TURBULENCE IN STRATIFIED ROTATING FLUIDS

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Collaborations:

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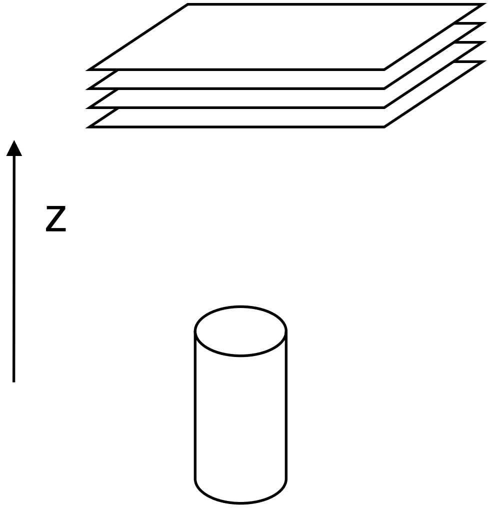
OVERVIEW

1) Laboratory study of stratified rotating turbulence
2) Statistical mechanics and the emergence of organised vortices
3) Competition between advection and straining

Antagonistic effects

stratification

rotation



Froude number:
$Fr = U/(NL)$

$N^2 = -g(d\rho/\rho dz)$

Rossby number:
$Ro = U/(fL)$

$f = 2\Omega$

Quasi-geostrophic model

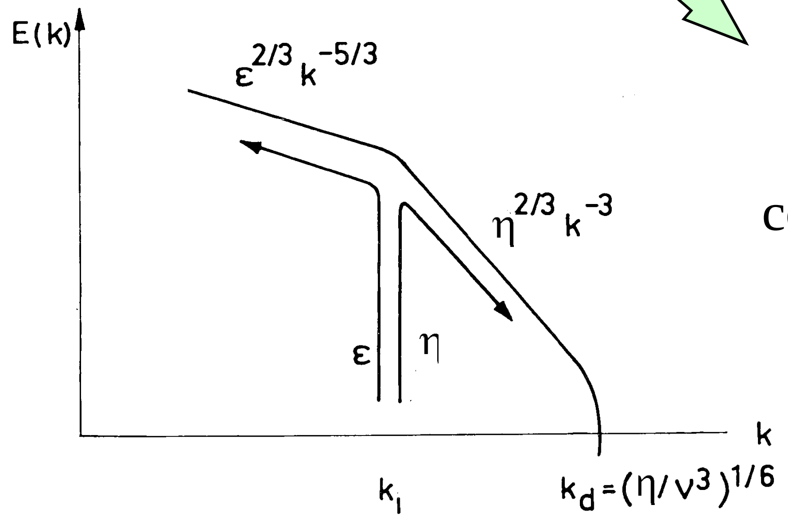
Charney (1971)

Fr << 1 Non-divergent horizontal flow at each *z*,

Ro << 1 $PV = -\Delta_h \psi - (N/f)^2 \partial^2 \psi / \partial z^2$

Formal analogy with two dimensional turbulence (conservation of PV)

