THE COUPLED LES - SUBGRID STOCHASTIC ACCELERATION MODEL (LES-SSAM) OF A HIGH REYNOLDS NUMBER FLOWS

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OBJECTIVE OF STUDY

- **Take into account in LES computation of a high Re flow intermittency (Re number dependent) at sub-grid (non-resolved) scales**
- **Concern: impact of intermittency on combustion in engines**
  - **spontaneous extinction and ignition**
  - **evaporation, coagulation and bursting of liquid drops, etc**

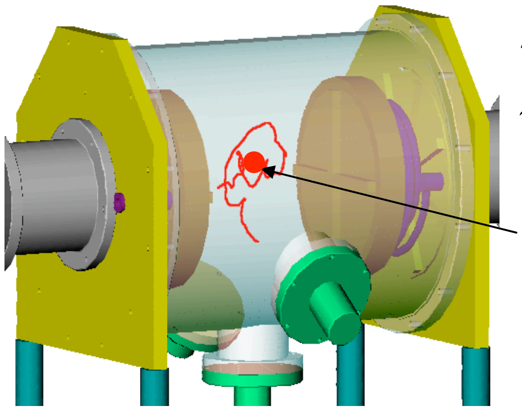
CONTENT

- **Impetus**
- **Residual acceleration: order estimation**
- **New approach. Principal points**
- **Formulation of LES-SSAM**
  - **starting point**
  - **assumptions**
  - **stochastic model of residual acceleration**
- **Validation tests**
  - **stationary isotropic turbulence**
  - **decaying turbulence**
- **Conclusions**

IMPETUS:

observations of a high Reynolds number stationary homogeneous turbulence (Mordant et al., 2001; La Porta et al., 2001; Voth et al.,1998, 2002):

“washing machine” by Mordant et al. PRL, 214501 (2001)



$Re_\lambda = 740$

$u' \approx 1 m/s$

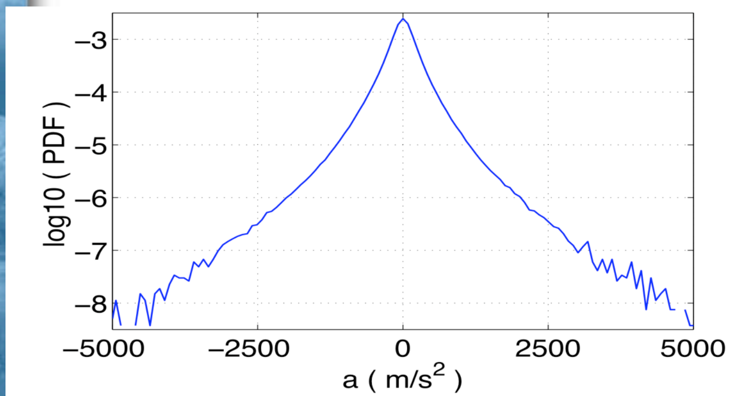
$\eta = 14 \mu m$

$\tau_\eta = 0,2\, ms$

$\rho_p / \rho_{eau} = 1{,}06$

$d_p = 250 \mu m$

PDF of acceleration (one component)



- **measurements of Lagrangian acceleration revealed a strong intermittency at small length /time scales**
- **non-Gaussian shape with stretched tails in distribution**
- **with increasing of time lag, the Lagrangian acceleration was rapidly decorrelated**
- **however, autocorrelation of its norm exposed a “long memory” (of order of few integral times)**
- **for higher turbulent Reynolds number, the intermittency was stronger manifested**

RESIDUAL ACCELERATION: ORDER ESTIMATION

- **Δ - the filter width**
- **L - the integral turbulent scale, η - Kolmogorov scale, σ_u - rms velocity**

$$u_i' u_i' \approx \sigma_u^2 (\Delta / L)^{2/3}$$

- $$a_i' a_i' / \overline{a}_k \overline{a}_k \approx (\Delta / \eta)^{2/3}, \quad \eta = \frac{L}{\mathrm{Re}_L^{3/4}}; \quad \mathrm{Re}_L = \frac{\sigma_u L}{\nu}$$

- **it follows:**

$$a_i' a_i' / \overline{a}_k \overline{a}_k >> 1, \quad \text{if} \quad \mathrm{Re}_L >> 1$$

