Mean-Field Effects: from Passive Scalar to Magnetic Field and Convection

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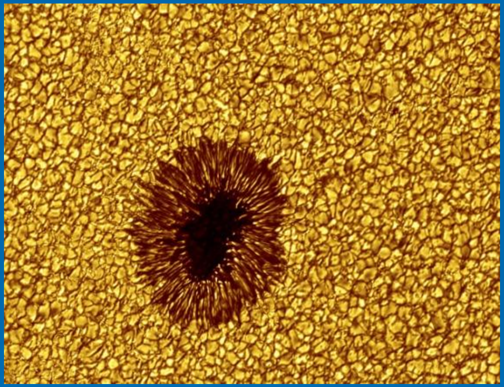
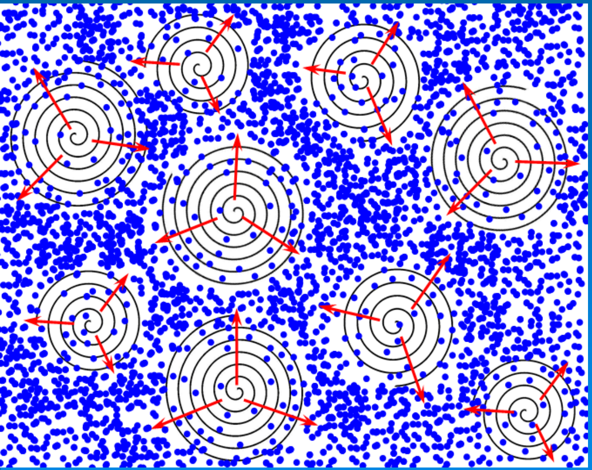
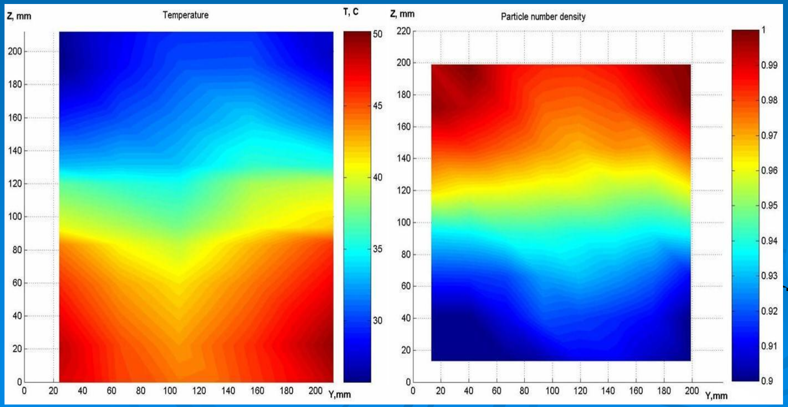
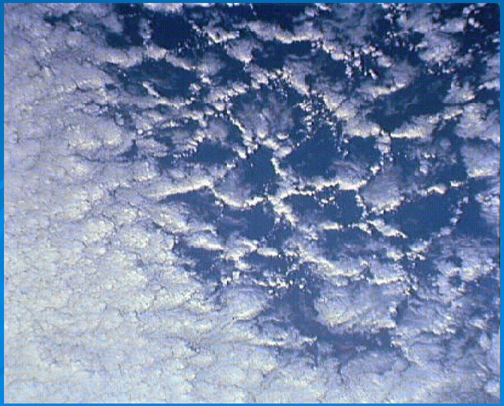




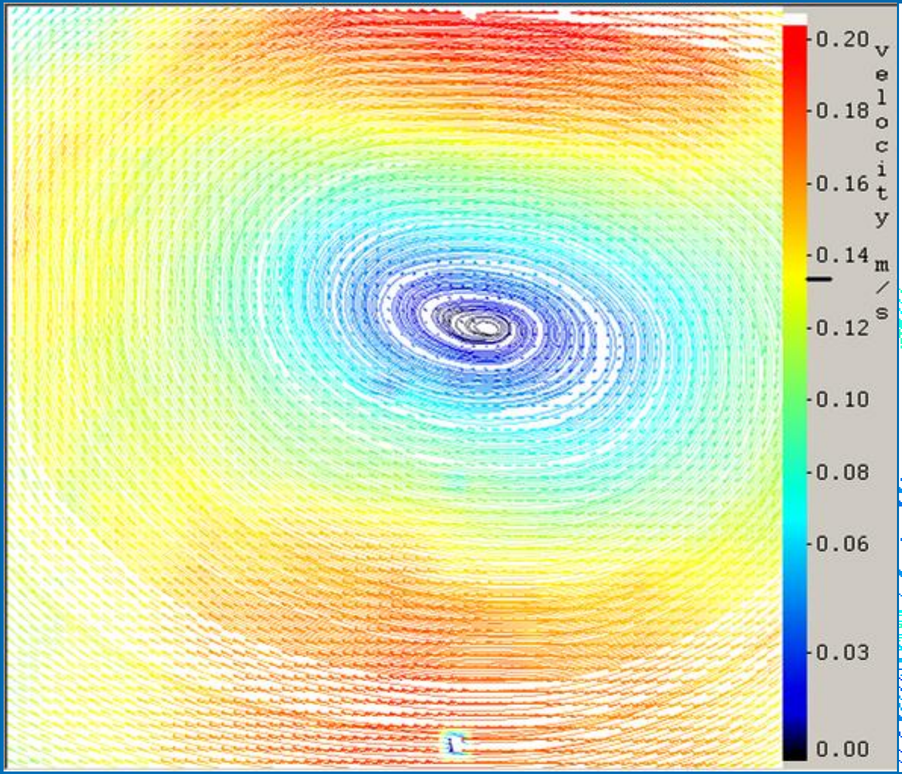


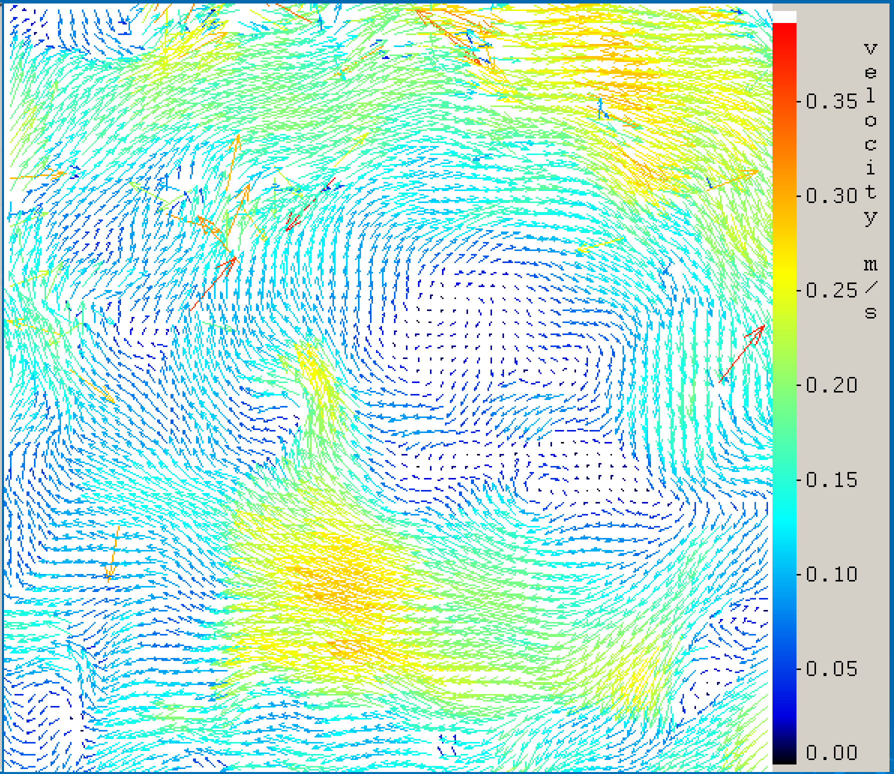


Outline

- **Examples of Mean-Field Effects**
- **Mean-Field Approach: Methods and Assumptions**
- **New Mean-Field Effect in Turbulent Transport of Particles:**
    - **theory of the new phenomenon of turbulent thermal diffusion**
    - **experimental detection of turbulent thermal diffusion**
    - **atmospheric applications**
- **Conclusions**