Non dissipative dynamics

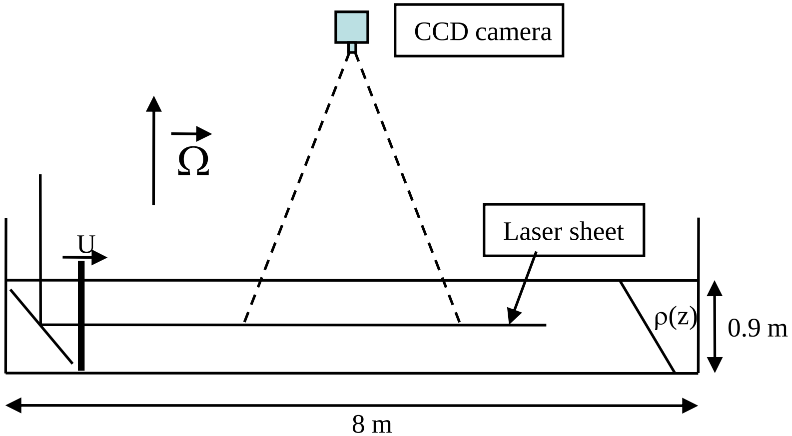


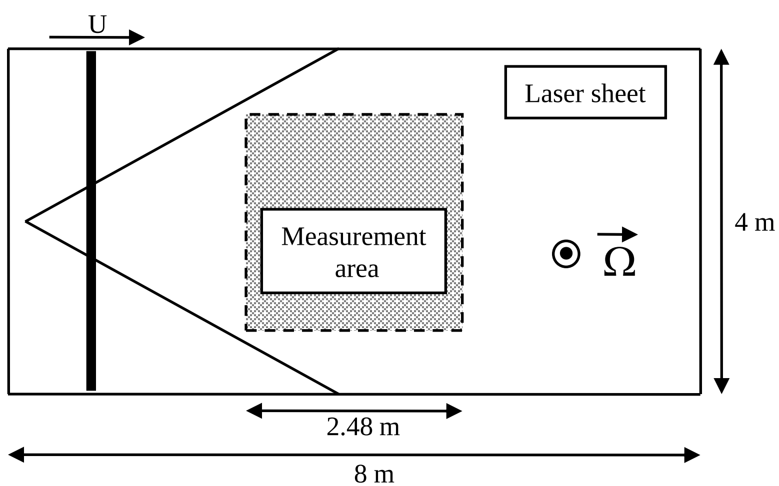
Emergence of coherent structures

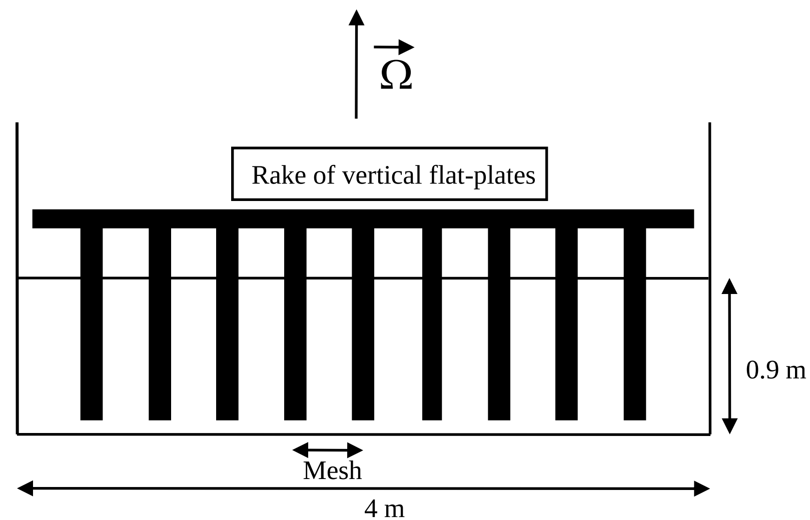
Jupiter atmosphere

Coriolis rotating platform

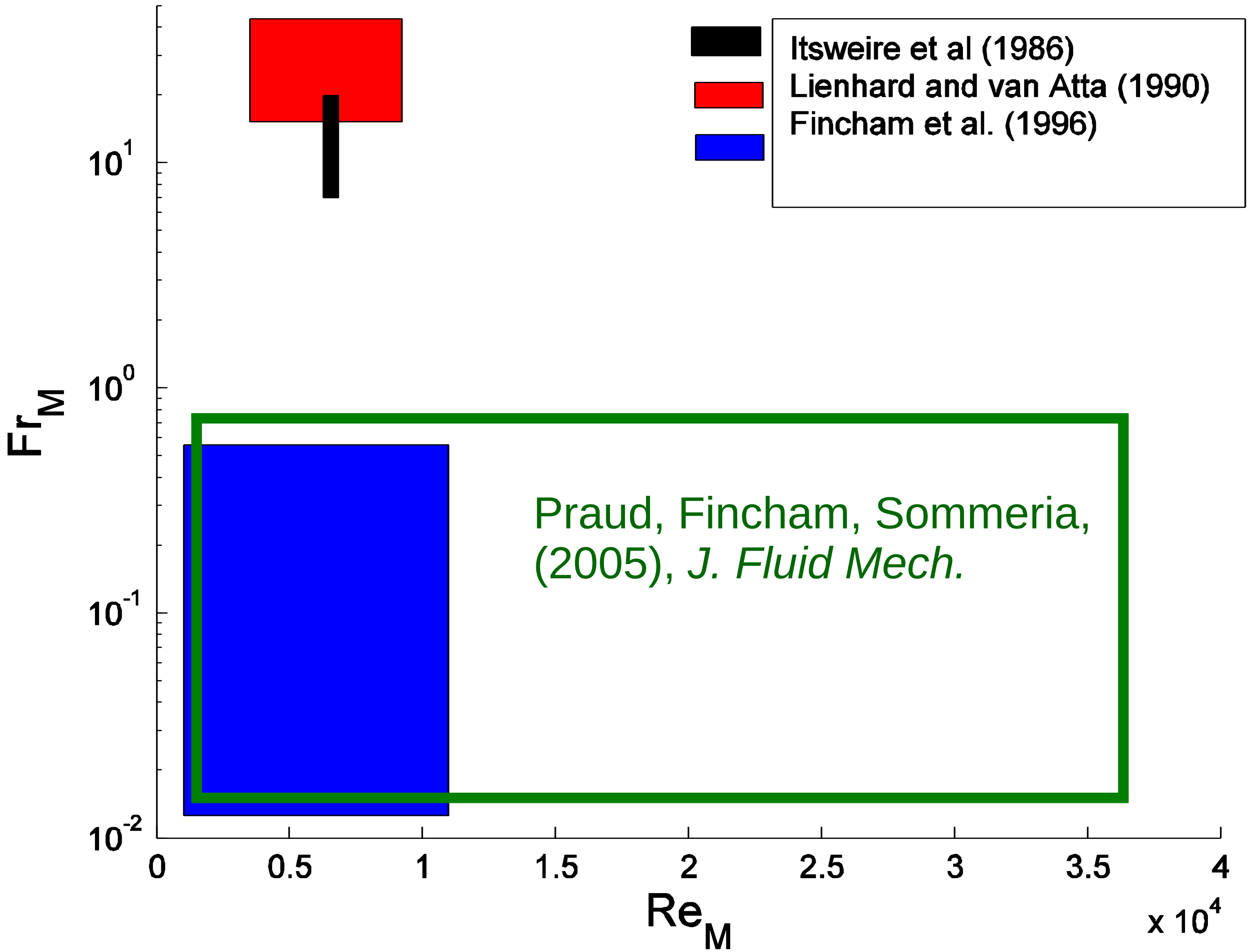
Grid turbulence





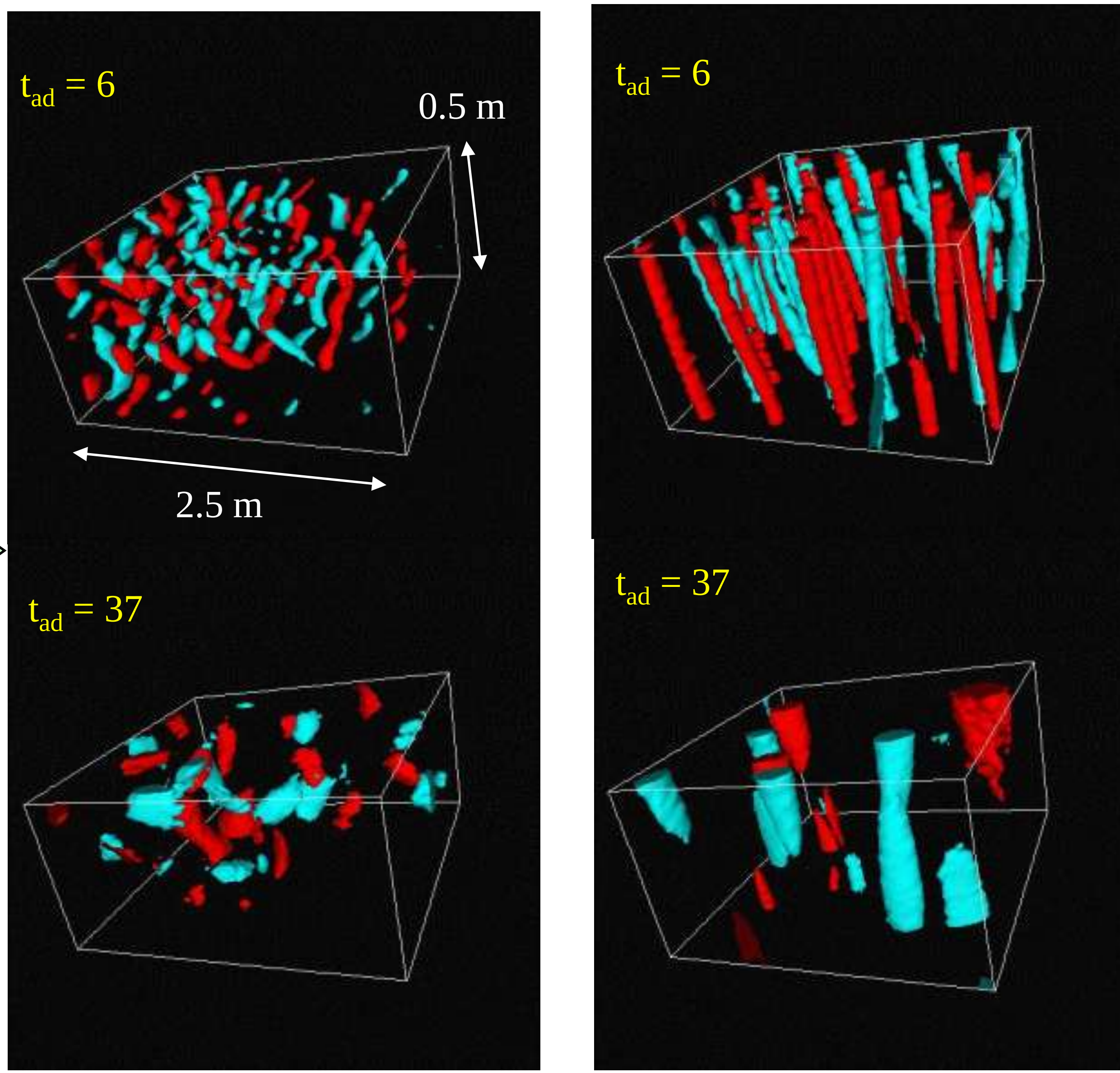


Experimental parameters (no rotation)



