Numerical Gyrokinetics:
Some basic issues associated with gyrokinetic simulations of plasma turbulence

William Dorland
Center for Multiscale Plasma Dynamics
Department of Physics
University of Maryland
Wolfgang Pauli Institute
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In collaboration with

Ingmar Broemstrup

Michael Barnes

Kyle Gustafson

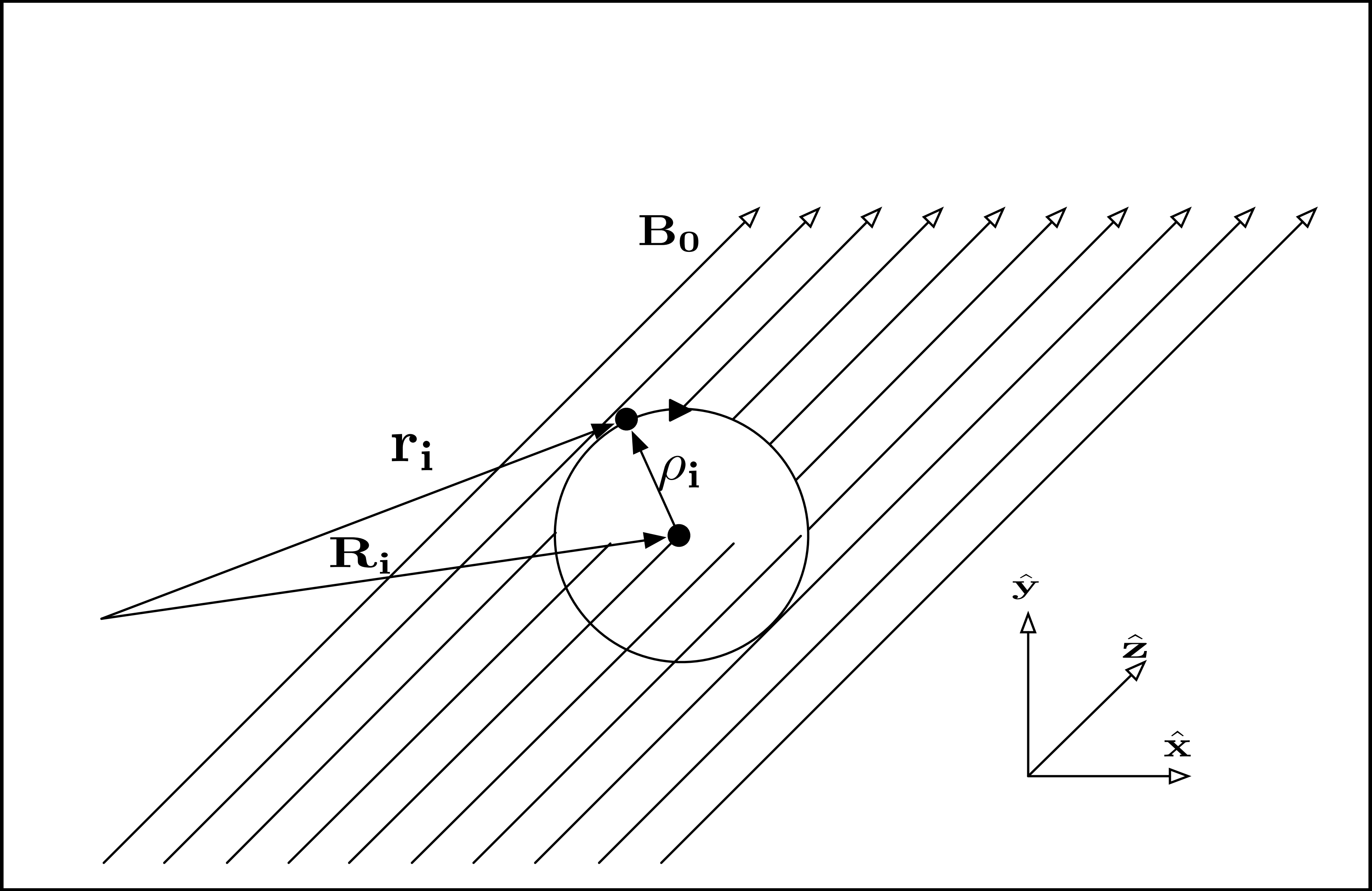
Ryusuke Numata

Tomoya Tatsuno

Overview

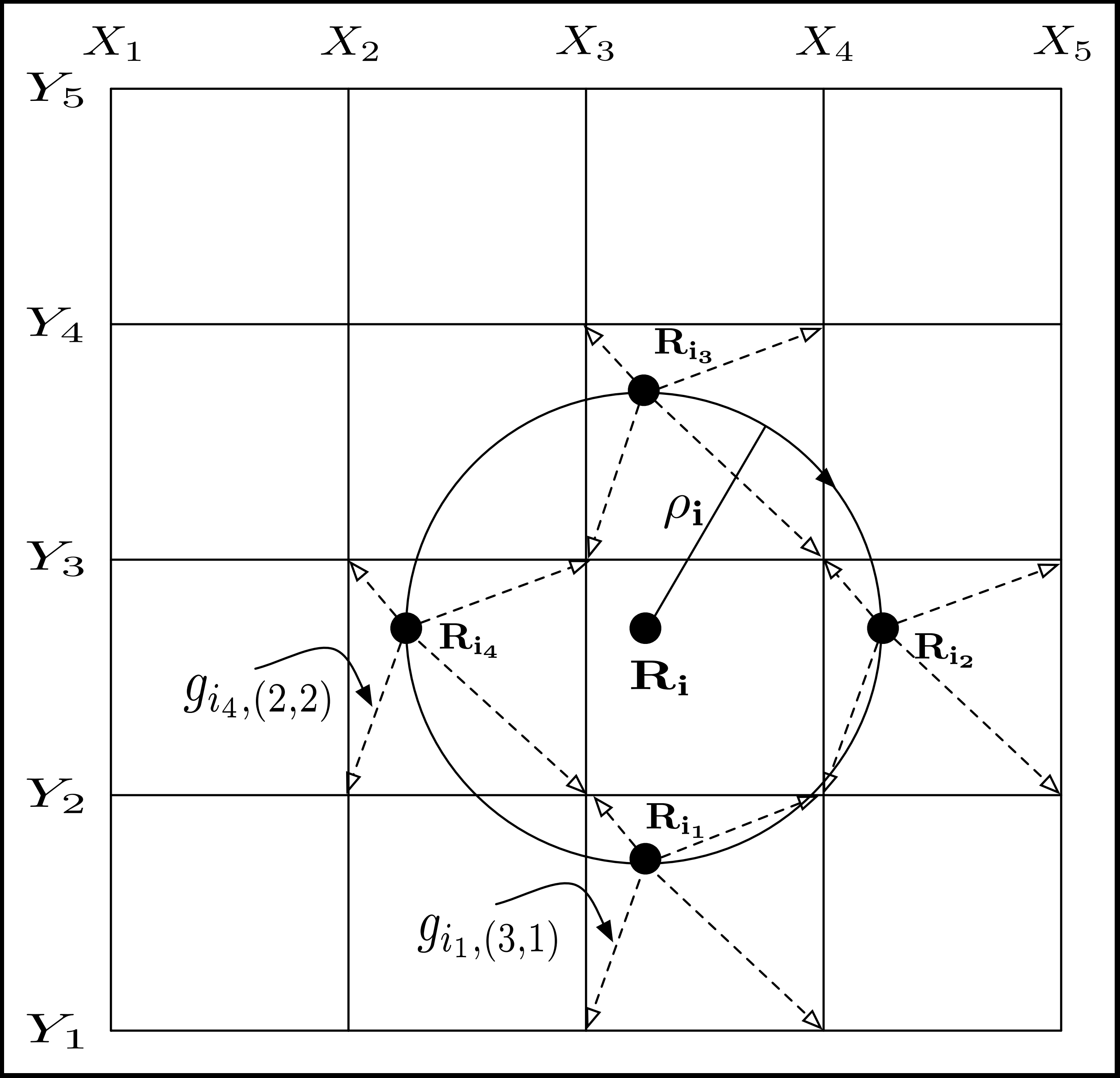
- Nonlocality of gyrokinetic equations
- Fluid vs kinetic description
- Eulerian, Lagrangian, Semi-Lagrangian
- Flux-tube vs global
- Hamiltonian vs dissipative
- Diffusive vs non-diffusive transport