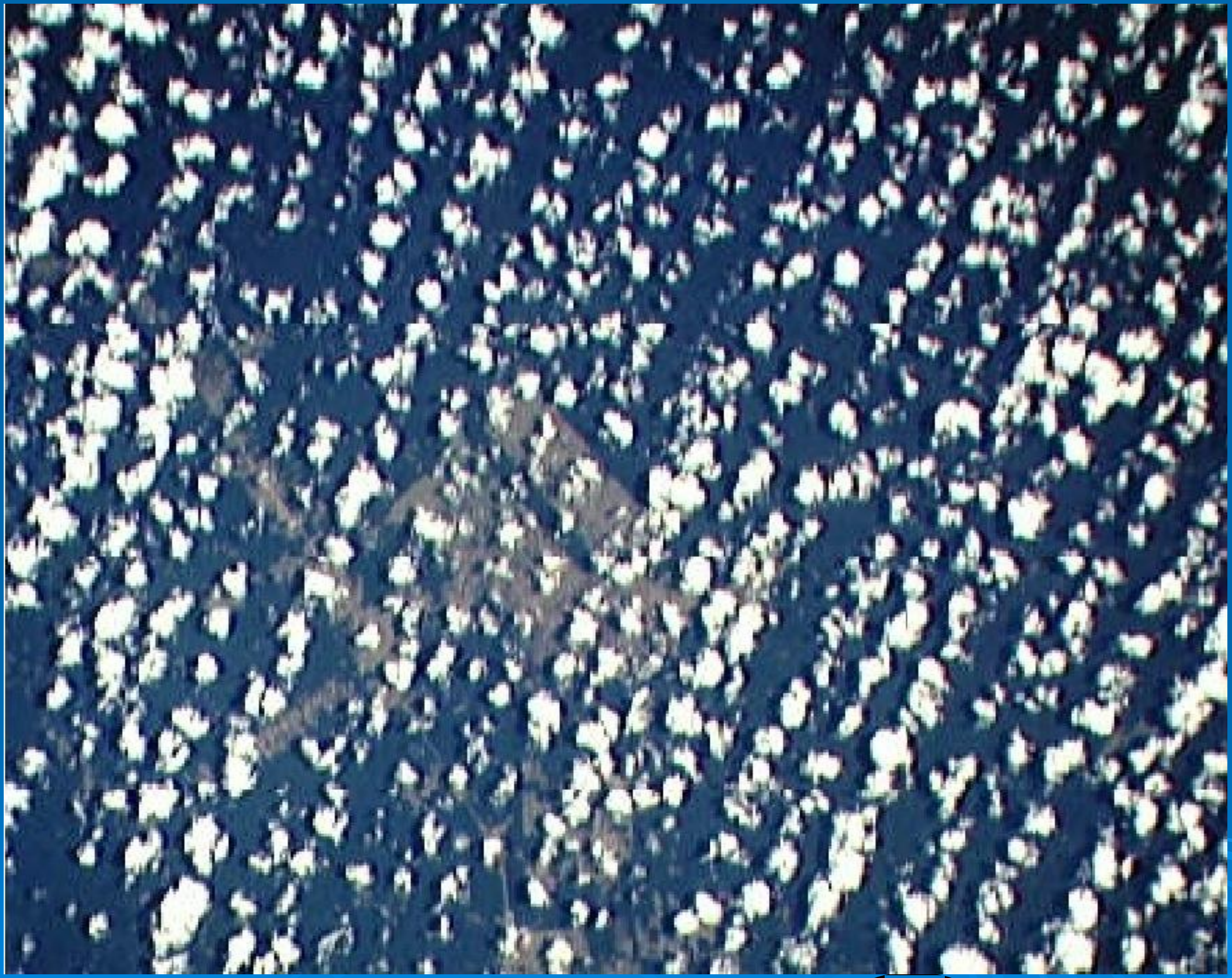
Velocity Fields



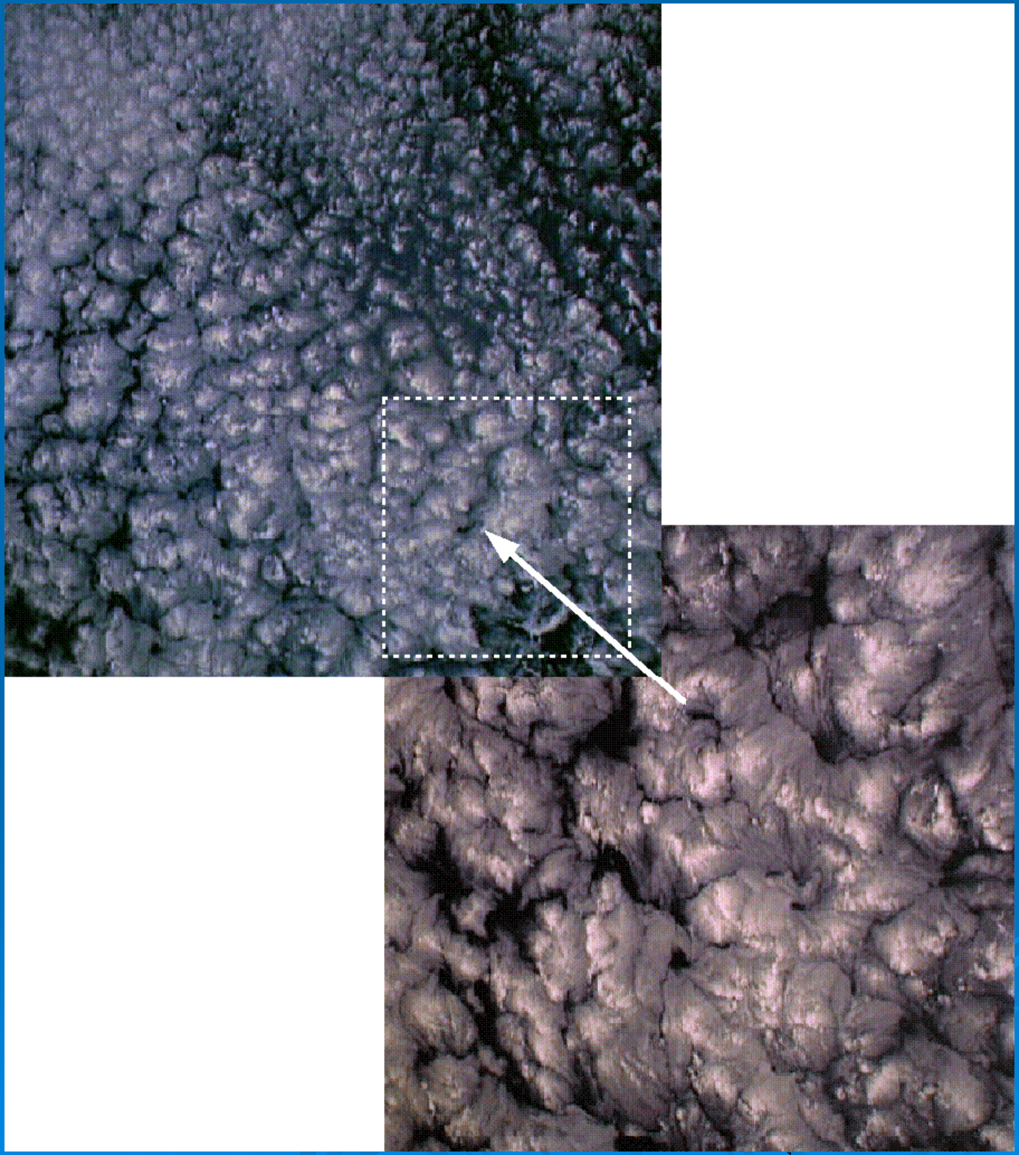


Cloud “streets” over Indian ocean

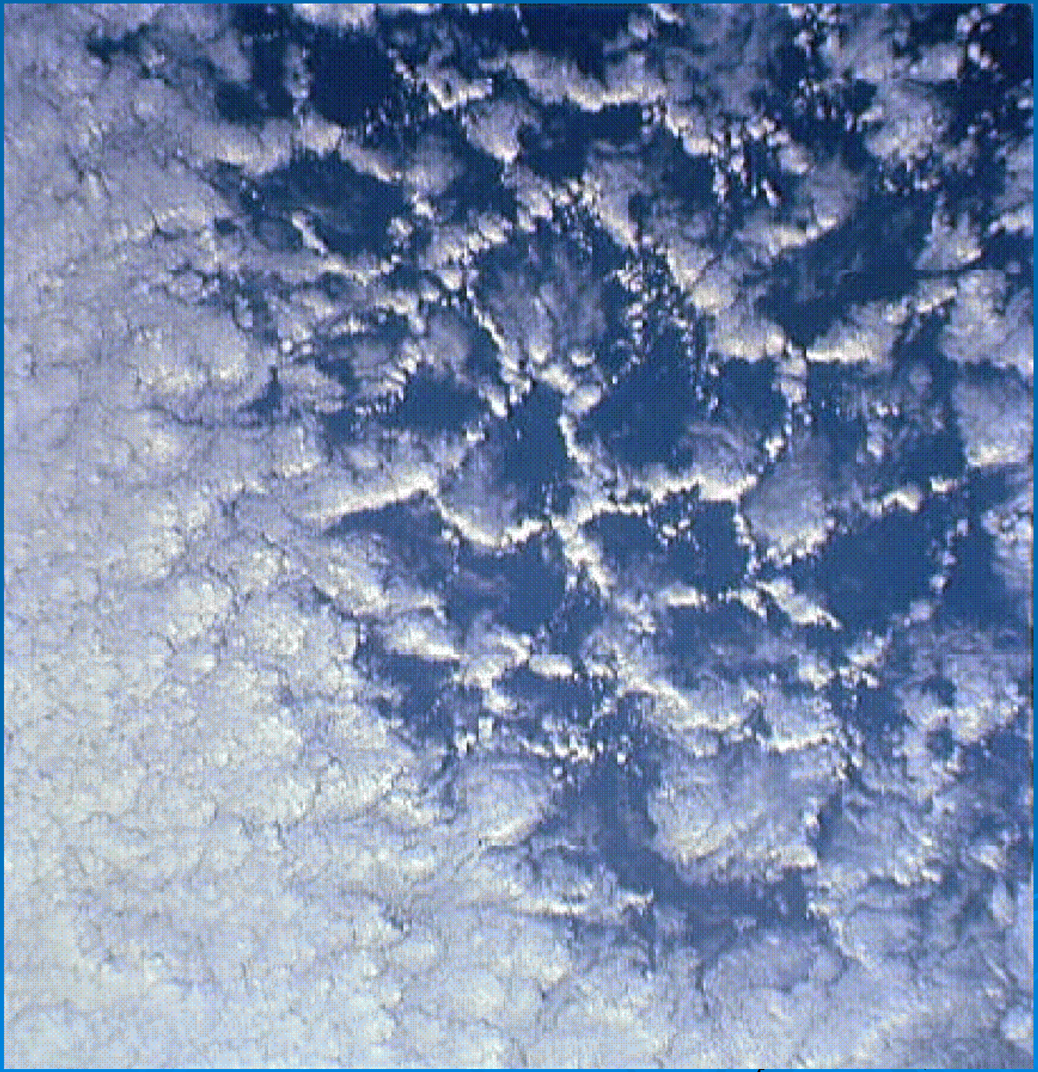
Cloud “streets” over the Amazon River



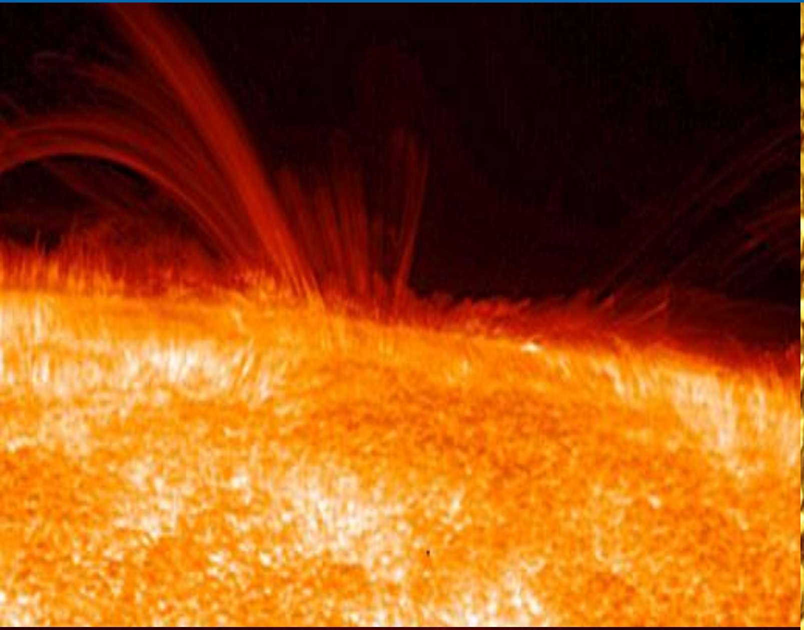
Closed cloud cells over the Atlantic Ocean

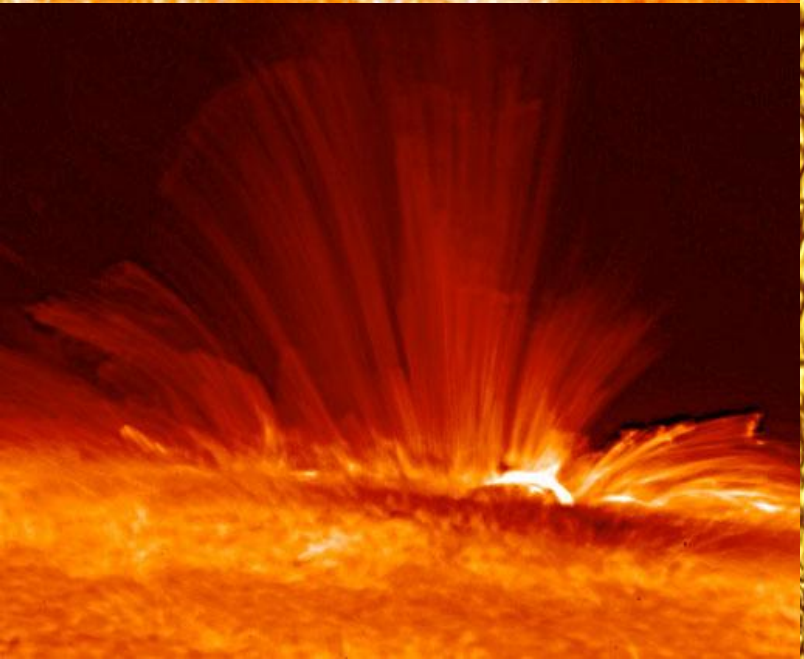


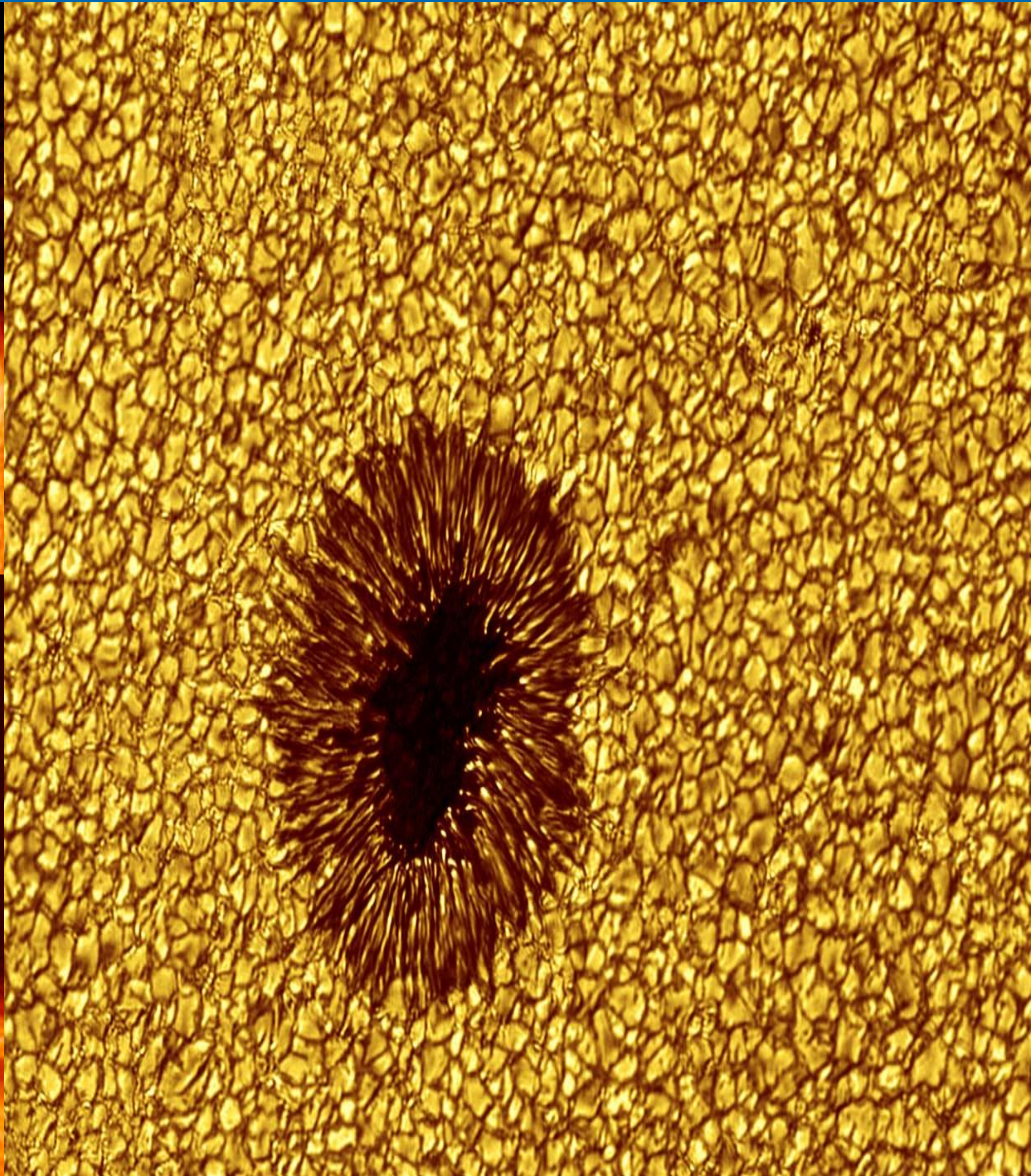
Open cloud cells over the Pacific Ocean



Solar Convection







FORMATION OF AEROSOL LAYERS

Smog cloud over Santiago

Mean-Field Approach

- **Induction equation for mean magnetic field:**

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times \left(\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \langle \mathbf{u} \times \mathbf{b} \rangle - \eta \nabla \times \overline{\mathbf{B}} \right)$$

- **Electromotive force:**

$$\boldsymbol{\varepsilon} \equiv \langle \mathbf{u} \times \mathbf{b} \rangle$$

Mean field equations

$$\left(\frac{\partial}{\partial t}+\overline{\mathbf{U}}\cdot\nabla\right)\overline{U}_i=-\nabla_i\left(\frac{P}{\rho_0}\right)-\nabla_j\left\langle u_i\,u_j\right\rangle-g_i\,\overline{\Theta}+\nu\,\Delta\overline{U}_i$$

$$\left(\frac{\partial}{\partial t}+\overline{\mathbf{U}}\cdot\nabla\right)\overline{\Theta}=-\nabla_j\left\langle\theta\,u_j\right\rangle+\kappa\,\Delta\,\overline{\Theta}$$

$\left\langle\theta\,\mathbf{u}\right\rangle$ is the heat flux

$\left\langle u_i\,u_j\right\rangle$ are the Reynolds stresses

Methods and Approximations

- **Second-Order Correlation Approximation** (SOCA) or First-Order Smothing Approximation (FOSA)

  (a) Re << 1 (b) Rm << 1

  H. K. Moffatt (1978); F. Krause and K. H. Raedler (1980)

- **Path-Integral Approach** (delta-correlated in time random velocity field or short yet finite correlation time)

  R. H. Kraichnan, Phys. Fluids 11, 945 (1968)

- **Tau-approaches** (spectral tau-approximation, minimal tau-approximation) – third-order or high-order closure

  Re >> 1 and Rm >> 1

  A. Pouquet, U. Frisch, and J. Leorat, J. Fluid Mech. 77, 321 (1976)

- **Renormalization Procedure** (renormalization of viscosity, diffusion, heat conductivity and other turbulent transport coefficients) -- no separation of scales

  H. K. Moffatt, Rep. Prog. Phys. 46, 621 (1983)

Tau Approach

Equations for the correlation functions for:

- The velocity fluctuations $\left(M_{ij}^{(II)}(\mathbf{k})\right)_u = \langle u_i \, u_j \rangle$
- The magnetic fluctuations $\left(M_{ij}^{(II)}(\mathbf{k})\right)_b = \langle b_i \, b_j \rangle$
- The cross-helicity tensor $\left(M_{ij}^{(II)}(\mathbf{k})\right)_\chi = \langle b_i \, u_j \rangle$

The spectral τ-approximation (the third-order closure procedure)

$$\hat{D}M^{(III)}(\mathbf{k}) - \hat{D}M_0^{(III)}(\mathbf{k}) = -\frac{M^{(II)}(\mathbf{k}) - M_0^{(II)}(\mathbf{k})}{\tau_c(\mathbf{k})}$$

$$\left(M_{ij}^{(II)}(\mathbf{k})\right)_u = -\langle u_i \, (\mathbf{u}\cdot\nabla) u_j \rangle - \langle u_j \, (\mathbf{u}\cdot\nabla) u_i \rangle$$

Tau Approach

Equations for the correlation functions for:

- The velocity fluctuations $$\left(M_{ij}^{(II)}(\mathbf{k})\right)_u = \langle u_i \, u_j \rangle$$
- The temperature fluctuations $$\left(M^{(II)}(\mathbf{k})\right)_\theta = \langle \theta \, \theta \rangle$$
- The heat flux $$\left(M_i^{(II)}(\mathbf{k})\right)_\Phi = \langle \theta \, u_i \rangle$$

The spectral τ-approximation (the third-order closure procedure)

$$\hat{D}M^{(III)}(\mathbf{k}) - \hat{D}M_0^{(III)}(\mathbf{k}) = -\frac{M^{(II)}(\mathbf{k}) - M_0^{(II)}(\mathbf{k})}{\tau_c(\mathbf{k})}$$

$$\left(M_{ij}^{(II)}(\mathbf{k})\right)_u = -\langle u_i \, (\mathbf{u} \cdot \nabla) u_j \rangle - \langle u_j \, (\mathbf{u} \cdot \nabla) u_i \rangle$$

Methods and Approximations

- **Second-Order Correlation Approximation** (SOCA) or First-Order Smothing Approximation (FOSA)

  (a) Re << 1 (b) Rm << 1

  H. K. Moffatt (1978); F. Krause and K. H. Raedler (1980)

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  Re >> 1 and Rm >> 1

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- **Renormalization Procedure** (renormalization of viscosity, diffusion, heat conductivity and other turbulent transport coefficients) -- no separation of scales

  H. K. Moffatt, Rep. Prog. Phys. 46, 621 (1983)

Renormalization Procedure

- The first step is the averaging over the scale that is inside the inertial range of turbulence.
- The next stage of the renormalization procedure comprises a step-by-step increase of the scale of the averaging up to the maximum scale of turbulent motions.
- This procedure allows the derivation of equations for the turbulent transport coefficients: eddy viscosity, turbulent diffusion, turbulent heat conductivity, etc.
- To apply this procedure an equation invariant under the renormalization of the turbulent transport coefficients must be determined.

Passive scalar

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho\, \mathbf{u}) = 0 \qquad (1)$$

$$\frac{\partial n}{\partial t} + \mathrm{div}(n\, \mathbf{u}) = D\Delta\, n \qquad (2)$$

Why does not turbulent diffusion arise in averaged equation (1) for the fluid density in a turbulent flow, while averaged equation (2) does contain the turbulent diffusion?

$$\frac{\partial \overline{\rho}}{\partial t} + \mathrm{div}\left(\overline{\rho}\, \overline{\mathbf{V}}\right) = 0$$

$$\frac{\partial \overline{N}}{\partial t} + \mathrm{div}\left(\overline{N}\, \overline{\mathbf{V}}\right) = \left(D + D_T\right)\Delta\, \overline{N}$$

Turbulent Diffusion

Taylor (1921)

$$D_T \approx l\,u \gg D$$

$$\frac{\partial \overline{N}}{\partial t} + \mathrm{div}\left(\overline{N}\,\overline{\mathbf{V}}\right) = D_T \Delta \overline{N}$$

- Turbulence results in a sharp increase of the diffusion coefficient (Taylor, 1921).

- Turbulence causes a decay of particle inhomogeneities.