Influence of rotation

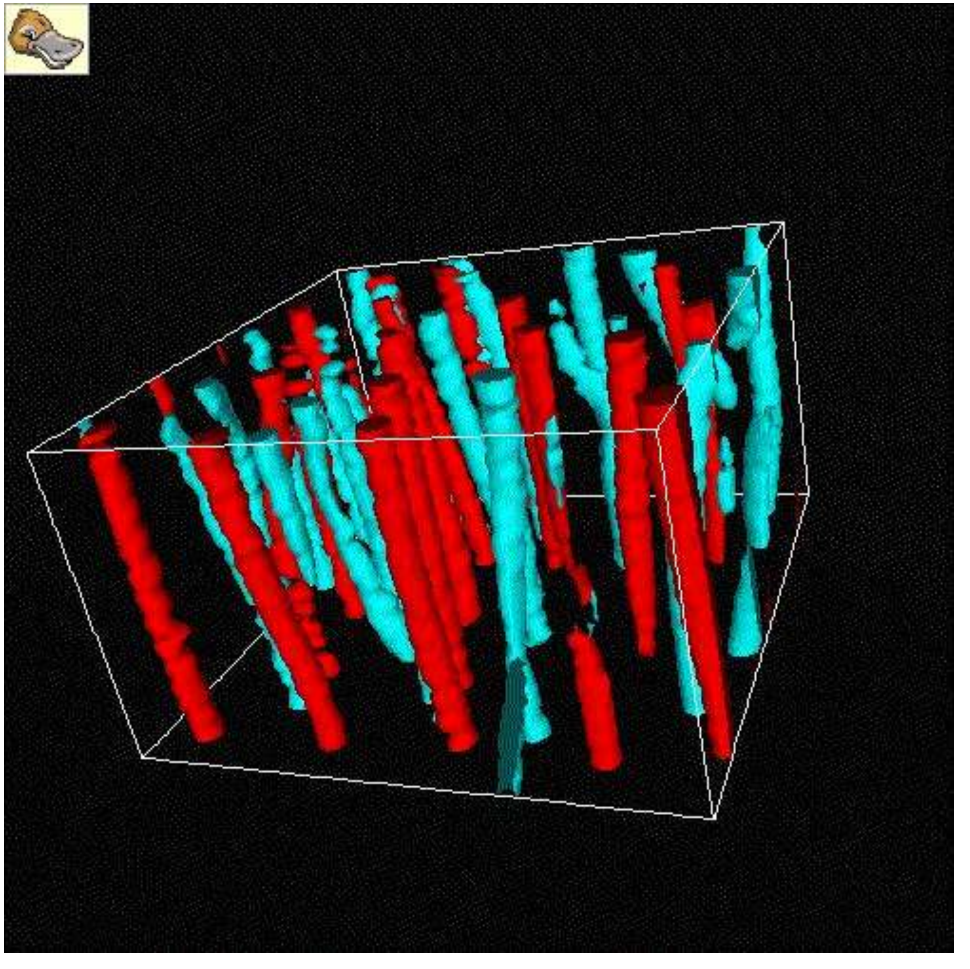
Re_M=4500
Fr_M=0.09



f = 0

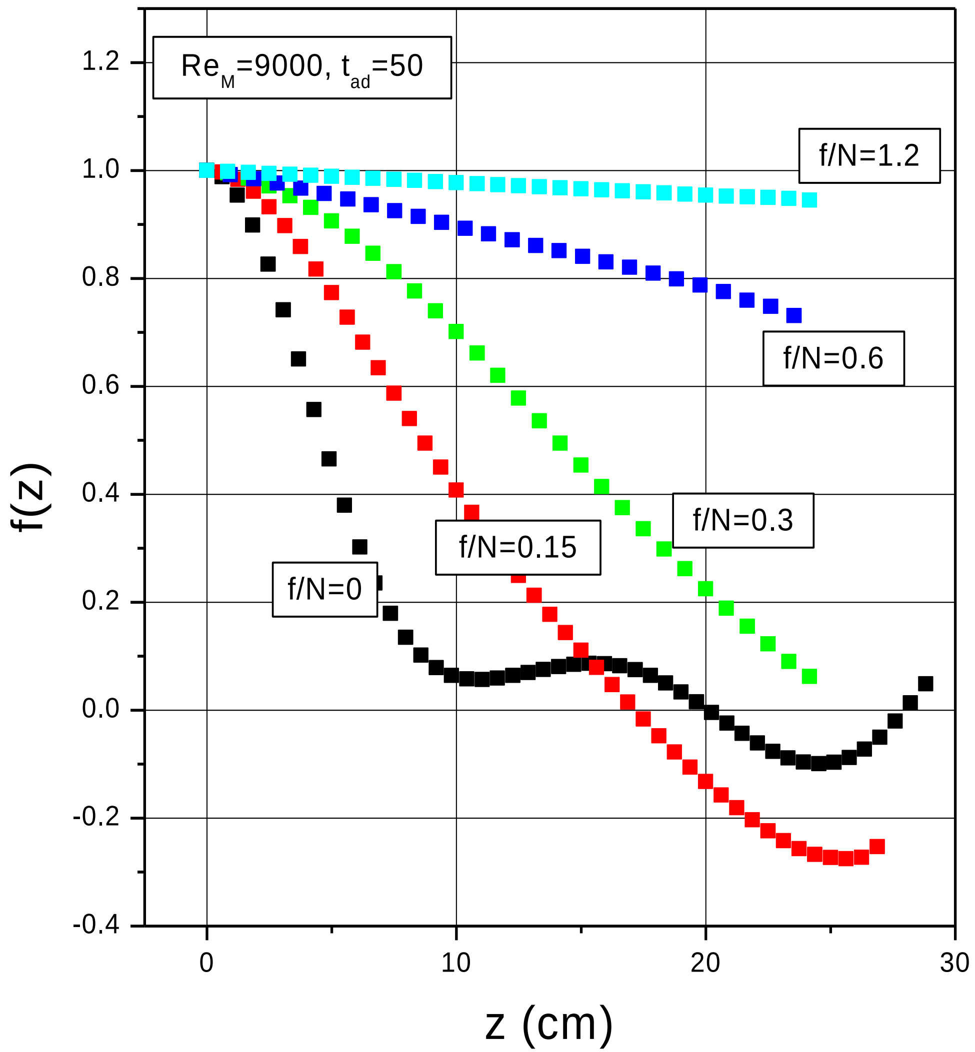
f / N = 1.2

Strong rotation f / N =1.2



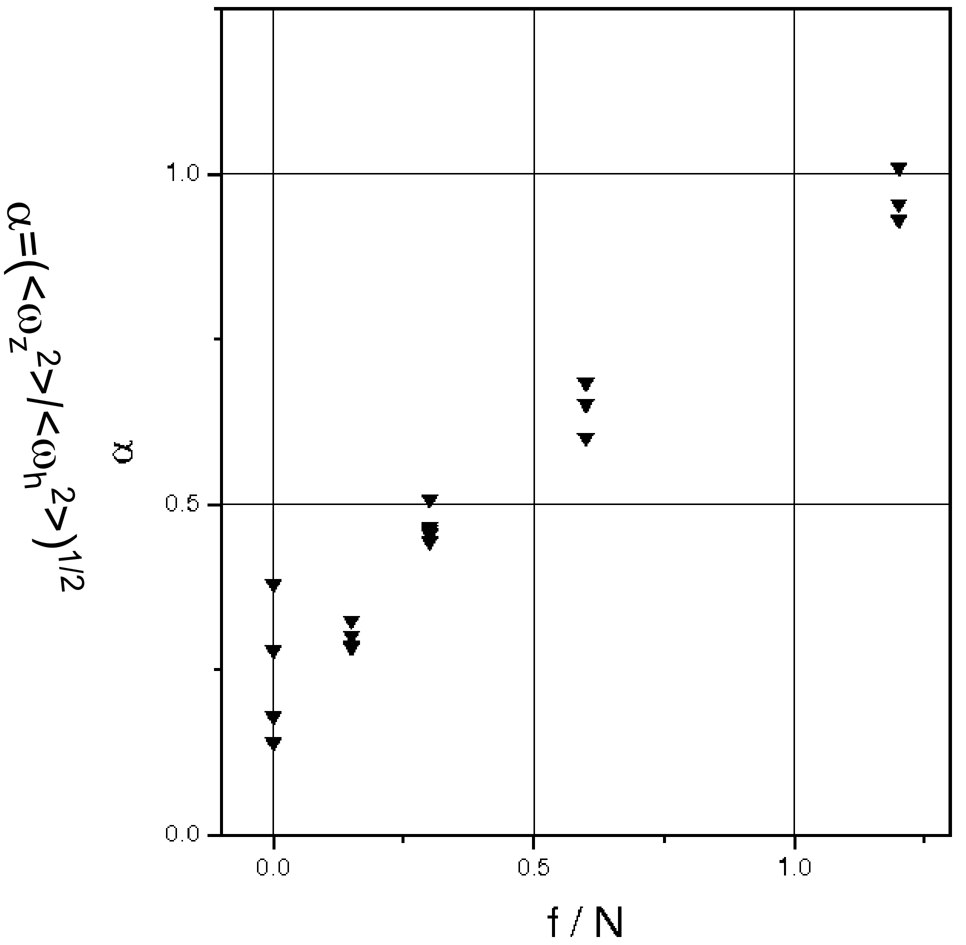
Vertical correlation

Vertical correlation function



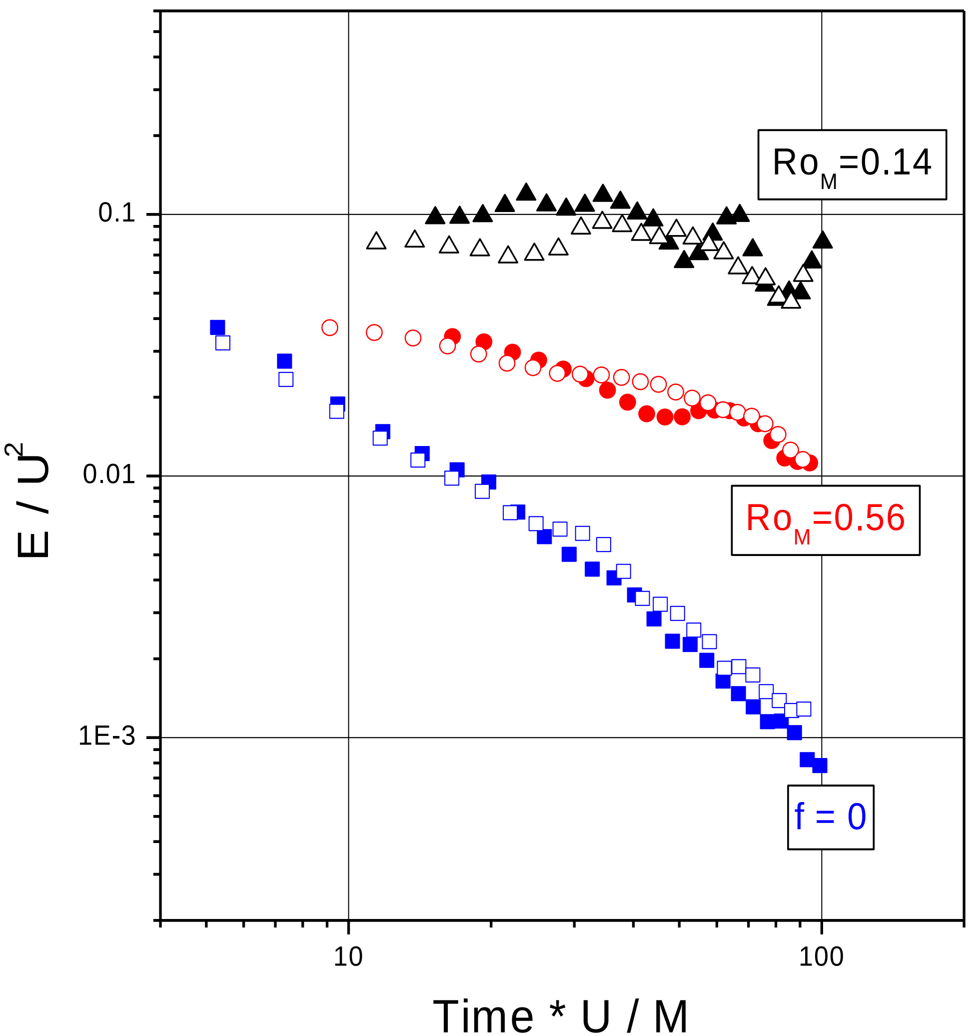
- The vertical scale increases with f/N
- Geostrophic adjustment of the vortices