Nonlocality of gyrokinetics

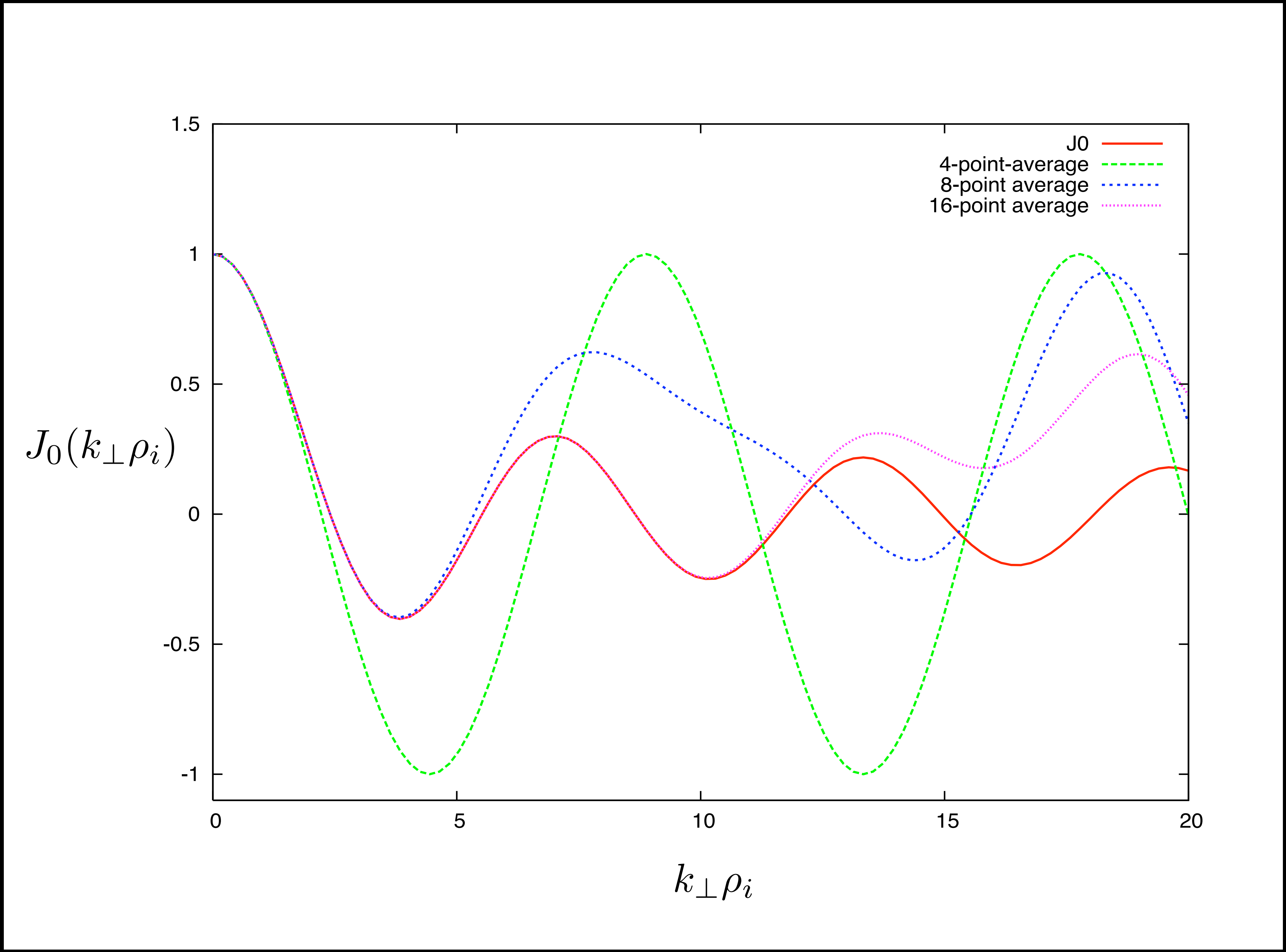


Rapid gyration around gyro-center is assumed to be infinitely fast, so particles drift according to the spatially averaged fields. Fields have structure larger and smaller.

