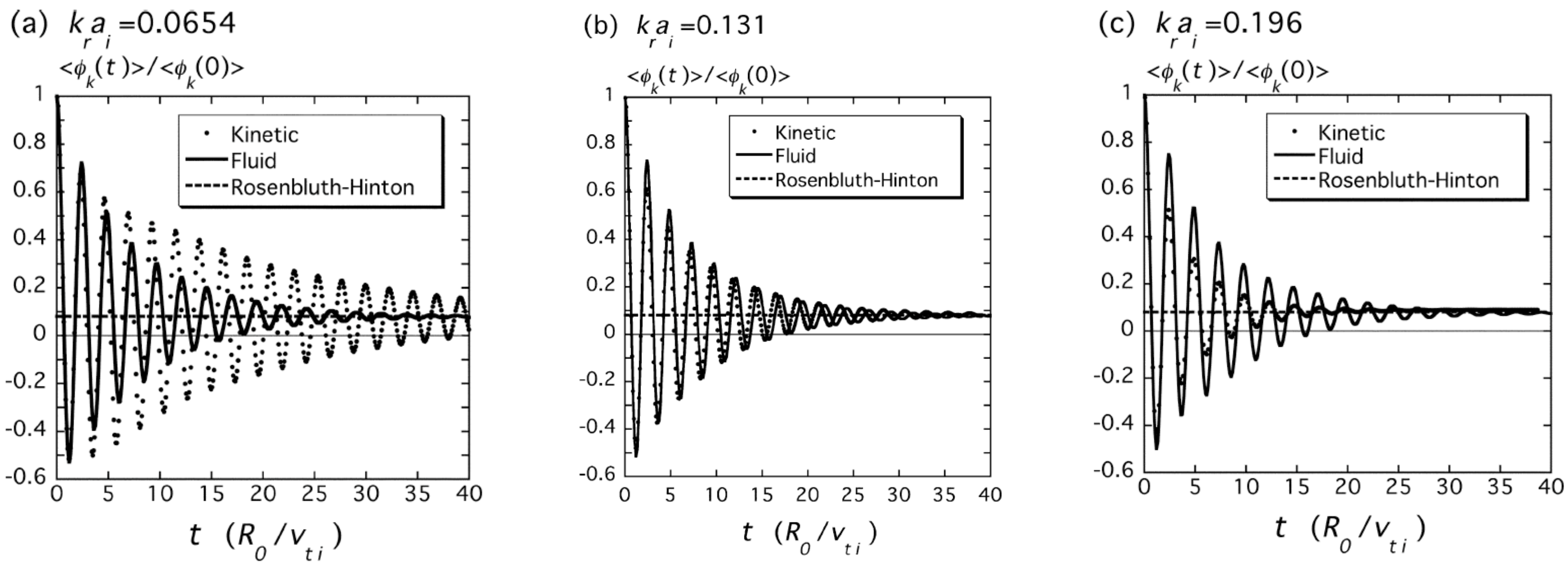
**Gyrofluid simulation shows a GAM damping process toward the same residual zonal-flow level as given by gyrokinetic simulation and the Rosenbluth-Hinton theory.**




**Rosenbluth-Hinton formula**

$$K_{\mathrm{R-H}} = 1/(1+1.6q^2/\varepsilon_t^{1/2})$$

# Radial Wavenumber Dependence of ITG-Mode-Driven Zonal Flow




**GAM dispersion relation from the fluid model :**

$$\omega = \frac{v_{ti}}{R_0}\left[\pm\sqrt{\frac{7+4\tau_e}{2}} - \frac{i}{q}\left(\frac{\chi_1+\chi_2/2}{7+4\tau_e}\right)\right]$$

$\tau_e \equiv T_e/T_i$

$\chi_1 \equiv 2\sqrt{2/\pi}$

$\chi_2 \equiv \sqrt{2/\pi}$

**Re($\omega_{\mathrm{GAM}}$) : good agreement with the kinetic GAM frequency**

**Im($\omega_{\mathrm{GAM}}$) : fails in reproducing the $k_r$-dependence of the kinetic GAM damping rate because the correct GAM phase velocity is not taken into account by the present closure model.**