Aspect ratio



Inhibition of the energy decay

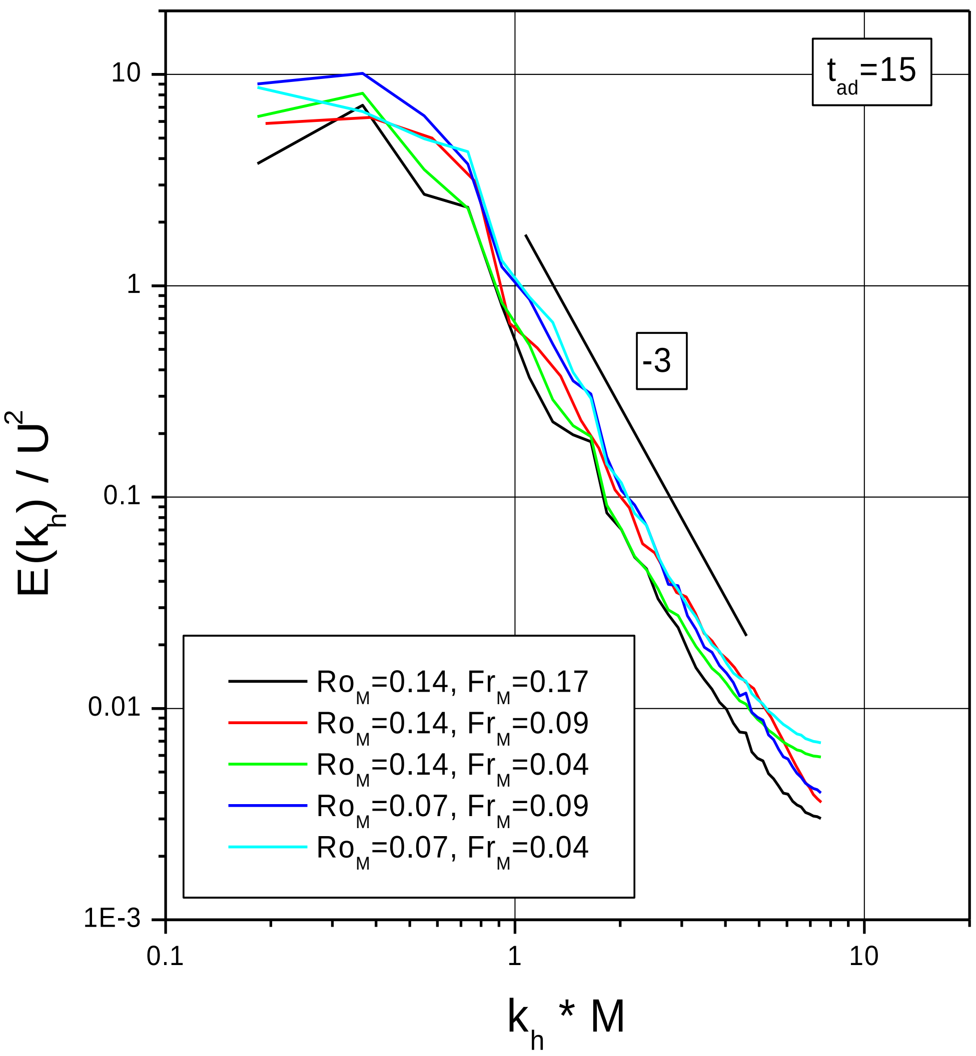
Kinetic energy decay



- Inhibition of the kinetic energy decay with rotation
- Conservation for Ro < 0.2
- Little influence of stratification

Energy spectra

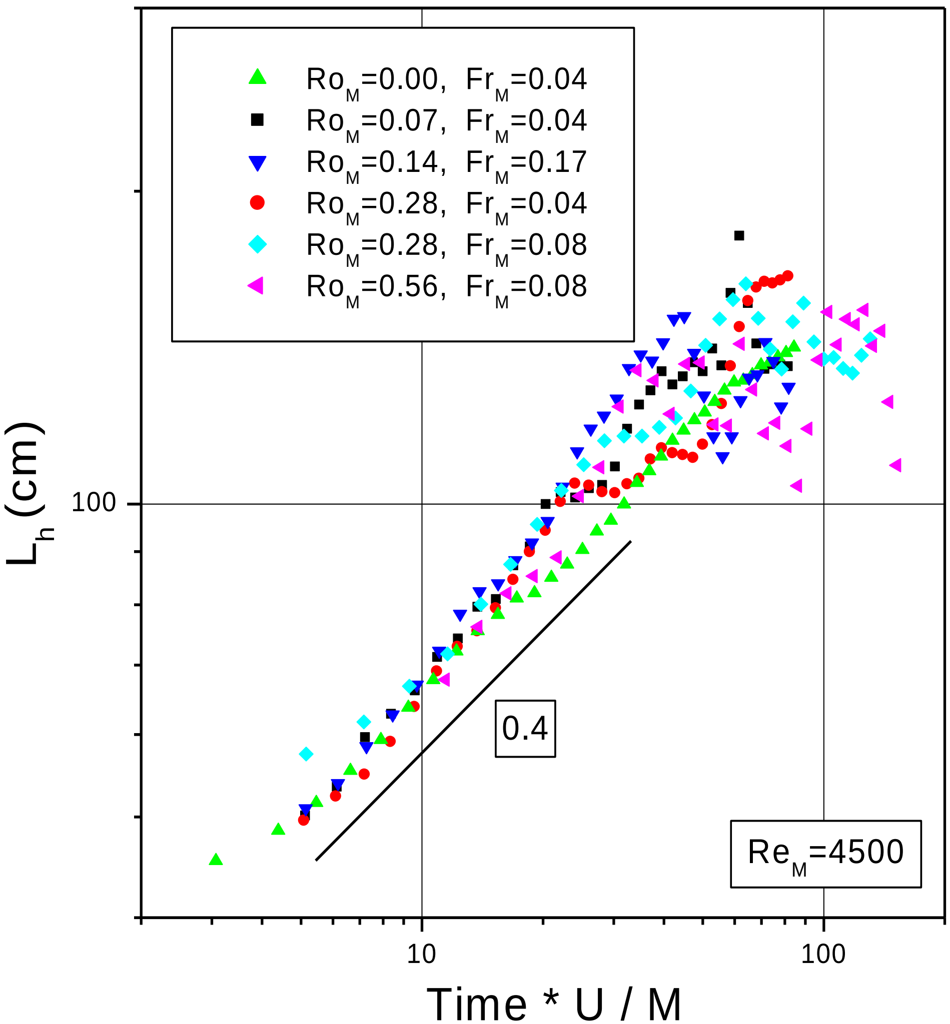
Horizontal energy spectra



- k_h^{-3} energy spectra in agreement with Charney's predictions
- The flow dynamics is quasi-geostrophic