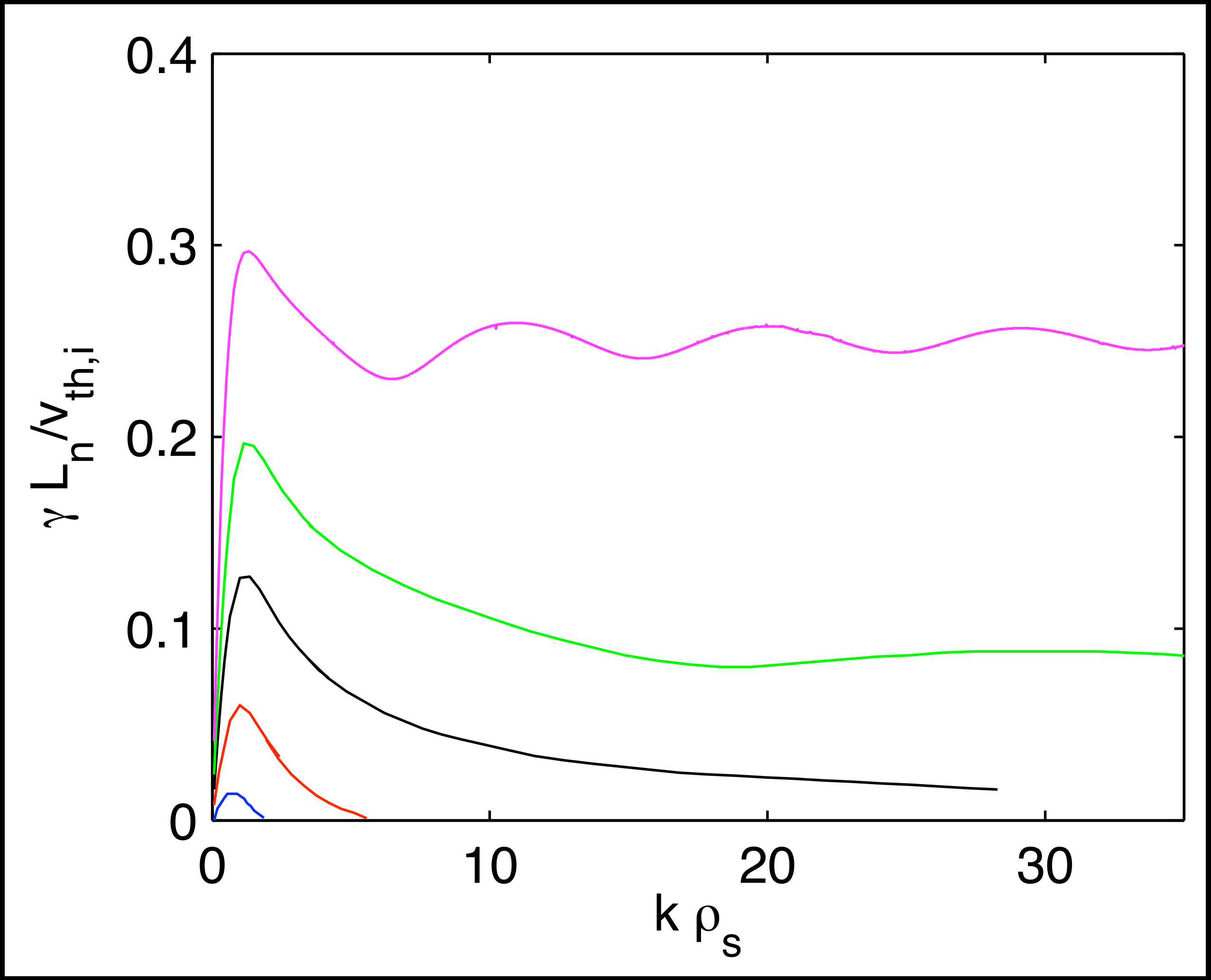
Discretization of average in PIC codes



Accuracy limited at short wavelengths



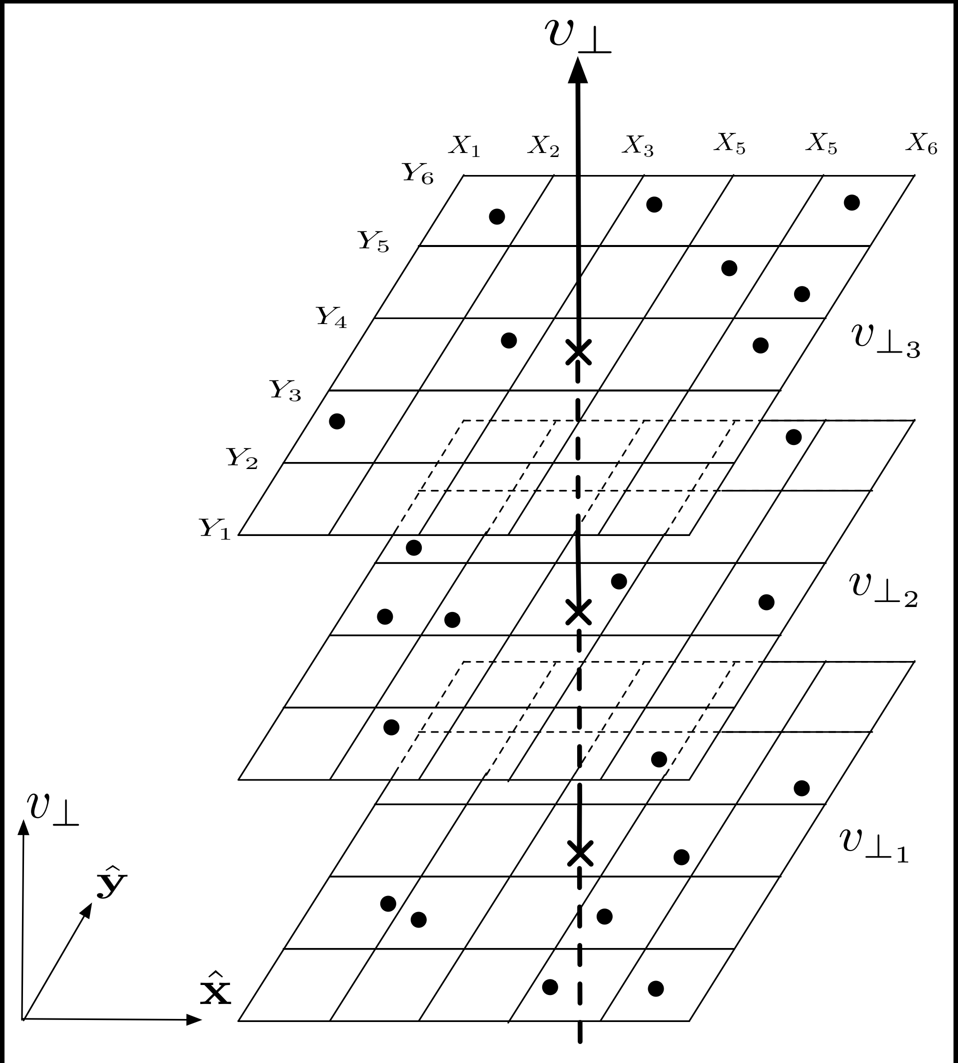
But small scales can be unstable!



- Growth rates shown are for a curvature driven instability such as discussed by Hammett this morning; details in recent papers by Ricci, *et al.*, and by Simakov and Catto.
- Faster growth rates correspond to steeper gradients.