- **However, the opposite process, the large-scale preferential concentration of particles in turbulent flows is still poorly understood.**

Turbulent thermal diffusion of particles

$$\boxed{\frac{\partial n}{\partial t} + \mathrm{div}\left(n\,\mathbf{v}_p\right) = D\,\Delta\,n}$$

Averaging over turbulent velocity field

$$\boxed{\frac{\partial \overline{N}}{\partial t} + \mathrm{div}\left(\overline{N}\,\overline{\mathbf{V}}_p + \overline{N}\,\mathbf{V}_{eff}\right) = (D + D_T)\,\Delta\,\overline{N}}$$

$$\overline{N} = \langle n \rangle$$

$$\mathbf{v}_p = \overline{\mathbf{V}}_p + \mathbf{u}$$

$$\overline{\mathbf{V}}_p = \langle \mathbf{v}_p \rangle$$

$$\boxed{\overline{\mathbf{J}}_T = \overline{N}\,\mathbf{V}_{eff} - D_T \nabla \overline{N}}$$

$$\boxed{\mathbf{V}_{eff} = -\tau \langle \mathbf{u}\,\mathrm{div}\,\mathbf{u} \rangle}$$

Turbulent thermal diffusion of non-inertial particles

$$\mathbf{v}_p = \mathbf{u}$$

$$\rho \, \mathrm{div} \, \mathbf{u} + (\mathbf{u} \cdot \nabla) \rho \approx 0$$

$$\mathrm{div} \, \mathbf{u} \approx -\mathbf{u} \cdot \frac{\nabla \rho}{\rho}$$

Equation of state for ideal gas yields:

$$\frac{\nabla \overline{\rho}}{\overline{\rho}} \approx -\frac{\nabla \overline{T}}{\overline{T}}$$

$$\frac{\partial \overline{N}}{\partial t} + \mathrm{div}\left(\overline{N} \, \overline{\mathbf{V}} + \overline{N} \, \mathbf{V}_{eff} - D_T \nabla \overline{N} \right) = 0$$

$$\mathbf{V}_{eff} = -\tau \left\langle \mathbf{u} \, \mathrm{div} \, \mathbf{u} \right\rangle$$

$$-\tau \left\langle u_i \, \mathrm{div} \, \mathbf{u} \right\rangle = \tau \left\langle u_i \, u_j \right\rangle \frac{\nabla_j \overline{\rho}}{\overline{\rho}} = D_T \, \frac{\nabla_i \overline{\rho}}{\overline{\rho}}$$

$$\boxed{\mathbf{V}_{eff} = D_T \, \frac{\nabla \overline{\rho}}{\overline{\rho}} = -D_T \, \frac{\nabla \overline{T}}{\overline{T}}}$$

Turbulent flux of particles

$$\frac{\partial n}{\partial t} + \mathrm{div}\left(n\,\mathbf{u}\right) = D\,\Delta\,n \qquad n = \overline{N} + n' \qquad \frac{\partial \overline{N}}{\partial t} + \mathrm{div}\left(\langle n'\mathbf{u}\rangle\right) = D\,\Delta\,\overline{N}$$

fluctuations of particles number density

$$\frac{\partial n'}{\partial t} - D\,\Delta\,n' + \mathrm{div}\left(n'\mathbf{u} - \langle n'\mathbf{u}\rangle\right) = -\mathrm{div}\left(\overline{N}\,\mathbf{u}\right)$$

$$n' \sim -\tau\,\overline{N}\,\mathrm{div}\,\mathbf{u} - \tau\,(\mathbf{u}\cdot\nabla)\,\overline{N}$$

$$\boxed{\overline{\mathbf{J}}_T \equiv \langle \mathbf{u}\,n'\rangle \sim -\tau\,\overline{N}\langle \mathbf{u}\,\mathrm{div}\,\mathbf{u}\rangle - \tau\langle \mathbf{u}\,(\mathbf{u}\cdot\nabla)\rangle\overline{N}}$$

$D_T \equiv D_{ij} = \tau\langle u_i\,u_j\rangle$ - turbulent diffusion tensor

$\boxed{\mathbf{V}_{eff} = -\tau\langle\,\mathbf{u}\,\mathrm{div}\,\mathbf{u}\rangle}$ - effective velocity

$\boxed{\overline{\mathbf{J}}_T = \overline{N}\,\mathbf{V}_{eff} - D_T\nabla\overline{N}}$ - turbulent flux of particles

Turbulent thermal diffusion of inertial particles

$$\frac{d\,\mathbf{v}_p}{d\,t} = -\frac{\mathbf{v}_p - \mathbf{u}}{\tau_p}$$

$$\mathbf{v}_p = \mathbf{u} - \tau_p \frac{d\,\mathbf{u}}{d\,t} + O(\tau_p^2)$$

$$\mathbf{V}_{eff} = -\tau \left\langle \mathbf{v}_p \,\mathrm{div}\, \mathbf{v}_p \right\rangle$$

$$\mathrm{div}\,\mathbf{v}_p = \mathrm{div}\,\mathbf{u} + \tau_p \frac{\Delta P}{\rho} + O(\tau_p^2)$$

$$\boxed{\mathbf{V}_{eff} = -D_T \alpha \frac{\nabla \bar{T}}{\bar{T}}}$$

$$\alpha \approx 1 + \left(\frac{m_p}{m_\mu}\right)\left(\frac{\bar{T}}{T_*}\right)\frac{\ln(\mathrm{Re})}{\mathrm{Pe}}$$

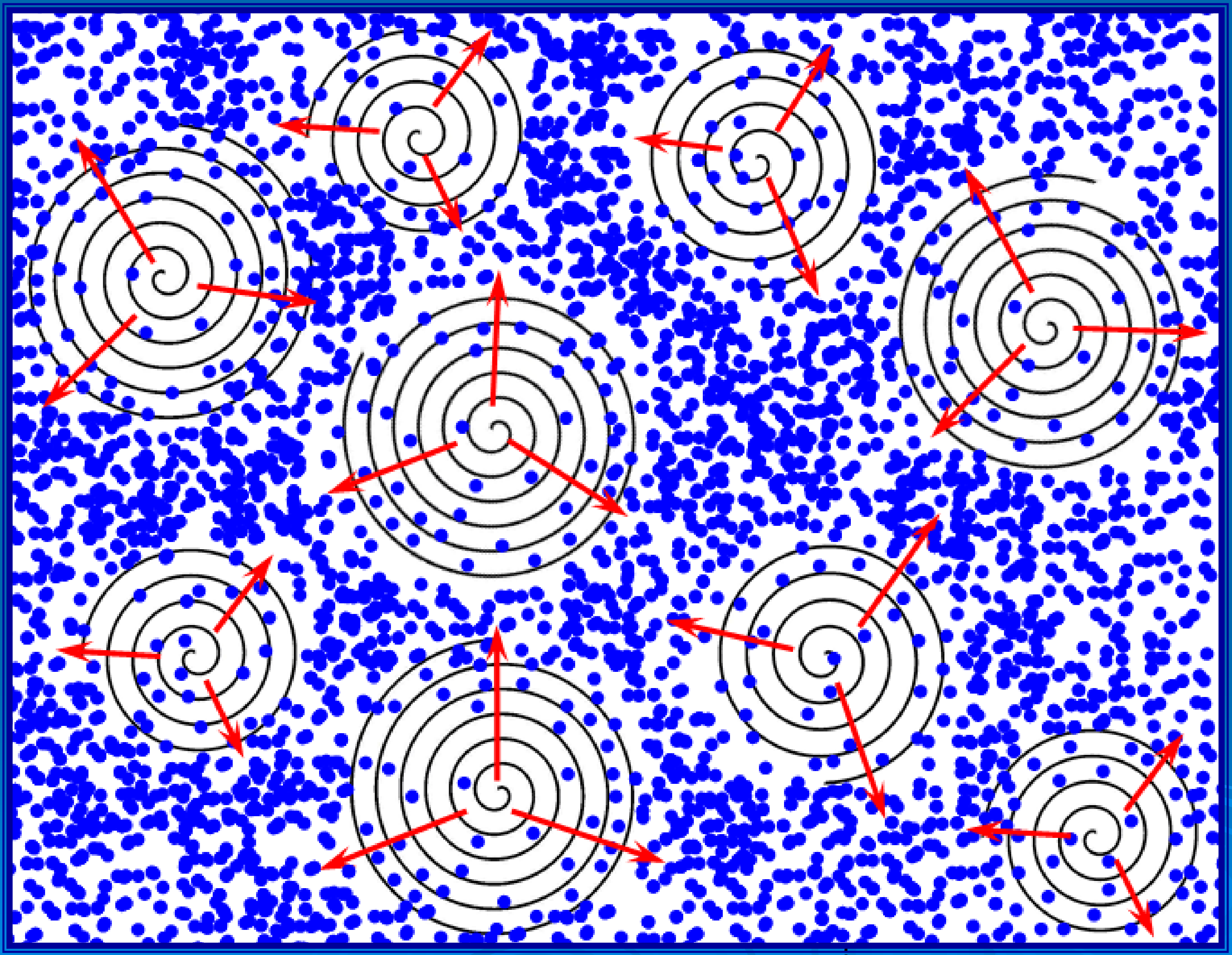
$$\bar{\mathbf{J}}_T = -D_T\, k_T \frac{\nabla \bar{T}}{\bar{T}} - D_T \nabla \bar{N}$$

- turbulent flux of particles

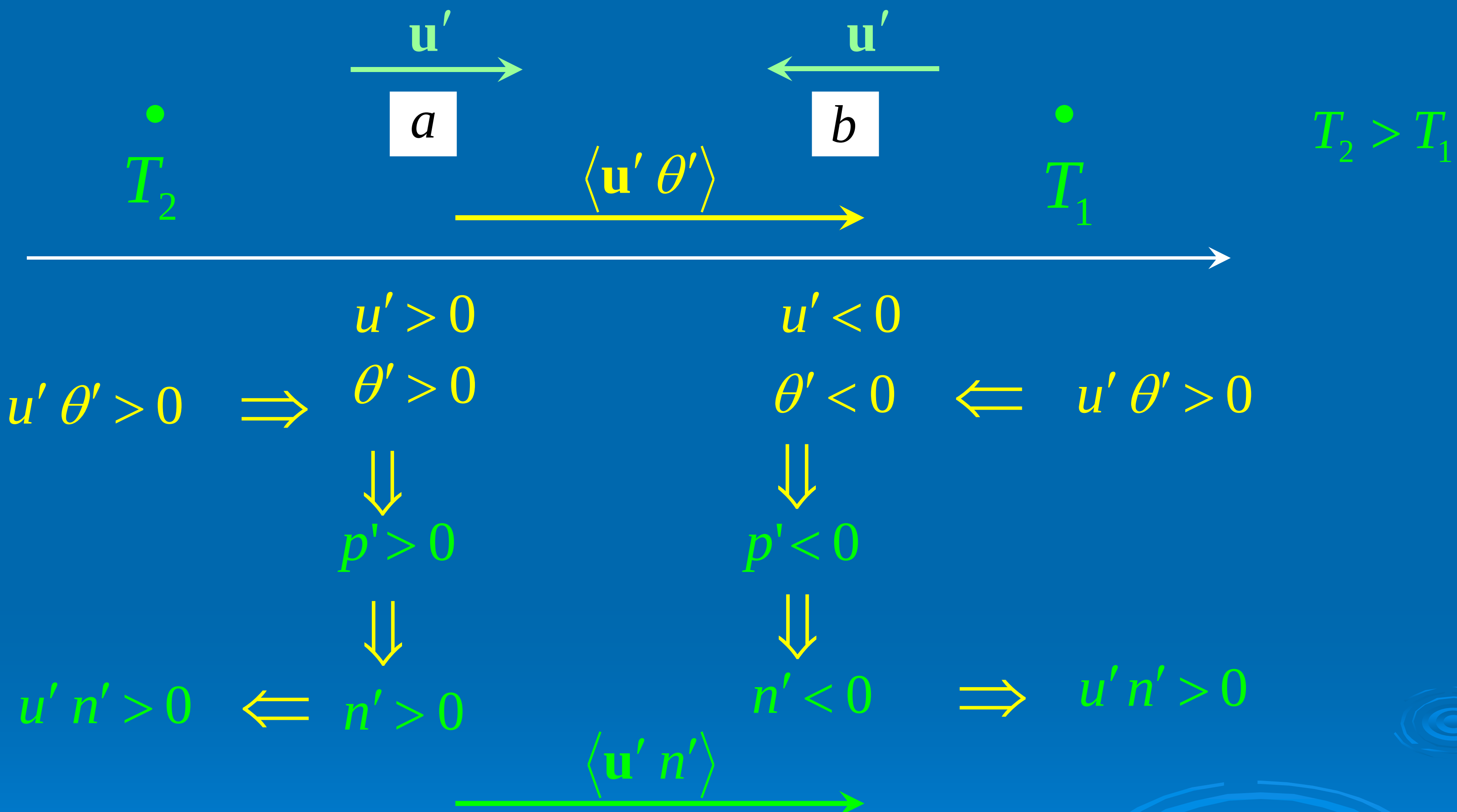
$$k_T = \alpha\, \bar{N}$$

- turbulent thermal diffusion ratio

Particle Inertia Effect



Turbulent Thermal Diffusion



$\mathbf{u}'$ $\mathbf{u}'$

a b

T_2 $\langle \mathbf{u}'\,\theta' \rangle$ T_1 $T_2 > T_1$

$u' > 0$ $u' < 0$

$u'\,\theta' > 0 \quad \Rightarrow \quad \theta' > 0$ $\theta' < 0 \quad \Leftarrow \quad u'\,\theta' > 0$

$\Downarrow$ $\Downarrow$

$p' > 0$ $p' < 0$

$\Downarrow$ $\Downarrow$

$u'\,n' > 0 \quad \Leftarrow \quad n' > 0$ $n' < 0 \quad \Rightarrow \quad u'\,n' > 0$

$\langle \mathbf{u}'\,n' \rangle$

Non-diffusive mean flux of particles is in the direction of the mean heat flux (i.e., in the direction of minimum fluid temperature).

Derivation of the effect of turbulent thermal diffusion

- Path integral approach (delta-correlated in time velocity field and finite correlation time)
- **The spectral tau approximation**

T. Elperin, N. Kleeorin and I. Rogachevskii

- Physical Review Letters **76**, 224 (1996)
- Physical Review E **55**, 2713 (1997)
- Physical Review Letters **80**, 69 (1998)
- Intern. Journal of Multiphase Flow **24**, 1163 (1998)
- Atmospheric Research **53**, 117 (2000)

T. Elperin, N. Kleeorin, I. Rogachevskii and D. Sokoloff

- Physical Review E **61**, 2617 (2000)
- Physical Review E **64**, 026304 (2001)

R.V.R. Pandya and F. Mashayek, Physical Review Letters **88**, 044501 (2002)

M.W. Reeks, Intern. Journal of Multiphase Flow **31**, 93 (2005)

Paradox

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho\,\mathbf{u}) = 0 \qquad (1)$$

$$\frac{\partial n}{\partial t} + \mathrm{div}(n\,\mathbf{u}) = D\Delta\, n \qquad (2)$$

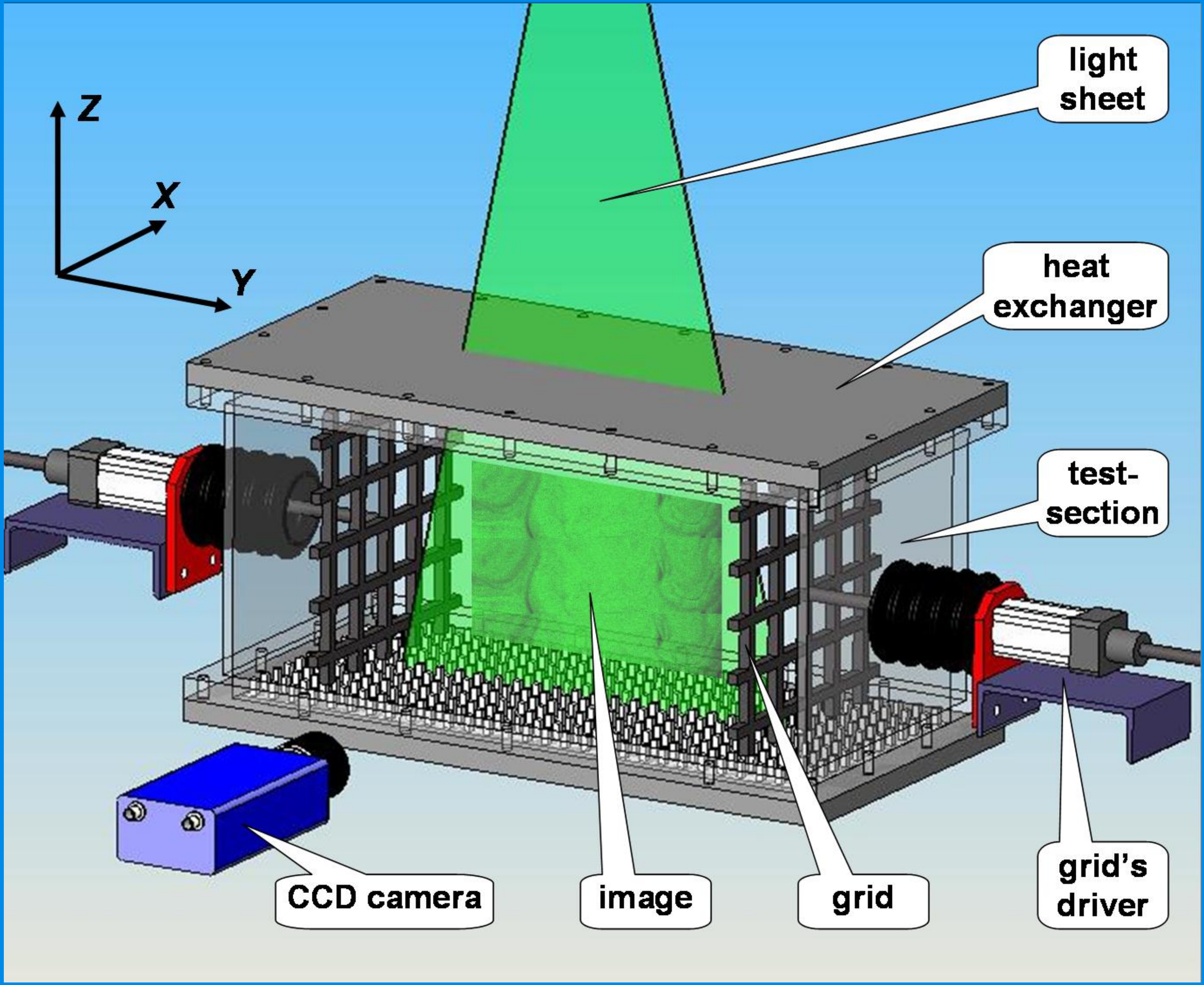
$$\frac{\partial \bar{N}}{\partial t} + \mathrm{div}\left(\bar{N}\,\bar{\mathbf{V}} + \bar{N}\,\mathbf{V}_{eff} - D_T \nabla \bar{N}\right) = 0$$