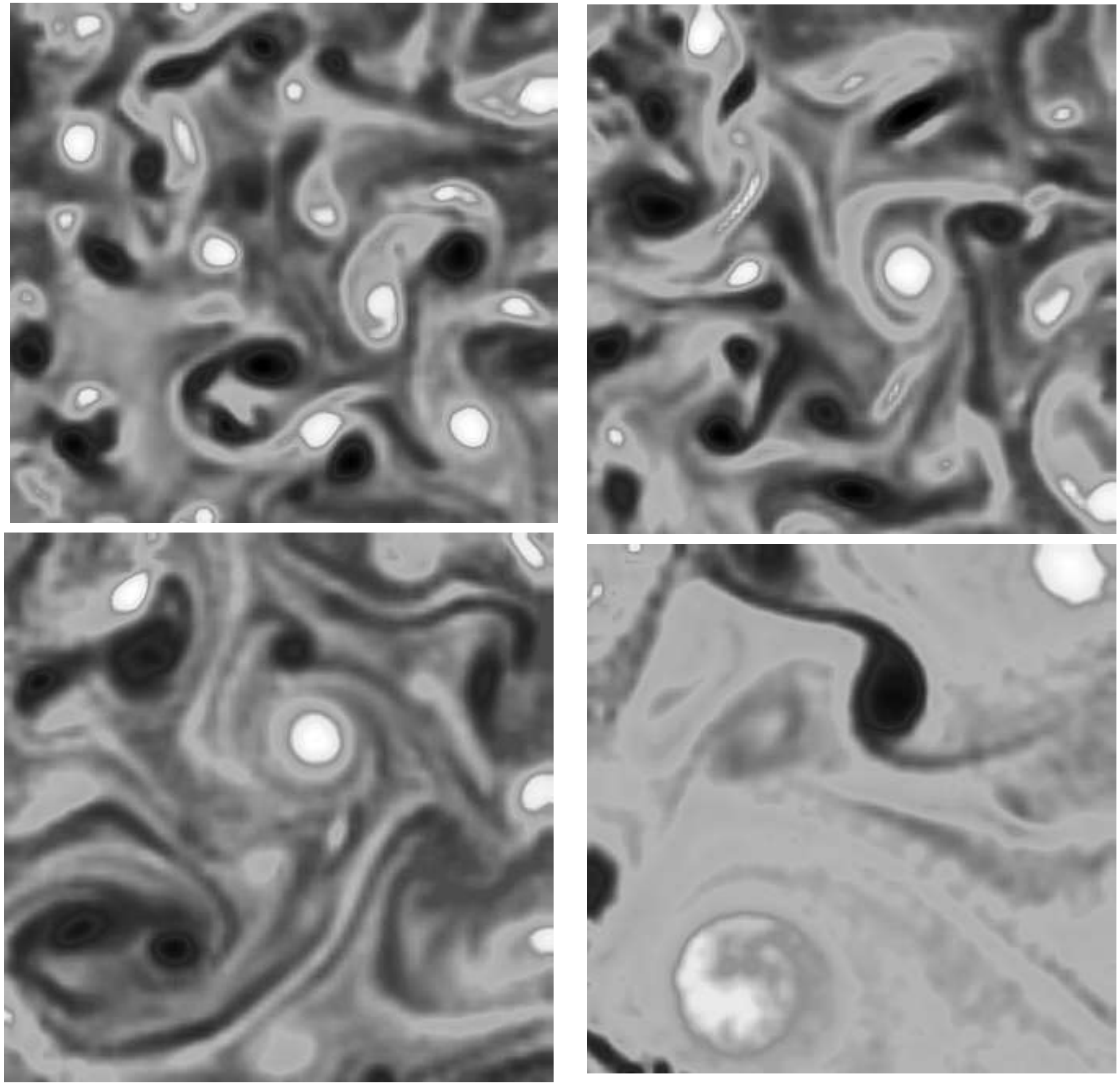
Horizontal length scale

Horizontal integral scale L_h



- Very little influence of rotation and stratification on the horizontal length scale

Emergence of vortices



t=377 s

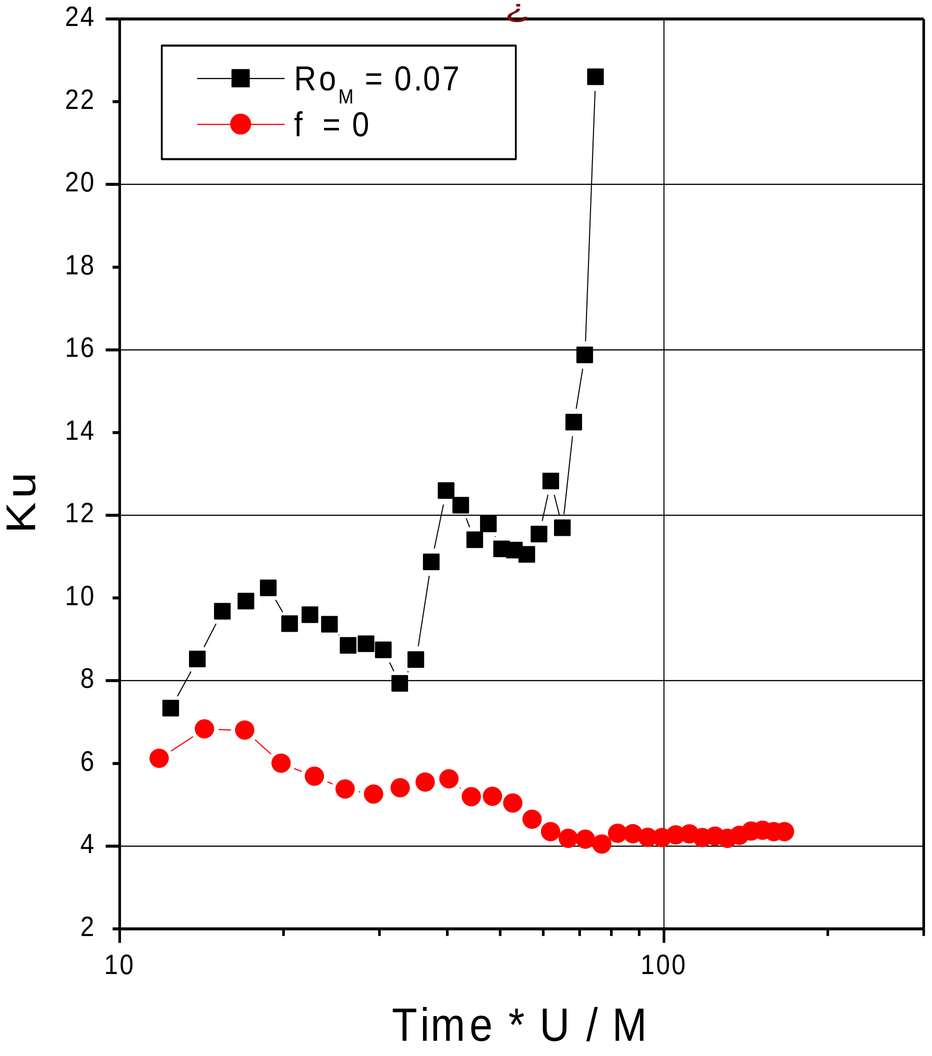
t=619 s

t=915 s

t=3000 s

Intermittency

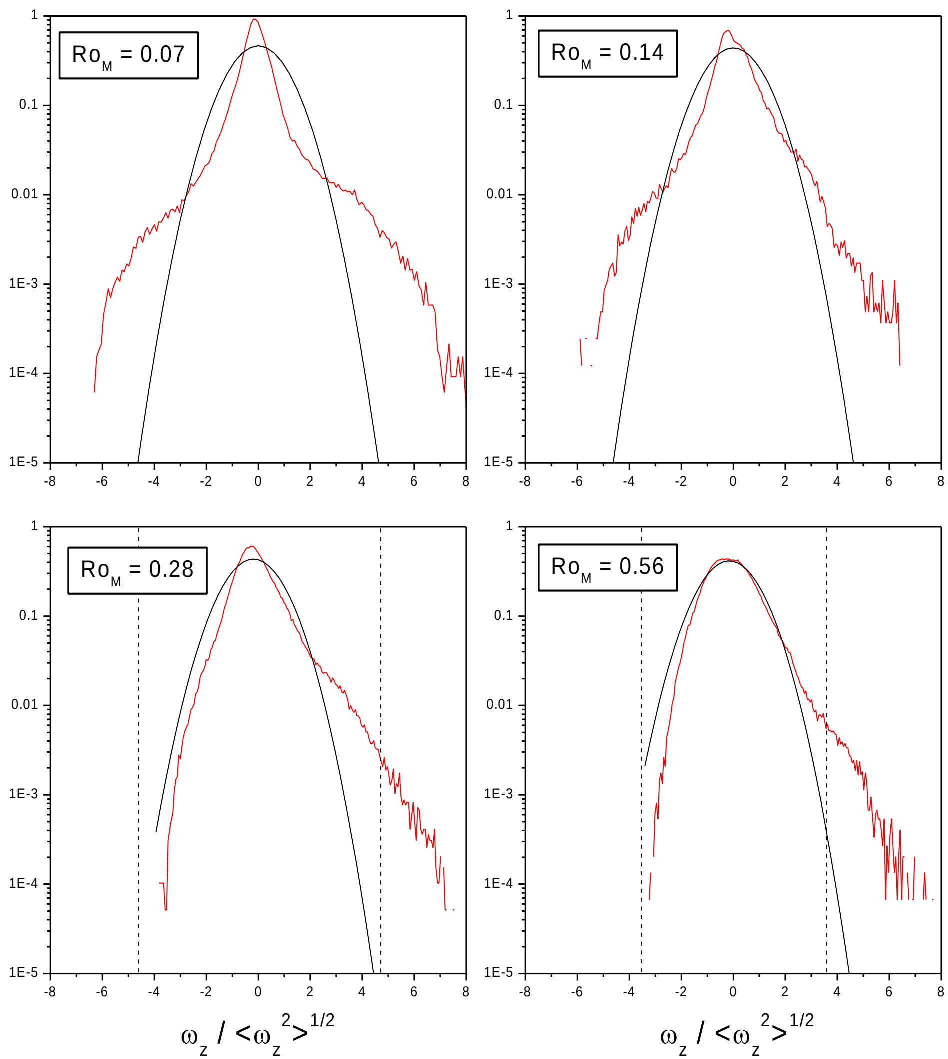
$Ku = < \omega_z^4 \square ¿ \square \omega_z^2 \square^2 ¿$
¿



- Strong intermittency for Ro < 0.2
- Quasi Gaussian distribution in absence of rotation

Intermittency and symmetry

P.D.F. of vertical vorticity



- Intermittency and symmetry for $Ro_M < 0.2$
- Domination of cyclones for $Ro_M > 0.2$
- Anticyclones limited by Ro=1

Conclusions

- Stratified turbulence without rotation has a 3D dynamics (strong decay)
- Stratified turbulence submitted to strong rotation (Ro < 0.2) has a quasi-geostrophic dynamics:
  - Energy conservation
  - Energy spectra in k^{-3}
  - Formation of coherent vortices (with symmetry cyclone-anticyclone)

  (similar to 2D turbulence, but with a z dependency)
- For moderate rotation (Ro close to 1): departure from quasi-geostrophic model:

  - Suppression of anticyclonic vortices such that ω/f <-1

  -Other possible ageostrophic effects: front formation, wave generation?