- **consequently: when intermittency is target in SGS model,**

the SGS model should comprise the non-resolved acceleration dependent of Reynolds number

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NEW APPROACH – PRINCIPAL POINTS

- focusing on acceleration of fluid particle
- reconstruction of non-resolved velocity by instantaneous model-equation, in which the acceleration comprises two parts:
  - the filtered total acceleration corresponding to classical LES
  - the sub-grid non-resolved (residual) acceleration of fluid particle
- the non-resolved acceleration is modelled stochastically in a way that intermittency at sub-grid scales is taken into account

REMARK:

Our LES-SSAM looks similar to " A LES-Langevin model for turbulence ", Eur. Phys. J. B 49, 471-481 (2006), J.-P. Laval and Dubrulle. But the physics and the way of simulating the non-resolved scales are quite different.

FORMULATION OF LES-SSAM (1/3)

1. Starting step

- **Navier-Stokes equations:**

$$a_i = \frac{d\,u_i}{d\,t} = -\frac{\partial\,P}{\partial\,x_i} + \nu\,\Delta u_i\,, \quad \frac{\partial\,u_k}{\partial\,x_k} = 0$$

- **By filtering the Navier-Stokes equations, we have two sets:**

$$\overline{a}_i = \overline{\left(\frac{d\,u_i}{d\,t}\right)} = -\frac{\partial\,\overline{P}}{\partial\,x_i} + \nu\,\Delta\overline{u}_i\,, \qquad \frac{\partial\,\overline{u}_k}{\partial\,x_k} = 0;$$

$\overline{a}_i$ **- is the resolved total acceleration**

$$a_i' = \left(\frac{d\,u_i}{d\,t}\right)' = -\frac{\partial\,P'}{\partial\,x_i} + \nu\,\Delta u_i'\,, \qquad \frac{\partial\,u_k'}{\partial\,x_k} = 0;$$

a_i' **- is the non-resolved acceleration**

$$a_i = \overline{a}_i + a_i'$$

- corresponds to original Navier-Stokes equation

FORMULATION OF LES-SSAM (2/3)

2. Assumptions

- **the original equation for residual acceleration**

$$a_i' = \left(\frac{d\,u_i}{d\,t}\right)' = -\frac{\partial P'}{\partial x_i} + \nu\,\Delta u_i',$$

is replaced by a model equation:

$$\left(\frac{d\,u_i}{d\,t}\right)' = -\frac{\partial \hat{P}'}{\partial x_i} + \hat{a}_i'$$

from presumed stochastic process

guarantee the velocity vector to be solenoidal

- **the original filtered equation is described by classical LES with the Smagorinsky model**

$$\overline{a}_i = \frac{d_\Delta \overline{u}_i}{d\,t} = -\frac{1}{\rho}\frac{\partial \overline{P}}{\partial x_i} + \nabla\left((\nu + \nu_t)\nabla \overline{u}_i\right), \quad \frac{d_\Delta}{d\,t} = \frac{\partial}{\partial t} + \overline{u}_k \frac{\partial}{\partial x_k},$$

- **unfiltered "hypothetical" velocity is represented by a model equation:**

$$\hat{a}_i = \frac{d\,\hat{u}_i}{d\,t} = -\frac{1}{\rho}\frac{\partial \hat{P}}{\partial x_i} + \frac{\partial}{\partial x_k}\nu_{tur}\left(\frac{\partial \hat{u}_i}{\partial x_k} + \frac{\partial \hat{u}_k}{\partial x_i}\right) + \hat{a}_i', \quad \frac{\partial \hat{u}_k}{\partial x_k} = 0$$

FORMULATION OF LES-SSAM (3/3)

3. Stochastic model of residual acceleration

- **Kolmogorov – Oboukhov, 1962:**

$$\langle a_i a_j | \varepsilon \rangle = const\ \varepsilon^{3/2} / \nu^{1/2} \delta_{ij}$$

