**Gyrokinetic simulations**

Aim to describe turbulence in a tokamak plasma, thus giving information about heat and particle transport and confinement related phenomena.

Several theoretical approaches and numerical methods (Vlasov, PIC, full-f, delta-f, etc.) have been and are still used.

**PIC method and ELMFIRE**

Key ingredients for PIC simulation model:

- Equations of motion

$$\frac{d\mathbf{R}}{dt} = U\mathbf{b} + \frac{\mu}{e} \frac{\mathbf{b}}{B} \times \mathbf{B} + \frac{1}{\Omega} \left[ \mathbf{b} \times \nabla \phi \right]$$

- The Poisson equation

$$-\frac{1}{4\pi} \nabla \phi = \int d^3Z \left[ F + \Omega B \frac{\partial}{\partial \mu} \left( \phi - \langle \phi \rangle \right) \right]$$

The equations above represent the ones ELMFIRE is based on.

In the standard set, the polarization drift is not included in the equations of motion and the $A^0$ and $5^0$ terms in the Poisson equation are not present.

Both sets of equations should in principle form a consistent description of the electrodynamical system comprised of charged particles.

In both approaches, $\Delta \phi = 0$ and the terms involving gradient of distribution function are also small due to long scale length of equilibrium density and temperature.

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**The Particle Lagrangian**

$$L = p - H = \frac{1}{2} m \left( \mathbf{u} \cdot \mathbf{u} \right) + \frac{e}{c} \mathbf{A} \cdot \mathbf{u} - e\phi$$

$$\mathbf{q} = \mathbf{x}, \quad p = \frac{\partial L}{\partial \dot{\mathbf{x}}} = \frac{\mathbf{u}}{c} = \mathbf{A} + m \dot{\mathbf{x}}$$

The Lagrangian divided to different orders is

$$\gamma_1 = \frac{m u_x c}{e} \mathbf{u} \cdot \mathbf{b} dt$$

$$\gamma_2 = \frac{1}{2} m u_x^2 + \mu B$$

In order to calculate the gyrocenter one form we proceed in a similar fashion as in the standard derivation.

The difference comes from the first order generating functions

$$g^0_x = \frac{1}{m} \left( S_x + (u_x - \langle u_x \rangle) \frac{m e B}{c B^2} U \mathbf{b} \right)$$

$$g^0 = \mathbf{b} \left( \frac{1}{m} \frac{\partial S}{\partial \mu} \right) + \frac{c}{e B^2} m \mathbf{F} \cdot \nabla S_x$$

Action principle was used to find the Poisson equation

$$\int d\mathbf{x} \int d^3 \mathbf{b} \mathbf{F} \cdot \mathbf{b} H_p \rightarrow \int d^3Z \phi \left( \frac{\partial}{\partial \mu} \left( \phi - \langle \phi \rangle \right) \right)$$

**References**