Statistical mechanics of vorticity

Miller(1989), Robert and Sommeria (1990)

Properties of the 2D Euler equations:

-Conservation of vorticity $\omega(x,y)$ for fluid elements

-Long range interactions: $-\Delta\psi=\omega$, energy $E=\int\psi\omega\, dxdy$

Mean field statistical equilibrium:

$-\Delta\psi=f(\psi)$, selects a steady solution

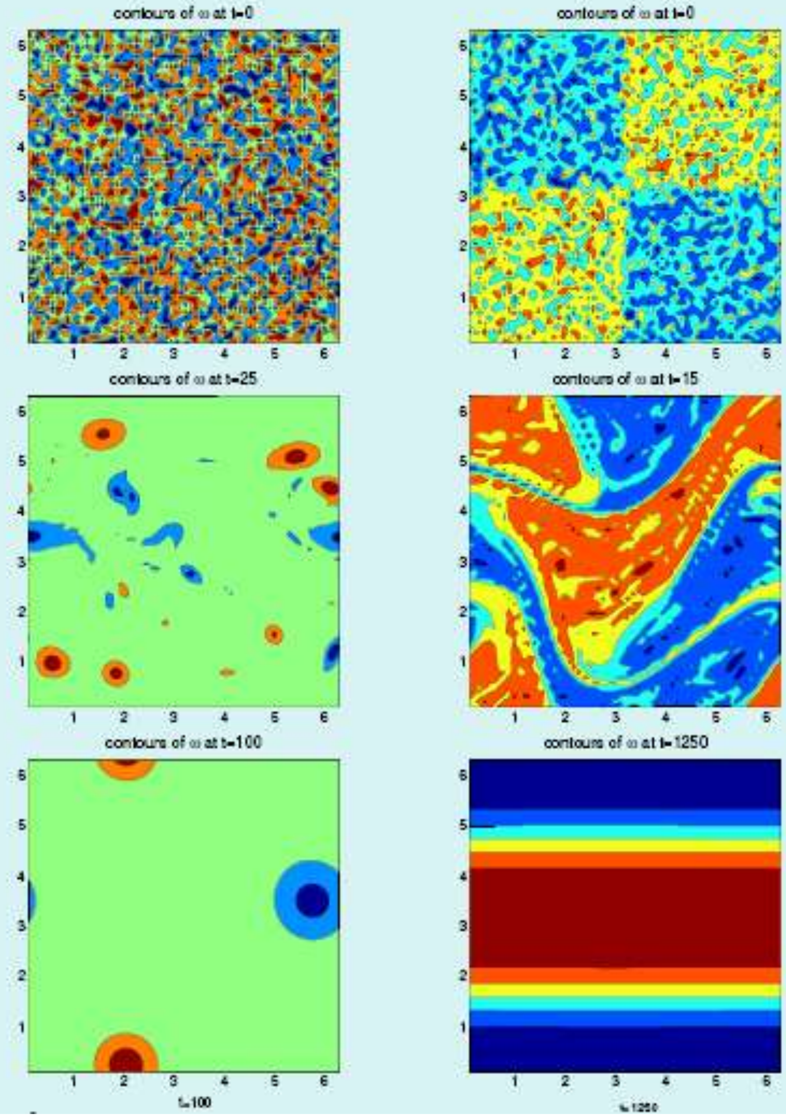
The function f depends on the global pdf of vorticity (given by the initial condition):

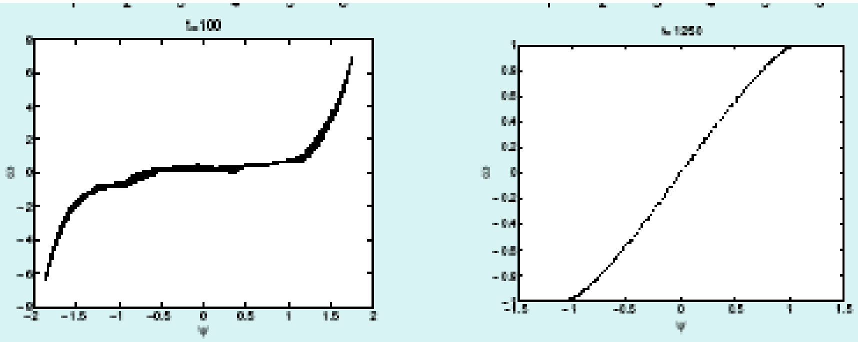
sinh for intense vortices

tanh for wide patches

Z. Yin, D.C. Montgomery, and H.J.H. Clercx
"Alternative statistical-mechanical descriptions of decaying two-dimensional turbulence in terms of 'patches' and 'points'"
Physics of Fluids **15**, 1937-1953 (2003).

Two numerical simulations of decaying turbulence are carried out with a 2D pseudospectral code which solves the Navier-Stokes equation. For both runs (the left column and the right column): $Re = 5000$; Resolution - 512^2; $\Delta t = 0.0005$.





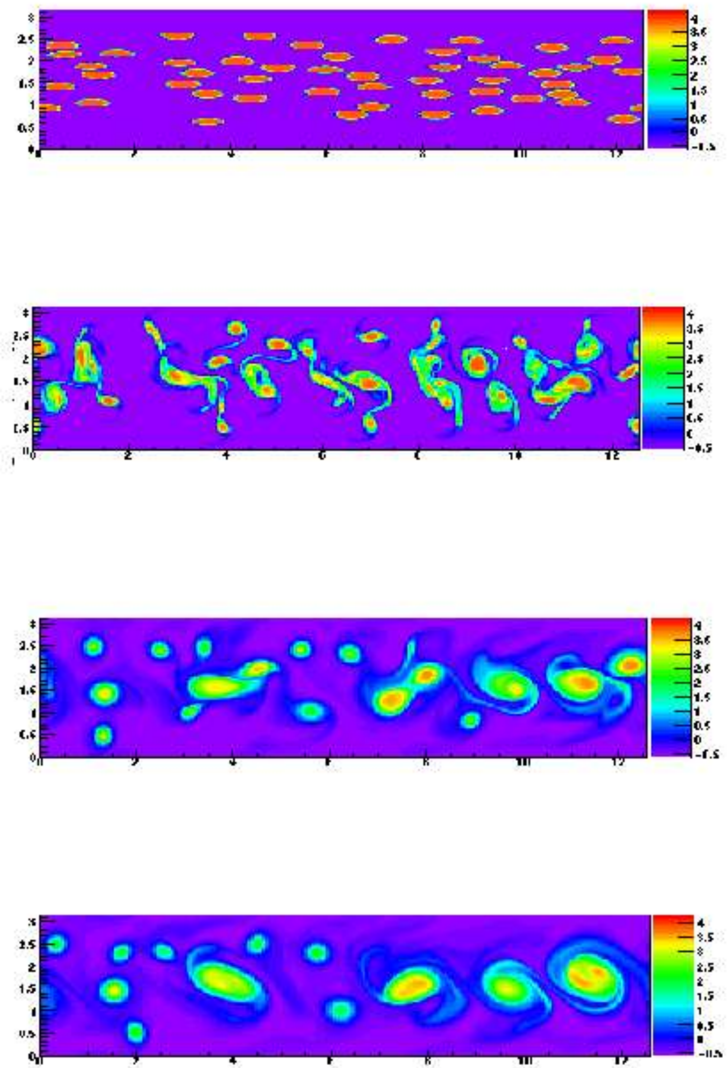
Not only the final states are predicted by the theories, but also the $\omega - \psi$ plots of the late time correspond to the sinh- and tanh-poission results, respectively.