- **Pope-Chen's (1990) log-normal stochastic process for** ε

- **Stochastic equation of the norm of acceleration** $\hat{a} = |\hat{a}'_i|$ **(by Ito transformation):**

$$d\hat{a} = -\hat{a}\left(\ln \frac{\hat{a}}{a_\eta} - \frac{3}{16}\sigma_\chi^2 \right) T_\chi^{-1}\, dt + \frac{3}{4}\hat{a}\sqrt{2\sigma_\chi^2 T_\chi^{-1}}\, dW(t); \quad \langle dW(t) \rangle = 0; \quad \langle dW(t)^2 \rangle = dt$$

$$T_\chi = \frac{\nu_{tur}}{\Delta^2}; \quad \mathrm{Re}_\Delta = \frac{\nu_t}{\nu}; \quad a_\eta = \frac{\langle \varepsilon \rangle^{3/4}}{\nu^{1/4}}; \qquad \sigma_\chi^2 = A + \mu \ln \mathrm{Re}_\Delta^{3/4}; \qquad A \approx -0.863, \quad \mu \approx 0.25$$

(values of constants according A.G. Lamorgese *et al.* JFM (2007))

- **components of acceleration are computed by introducing the unit vector $e_i(t)$ with random direction in time (this hypotheses can be modified):**

$$\hat{a}'_i(t) = \hat{a}(t) e_i(t)$$

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VALIDATION TESTS

1. Stationary (forced) homogeneous isotropic turbulence
2. Decaying homogeneous isotropic turbulence

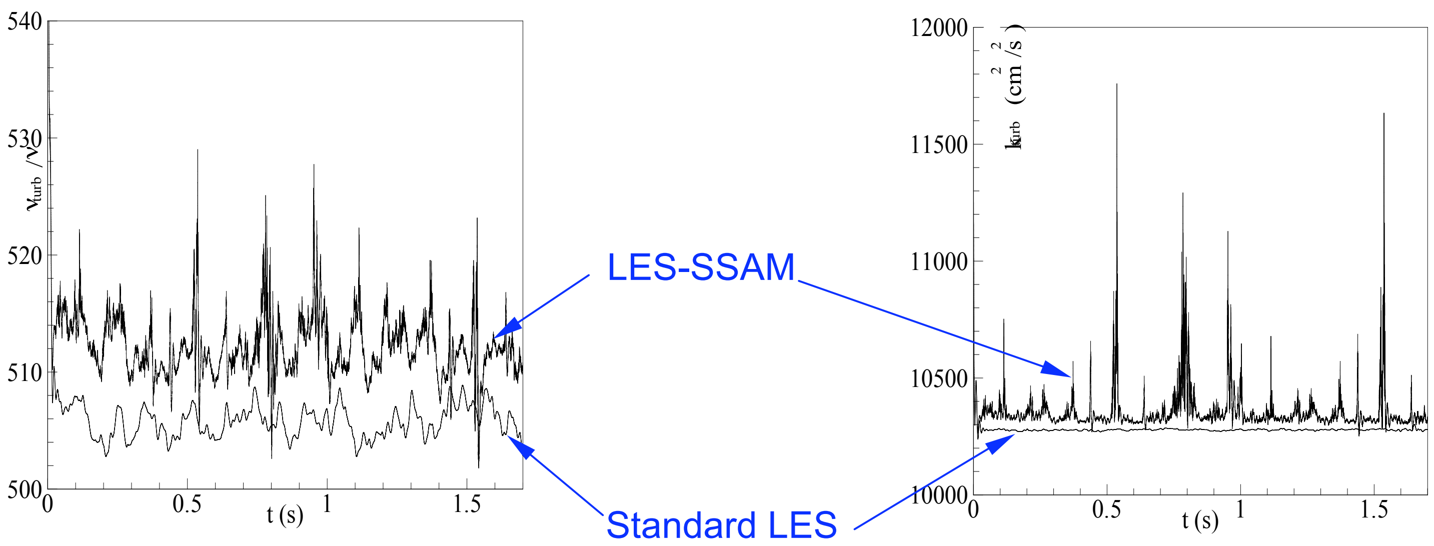
Parameters:

- **Integration time step Δt : in both cases is about of the Kolmogorov time scale τ_η**
- **grid resolution: stationary turbulence – 32^3, decaying turbulence - 64^3**
- **Smagorinsky constant was adjusted to obtain a correct energy dissipation rate (1) and energy decay (2) – part of energy dissipation is due to stochastic acceleration term (in current version of SSAM)**

STATIONARY ISOTROPIC TURBULENCE (1/6)

a) Evolution of the Reynolds number (averaging over a computational domain)

b) Evolution of the turbulent energy (averaging over a computational domain)