$$\mathbf{V}_{eff} = D_T \frac{\nabla \bar{\rho}}{\bar{\rho}}$$

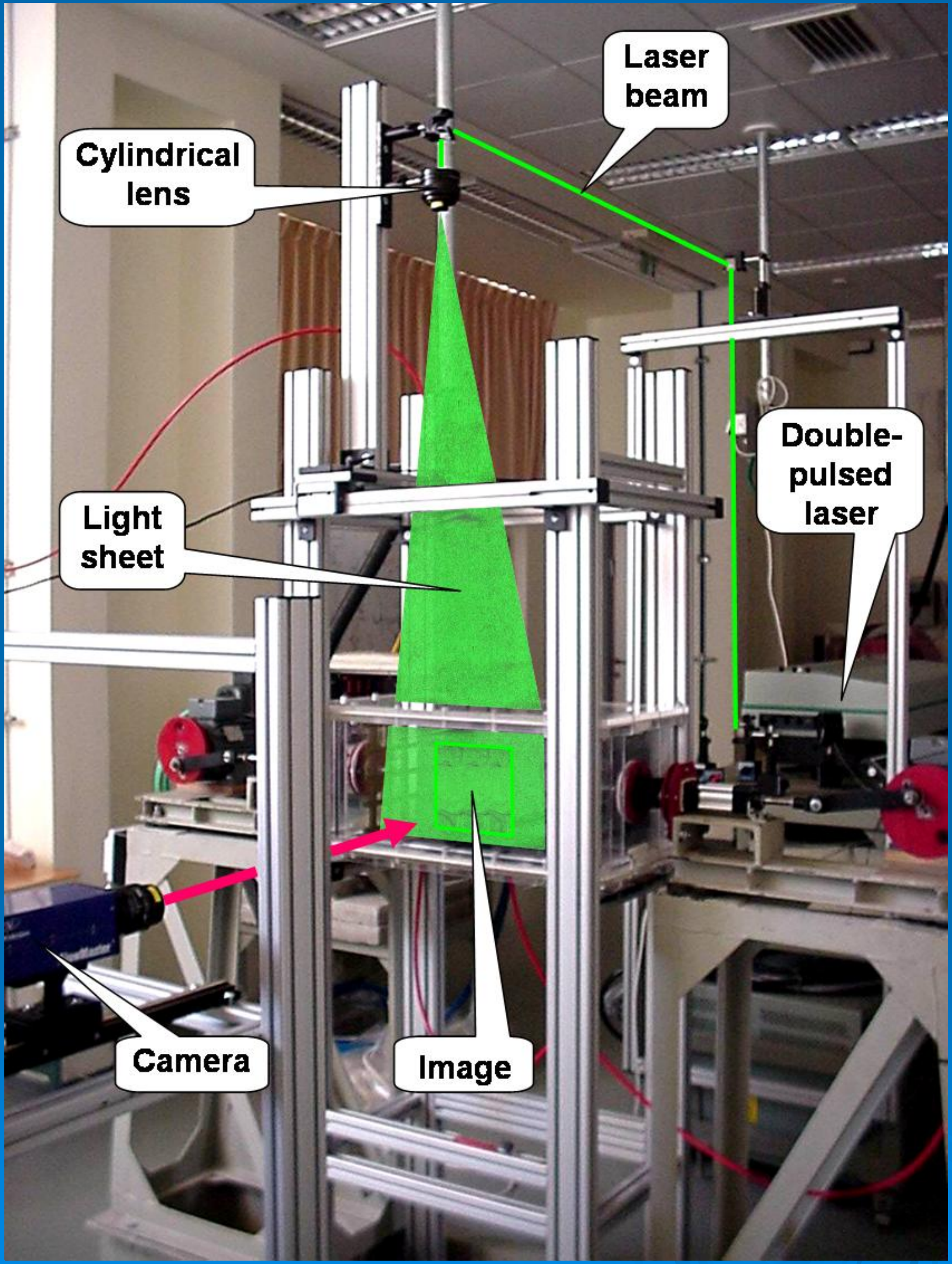
$$\frac{\partial \bar{N}}{\partial t} + \mathrm{div}\left(\bar{N}\,\bar{\mathbf{V}} + D_T \frac{\nabla \bar{\rho}}{\bar{\rho}} \bar{N} - D_T \nabla \bar{N}\right) = 0$$

$$\bar{N} = \bar{\rho} \quad \Rightarrow \qquad \frac{\partial \bar{\rho}}{\partial t} + \mathrm{div}\left(\bar{\rho}\,\bar{\mathbf{V}}\right) = 0$$

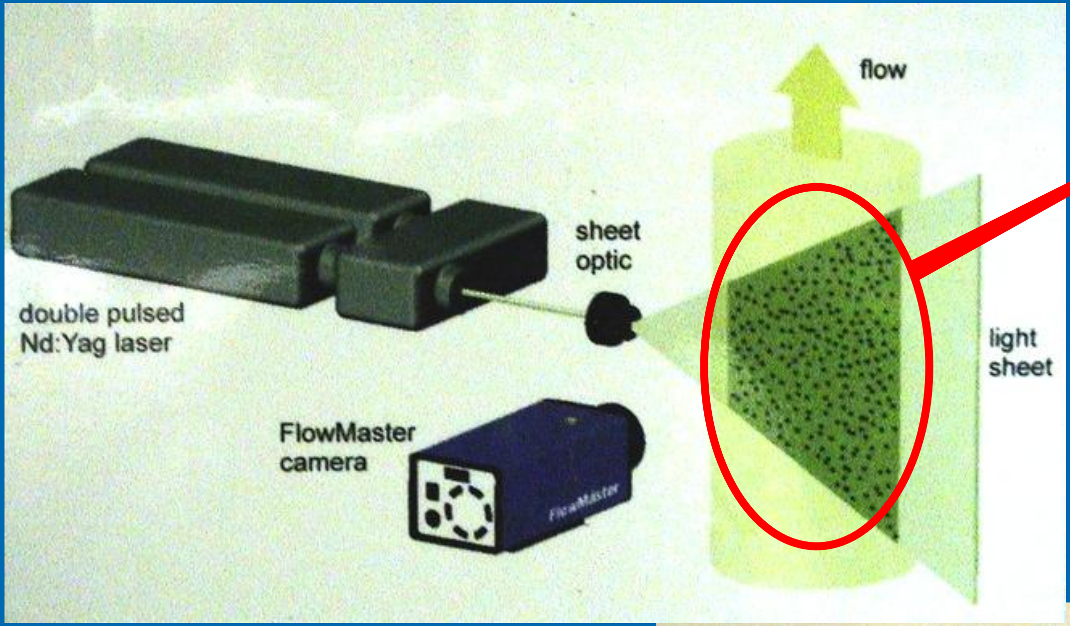
Experimental Set–up

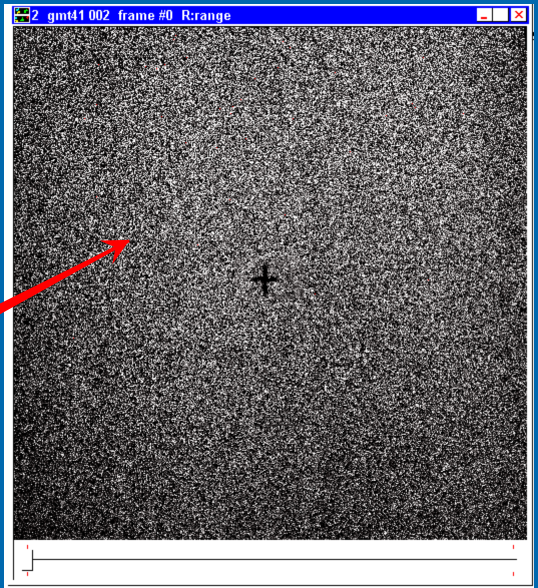


Experimental set - up: oscillating grids turbulence generator and particle image velocimetry system



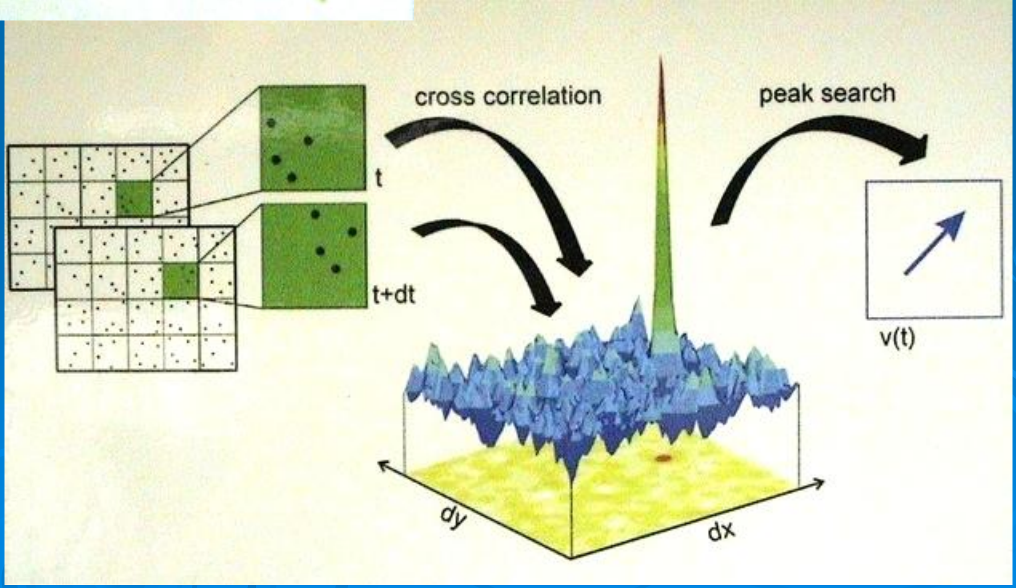
Particle Image Velocimetry System



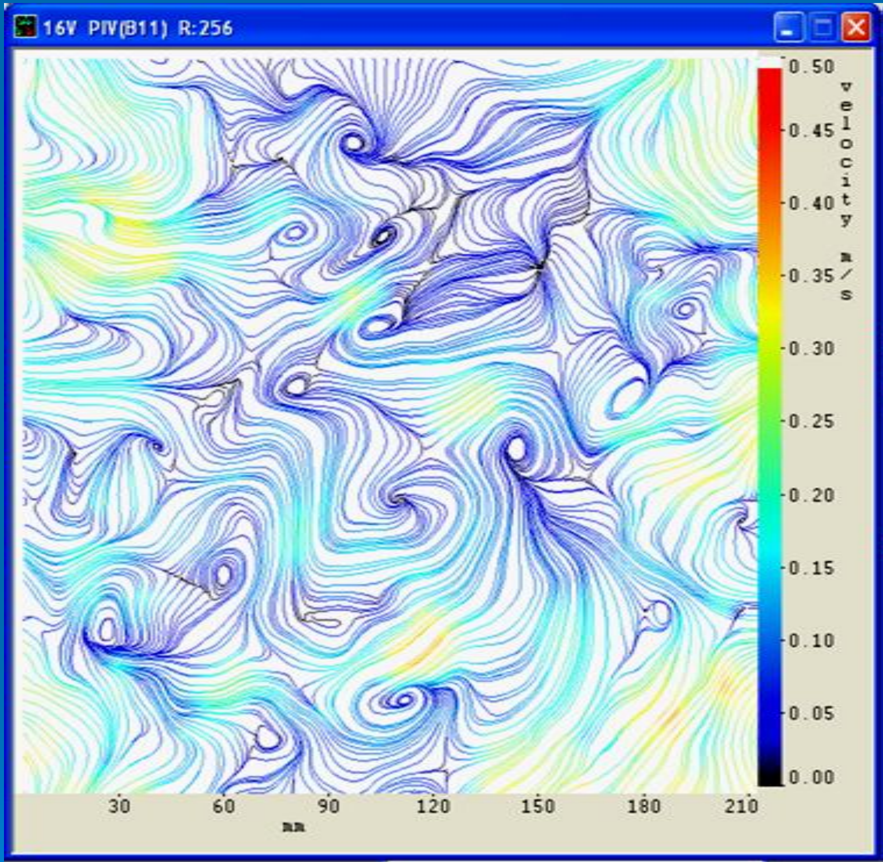


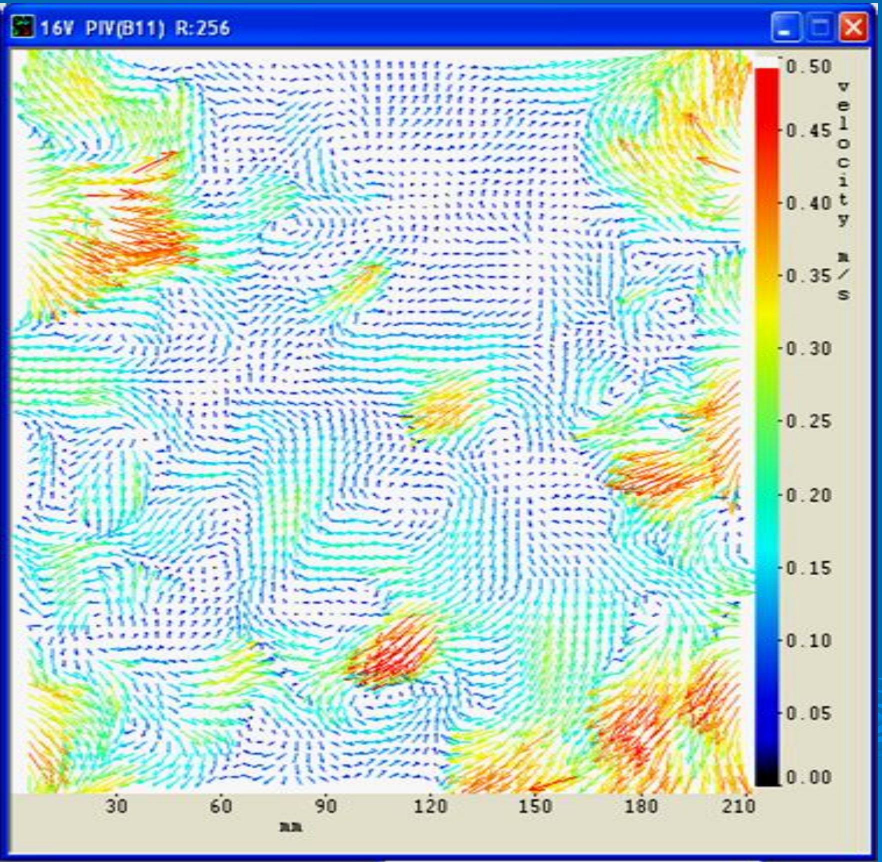
Raw image of the incense smoke tracer particles in oscillating grids turbulence