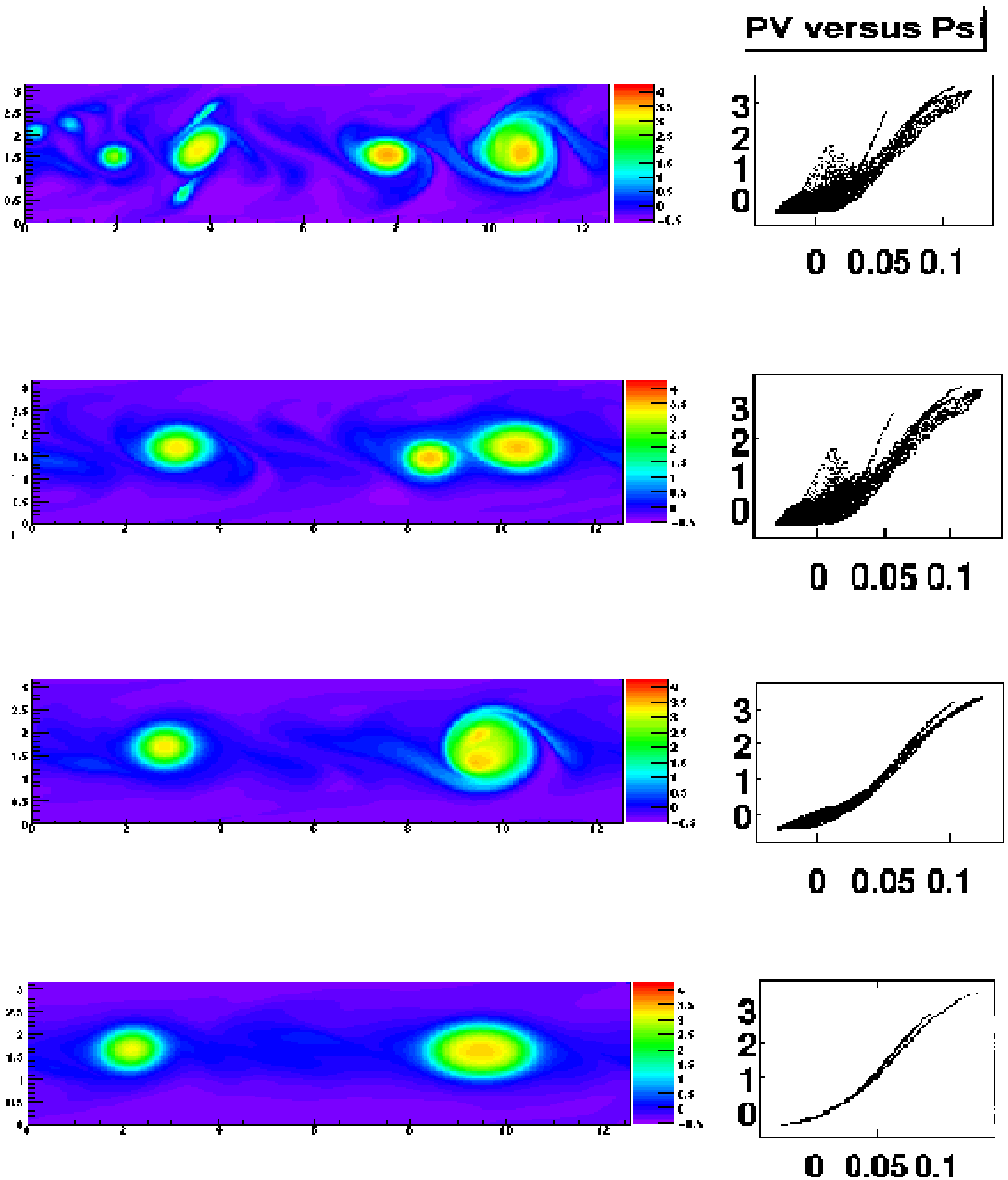
Application to the quasi-geostrophic model (Great Red Spot)

Relaxation toward statisitical equilibrium

R. Robert and J. Sommeria (1991).

F. Bouchet and T.Dumont (2004): QG model for a shallow layer over a deep imposed shear flow




PV versus Psi

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Statistical mechanics of the shallow water system

P. H. Chavanis[1] and J. Sommeria[2]

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{u}) = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\boldsymbol{\omega} + 2\boldsymbol{\Omega}) \times \mathbf{u} = -\nabla B. \tag{2}$$

$$B = gh + \frac{\mathbf{u}^2}{2},$$

Statistical equilibrium, with the assumption *h* smooth:

$B=B(\psi)$

$q \equiv (\omega+2\Omega)/h = -dB/d\psi$

Steady solution of the shallow water equations

Fundamental question: excitation of waves ‘thermodynamically’ possible

Competition with vorticity straining

Scale $l \sim L_0 \exp(-st)$, s rate of strain

viscous time $l^2/\nu = (L^2{}_0/\nu) \exp(-2st)$,

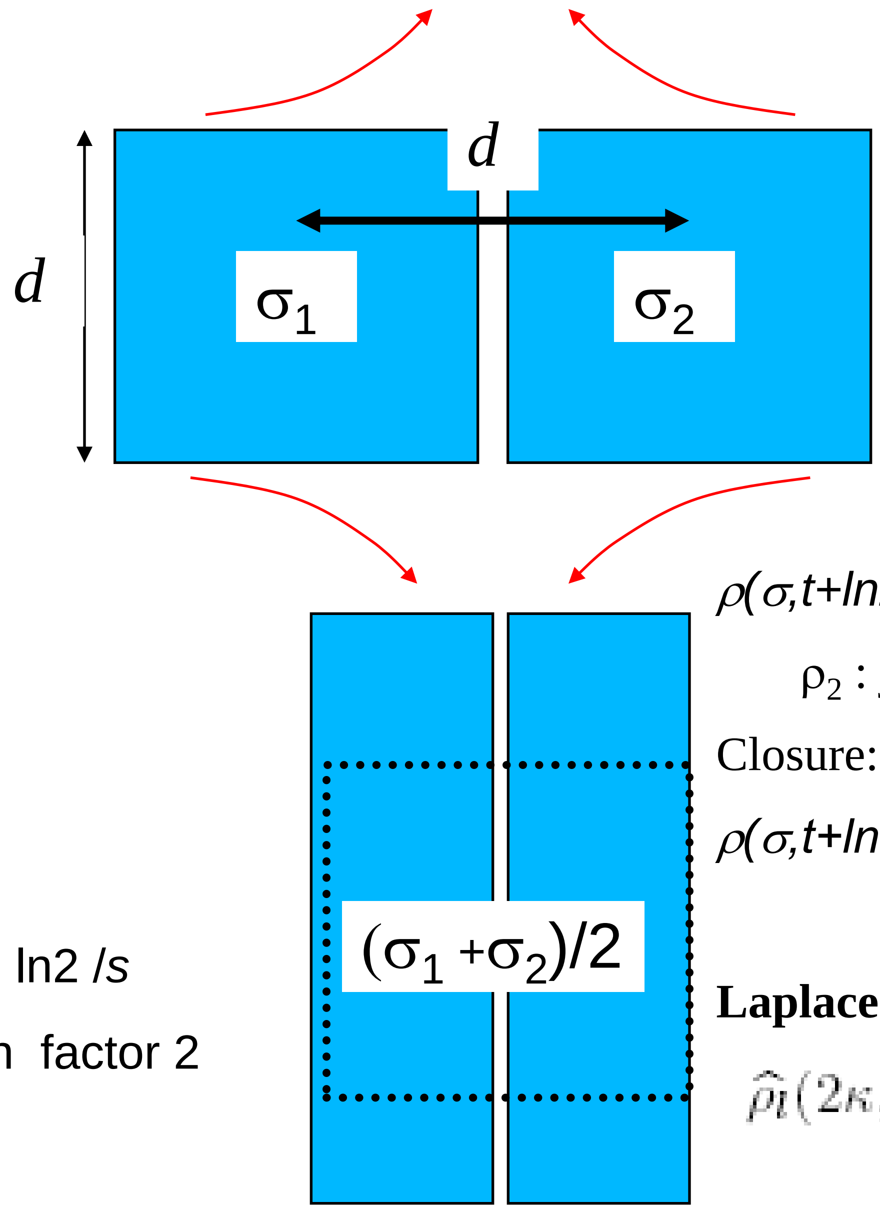
viscous effect ~ advection time ω^{-1}

for $t = \ln(L^2{}_0 \nu/\omega)/(2\,s)$ ~ ln (Re)

Navier-Stokes converges to Euler very slowly with increasing Re

Effect of strain on a scalar

Venaille and Sommeria, Phys. Fluids 2006, submitted PRL 2007



time t

rate of strain s

time t+ ln2 /s

reduction factor 2

$\rho(\sigma, t+\ln 2/s) = \int \rho_2(\sigma-\delta, \sigma+\delta, t)\, d\delta$

ρ_2 : joint probability for pairs separated by d

Closure: independence of fluctuations assumed

$\rho(\sigma, t+\ln 2/s) = 2\int \rho(\sigma', t)\, \rho(\sigma-\sigma', t)\, d\sigma'$

(self-convolution)

Laplace transform:

$$\widehat{\rho}_l(2\kappa, t+\Delta t_{1/2}) = [\widehat{\rho}_l(\kappa, t)]^2$$

