Gyrokinetic models:

some open mathematical problems

François Golse
Ecole polytechnique
Centre de Mathématiques L. Schwartz
golse@math.polytechnique.fr

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1) Derivation of gyrokinetic models

a) Rigorous derivation of gyrokinetic models what is known:

Frénod-Sonnendrucker (2001), D. Han-Kwan (2008)

M. Bostan (2008) — abstract framework

b) What remains to be done (to some extent): complete treatment of varying/self-consistent magnetic fields

H. Sugama's derivation based on Lagrangian formulation of VPA system

M. Bostan (2008) [it remains to translate the abstract result in the context of the gyrokinetic plasma]

c) Interaction of high magnetic field limit with quasi-neutrality

●Difficulties: it is not known to this date whether Vlasov + EM field has classical solutions for all positive times

a) Vlasov-Darwin: existence of global weak solutions OK (C. Pallard, 2007); local existence of classical C^1 solutions

b) Vlasov-Maxwell: existence of global weak solutions (DiPerna-Lions 1989); local existence of classical C^1 solutions (Glassey-Strauss, 1986); blow-up criterion (blow up infinite time iff particles reach infinite momentum in finite time: due to Glassey-Strauss, 1986)

2) Long time asymptotics/effect of collisions

•arrive at a clear understanding of the role of $\mathcal{C}$, especially

a) its size (scaling)

b) the influence of $\mathcal{C}$ on transport coefficients

[In the case of the derivation of hydrodynamics from the Boltzmann equation, the specifics of the collision integral influence the formulas for the viscosity and thermal diffusivity somewhat implicitly, via the solution of a Freholm integral equation]

c) In the context of fluid mechanics, using reduced models can create problems

[Example: replacing Boltzmann's collision integral with BGK relaxation model leads to Prandtl number=1]

●<u>For instance:</u> see S. Cowley's presentation of the ion Fokker-Planck equation, with the appropriate scaling:

$$\begin{aligned} \epsilon^2 \partial_t F_0 + \epsilon \partial_t \delta f_\epsilon + v \cdot \nabla_x F_0 + v_\perp \cdot \nabla_x \delta f + \epsilon v_{||} \partial_{||} \delta f \\ + \left(E + \frac{1}{\epsilon} v \wedge B \right) \cdot \nabla_v (F_0 + \epsilon \delta f) + \epsilon^{3/4} \left(E + \frac{1}{\epsilon} v \wedge B \right) \cdot \nabla_{\tilde{v}} \delta f \\ = \mathcal{C}(F_0, F_0) + \epsilon \left(\mathcal{C}(F_0, \delta f) + \mathcal{C}(\delta f, F_0) \right) + \epsilon^2 \mathcal{C}(\delta f, \delta f) \end{aligned}$$

$\Rightarrow$ derive asymptotic limit of that equation as $\epsilon \to 0$

<u>Mathematical difficulties:</u>

●In the case of fluid mechanics, building the Hilbert or Chapman-Enskog expansion is based on solving a Fredholm integral equation for each order in the expansion. Stability implied by the entropy production term in the H-Theorem.

$\Rightarrow$ need for a systematic study in the gyrokinetic plasma context.

●Hilbert or Chapman-Enskog type expansions, as truncated expansions may lead to number densities that are not everywhere nonnegative.

●Using moment methods with rigorously justified closure relations avoids this type of difficulty — however, little control on the size of the error