Particle Image Velocimetry Data Processing

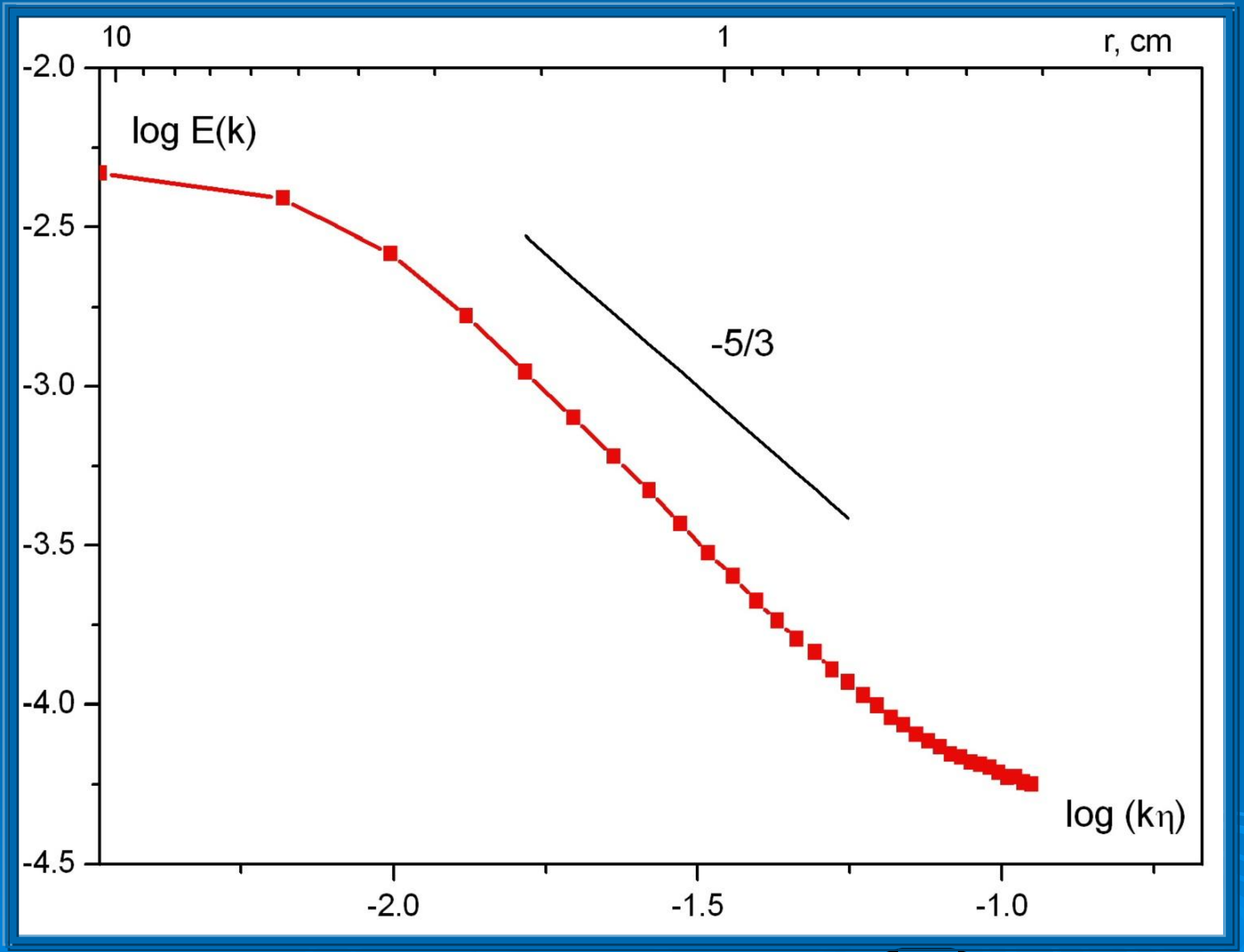


Instantaneous Streamlines of the Flow and Velocity Map

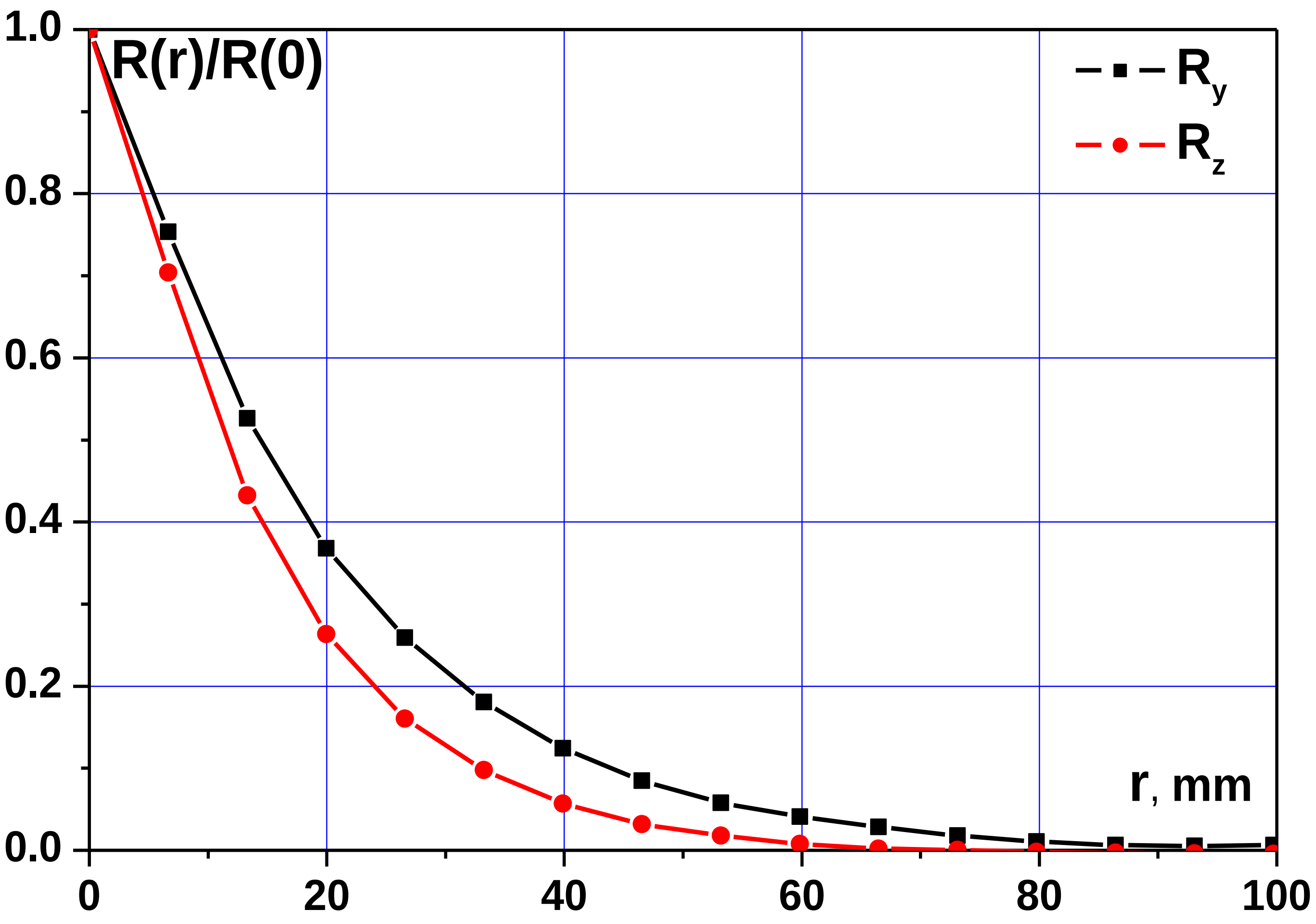




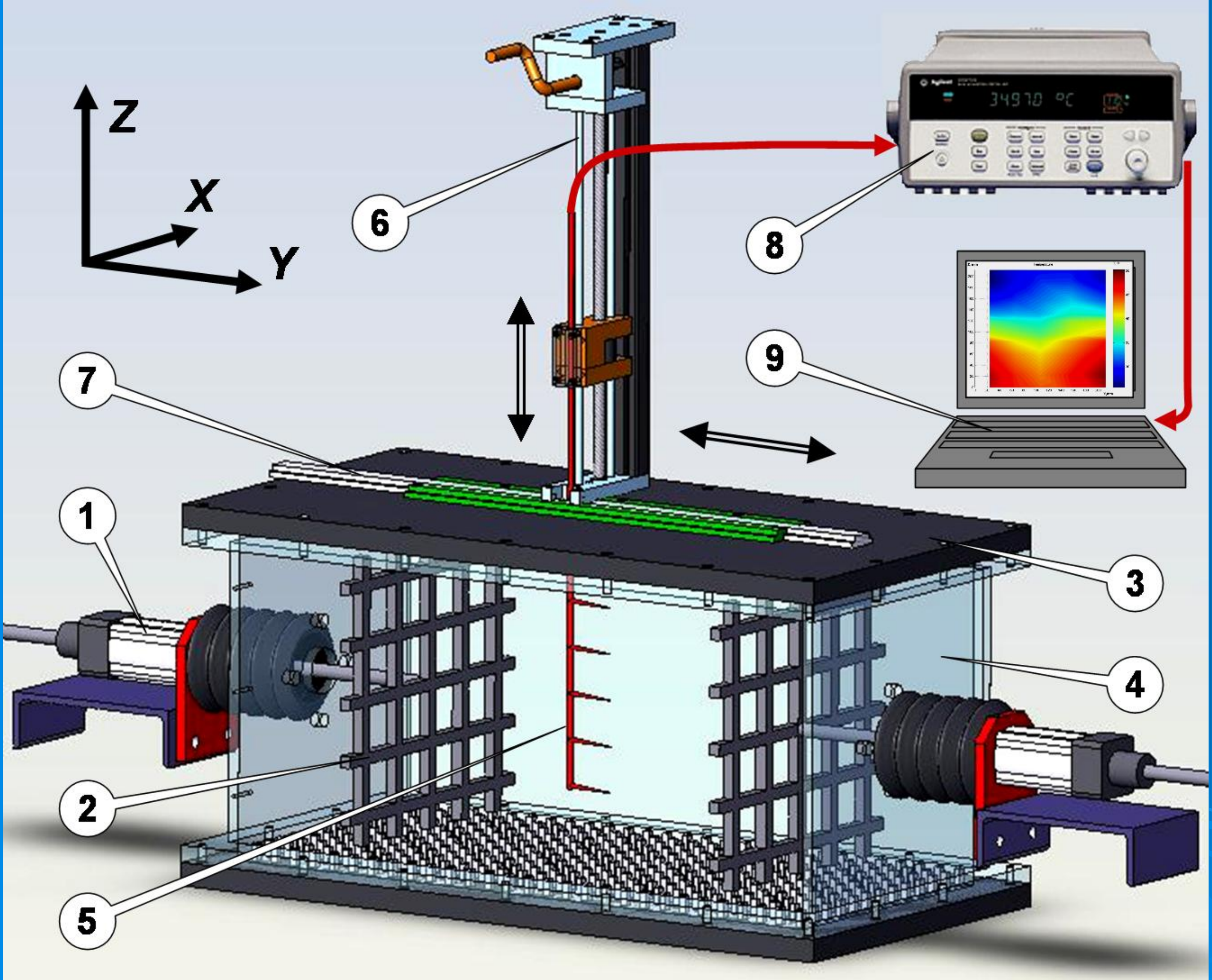
Turbulent Energy Spectrum



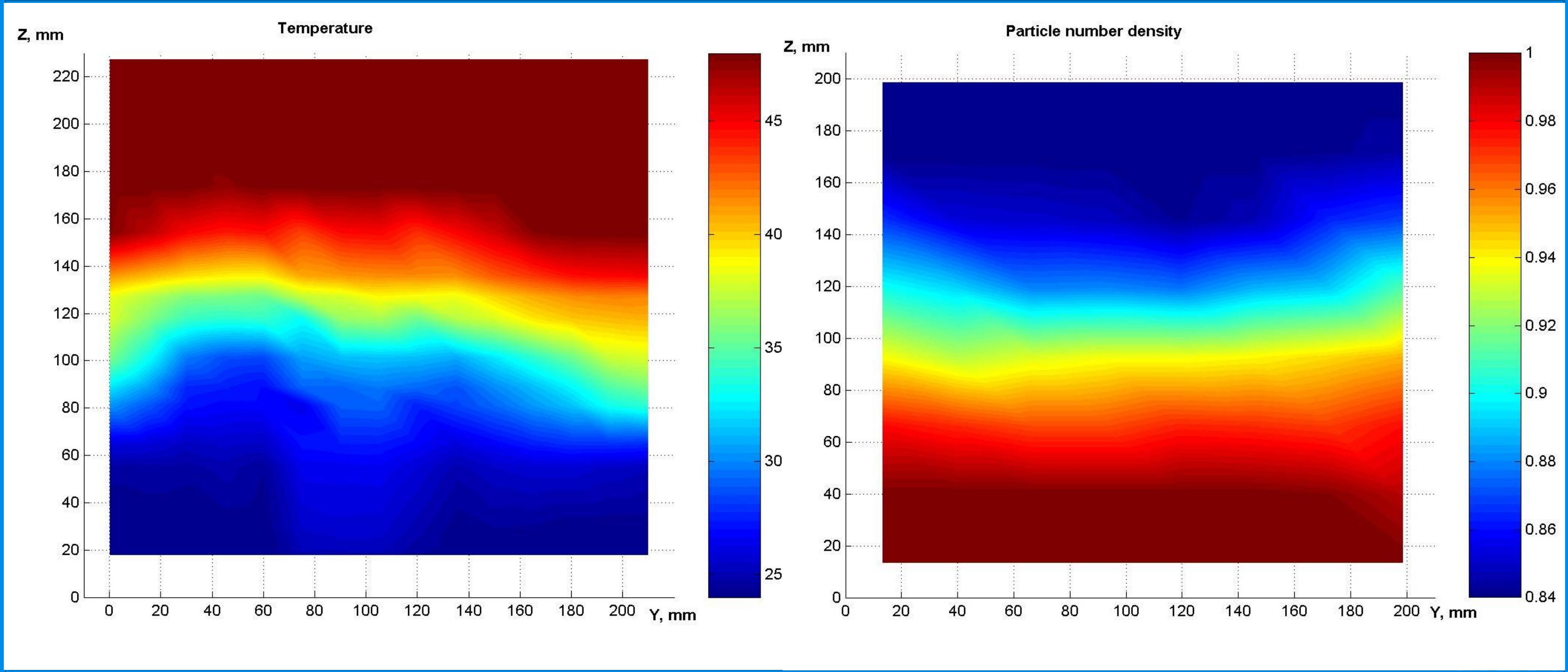
Longitudinal Correlation Functions



Experimental Set–up for Temperature Measurements



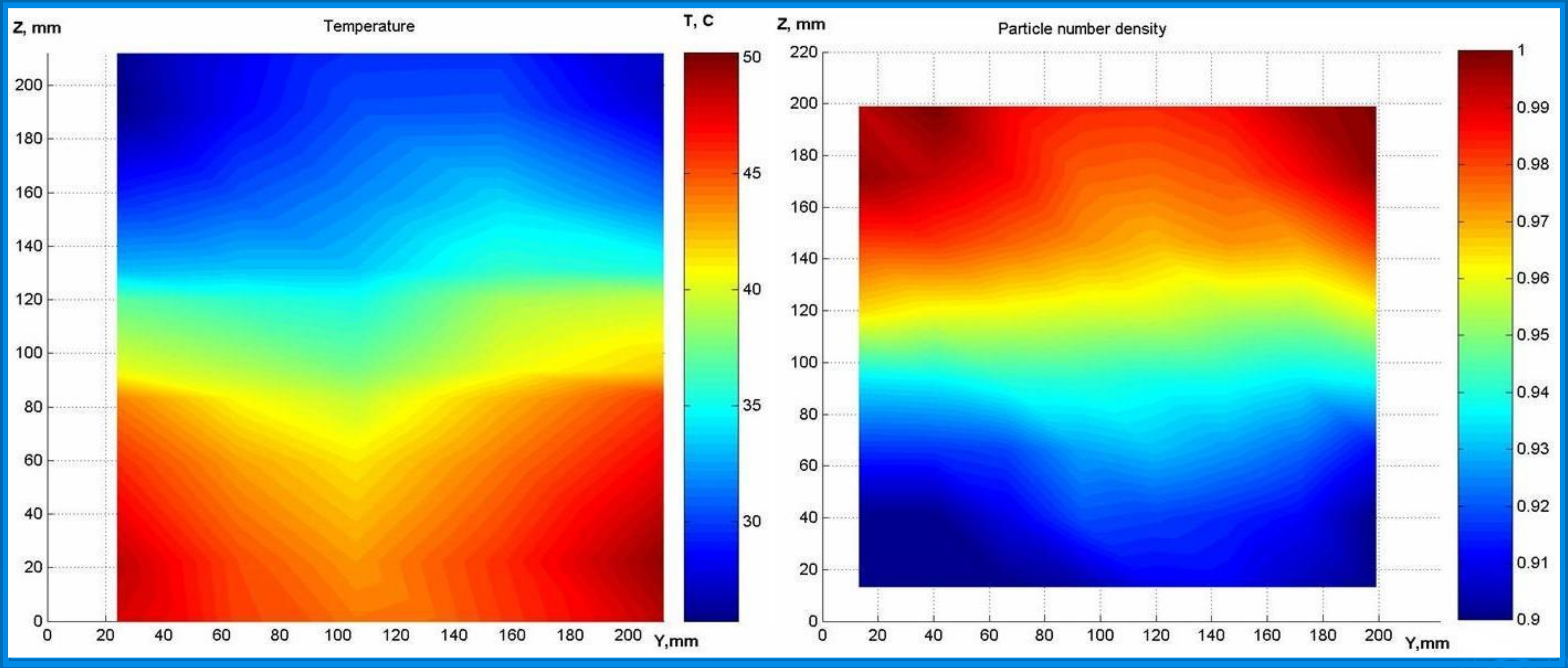
Temperature and Particle Number Density Fields. Stable Stratification



$\bar{T}(y,z)$

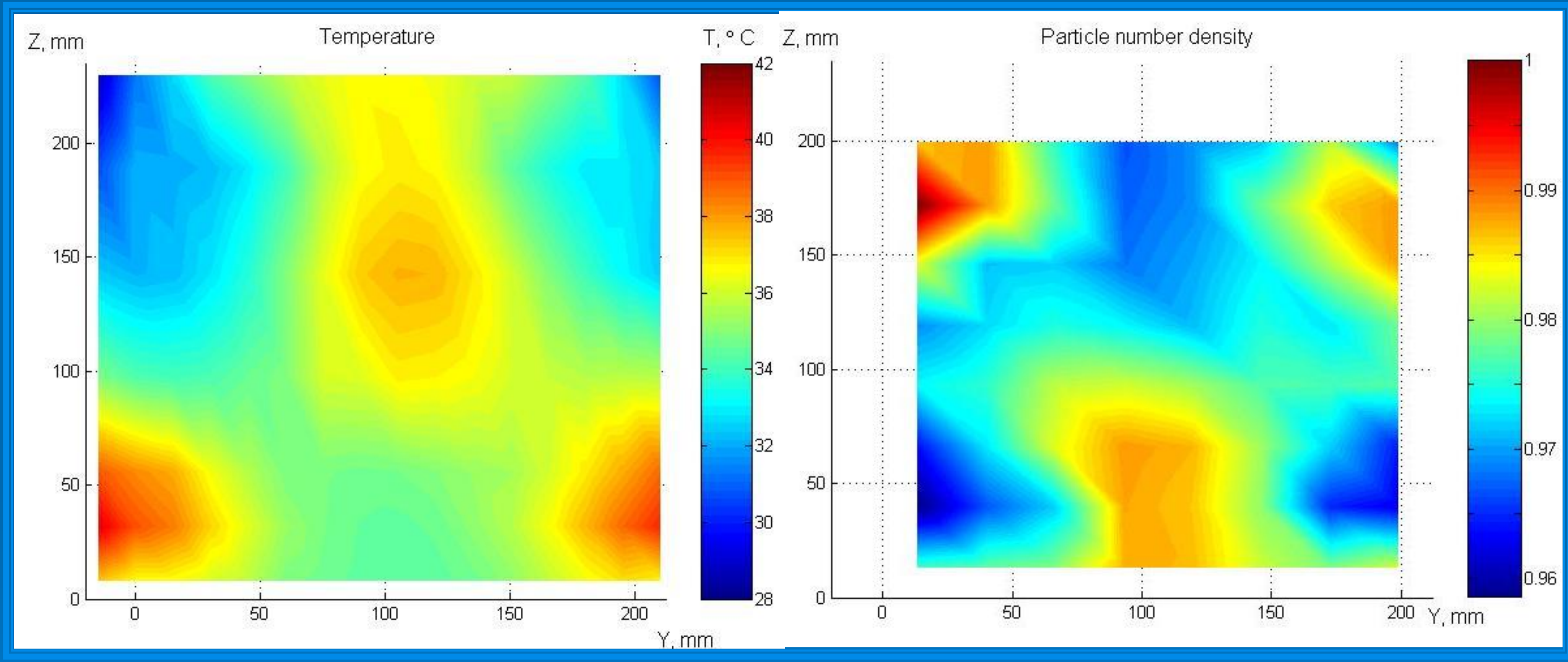
$\bar{N}(y,z)$

Temperature and Particle Number Density Fields. Unstable Stratification, f = 10.5 Hz