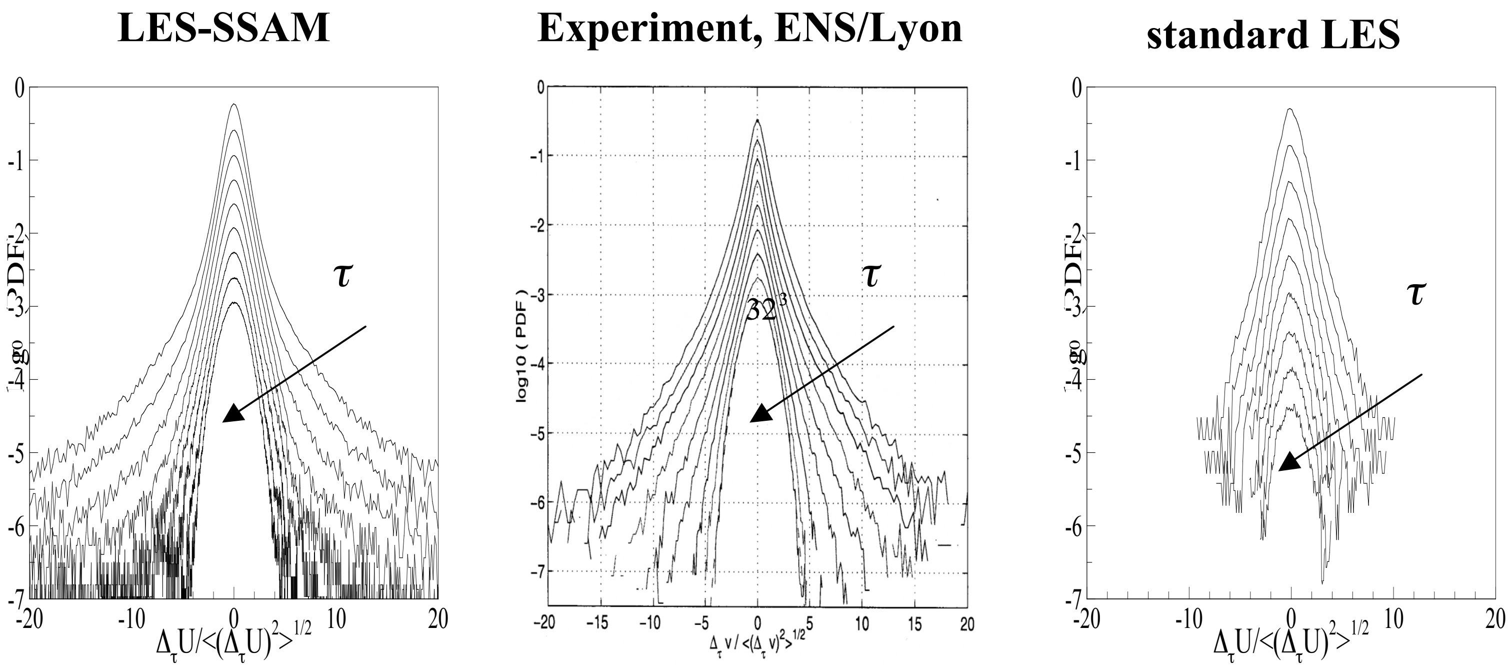
LES-SSAM:

- **reproduces "spotty" intermittent turbulent fields**

STATIONARY ISOTROPIC TURBULENCE (2/6)

PDF of the Lagrangian velocity increment at different time lag

τ = 0; 0.15; 0.3; 0.6; 1.2; 2.5; 4.9; 9.8; 20 et 39 ms (initial conditions from Mordant et al. PRL, 214501, 2001)



LES-SSAM:

- **gaussian distribution at time lags of order of integral time scale (≈ 45 ms)**
- **strong non-gaussianity with stretched tails at small time lags**

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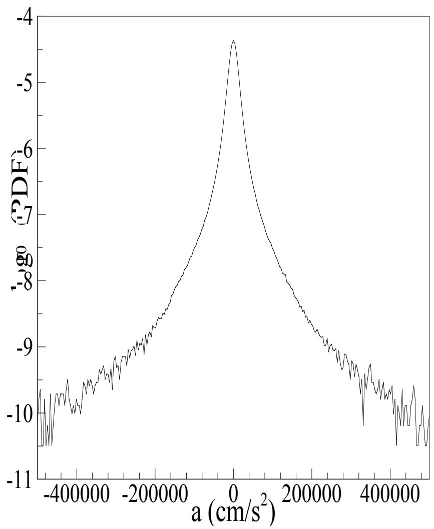
STATIONARY ISOTROPIC TURBULENCE (3/6)