$K_{\mathrm{R-H}} = 1/(1+1.6q^2/\varepsilon_t^{1/2})$ $\Longrightarrow$ **no $k_r$-dependence of** $\langle\phi(t=\infty)\rangle/\langle\phi(t=0)\rangle$

# ETG-Mode-Driven Zonal Flow

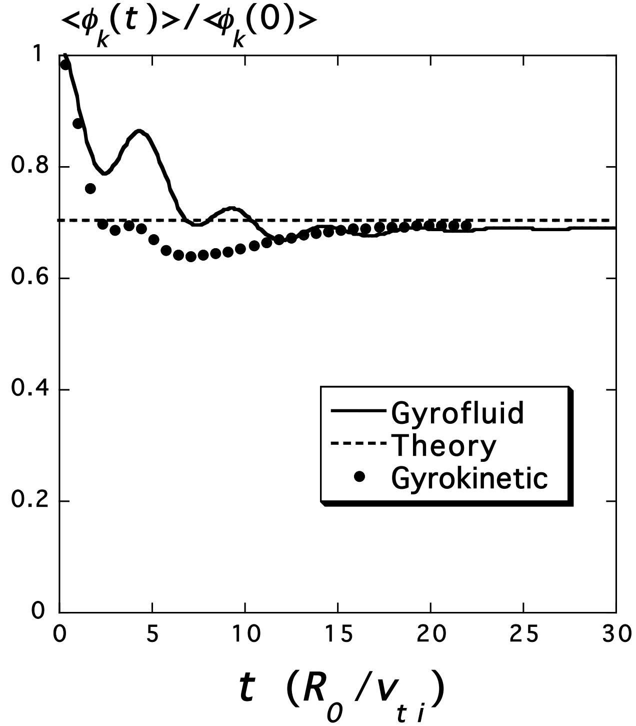
**Gyrofluid equations for electrons combined with the Poisson equation**

$$e^{-b_e/2}\left(\frac{\delta n_{e\mathbf{k}_\perp}^{(\mathrm{g})}}{n_0}-\frac{b_e}{2}\frac{\delta T_{e\perp\mathbf{k}_\perp}}{T_e}\right)+\frac{e\phi_{\mathbf{k}_\perp}}{T_e}\left[1-\Gamma_0(b_e)+k_\perp\lambda_{De}^2\right] = -\frac{e\phi_{\mathbf{k}_\perp}}{T_i}$$

**Gyrofluid simulation shows the same residual zonal-flow level as given by gyrokinetic simulation and the analytical theory.**

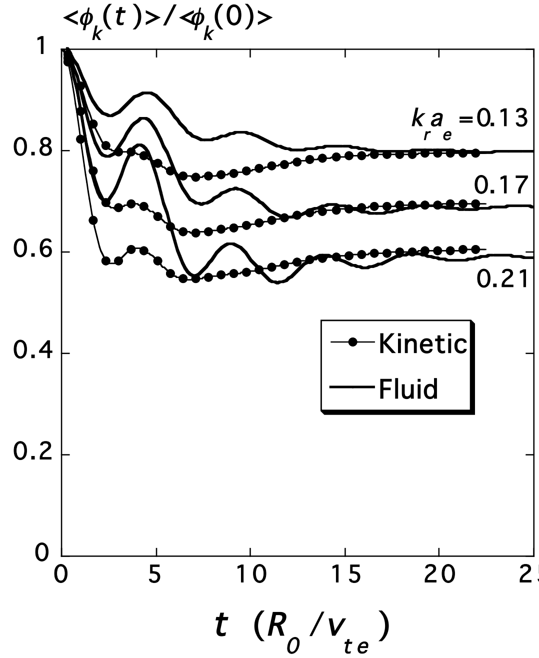
$$\boxed{\phi_{\mathbf{k}_\perp}(t)=\frac{T_e/T_i+k_\perp^2(a_e^2+\lambda_{De}^2)}{T_e/T_i+k_\perp^2 a_e^2[1+1.6(1+T_e/T_i)q^2/\varepsilon^{1/2}]+k_\perp^2\lambda_{De}^2}\phi_{\mathbf{k}_\perp}(0)}$$