$\bar{T}(y,z)$ $\bar{N}(y,z)$

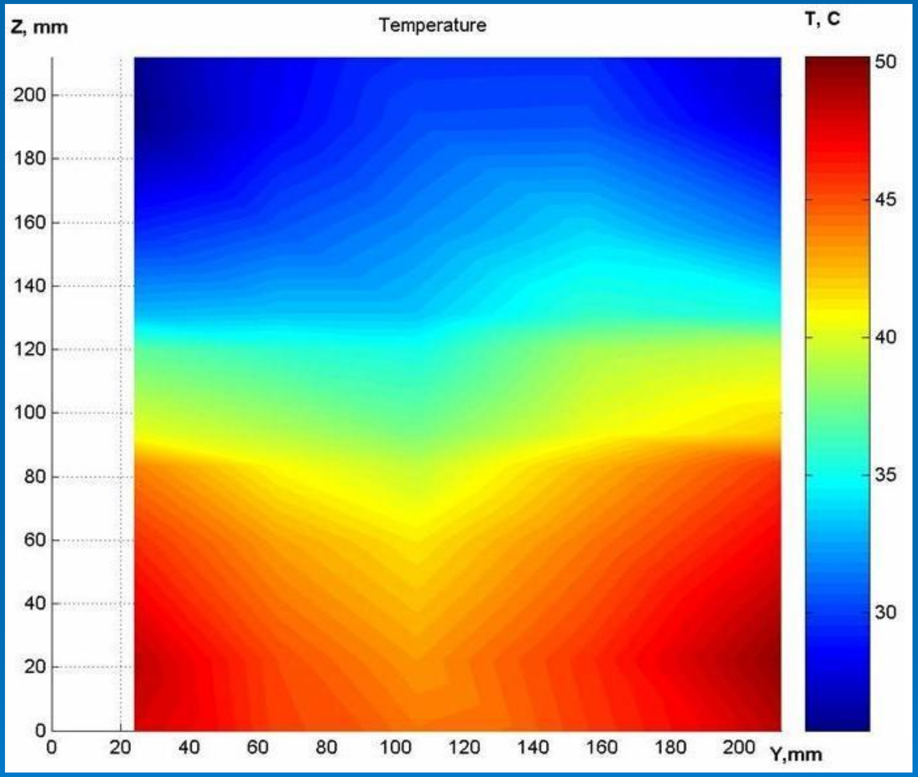
Temperature and Particle Number Density Fields.
Unstable Stratification,   f = 4.4 Hz

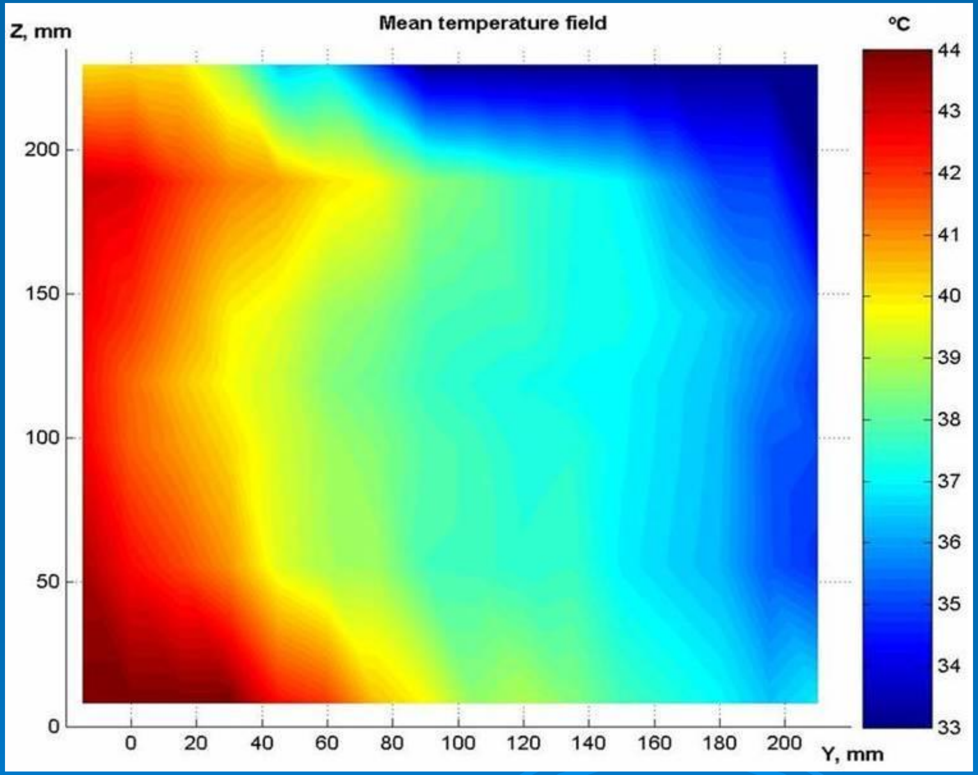


$\bar{T}(y,z)$

$\bar{N}(y,z)$

Temperature Field in Forced and Unforced Turbulent Convection

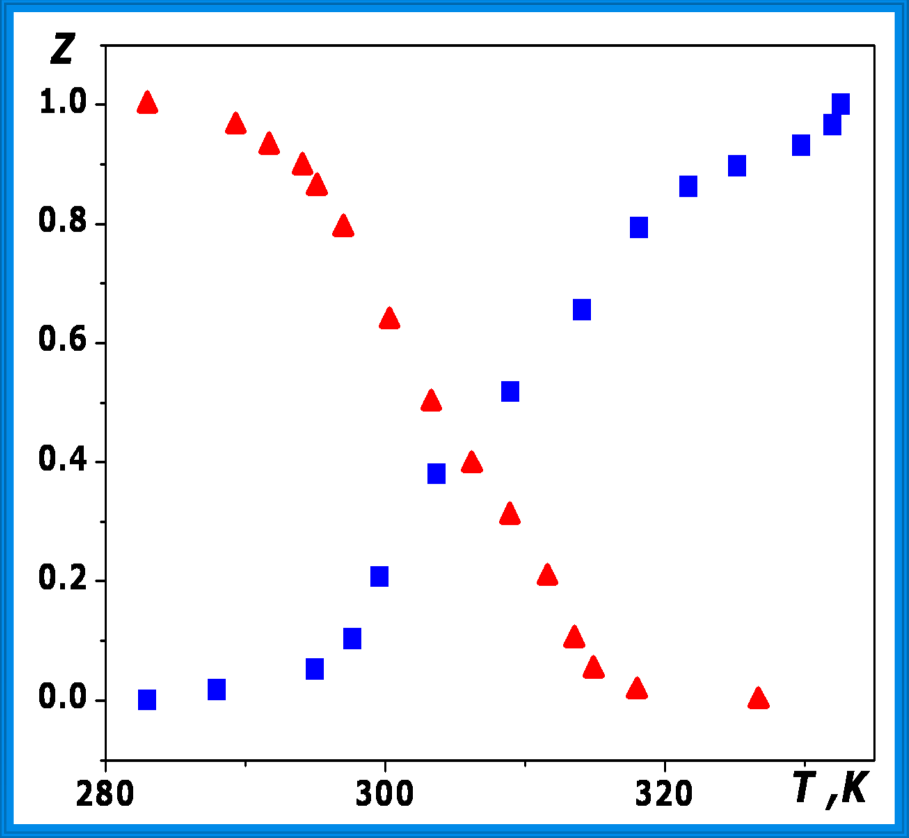


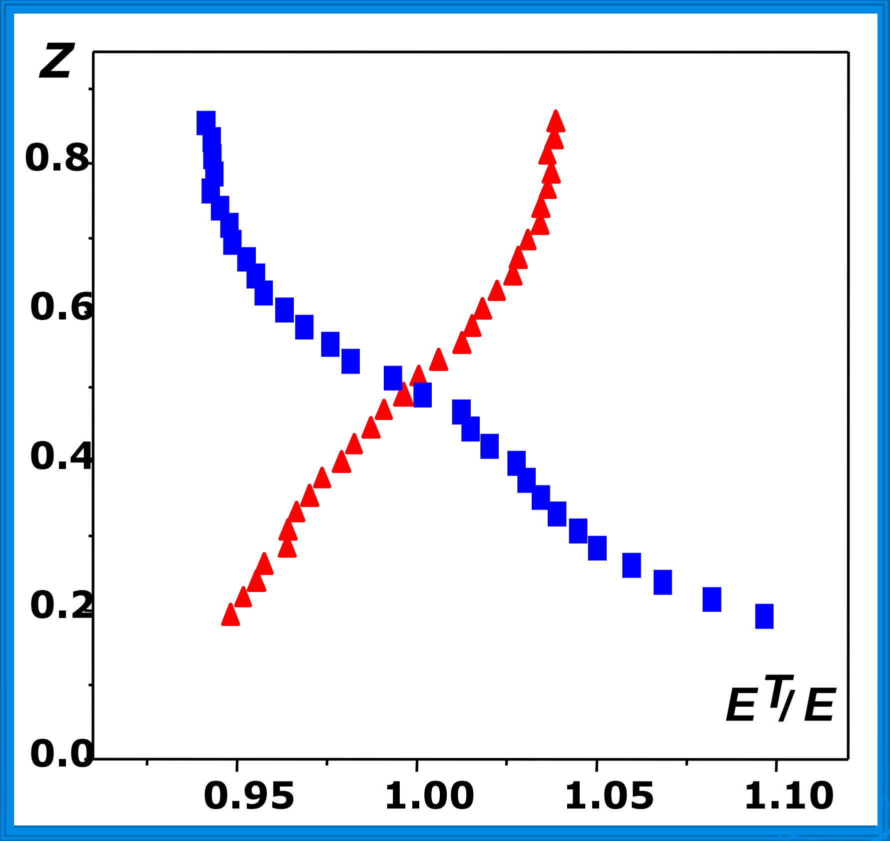


Forced turbulent convection (two oscillating grids)

Unforced convection

Temperature and Particle Spatial Distributions





$$\bar{T}(y,z)$$

$$\bar{N}(y,z)$$

■ - stable stratification

▲ - unstable stratification

Turbulent Thermal Diffusion

$$\frac{\partial \overline{N}}{\partial t} + \mathrm{div}\left(\overline{N}\,\mathbf{V}_{eff} - D_T \nabla \overline{N}\right) = 0$$

$$\boxed{\mathbf{V}_{eff} = -D_T \alpha \frac{\nabla \overline{T}}{\overline{T}}}$$

$\alpha = 1$ for non-inertial particles

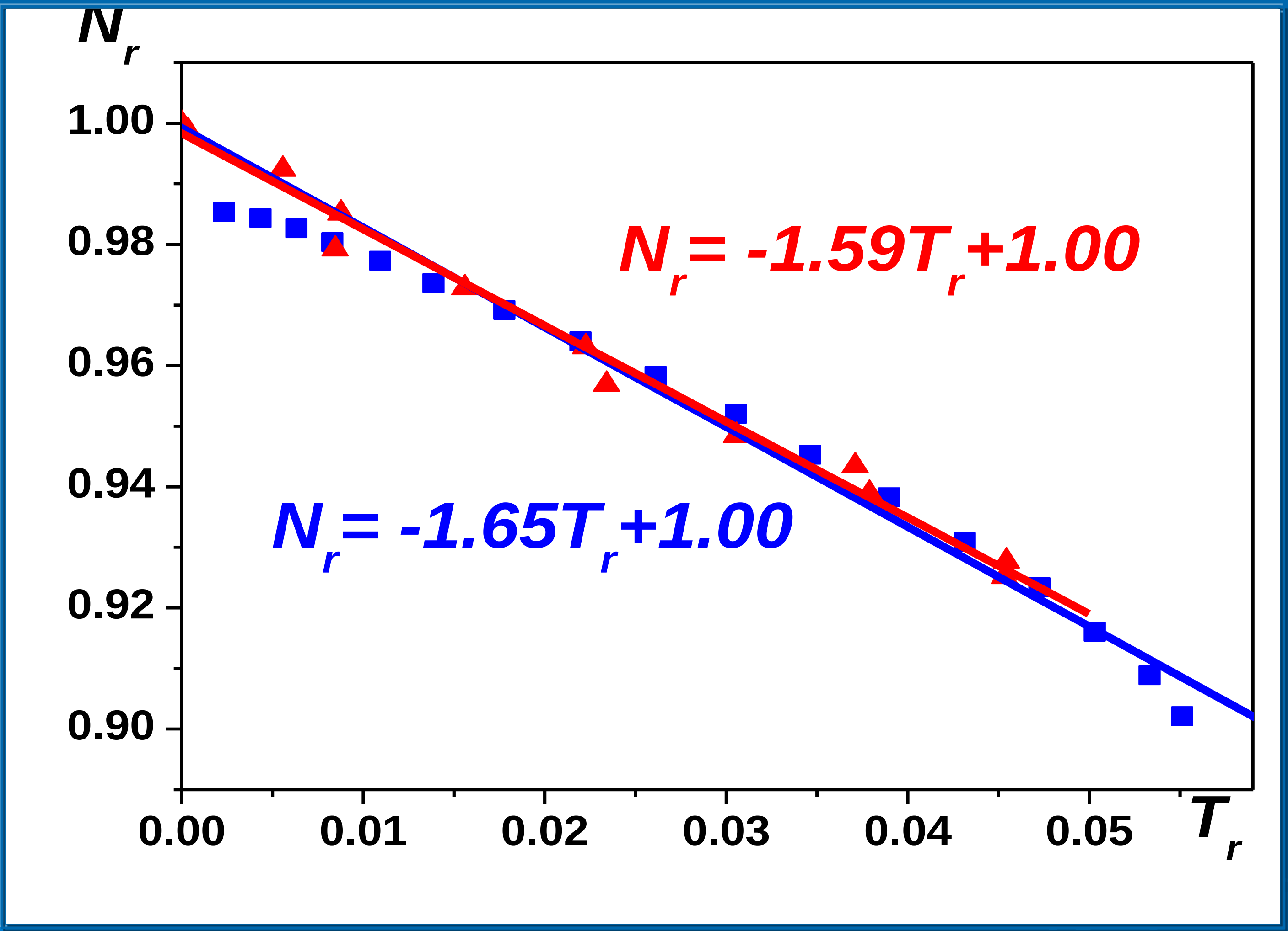
Steady state:

$$\frac{\nabla \overline{N}}{\overline{N}} = -\alpha \frac{\nabla \overline{T}}{\overline{T}}$$

$$\frac{\overline{N} - \overline{N}_0}{\overline{N}_0} = -\alpha \frac{\overline{T} - \overline{T}_0}{\overline{T}_0}$$