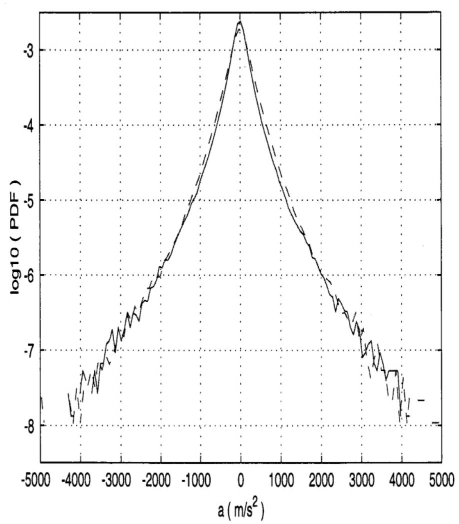
PDF of the Lagrangian acceleration

(initial conditions from Mordant et al. PRL, 214501, 2001)

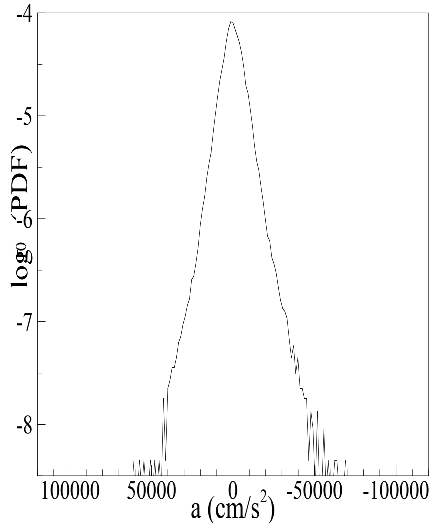
LES-SSAM



Experiment, ENS/Lyon



Standard LES



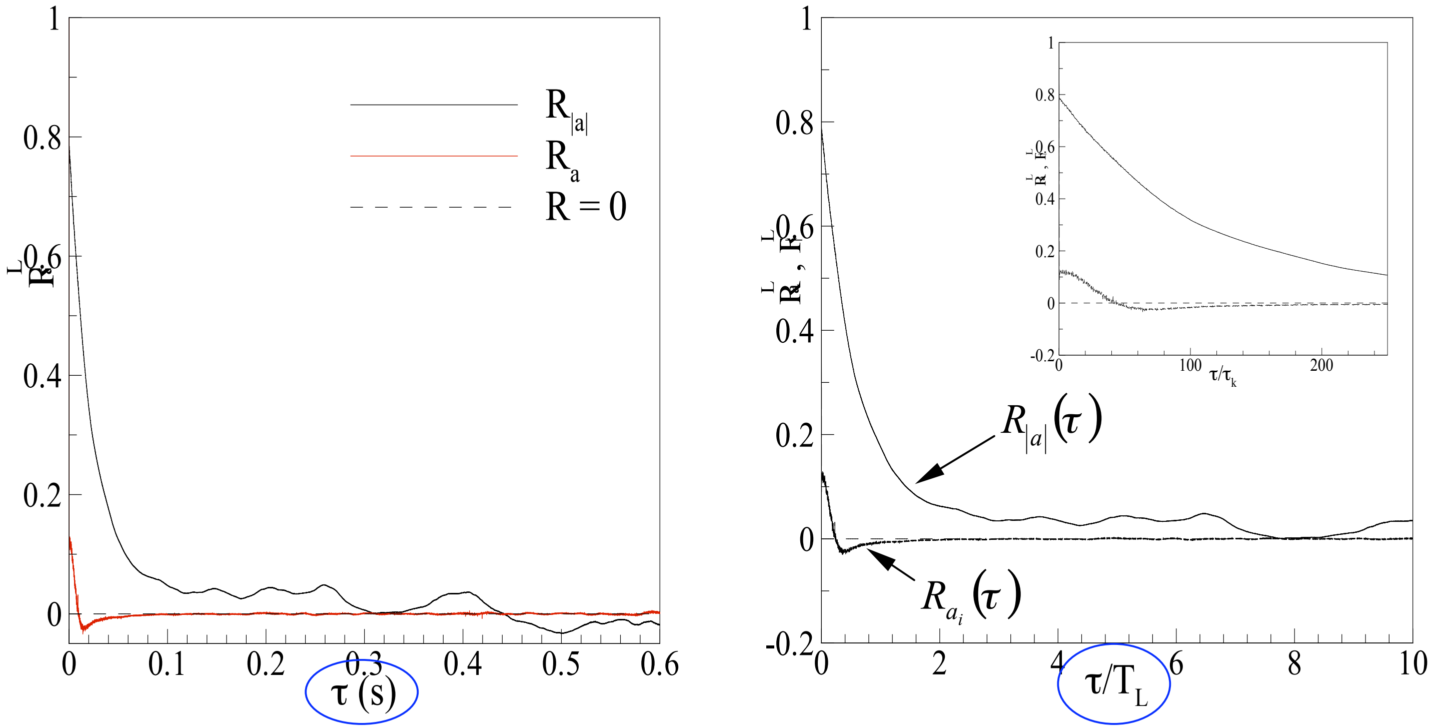
LES-SSAM:

- **the small amplitude acceleration events are alternating with events of very large acceleration**

STATIONARY ISOTROPIC TURBULENCE (4/6)

Auto-correlation of acceleration $R_{a_i}(\tau)$ **and of its norm** $R_{|a|}(\tau)$

(initial conditions from Mordant et al. PRL, 214501, 2001)



LES-SSAM:

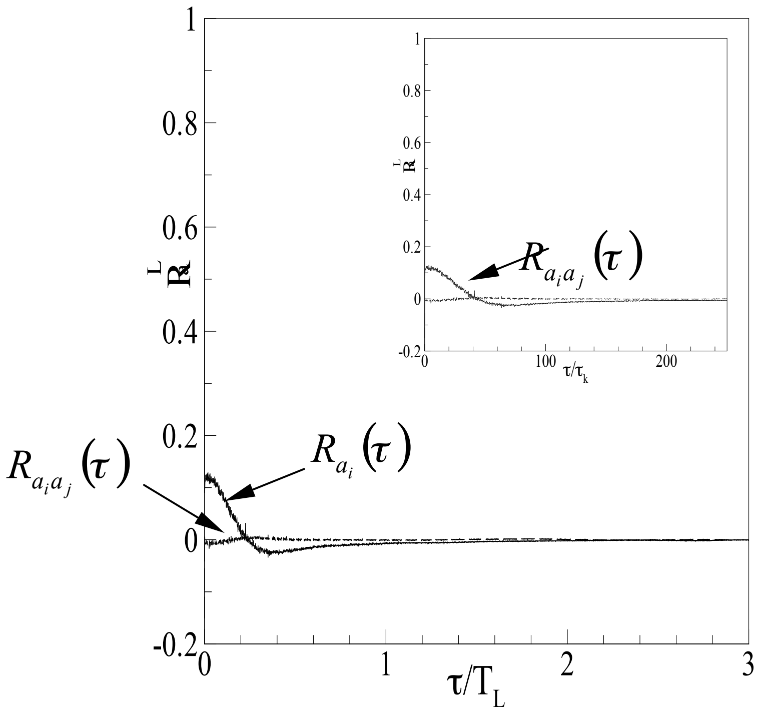
- **rapid decorrelation of acceleration vector component**
- **"long memory" of acceleration norm**
- **qualitative reproduction of effects observed in experiment**

STATIONARY ISOTROPIC TURBULENCE (5/6)