Equation for the coarse-grained scalar pdf

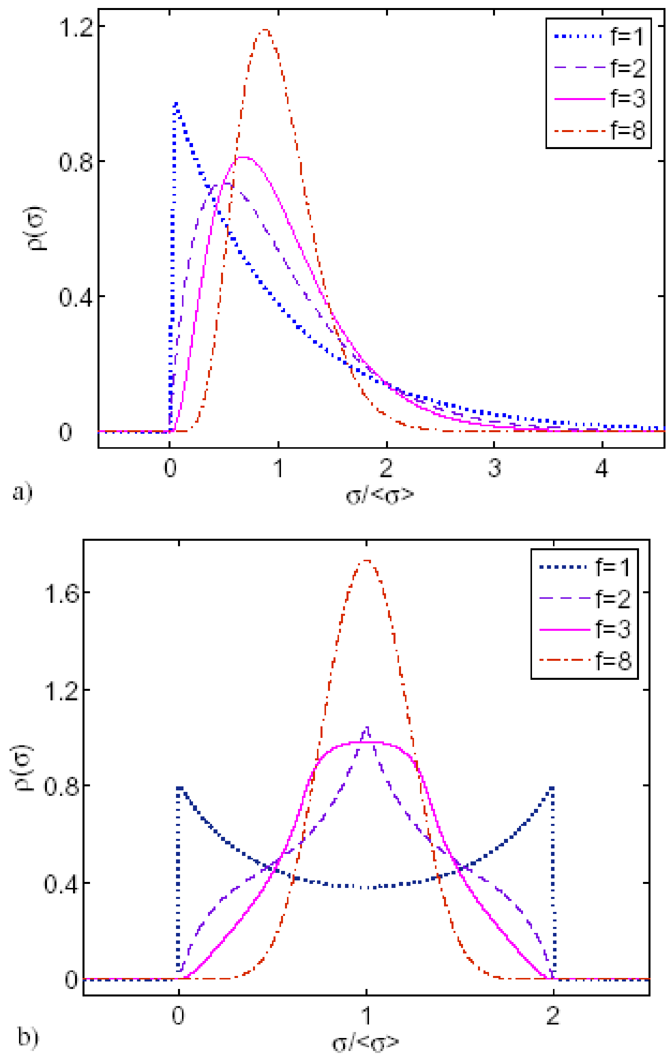
-n self-convolutions: transformed in a product by Laplace transform

$$\widehat{\rho}_l(n\kappa, t + \Delta t_{1/n}) = [\widehat{\rho}_l(\kappa, t)]^n.$$

-infinitesimal limit n =1+ε:

$$\partial_t \widehat{\rho}_l = s(t)[\widehat{\rho}_l \ln \widehat{\rho}_l - \kappa \partial_\kappa \widehat{\rho}_l]$$

relaxation toward a Gaussian with decreasing variance (symmetric case), or through gamma pdf.



Is turbulent mixing a self-convolution process ?

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Coriolis-LEGI 21 rue des martyrs 38000 Grenoble France
(Dated: November 29, 2007)

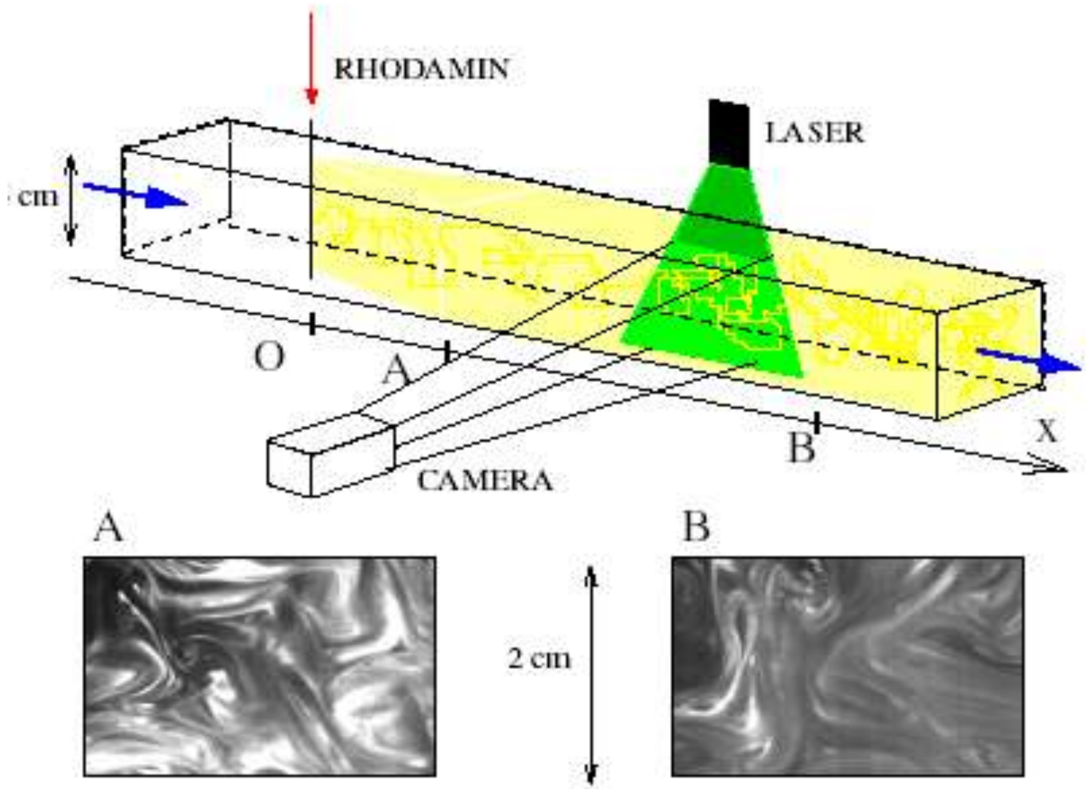

FIG. 1: Experimental set up

Self-convolution model vs experiment

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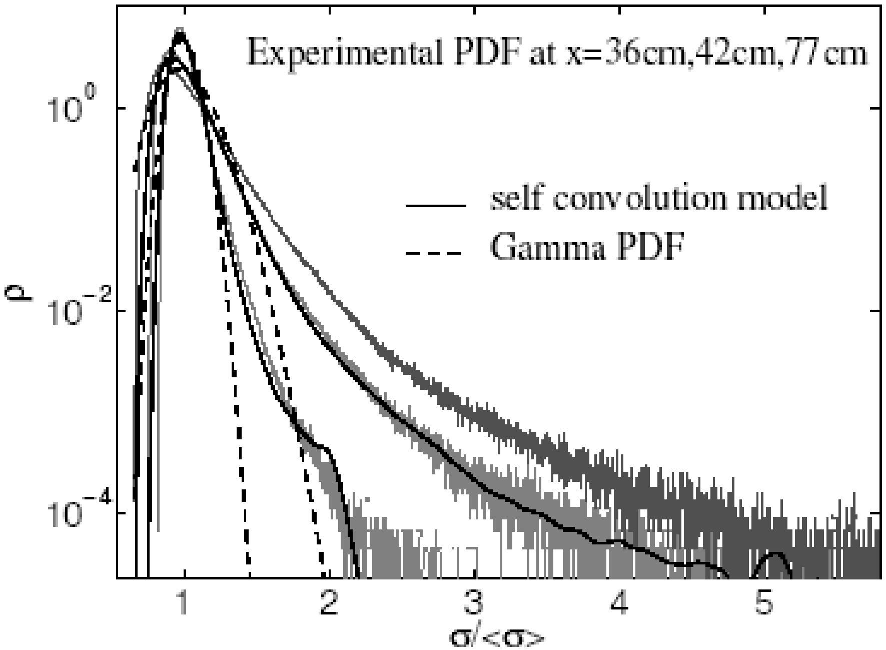

FIG. 2: The experimental PDF at $x = 36cm$ and variances at $x = 42cm$ and $x = 77cm$ are used for the prediction of the self-convolution model. The fit with Gamma-distribution requires only the variance value.

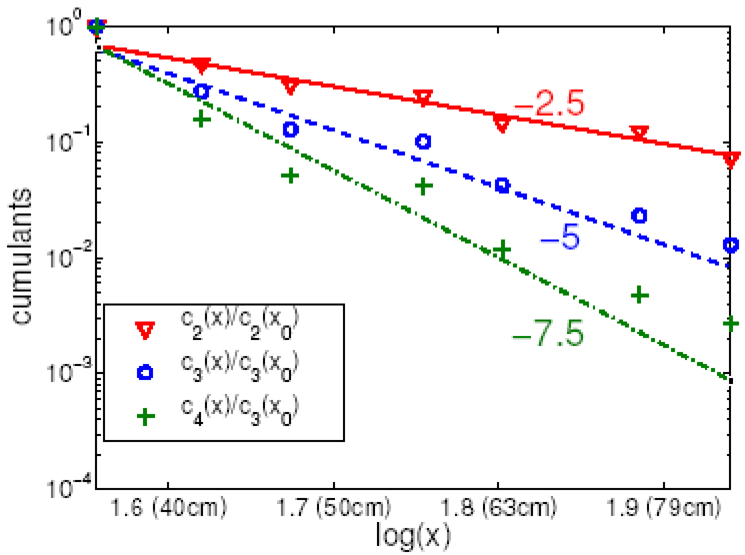

FIG. 4: Evolution of the first cumulants at increasing distance from the injector. The dashed and dot-dashed line are prediction of the self-convolution model, by supposing $f \sim x^{\alpha}$, where $\alpha = 2.6$ is obtained from the experimental variance decay

SOME ‘CONCEPTUAL’ QUESTIONS

- Is the statistical equilibrium the true asymptotic limit of the 2D Euler equations?
- How the theory extends to balanced states beyond the quasi-geostrophic model ?
- Can we derive a kinetic model combining all the requested fundamental properties?

(relaxation to statistical equilibrium, galilean invariance…)