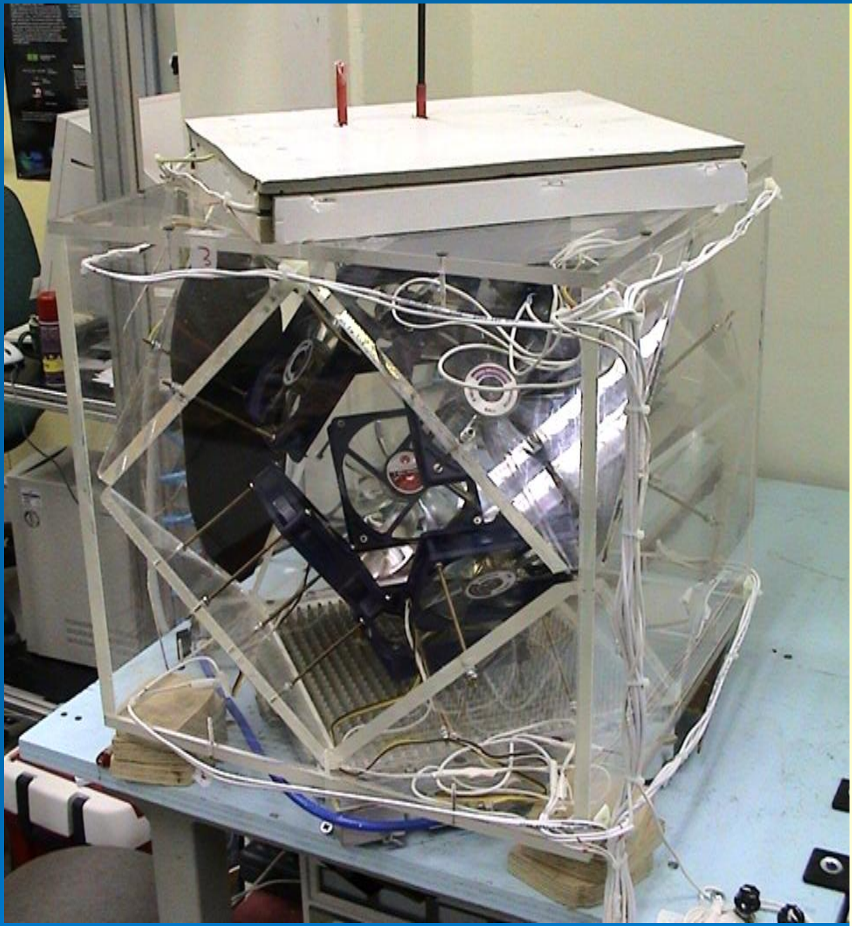
$$\boxed{\frac{\overline{N}}{\overline{N}_0} = -\alpha \frac{\overline{T} - \overline{T}_0}{\overline{T}_0} + 1}$$

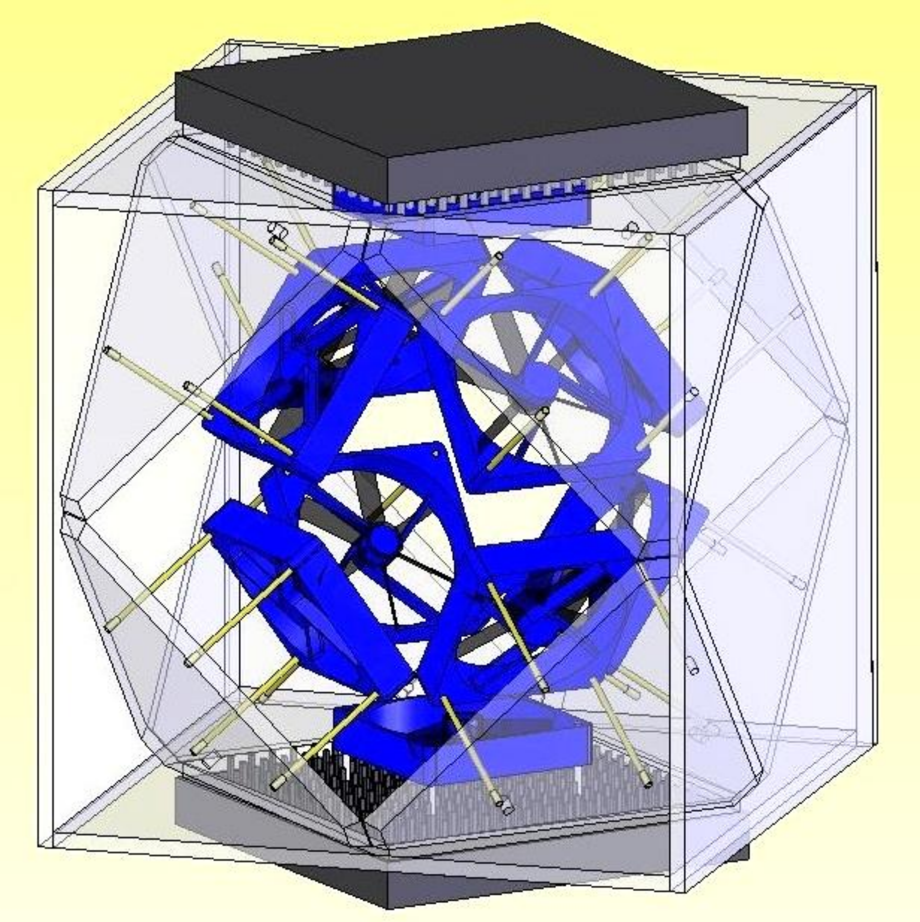
Turbulent Thermal Diffusion



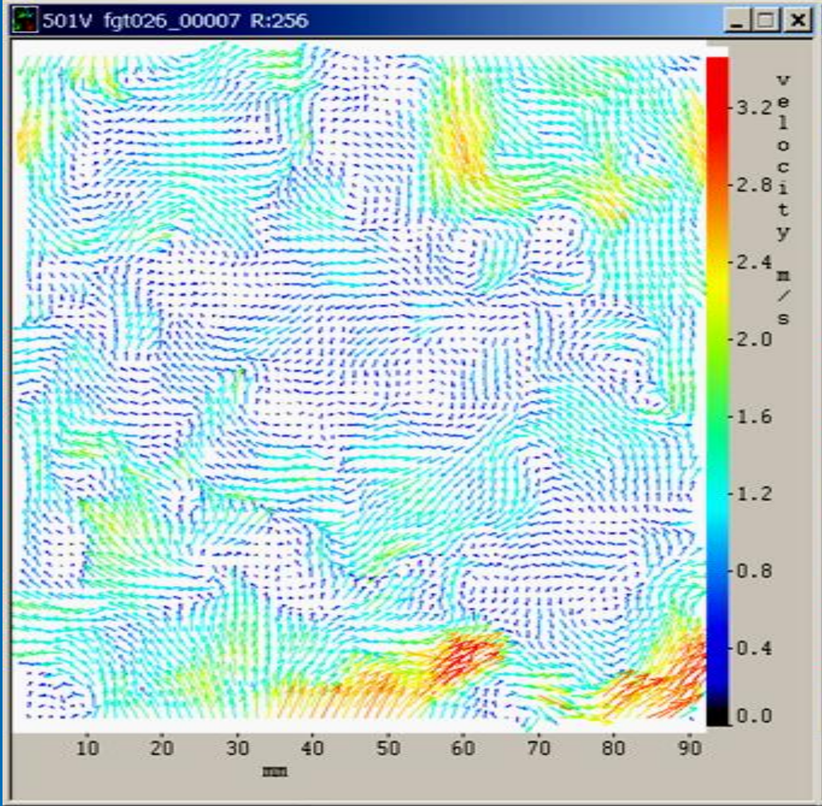
Normalized mean particle number density $N_r = \overline{N}/\overline{N}_0$ vs. normalized temperature gradient $T_r = (\overline{T} - \overline{T}_0)/\overline{T}_0$: ■ - stable stratification, ▲ - unstable stratification.

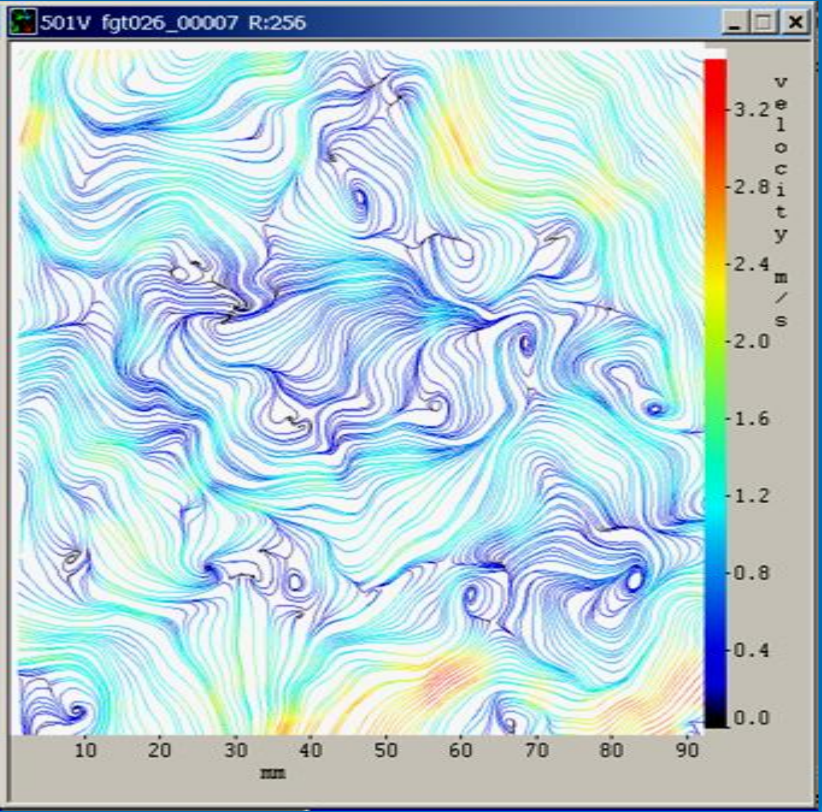
Experimental set–up with ten fans



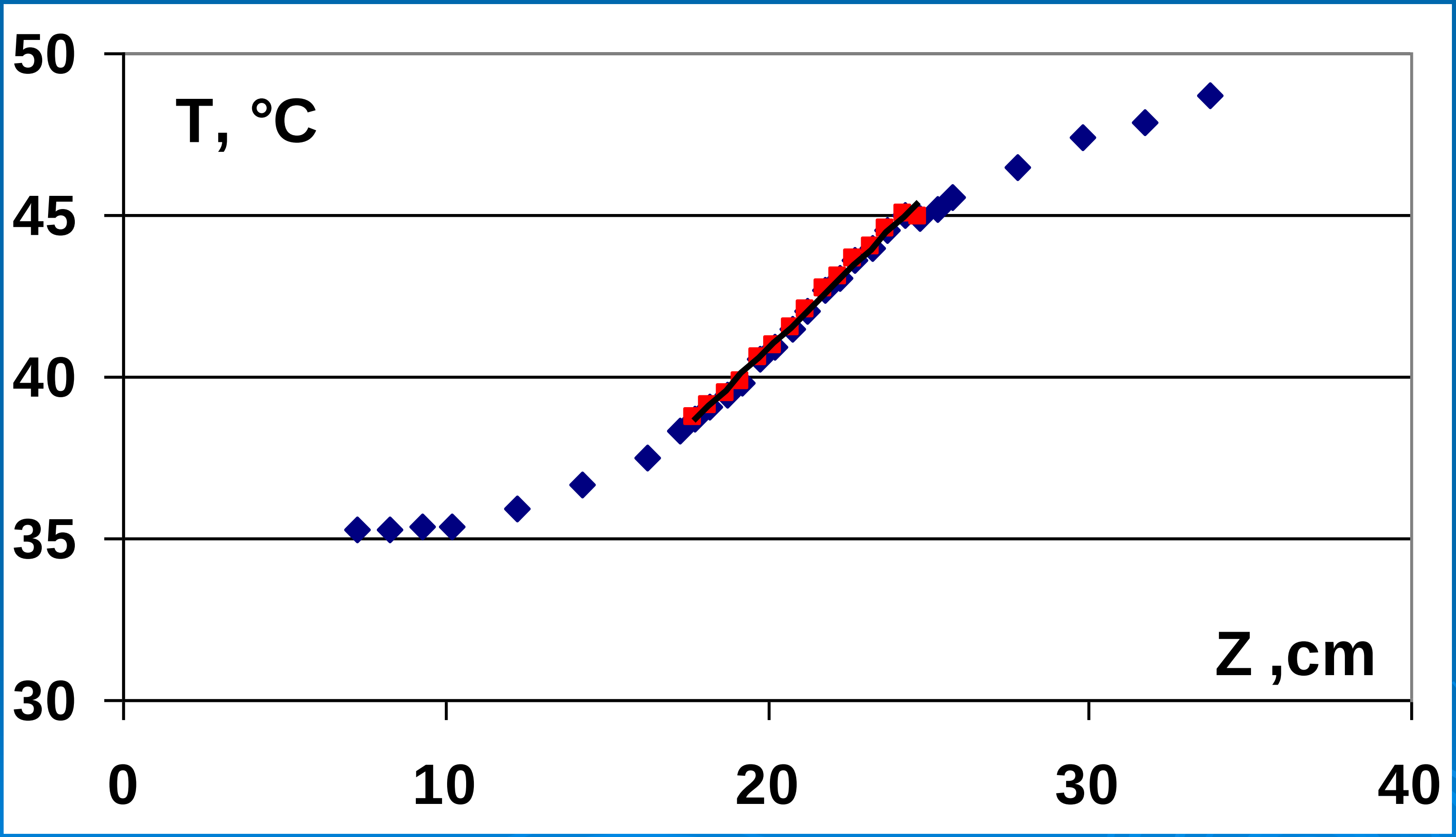


Instantaneous vector map and streamlines of flow in FTG

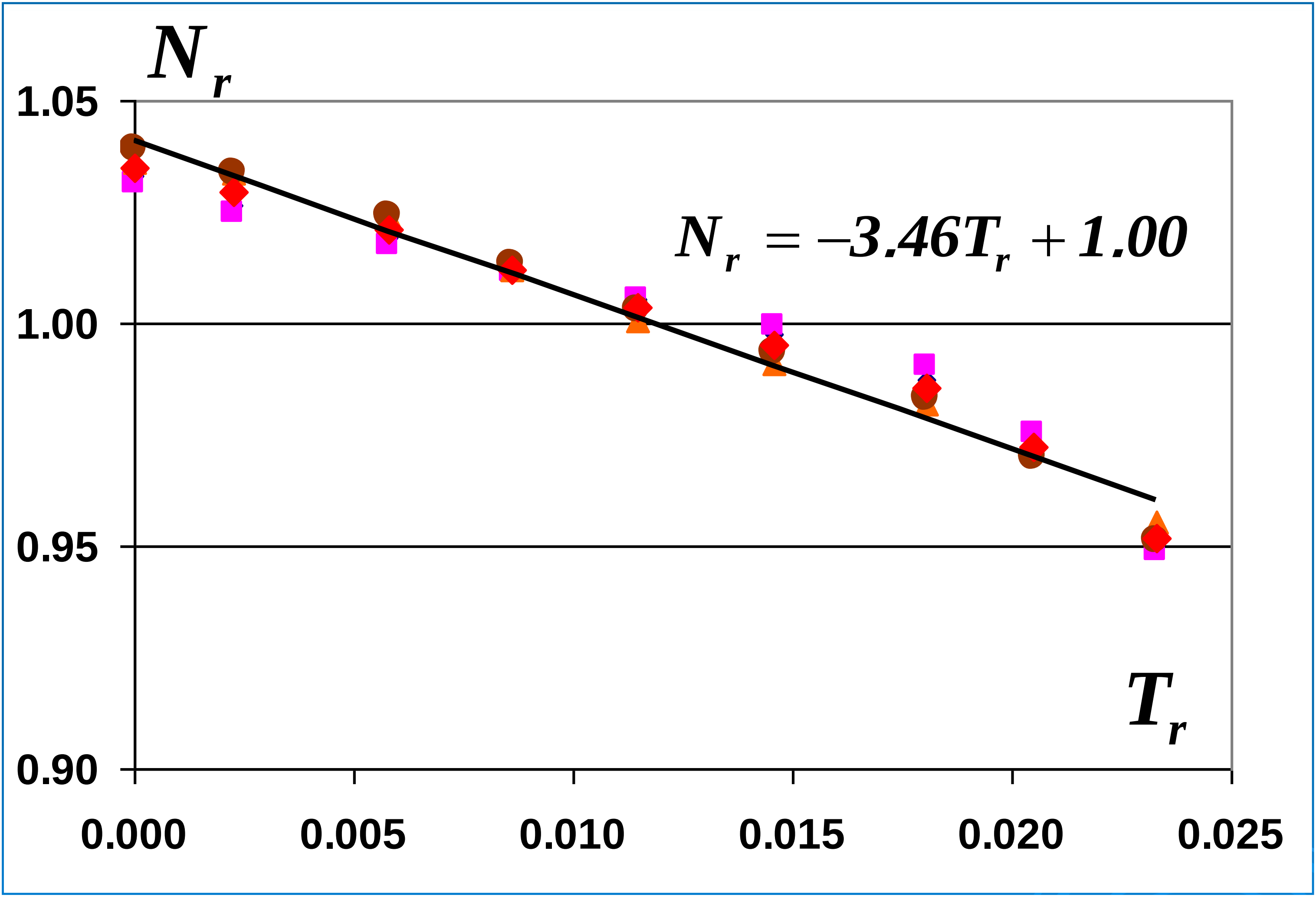




Mean temperature distribution in FGT



Turbulent Thermal Diffusion



Normalized mean particle number density $N_r = \overline{N}/\overline{N}_0$ vs. normalized temperature gradient $T_r = (\overline{T} - \overline{T}_0)/\overline{T}_0$ in FGT.