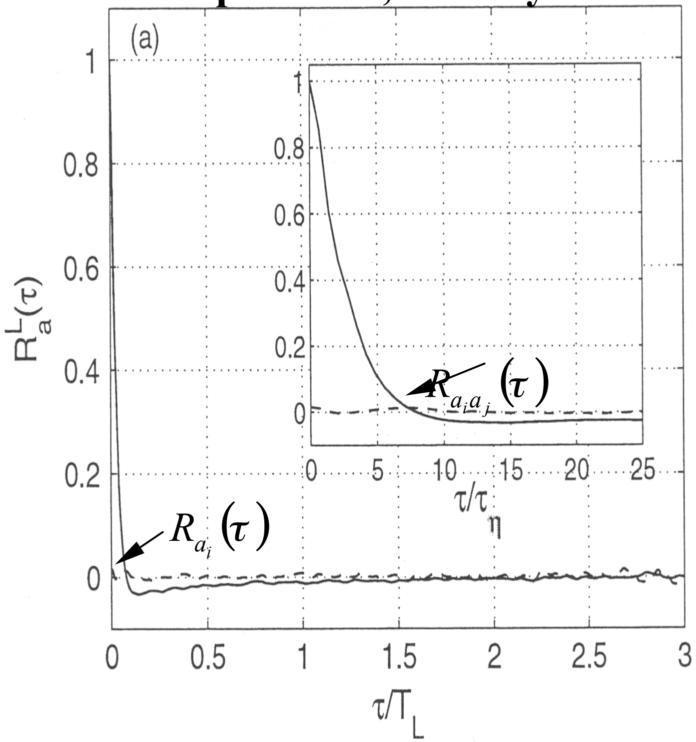
Cross-correlation $R_{a_i a_j}(\tau)$ and auto-correlation $R_{a_i}(\tau)$ of acceleration

(initial conditions from Mordant et al. PRL, 214501, 2001)

LES-SSAM



Experiment, ENS/Lyon



LES-SSAM:

- **immediate decorrelation between cross-components of acceleration vector**
- **qualitative agreement with experiment**

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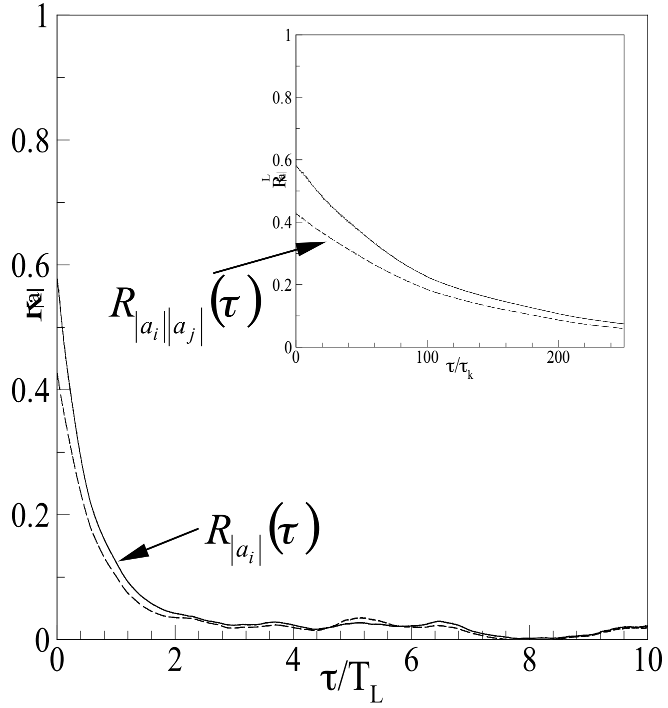
STATIONARY ISOTROPIC TURBULENCE (6/6)

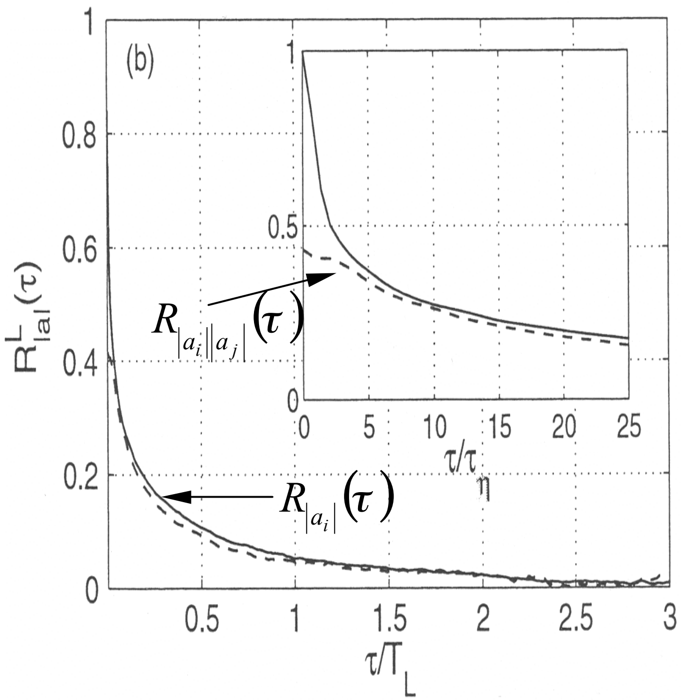
Cross-correlation $R_{|a_i||a_j|}(\tau)$ and auto-correlation $R_{|a_i|}(\tau)$ of modulus of acceleration