- Very typical behavior in tokamaks: short wavelengths are unstable, and in some cases, tend to saturate at high levels.

Alternative averaging scheme required



- Broemstrup's scheme is similar to that used in continuum codes.
- Basic idea: find contributions to fields from groups of particles with (roughly) the same gyroradius one at a time
- Easy to implement if Fourier modes are easily obtained (such as in flux-tube sims)

$$\Phi \sim \int J_0\left(\frac{k_\perp v_\perp}{\Omega}\right) f_{\mathbf{k}}(v_\perp, v_\parallel, x, y, z) v_\perp \, dv_\perp dv_\parallel$$

Fluid vs kinetic descriptions

- **Fully kinetic:** equations for background and fluctuations all kinetic.
    - Edge, full-*F*, and/or global codes. Sonnendrucker, Leerink, Parker, *et al.*
    - Wasteful (or wrong) for realistic collisionality in Cowley's ITER ordering, but perhaps necessary for edge, where separation of scales is not as extreme
- **Hierarchical:** fluid equations for background and kinetic equations for fluctuations
    - Focus of Maryland/UCLA/CMPD effort. See posters by Barnes and Parra here; talks by Catto, Cowley.
- **Fully fluid:** equations for background and fluctuations all fluid
    - Gyrofluid closure approach. T Passot will discuss this afternoon.

Eulerian, Lagrangian, Semi-Lagrangian

- **In each case, real-space grid is fixed in time.**
- **Eulerian:** Continuum methods, familiar from CFD

$$f = f(\mathbf{x}, \mathbf{v}; t) \qquad \bar{\chi} = \bar{\chi}(\mathbf{x}, \mathbf{v}; t)$$

- **Lagrangian:** PIC methods -- Method of characteristics

$$f_i = f_i(\mathbf{x}_i(t), \mathbf{v}_i(t); t) \qquad \bar{\chi}_i = \bar{\chi}_i(\mathbf{x}, \mathbf{v}_i(t); t)$$

- **Semi-Lagrangian:** Previous talk: aim for best combination of above methods.
    - Avoid timestep limitations for perpendicular, weakly-sheared flows (drifts).
- Boundary conditions important in each case.

Flux-tube vs Global

- **Flux-tube**
  - Scale perpendicular dimensions of simulation domain to the gyroradius and take GK expansion parameter equal to zero.
  - No variation of equilibrium temperature, density, scale lengths, *etc.* across domain.
  - Use periodic boundary conditions for fluctuations.
  - Find surface-averaged transport and heating.
  - My view: Rely on asymptotics to separate time and space scales, easing numerics.
- **Global**
  - Scale overall domain size to gyroradius; use finite value of GK expansion parameter.
  - Equilibrium distribution function varies across domain. Collisions req'd to get Maxwellian.
  - Boundary conditions should reflect separatrix, walls, magnetic axis (although not usually done).
  - Require sources to prevent profile flattening.
  - My view: Numerics must be extraordinary for this approach to be credible at small rho star.

Hamiltonian vs Diffusive

- **Hamiltonian**
  - Take the Vlasov equation as starting point.
  - Emphasis on maintaining Hamiltonian character leads to keeping higher order terms in dynamical equations.
  - Problem: How should physical diffusion appear on dynamical time scales?
- **Diffusive**
  - Take the Fokker-Planck equation as starting point.
  - Higher-order terms appear at transport space and time scales (knocked down an additional factor of epsilon by averages).
  - Opportunity: Use physically motivated models of diffusion on dynamical time scales.

Diffusive vs. Non-diffusive

- **Diffusive**
  - Transport is well-described by random walk with normally distributed step sizes and Poisson-distributed step times.
  - Implies that fluxes are locally determined, *e.g.,* by gradients.
  - Expected to characterize core tokamak turbulence.
- **Non-diffusive**
  - Transport levels are heavily influenced by rare or long-distance events.
  - May imply, *e.g.,* radial non-locality to transport fluxes.
  - May be particularly important for transport across narrow barriers.

The end