References (Experimental study)

- A. Eidelman, T. Elperin, N. Kleeorin, A. Krein, I. Rogachevskii, J. Buchholz and G. Grünefeld. *Nonlinear Processes in Geophysics*, **11**, 343-350, 2004.
- J. Buchholz, A. Eidelman, T. Elperin, G. Grünefeld, N. Kleeorin, A. Krein and I. Rogachevskii. *Experiments in Fluids*, **36**, 879-887, 2004.
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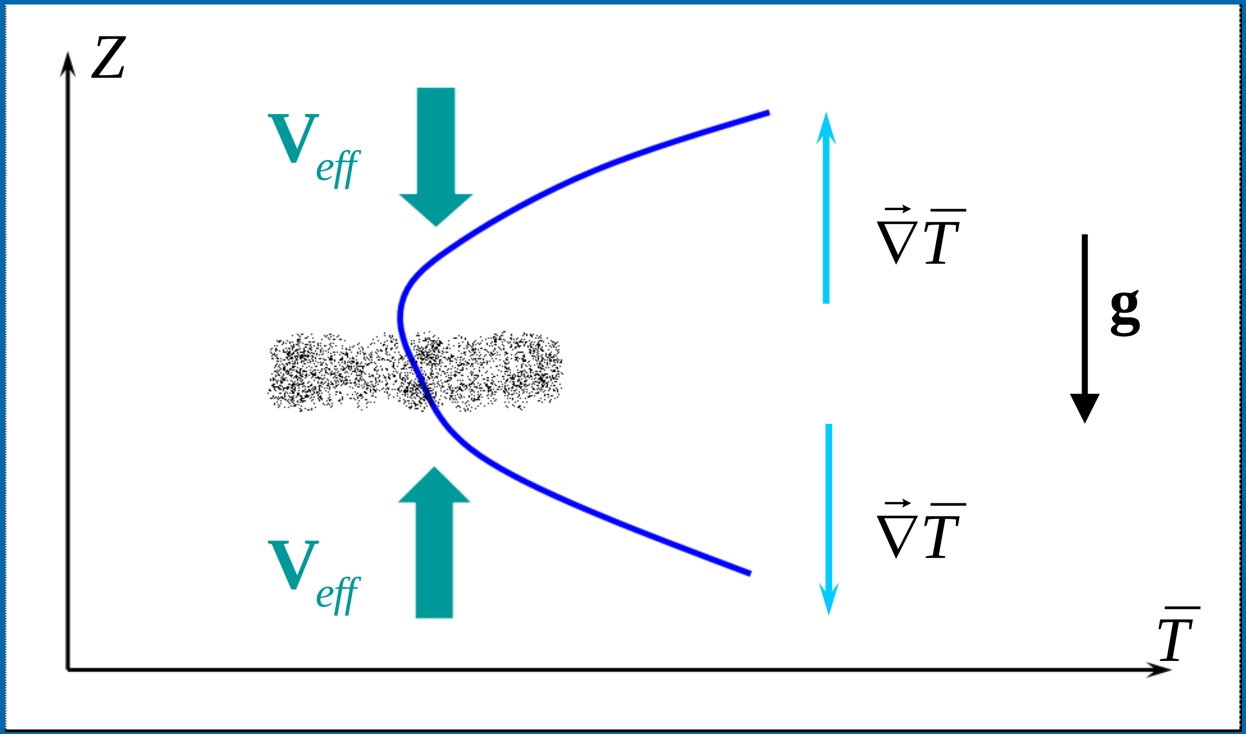
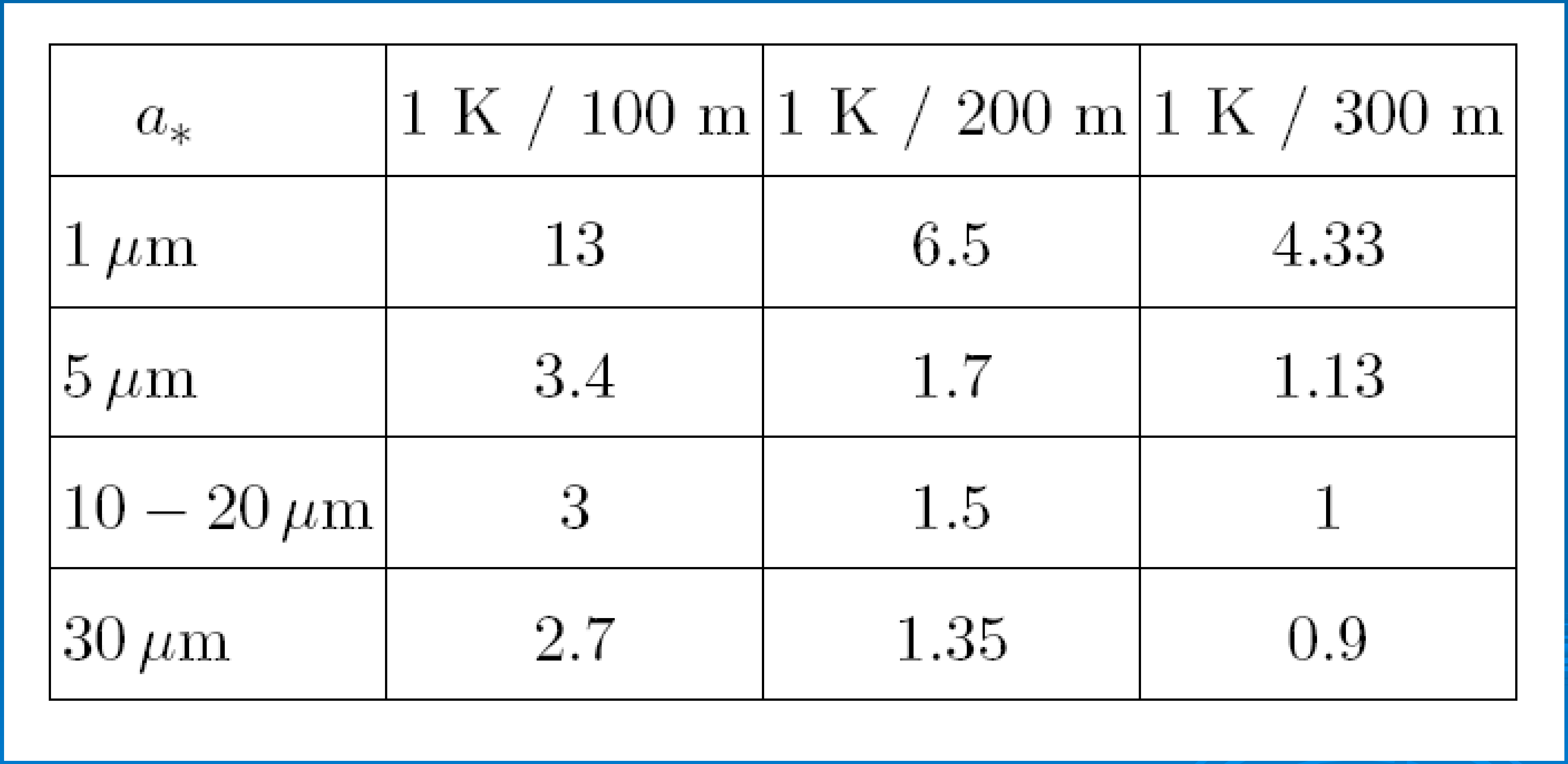
- A. Eidelman, T. Elperin, N. Kleeorin, A. Markovich, I. Rogachevskii. *Nonlinear Processes in Geophysics*, **13** , 109-117, 2006.

Turbulent Thermal Diffusion



$$\mathbf{V}_{eff} = -D_T\left(1+\left(\frac{m_p}{m_\mu}\right)\left(\frac{\bar{T}}{T_*}\right)\frac{\ln(\mathrm{Re})}{\mathrm{Pe}}\right)\frac{\nabla\bar{T}}{\bar{T}}$$

The ratio $\left| V_{eff} / W \right|$ for typical atmospheric parameters (different temperature gradients and different particle sizes)



a_*	1 K / 100 m	1 K / 200 m	1 K / 300 m
$1\,\mu\mathrm{m}$	13	6.5	4.33
$5\,\mu\mathrm{m}$	3.4	1.7	1.13
$10 - 20\,\mu\mathrm{m}$	3	1.5	1
$30\,\mu\mathrm{m}$	2.7	1.35	0.9

The ratio $|V_{eff}/W|$ for typical atmospheric parameters (different temperature gradients and different particle sizes)

	1 K/100 m	**1 K/200 m**	**1 K/300 m**	**1 K/1000 m**
$a_* = 30$ μm	2.7	1.35	0.9	0.27
$a_* = 50$ μm	2.43	1.22	0.81	0.243
$a_* = 100$ μm	2.06	1.03	0.687	0.206
$a_* = 200$ μm	1.7	0.85	0.567	0.17
$a_* = 300$ μm	1.5	0.75	0.5	0.15
$a_* = 500$ μm	1.2	0.6	0.4	0.12

T. Elperin, N. Keeorin, I. Rogachevskii, *Atmospheric Research*, **53**, 117 (2000).

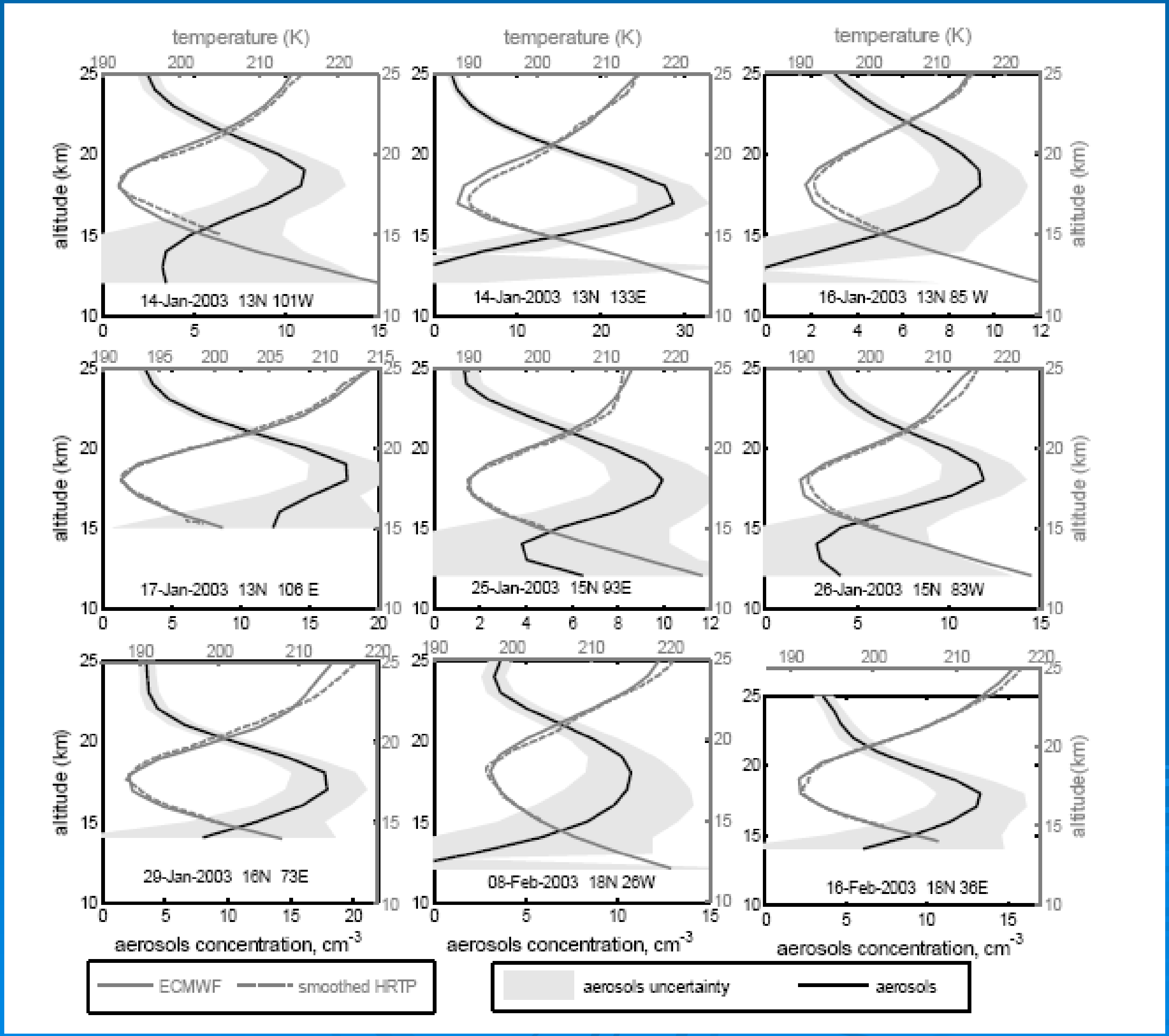
Time of Formation of Aerosol Layers

	1 K/100 m	1 K/200 m
$a_* = 30\ \mu\text{m}$	11 min	105 min
$a_* = 100\ \mu\text{m}$	1 min	120 min

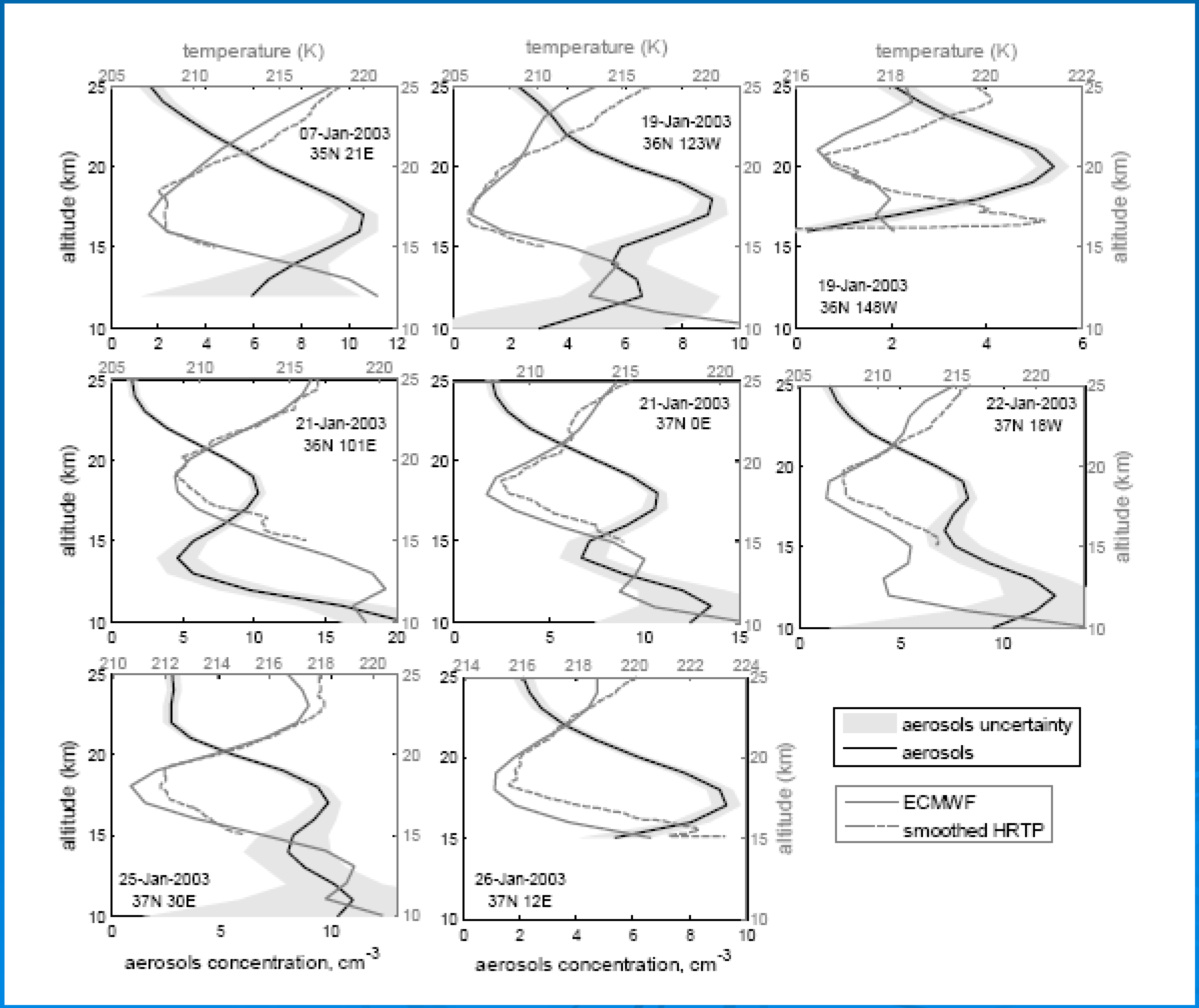
$$t_T \propto \frac{L_T}{|\mathbf{V}_{eff} - \mathbf{W}|}$$

T. Elperin, N. Keeorin, I. Rogachevskii, *Atmospheric Research*, **53**, 117 (2000).

Distribution of Number Density of Aerosols (black) and Mean Temperature Distribution (gray) (Satellite Gomos Data)



Distribution of Number Density of Aerosols (black) and Mean Temperature Distribution (grey) (Satellite GOMOS Data)



Conclusion

Mean-Field Effects

- A new phenomenon of turbulent thermal diffusion associated with turbulent transport of particles in the atmosphere and in laboratory experiments has been found.

- The essence of this phenomenon is the appearance of a non-diffusive mean flux of particles in the direction of the mean heat flux, which results in the formation of large-scale inhomogeneous structures in the spatial distribution of particles. Particles accumulate in regions of minimum mean temperature of the surrounding fluid.

- The effect of turbulent thermal diffusion has been detected experimentally: in oscillating grids turbulence generator and in a multi-fan turbulence generator in two directions of the imposed vertical mean temperature gradient (stable and unstable stratifications).

- Turbulent thermal diffusion can explain the large-scale aerosol layers that form inside atmospheric temperature inversions.

THE END