(initial conditions from Mordant et al. PRL, 214501, 2001)

LES-SSAM



Experiment, ENS/Lyon



LES-SSAM:

- **long-time correlation of modulus of cross-components of acceleration**
- **qualitative agreement with experiment**

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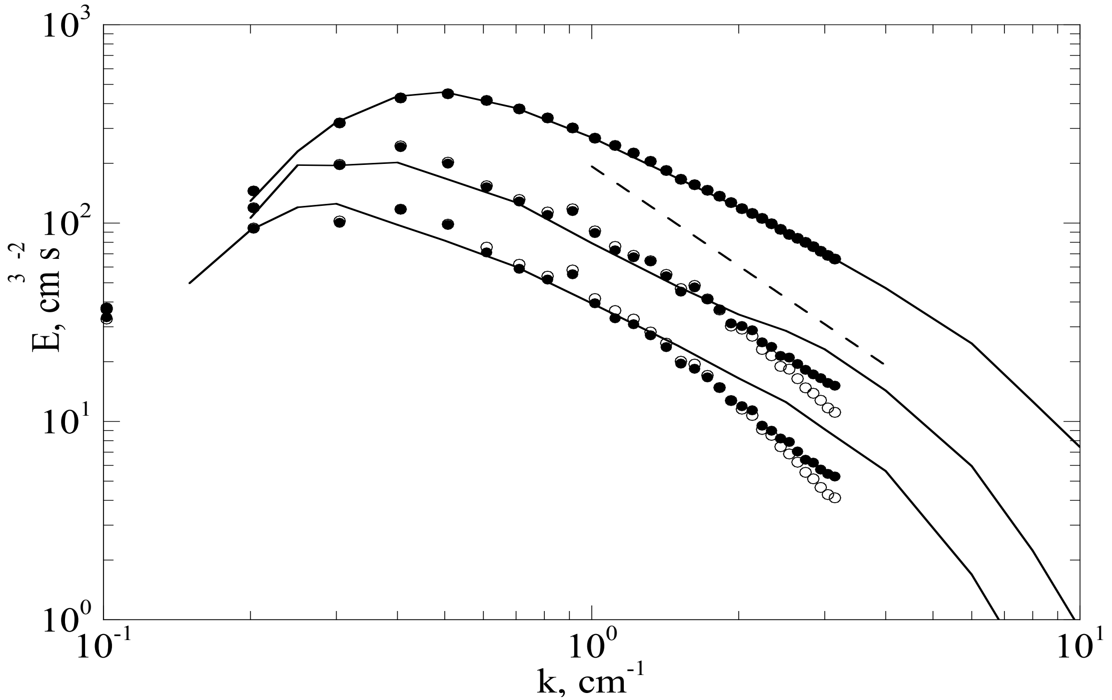
DECAYING TURBULENCE



(From the measured spectra at 0.21s (upper line), the initial (gaussian) velocity field is computed and then the energy spectra are compared at 0.284s and 0.655s)

- **LES-SSAM: filled symbols**
- **standard LES: empty symbols**
- **Comte-Bellot & Corrsin: continuous lines**
- **–5/3 law: discontinuous lines**

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CONCLUSIONS

- Novel coupled LES – Subgrid Stochastic Acceleration Model (LES-SSAM) is constructed
- LES-SSAM – targeting the small scale turbulence structure
- Residual acceleration is as a key variable of the SGS model. It is modelled by stochastic process
- The effects of intermittency, observed recently by measurements of the Lagrangian acceleration in a high Reynolds number turbulence,
  are reproduced by the model proposed

WORK IN PROGRESS

- **Elaboration of SSAM :**

  - to control the turbulence energy balance

  - direct modelling spatial non-resolved acceleration correlations

- **Adaptation of SSAM for simulation of intermittency only in inertial range – to perform calculation with larger time step and low computational cost**

Thank you

Questions ?