Numerical Gyrokinetics:
Some basic issues associated with gyrokinetic simulations of plasma turbulence

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Overview

• Nonlocality of gyrokinetic equations
• Fluid vs kinetic description
• Eulerian, Lagrangian, Semi-Lagrangian
• Flux-tube vs global
• Hamiltonian vs dissipative
• Diffusive vs non-diffusive transport
Rapid gyration around gyro-center is assumed to be infinitely fast, so particles drift according to the spatially averaged fields. Fields have structure larger and smaller.
Discretization of average in PIC codes

\[ \rho_i \]

\[ Y_1 \quad Y_2 \quad Y_3 \quad Y_4 \quad Y_5 \]

\[ X_1 \quad X_2 \quad X_3 \quad X_4 \quad X_5 \]

\[ g_{i_1}, (3,1) \]

\[ g_{i_4}, (2,2) \]

\[ R_{i_1} \]

\[ R_{i_2} \]

\[ R_{i_3} \]

\[ \rho_i \]
Accuracy limited at short wavelengths

\[ J_0(k_{\perp} \rho_i) \]

\[ k_{\perp} \rho_i \]
But small scales can be unstable!

- Growth rates shown are for a curvature driven instability such as discussed by Hammett this morning; details in recent papers by Ricci, et al., and by Simakov and Catto.

- Faster growth rates correspond to steeper gradients.

- Very typical behavior in tokamaks: short wavelengths are unstable, and in some cases, tend to saturate at high levels.
Alternative averaging scheme required

- Broemstrup's scheme is similar to that used in continuum codes.
- Basic idea: find contributions to fields from groups of particles with (roughly) the same gyroradius one at a time
- Easy to implement if Fourier modes are easily obtained (such as in flux-tube sims)

\[ \Phi \sim \int J_0 \left( \frac{k_{\perp} v_{\perp}}{\Omega} \right) f_k(v_{\perp}, v_{\parallel}, x, y, z) v_{\perp} dv_{\perp} dv_{\parallel} \]
Fluid vs kinetic descriptions

- **Fully kinetic**: equations for background and fluctuations all kinetic.
  - Edge, full-$F$, and/or global codes. Sonnendrucker, Leerink, Parker, *et al.*
  - Wasteful (or wrong) for realistic collisionality in Cowley’s ITER ordering, but perhaps necessary for edge, where separation of scales is not as extreme

- **Hierarchical**: fluid equations for background and kinetic equations for fluctuations
  - Focus of Maryland/UCLA/CMPD effort. See posters by Barnes and Parra here; talks by Catto, Cowley.

- **Fully fluid**: equations for background and fluctuations all fluid
  - Gyrofluid closure approach. T Passot will discuss this afternoon.
Eulerian, Lagrangian, Semi-Lagrangian

- In each case, real-space grid is fixed in time.
- **Eulerian**: Continuum methods, familiar from CFD
  \[ f = f(x, v; t) \quad \bar{\chi} = \bar{\chi}(x, v; t) \]
- **Lagrangian**: PIC methods -- Method of characteristics
  \[ f_i = f_i(x_i(t), v_i(t); t) \quad \bar{\chi}_i = \bar{\chi}_i(x, v_i(t); t) \]
- **Semi-Lagrangian**: Previous talk: aim for best combination of above methods.
  - Avoid timestep limitations for perpendicular, weakly-sheared flows (drifts).
- Boundary conditions important in each case.
Flux-tube vs Global

- **Flux-tube**
  - Scale perpendicular dimensions of simulation domain to the gyroradius and take GK expansion parameter equal to zero.
  - No variation of equilibrium temperature, density, scale lengths, etc. across domain.
  - Use periodic boundary conditions for fluctuations.
  - Find surface-averaged transport and heating.
  - My view: Rely on asymptotics to separate time and space scales, easing numerics.

- **Global**
  - Scale overall domain size to gyroradius; use finite value of GK expansion parameter.
  - Equilibrium distribution function varies across domain. Collisions req’d to get Maxwellian.
  - Boundary conditions should reflect separatrix, walls, magnetic axis (although not usually done).
  - Require sources to prevent profile flattening.
  - My view: Numerics must be extraordinary for this approach to be credible at small rho star.
Hamiltonian vs Diffusive

**Hamiltonian**
- Take the Vlasov equation as starting point.
- Emphasis on maintaining Hamiltonian character leads to keeping higher order terms in dynamical equations.
- Problem: How should physical diffusion appear on dynamical time scales?

**Diffusive**
- Take the Fokker-Planck equation as starting point.
- Higher-order terms appear at transport space and time scales (knocked down an additional factor of epsilon by averages).
- Opportunity: Use physically motivated models of diffusion on dynamical time scales.
Diffusive vs. Non-diffusive

• **Diffusive**
  
  • Transport is well-described by random walk with normally distributed step sizes and Poisson-distributed step times.
  
  • Implies that fluxes are locally determined, *e.g.*, by gradients.
  
  • Expected to characterize core tokamak turbulence.

• **Non-diffusive**
  
  • Transport levels are heavily influenced by rare or long-distance events.
  
  • May imply, *e.g.*, radial non-locality to transport fluxes.
  
  • May be particularly important for transport across narrow barriers.
The end