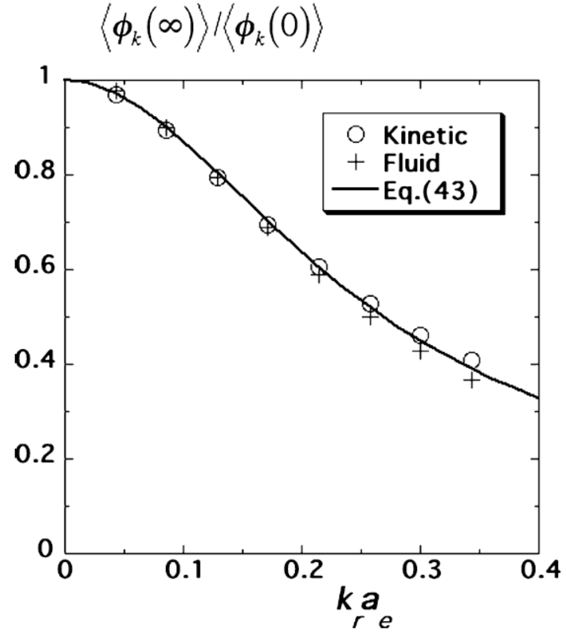
(b) $k_r a_e = 0.172$



# Radial Wavenumber Dependence of ETG-Mode-Driven Zonal Flow

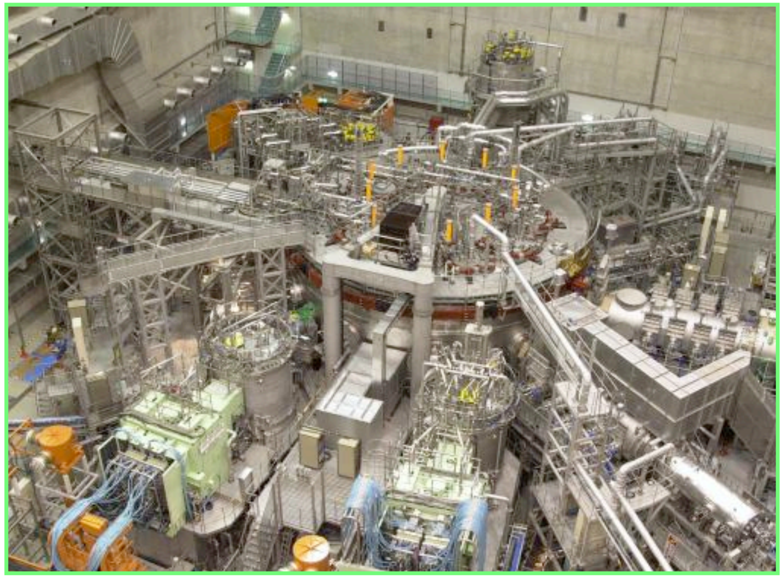
$$\phi_{\mathbf{k}_\perp}(t) = \frac{T_e/T_i + k_\perp^2(a_e^2 + \lambda_{De}^2)}{T_e/T_i + k_\perp^2 a_e^2[1 + 1.6(1 + T_e/T_i)q^2/\varepsilon^{1/2}] + k_\perp^2\lambda_{De}^2}\phi_{\mathbf{k}_\perp}(0)$$




**Gyrofluid simulation successfully reproduces the $k_r$-dependence of the residual zonal flow given by gyrokinetic simulation and the analytical theory.**

# Zonal Flows and ITG Turbulence in Helical Plasmas



Large Helical Device (LHD)